Stars in Motion

A Yale symposium in honor of Bill Van Altena

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M67 most probable members and age estimation using BASTI isochrones

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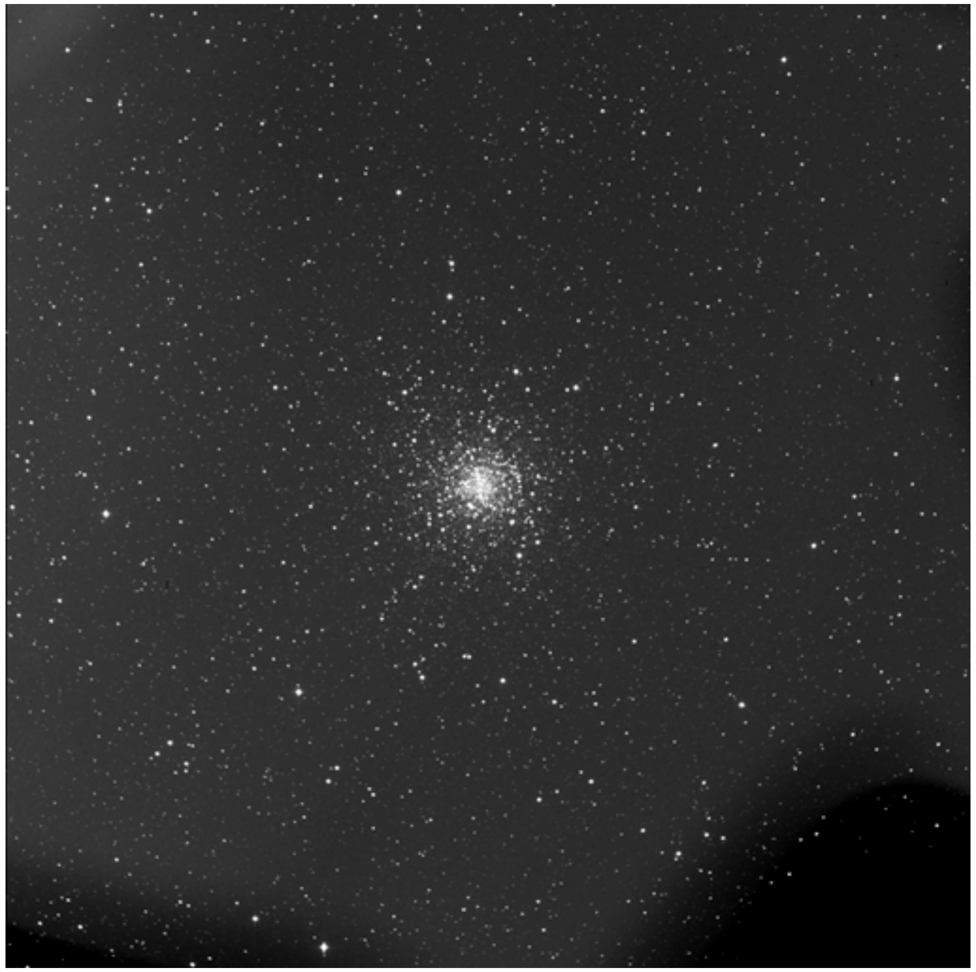
Open Clusters in galactic astronomy

- Data analysis in astrostatistics
  - Data Bases
- Membership in open clusters
- Distance Moduli
- Galactic extinction
- Age of galactic clusters

Open Cluster M67

- Position
  - α_{2000}= 8h 50m 26.1s
  - δ_{2000}= 11° 48´ 46´´
- Distance 870 pc



