Radian measure

Angles are most often measured in degrees, arcminutes and arcseconds.

1 degree (°) is 1/360 of a complete circle.
1 arcminute = 1/60 of a degree
1 arcsecond = 1/60 of a minute = 1/3600 of a degree

A circle has a circumference $C = \pi r$ so the distance around half a circle is $\frac{\pi r}{2}$ and the distance around a quarter of a circle is $0.5\pi r$ etc..

Let distance $D$ be the distance around a circle spanned by $\alpha^\circ$.
$D$ is a fraction of the circumference $C$

$$D = \frac{\alpha}{360}$$

As $C = 2\pi r$
$$\frac{D}{2\pi r} = \frac{\alpha}{360}$$

So we can rearrange this formula to get
$$D = \frac{2\pi}{360} r \alpha$$

If we define a new unit of length the radian
where \( \alpha^c = \frac{2\pi^c}{360^\circ} \times \alpha^o \)

then the D formula becomes

\[ D = r\alpha^c \]

The superscript 'c' can be used to denote radians.

NOTE: To use this formula \( \alpha \) MUST BE IN RADIANS.

**Small angle approximation**

The formula \( D = r\alpha^c \) provides a way of estimating distances in certain circumstances.

Suppose a pole is stuck vertically into the ground a distance \( d \) away from an observer (see diagram).

If the angle \( \alpha \) is small (less than 20 degrees) then the height \( h \) is very close to the distance \( D \) along the arc so that,

\[ d = r \]

\[ h \approx D = d\alpha^c \]

The smaller the angle is then the more accurate this approximation is.
Astrophysics Applications

In most astrophysical applications $\alpha$ is generally much less than 1 degree.

Instead of expressing $\alpha$ in radians, it is expressed in terms of arcseconds so a conversion factor is required.

As shown before, for $\alpha$ in degrees

$$D = \frac{2\pi}{360} d\alpha$$

$2\pi$ radians is equivalent to 360 degrees

360 degrees is equivalent to $360 \times 60 \times 60$ arcseconds

For $\alpha$ in arcseconds

$$D = \frac{2\pi}{360 \times 60 \times 60} d\alpha$$

$$D = \frac{\alpha d}{206265}$$

D and d must have same units

(e.g. m, km, A.U., light years, parsecs)