1. **Fitting the NFW Profile:** The Via Lactea simulation is a high resolution N-body simulation of a Milky Way-size dark matter halo (Diemand et al. 2008, *Nature*, 454, 735). How well does a spherical NFW model fit to this simulation? The 3-dimensional particle data is available on the class website. Each particle mass is equivalent to $2.8 \times 10^7$ Msun. Within 300 kpc, what is the total mass of this halo? Does this agree with the Milky Way mass estimated in the literature?

Compare the radial density profile from the simulation to a spherically symmetric NFW profile. You will first need to bin the 3D data, I suggest binning between 3 to 300 kpc with bins evenly spaced in log of distance. Fit for the free parameters in the NFW profile. Compare this to the best-fitting isothermal sphere (from HW#2). Overplot both profiles on the simulated density profile.

**Bonus #1:** Determine the gravitational potential for each particle (due to all other particles) in the 3D dataset and compare to the predicted potential at that location from your best NFW fit. Plot the residual differences in projected x-y and y-z space. What is the largest % difference between your model and the data? Compare this to the 1-3% residuals from a triaxial NFW fit.

**Undergraduates:** I will email you the radially binned data. Bonus if you decide to do the radial binning on your own.

2. **Lagrange points:** The origin of an x-y coordinate system is located at the center of mass of the Earth-Sun system. If both the Earth and Sun are positioned on the x-axis (y=0), then the first three Lagrange stability points are located at (x,y):

$$L1 : (R[1 - (\frac{\alpha}{3})^{1/3}], 0) \quad L2 : (R[1 + (\frac{\alpha}{3})^{1/3}], 0) \quad L3 : (-R[1 + (\frac{5\alpha}{12})^{1/3}], 0)$$

where $\alpha = (\frac{M_{\text{Earth}}}{M_{\text{Sun}} + M_{\text{Earth}}})$ and R is the Earth-Sun distance. The next generation James Webb Space Telescope (JWST), will be placed at the L2 point. Calculate the distance from Earth to the satellite. Please diagram the positions of the Sun-Earth-satellite, including the Earth’s moon for comparison. Discuss advantages and disadvantages to putting a telescope in this position. **Bonus #2:** Create an animated or 3D version of B+T Figure 3.14.

3. **The Local Group Timing Argument:** The Milky Way (MW) and Andromeda (M31) are moving towards each other with a relative velocity of 120 km/s and are separated by a distance 785 kpc (McConnachie et al. 2005). If we assume these two galaxies are an isolated, gravitationally bound system, we can determine their combined mass by assuming their motion is described by the two-body problem. In doing so, we assume that MW and M31 were co-located at the beginning of the universe ($t = 13.7$ Gyr).
Begin with the solution to the equations of motion for the Kepler potential, B+T Equations 3.28a and 3.28b, and determine the total mass of the MW+M31 by following the procedure discussed in Box 3.1. Assume the MW-M31 properties above and the eccentricity $e=1$. Explain what you have assumed by setting $e=1$. You will need to solve the resulting equation numerically. I did this by plotting the function $t/r \times dr/dt$ as a function of $\eta$ and finding the appropriate intercept.

What is the total mass of the Local Group (e.g., MW+M31) as implied by the timing argument? If M31 is twice the mass of the MW, what is the mass of the MW? If the circular velocity of the MW remains roughly 220 km/s until the edge of the mass distribution, how big must the MW be to include the mass inferred?