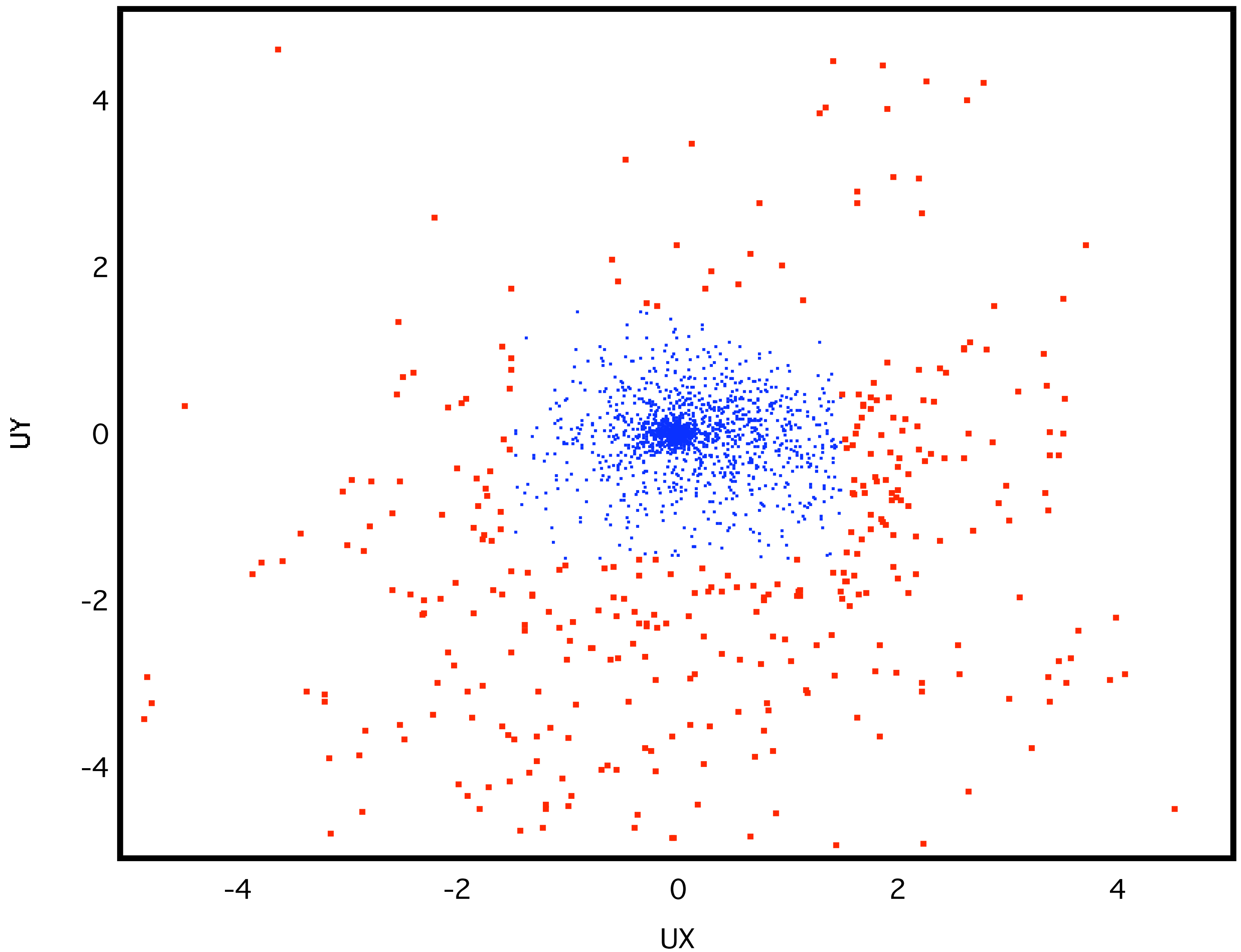
The importance of M67

- Age 4 Gyrs.
- Chemical composition.
- Solar type stars.

