Derivation of Newton’s form of Kepler’s 3rd law
(this will not be in exam)

Consider 2 masses M1 and M2 orbiting their stationary center

Though they may move at different speeds – they must both have the same sidereal period. Why?

So they must also have the same angular velocity $\omega$

$r_1 = \text{dist from M1 to stationary center}$

$r_2 = \text{dist from M2 to stationary center}$

$r_1 + r_2 = a = \text{semi-major axis of the orbit}$

The centrifugal force $F$ on an object in a circular orbit about its stationary center is

$F = m r \omega^2$

Where $r$ is distance to stationary center (center of mass)
Newton’s 3\textsuperscript{rd} law requires:

\[ m_1 r_1 \omega^2 = m_2 r_2 \omega^2 \]

Rearranging gives

\[ \frac{r_1}{r_2} = \frac{m_2}{m_1} \quad (1) \]

As \( r_1 + r_2 = a \) equation (1) can be written as

\[ r_2 = \frac{a}{1 + \frac{m_2}{m_1}} \quad (2) \]

The centrifugal force is produced by gravity

\[ \frac{G m_1 m_2}{a^2} = m_2 r_2 \omega^2 \quad (3) \]

Now substitute (2) into equation (3)

and use the definition \( \omega = \frac{2\pi}{P} \) (P=period)

Gives:

\[ \frac{P^2}{a^3} = \frac{4\pi^2}{G(m_1 + m_2)} \]