High precision cluster lensing modeling

Eric Jullo, Ana Acebron @ LAM and the CATS team
Questions?

• What is the modeling precision needed for which accuracy on a particular science topic?

• Is the modeling precision proportional to the number of multiple images to fit?
Strong lensing modeling strategies

1) Observationally motivated models
   – Decomposition into halos
   – Good fit with few constraints
   – Direct test of N-body derived models with evidence model ranking

2) Free-form models
   – Decomposition into RBF "pixels"
   – Better fit with lots of constraints
   – Good at
     • Detecting substructures
     • Testing Light Traces Mass assumption
Multiscale RBF model

Linearized weak lensing equations

\[ \mathbf{e} = \mathbf{M} \gamma \mathbf{v} + \mathbf{n} \]

With data \( \mathbf{e} = [\mathbf{e}_1, \mathbf{e}_2] \) and unknown \( \mathbf{v} = [\nu_1, \ldots, \nu_N] \)

\[ M_{ij} = D_{LSi}/D_{OSi} \, f_j(|\theta_i - \theta_j|) \]

\( f_j \) functions are derivatives of the TIMD potential (circular PIEMD)

Here the convergence is:

\[ f(R, s, t) = \frac{1}{2G} \frac{t}{t-s} \left( \frac{1}{\sqrt{s^2+R^2}} - \frac{1}{\sqrt{t^2+R^2}} \right) \]

The RBFs density can be matched to the light distribution
Galaxy scale components model

• Large scale cluster component + galaxy halo components (stars + DM):

\[ \phi_{tot} = \phi_{cluster} + \sum_i \phi_{halos} \]

Kneib et al 1996

• Need to scale the galaxy halo components, for example for a PIEMD mass distribution:

\[ \sigma = \sigma_*(\frac{L}{L_*})^{1/4} \]
\[ r_{cut} = r_{cut}^*(\frac{L}{L_*})^\eta \]

• Hence:

\[ \frac{M}{L} \propto L^{\eta - 1/2} \]
\[ \eta = 1/2 \]
\[ \eta = 0.8 \]

See also Mila’s poster
New modeling of the Cluster Members

Brimioulle et al. 2013

- CM modeled with NFW profiles and scaling relation:

\[
\left( \frac{M}{M^*} \right) = \left( \frac{L}{L^*} \right)^{\eta_M}
\]

- Best fit for SDSS red galaxies \( L^* = 1.6 \times 10^{10} \, h^{-2} \, L_{r,\text{sun}} \)

- \( \eta_{M200} = 1.05\pm0.12 \) and \( M^*_{200} = 18.6\pm0.8 \times 10^{11} \, h^{-1} \, \text{Msun} \)

- Best fit of the simulated cluster with \( L^* = 1.2 \times 10^{10} \, h^{-2} \, L_{K,\text{sun}} \)

- \( \eta_{M200} = 0.54\pm0.16 \) and \( M^*_{200} = 4.1\pm0.3 \times 10^{11} \, h^{-1} \, \text{Msun} \)
Errors due to galaxies modeling

D’Aloisio & Natarajan 2010

\[
\begin{align*}
\sigma_0 &= \sigma_0^* \left( \frac{L}{L^*} \right)^{1/4}, \\
\rho_{\text{core}} &= \rho_{\text{core}}^* \left( \frac{L}{L^*} \right)^{1/2}, \\
\rho_{\text{cut}} &= \rho_{\text{cut}}^* \left( \frac{L}{L^*} \right)^\alpha.
\end{align*}
\]

The total mass of a subhalo scales then as:

\[
M = \frac{\pi}{G} (\sigma_0^*)^2 r_{\text{cut}}^* (L/L^*)^{1/2+\alpha}.
\]

- For A1689
- Scatter in the scaling relations \( \sim 1'' \) \( \text{(Jullo+07)} \)

- 10% scatter
- 20% scatter
- 30% scatter

\( 1 \text{ arcsec} = 4 \text{ kpc} \)

\( \rightarrow \) Scatter for each image

\( \rightarrow \) Images are weighted in \( \chi^2 \)

INDIVIDUALLY

Meneghetti+07
Errors due to deflections by LOS structures
D’Aloisio & Natarajan 2010

For A1689

- 1" of scatter due to structures in the lens plane & along L.O.S.

Correlated LOS (infalling subclusters, filaments)

Uncorrelated LOS (primary contribution to the errors)

See also the talk by A. D’Aloisio
MACS J0416.1-2403

- 148 image constraints
- 57 systems
- 10 spec z
- 97 cluster members
Mixed parametric and free-form reconstruction to detect substructures

Joint SL+WL fit for grid-based model doesn’t work yet

→ Need to figure out SL and WL relative weights

Alternative, 2 step optimization
  – Reconstruction with SL data
  – WL reconstruction with prior parametric model
Impact of modeling technique
Strong lensing BLF Simulated cluster

- 44 image constraints
- 126 Cluster Members
- Optimized potential

- Cluster @ $z = 0.288$
- N-body-SAM simulation
- Skylens Ray tracing (Meneghetti et al. 2008)
- Simulated arcs from observed LF

$20'' = 86.5$ kpc
Density profiles Comparison

Model description | Best Chi2 (src) | Log (E) | RMSI
--- | --- | --- | ---
Mixed Param+Grid | 50 | -128 | 0.81
PIEMD, PIEMD sat | 109 | 8.61 | 0.57
PIEMD, NFW sat | 120 | 5.11 | 0.67
NFW, NFW sat | 872 | -326 | 1.25
Impact of missing data
Cosmography Methodology

\[ \alpha = \frac{D_{LS}}{D_{OS}} \nabla \varphi (\theta_I) \]

Cosmology

mass
Scaling with Cosmology or Mass??
Cosmography with Abell 1689

Mass model with 3 PIEMD potentials; 58 cluster galaxies
Bayesian optimization: 32 constraints, 21 free parameters;
**RMS = 0.6 arcsec**; 28 multiple images from 12 sources with spec z, flat Universe prior

\[0.1 \leq \Omega_M \leq 0.58; -1.57 \leq w_X \leq -0.85\]
Ares Cluster

- Image constraints
- Cluster Members
- Optimized potential

- 242 image constraints
- 122 systems
- 200 CM with magK<22
Hera Cluster

- 65 image constraints
- 19 systems
- 343 CM magK < 24

$10'' = 61.5$ kpc

N images

22 23 24 25 26 27 28 29
Magnitude cut
Errors on magnification

- In our model of ARES, we improve on statistical errors until $N_{\text{img}} < 50$
- Systematic errors dominate at $N_{\text{img}} > 50$

![Graph showing errors on magnification vs. number of images.](image)
Errors on magnification, Cont.
Errors on cosmological parameters in ARES
Errors on cosmological parameters in ARES
Source vs Image plane

- Optimization with 242 image constraints
- Same errors in the $\chi^2$ for all the images

➤ Image plane and source plane results are consistent
Impact of photometric redshifts

ARES simulation with 2/3 of the redshifts considered as zphot with $\sigma_z / (1 + z) = 0.02$
Conclusion

• We propose a new mixed "parametric" and free-form model to detect substructures in the outskirt of MACS0416
  – Good fit to the data but not justified in terms of Evidence
  – Not necessarily better in terms of RMSI

• Error bars saturate after N_{img} > 50 \(\Rightarrow\) models get dominated by systematic errors

• There is a lot of room for modeling improvement

• We not necessarily need more simulated clusters, but more informations the existing ones (ex: v_{disp} of galaxies, 3D distribution, 3D cluster shape, etc)