Membership study

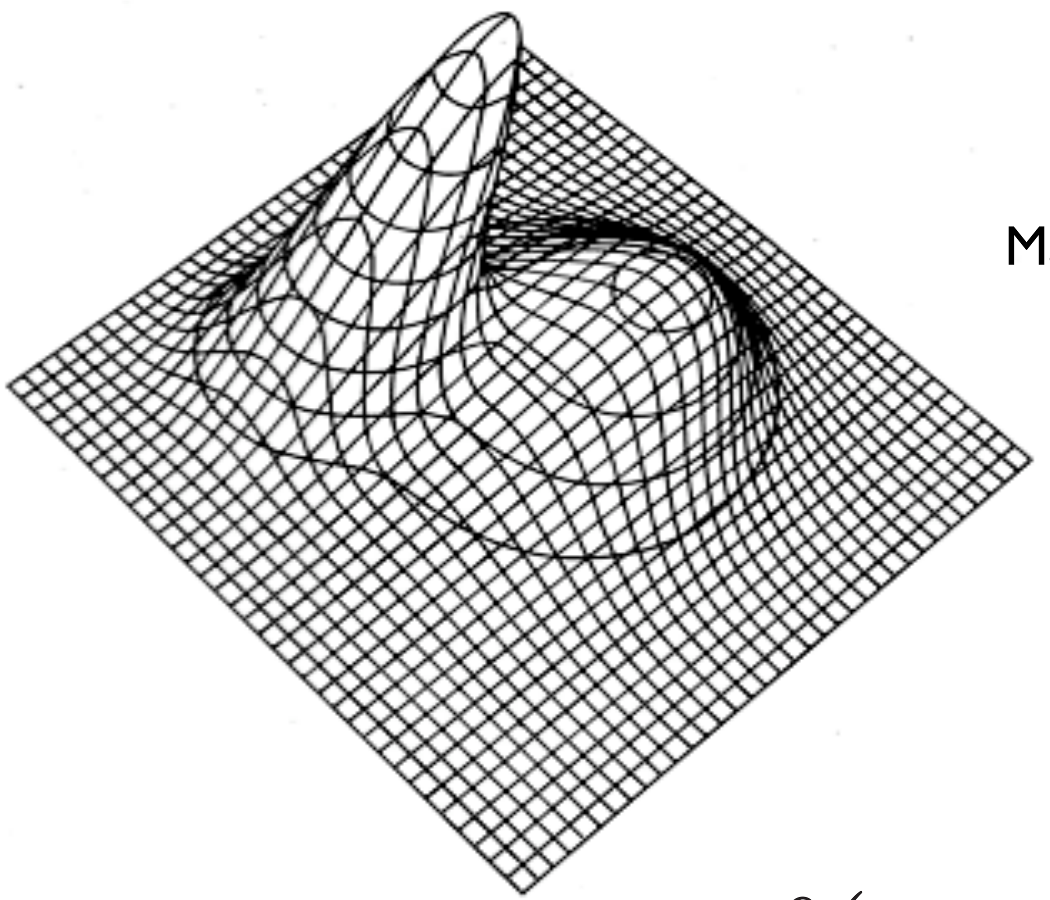
- Proper Motions from 1866 stars W. Sanders
- Photometry from Montgomery, Sanders, Girard
- Mixture Density Model
- Bivariate normal components
- Parameter estimation
- EM algorithm
- EMMIX software

PM vector point spread diagram



Arcsec/cent

PM bivariate normal components for the field and cluster stars



Mark H. Slovak

$$f(x_j; \Theta) = \sum_{i=1}^{2} \alpha_i f_i(x_j; \mu_i, \Sigma_i)$$

The EM algorithm

- This has been the statistical approach to estimate the parameters of the mixture model.
- This goal is reached because the EM algorithm allows to find a point of the parameter space that maximizes the log-likelihood function through the E and the M steps.

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UDC 524.45M67–16–17
Original scientific paper

THE EXPECTATION MAXIMIZATION ALGORITHM AS A POWERFUL TOOL TO SOLVE THE STELLAR MEMBERSHIP IN OPEN CLUSTERS. AN APPLICATION TO M67

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SUMMARY: The EM algorithm is a powerful tool to solve the membership problem in open clusters when a mixture density model overlaping two heteroscedastic bivariate normal components is built to fit the cloud of relative proper motions of the stars in a region of the sky where a cluster is supposed to be. A membership study of 1866 stars located in the region of the very old open cluster M67 is carried out via the Expectation Maximization algorithm using the McLachlan, Peel, Basford and Adams EMMIX software.

Key words. Methods: analytical – Methods: data analysis – Methods: statistical – open clusters and associations: individual: M67

The EMMIX software

http://www.maths.uq.edu.au/~gjm/emmix/emmix.html

Parameters estimation by EM

$$\hat{\alpha}_i^{(k+1)} = \frac{1}{n}\sum_{j=1}^{n} \hat{\tau}_{ji}(X_j, \hat{\Theta}^{(k)})$$

$$\hat{\mu}_i^{(k+1)} = \frac{1}{n\hat{\alpha}_i^{(k)}}\sum_{j=1}^{n} \hat{\tau}_{ji}(X_j, \hat{\Theta}^{(k)})X_j$$

$$\hat{\Sigma}_i^{(k+1)} = \frac{1}{n\hat{\alpha}_i^{(k)}}\sum_{j=1}^{n} \hat{\tau}_{ji}(X_j, \hat{\Theta}^{(k)})\left(X_j - \hat{\mu}_i^{(k+1)}\right)\left(X_j - \hat{\mu}_i^{(k+1)}\right)^T$$

The membership probabilities are found through a Bayesian rule

$$\hat{\tau}_{ji}(X_j, \Theta^{(k)}) = \frac{\hat{\alpha}_i^{(k)} f_i(X_j, \hat{\Theta}^{(k)})}{\sum_{i=1}^{2} \alpha_i^{(k)} f_i(X_j \hat{\mu}_i^{(k)}, \hat{\Sigma}_i^{(k)})}$$

