Problem Set #4
Due February 20, 2014

1. Epicycle Approximation: The velocity of the Sun perpendicular the Galactic plane is $v_z = +7 \text{ km s}^{-1}$, and $z_{\text{now}}$ is nearly zero. What is the Sun’s oscillation period in $z$? When will the next maximum above the plane occur? How high will the Sun get above the plane? Assume $\rho(z) = \text{constant} = 0.1 \text{ M}_\odot \text{ pc}^{-3}$. If we now assume that the number density of stars is symmetric in $z$ and proportional to:

$$n(z) \propto e^{-z/z_{\text{disk}}}$$

where $z_{\text{disk}} = 200 \text{ pc}$. How different will the night sky look to the unaided eye at maximum?

2. Oort Constants: It is possible to directly measure the Oort constants from the motions of stars in the solar neighborhood. Feast & Whitelock (1997) estimated these constants using Hipparcos observations to be: $A = 14.8 \pm 0.8 \text{ km s}^{-1} \text{ kpc}^{-1}$ and $B = -12.4 \pm 0.6 \text{ km s}^{-1} \text{ kpc}^{-1}$. Using these values, calculate the circular velocity of the Sun assuming it is 8 kpc from the Galactic Center. Please propagate errors. Again based these constants, what does this say about the shape of the Milky Way rotation curve near the solar neighborhood? Bonus: described observationally how the Oort constants are estimated.

3. The Jeans Equation: The collisionless Boltzmann equation in cylindrical coordinates $(r, \theta, z, v_r, v_\theta, v_z)$ is:

$$\frac{\partial f}{\partial t} + v_r \frac{\partial f}{\partial r} + \frac{v_\theta}{r} \frac{\partial f}{\partial \theta} + v_z \frac{\partial f}{\partial z} + \left( \frac{v_r^2}{r} + \frac{\partial \Phi}{\partial r} \right) \frac{\partial f}{\partial v_r} - \frac{1}{r} (v_r v_\theta + \frac{\partial \Phi}{\partial \theta}) \frac{\partial f}{\partial v_\theta} - \frac{\partial \Phi}{\partial z} \frac{\partial f}{\partial v_z} = 0$$

From this equation, derive the radial-component to the second moment Jeans Equation for an axially-symmetric system. One requirement on the distribution function is $f(v=\pm \infty) \Rightarrow 0$. What is the second requirement (hint use the Divergence Theorem, BT p776)? Describe in physical terms the meaning of each term in the equation. Remember that this is a momentum balance equation, so each term represents a momentum flow-- don’t use the word ‘energy’ in the answer.