M67 most probable members

Table 1. Proper motions, absolute magnitudes, colors and M67 cluster members

ID1	Sand	μ_x	μ_y	M_V	B-V	P1	P2
1	8	-0.045	0.055	5.170	0.710	95	94
2	144	0.006	0.052	4.470	0.880	95	94
3	145	0.036	0.066	3.200	0.440	94	93
4	218	-0.032	-0.022	3.570	0.730	96	95
5	277	0.083	0.022	2.580	0.520	93	92
6	362	0.027	-0.094	6.120	0.980	93	90
7	368	0.023	-0.070	3.020	0.570	95	93
8	395	0.021	0.053	3.660	0.570	95	94
9	465	-0.099	0.075	4.490	0.600	92	90
10	469	0.009	-0.003	4.860	0.620	96	95
• • •							
223	1271	0.041	-0.075	2.670	0.530	94	92
224	1272	0.054	-0.009	2.814	0.598	95	94
225	1274	0.025	-0.095	2.964	0.598	93	90
226	1275	-0.031	-0.001	2.862	0.593	96	95
227	1277	-0.004	-0.079	1.920	1.060	95	93
228	1278	-0.026	-0.106	4.701	0.775	93	90
229	1280	-0.019	0.108	2.557	0.260	93	90
230	1282	-0.027	0.061	3.633	0.564	95	94
231	1283	0.013	-0.004	4.415	0.640	96	95
232	1284	0.027	0.002	1.220	0.240	95	95
233	1287	-0.104	0.020	4.330	0.608	94	93

ID1	Sand	μ_x	μ_y	M_V	B-V	P1	P2
78	829	-0.109	0.021	4.670	0.590	94	93
79	830	0.030	-0.012	3.050	0.560	95	94
80	833	-0.094	-0.085	3.720	0.570	93	90
81	848	-0.023	0.072	3.720	0.620	95	93
82	853	-0.044	0.074	3.590	0.590	94	93
83	859	-0.073	0.033	2.970	0.660	89	94
84	871	-0.044	-0.095	2.920	0.600	94	91
85	911	-0.135	-0.050	4.350	0.830	91	90
86	912	0.064	-0.002	3.550	0.550	94	93
87	913	-0.027	-0.080	4.160	0.560	95	93
• • •							
303	1634	-0.004	0.003	3.490	0.580	96	95
304	1639	-0.008	0.072	2.960	0.580	90	93
305	1688	0.015	0.100	4.190	0.570	93	90
306	1713	0.004	-0.051	2.830	0.970	95	94
307	1792	-0.089	0.005	2.930	0.600	95	94
308	1912	-0.083	0.057	3.320	0.540	94	93
309	1983	0.078	0.021	4.690	0.640	93	92
310	2212	0.044	0.020	3.025	0.587	95	94
311	2213	-0.069	0.000	5.190	0.720	95	94
312	2219	0.023	-0.063	3.460	0.570	94	93
313	2221	0.026	0.065	3.680	0.580	90	93

Learning from Allan Sandage, Lori M. and Don VandenBerg

PUBLICATIONS OF THE ASTRONOMICAL SOCIETY OF THE PACIFIC, **115**:1187–1206, 2003 October

The Age of the Oldest Stars in the Local Galactic Disk from *Hipparcos* Parallaxes of G and K Subgiants[1]

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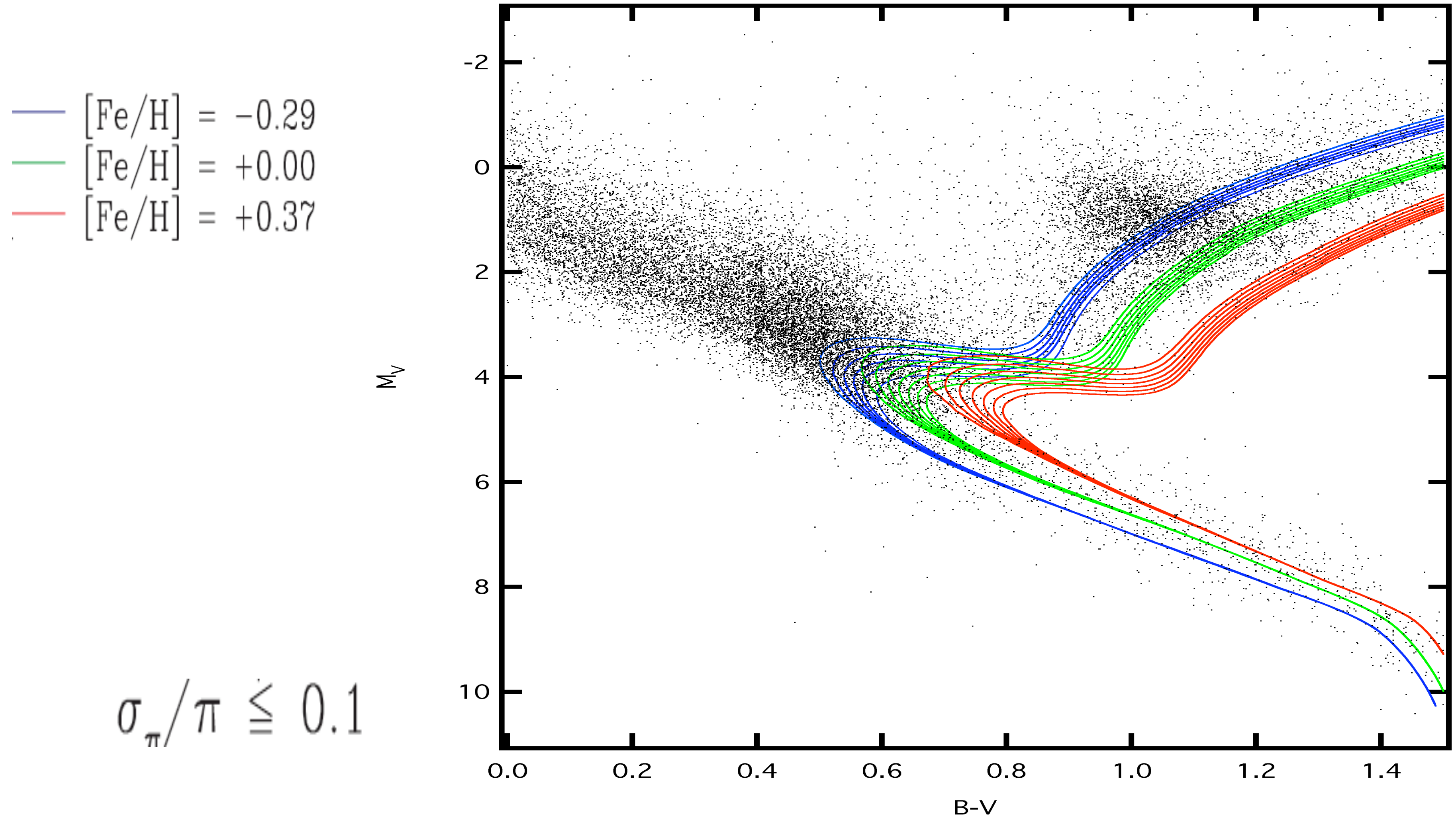
AND

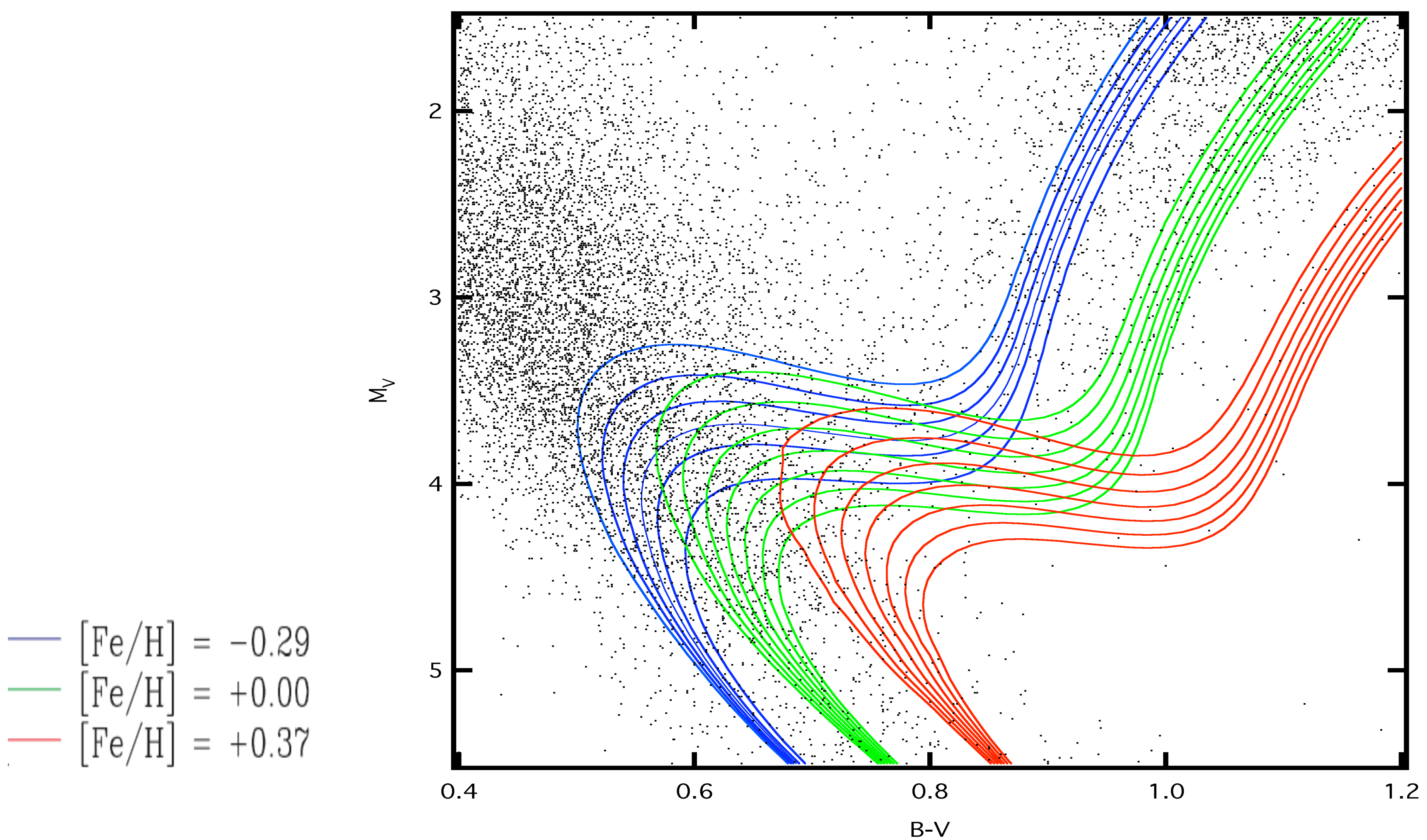
DON A. VANDENBERG

Main Hipparcos catalog cloud and isocrhones from Don VandenBerg Victoria Regina models



$$\sigma_\pi/\pi \leqq 0.1$$

Main Hipparcos catalog cloud and isocrhones from Don VandenBerg Victoria Regina models



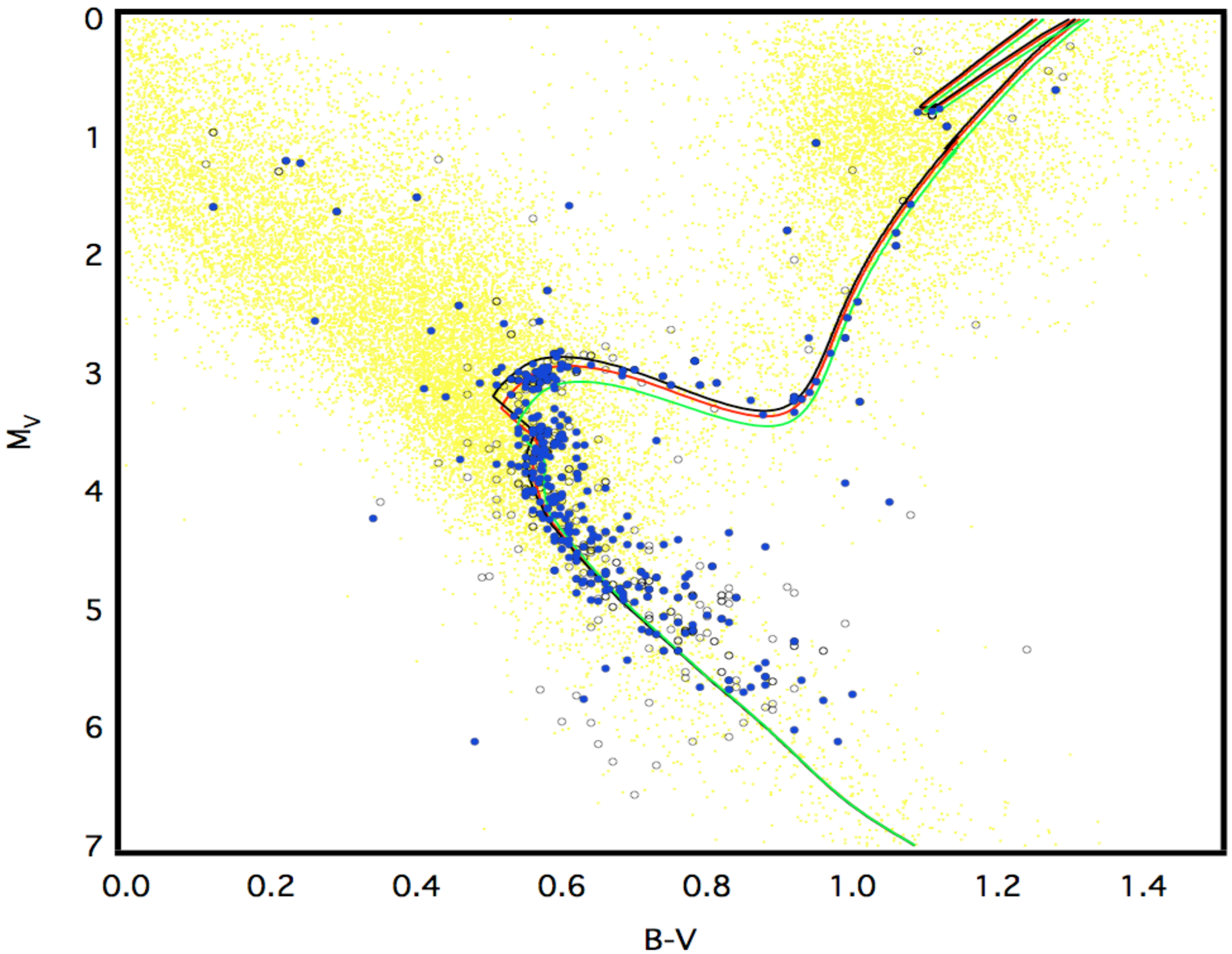
Isochrone by Pietrinferni-Cassisi-Salaris-Castelli 2006 Standard Model

- Scaled solar model & transformation (Castelli 1999)

(M/Mo)in	(M/Mo)	log(L/Lo)	logTe	Mv	(U-B)	(B-V)	(V-I)
0.500000	0.499987	-1.3886	3.58841	9.357	1.046	1.452	2.055
0.502945	0.502945	-1.38043	3.58913	9.329	1.049	1.452	2.046
0.505890	0.505890	-1.3723	3.58985	9.302	1.052	1.451	2.037
0.508834	0.508834	-1.36423	3.59057	9.275	1.054	1.45	2.028
0.511779	0.511779	-1.3562	3.59128	9.247	1.057	1.45	2.019
...							
1.366720	1.288992	2.86406	3.57999	-1.144	2.052	1.621	1.787
1.366728	1.288762	2.89547	3.57709	-1.168	2.072	1.635	1.817
1.366735	1.288500	2.92882	3.57387	-1.184	2.092	1.648	1.847
1.366743	1.288198	2.96353	3.57036	-1.19	2.11	1.662	1.876
1.366751	1.287849	3.00265	3.56632	-1.183	2.125	1.675	1.907

[Fe/H]=0.058 Z=0.0198 Y=0.273 t(Gyr)=4

M67 Color Magnitud Diagram and Age



M67 Stellar spectra by Allen L. et al.

THE ASTRONOMICAL JOURNAL VOLUME 109, NUMBER 3 MARCH 1995

MODERATE-RESOLUTION SPECTRAL STANDARDS FROM λ5600 TO λ9000 Å

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ABSTRACT

We present a grid of stellar classification spectra of moderate resolution ($R\sim1500$) in the range λλ5600–9000 Å, compiled from high signal-to-noise spectra of 275 stars, most in the open clusters Praesepe and M67. The grid covers dwarfs from types B8 through M5, giants from G8 through M7, and subgiants from F5 through K0. We catalog atomic and molecular absorption features useful for stellar classification, and demonstrate the use of luminosity-sensitive features to distinguish between late-type dwarf and giant stars. The entire database is made available in digital format on anonymous ftp and through the World Wide Web.

M67 Stellar spectra by Allen L. et al.

L. E. ALLEN AND K. M. STROM: SPECTRAL STANDARDS

TABLE 2. Spectral standards in M67.

ID[a]	α (1950)	δ (1950)	Adopted Sp. Type	m_V	B-V	V-I	%[b]	Notes[c,d,e]
471	8:47:37.6	11:52:09	F6 V	13.660	0.580	...	96	1
611	8:47:54.5	11:57:51	K0 V	15.550	0.750	...	76	1
621	8:47:49.2	12:00:17	G1 V	14.490	0.630	...	87	1
711	8:48:09.2	11:42:36	F5 V	13.340	0.650	...	96	1
724	8:48:22.6	11:47:15	G2 V	14.570	0.680	...	93	1
744	8:48:03.1	11:54:10	F9 V	14.190	0.580	...	79	1
...								
1319	8:49:04.6	12:08:10	K0 IV	12.950	0.950	...	99	1;F237,RGO481,SIII-75
1323	8:48:51.2	12:09:14	G2 IV	12.890	0.720	...	99	1; F192,RGO472,SIII-82
1415	8:49:09.5	11:49:52	F8 IV	12.710	0.570	...	99	1; F256,RGO66,SII-110
1438*	8:49:27.3	11:56:58	G5 IV	12.889	0.817	0.870	98	2; F281,RGO154,SII-129
1456	8:49:09.3	11:59:40	F8 IV	12.705	0.575	0.663	99	2; F255,RGO317
1463	8:49:12.0	12:01:34	K0 IV	12.910	1.000	...	98	1; F262,RGO319,SIII-57
1485	8:49:07.2	12:09:09	F8 IV	12.870	0.550	...	98	1; F248,RGO482,SIII-92
1506	8:49:08.5	12:15:38	F8 IV	12.730	0.570	...	98	1; F252,SIII-114
1508	8:49:15.6	12:16:18	G2 IV	12.730	0.700	...	96	1
1589	8:49:34.7	11:54:45	G0 IV	12.660	0.610	...	98	1; F287,RGO156,SII-139
2208	8:48:48.3	11:59:19	F5 IV	12.777	0.822	0.882	97	2; F181,RGO208,SII-6
2212	8:48:46.5	12:00:31	F9 IV	12.725	0.587	0.704	99	2; F174,RGO299,SIII-1
721	8:48:05.7	11:46:24	K1 III	11.240	1.100	1.070	98	2; RGO6
989	8:48:37.5	11:57:23	K1 III	11.450	1.060	...	99	1; F135,RGO192; K2III[Po]
1016*	8:48:33.0	11:59:33	K2 III	10.300	1.260	1.230	98	2; F105,RGO180; K2III[BB]

The age of M67

- CMD with 314 M67 most probable members
- Three Basti isochrones (3.75, 4 y 4.5) Gyr
- Isochrones are based on a scaled solar model with 2000 points
- [Fe/H] = 0.58, Z=0.0198, Y= 0.273
- Distance modulus = 9.7
- 24082 hipparchos stars from HMC

Membership results

- 1510
- 518 Membership probability > 0.5
- 314 Membership probability > 0.9
- 12 G2V solar type stars
- Correct allocation rate 0.937

Main references

[1] Barry D. C., & Cromwell R. H. 1974, *ApJ*, 187, 107

[2] Cabrera, J. & Alfaro E. 1985, *A&A*, 150, 298

[3] Demarque,P., Guenther D. B. & Green E. M. 1992, *AJ*, 103, 151

[4] Dempster A., Laird N. &Rubin D. J. 1977, *J. of the Royal Statistical Society, Series B*, 39, 1

[5] Girard T. M, Grundy W. M., Lopez C. E., & Van Altena W. F. 1989, *AJ*, 98, 227

[6] Giampapa M. S., Radick R. R., Hall J. C., Baliunas S. L. 2000, *BAAS*, 32, 832

[7] McLachlan G. & Krishnan T.1997 *The EM Algorithm and Extensions*, John Wiley

[8] McLachlan G. J., Peel D., Basford K. E. & Adams P. 1999, *J. of Statistical Software*, 4, 2

[9] McLachlan G. & Peel D. 2000, *Finite Mixture Models*, John Wiley

[10] Montgomery K.A., Marschall L. A. & Janes K. A. 1993, *AJ*, 106,181

[11] Pietrinferni A., Cassisi S., Salaris M. & Castelli F. 2006, *AJ*, 1642, 797

[12] Sandage A., Lubin L. & VandenBerg D. 2003, *PASP*, 115, 1187

[13] Sanders W.. 1971, *AA*,14, 226

[14] Sanders W.. 1977, *A&AS*,127, 89

[15] Sanders W.. 1989, *RMxAA*, 17, 31

[16] Stello D. & 21 coauthors 2006, *MNRAS*, 373, 1141

[17] Uribe A., Barrera-Rojas R. & Brieva E. 2007, *PASP, Conference Series*

[18] Uribe A., Barrera-Rojas R. & Brieva E. 2006, *Serbian Astronomical journal*

[19] Uribe A., García-Varela J.A., Sabogal-Marínez, B-E., Higuera G. M.A., & Brieva E. 2002, *PASP*, 114, 233

[20] Van den Berg M,Verbunt F. & Mathieu R. D. 2000, *Proceedings from ASP Conference*, 198, 503

[21] Wolfe, J. H. 1970, *Multivariate Behavioral Research*,, 5, 329.

[22] Zhao J.L., Tian K.P., Pan R.S., He Y.P. & Shi H.M. 1993, *A&AS*, 100, 24

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