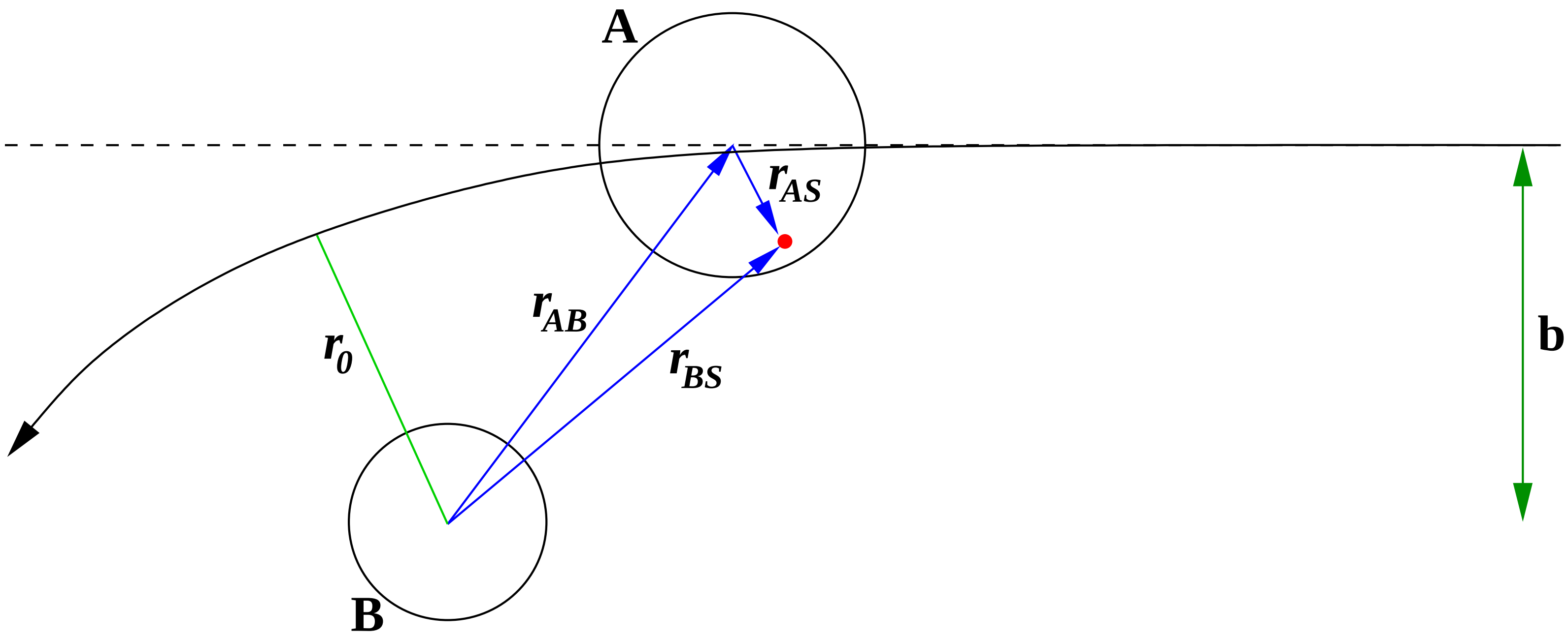
Collisions & Encounters I



Let A encounter B with an initial velocity v_∞ and an impact parameter b.

A star S (red dot) in A gains energy wrt the center of A due to the fact that the center of A and S feel a different gravitational force due to B.

Let $\vec{v}$ be the velocity of S wrt A then

$$\frac{dE_S}{dt} = \vec{v} \cdot \vec{g}[\vec{r}_{BS}(t)] \equiv \vec{v} \cdot \left(-\vec{\nabla}\Phi_B[\vec{r}_{AB}(t)] - \vec{\nabla}\Phi_B[\vec{r}_{BS}(t)] \right)$$

We define $\vec{r}_0$ as the position vector $\vec{r}_{AB}$ of closest approach, which occurs at time t_0.

Collisions & Encounters II

If we increase v_∞ then $|\vec{r}_0| \rightarrow b$ and the energy increase

$$\Delta E_S(t_0) \equiv \int_0^{t_0} \vec{v} \cdot \vec{g}[\vec{r}_{BS}(t)] \, \mathrm{d}t$$

dimishes, simply because t_0 becomes smaller. Thus, for a larger impact velocity v_∞ the star S withdraws less energy from the relative orbit between A and B.

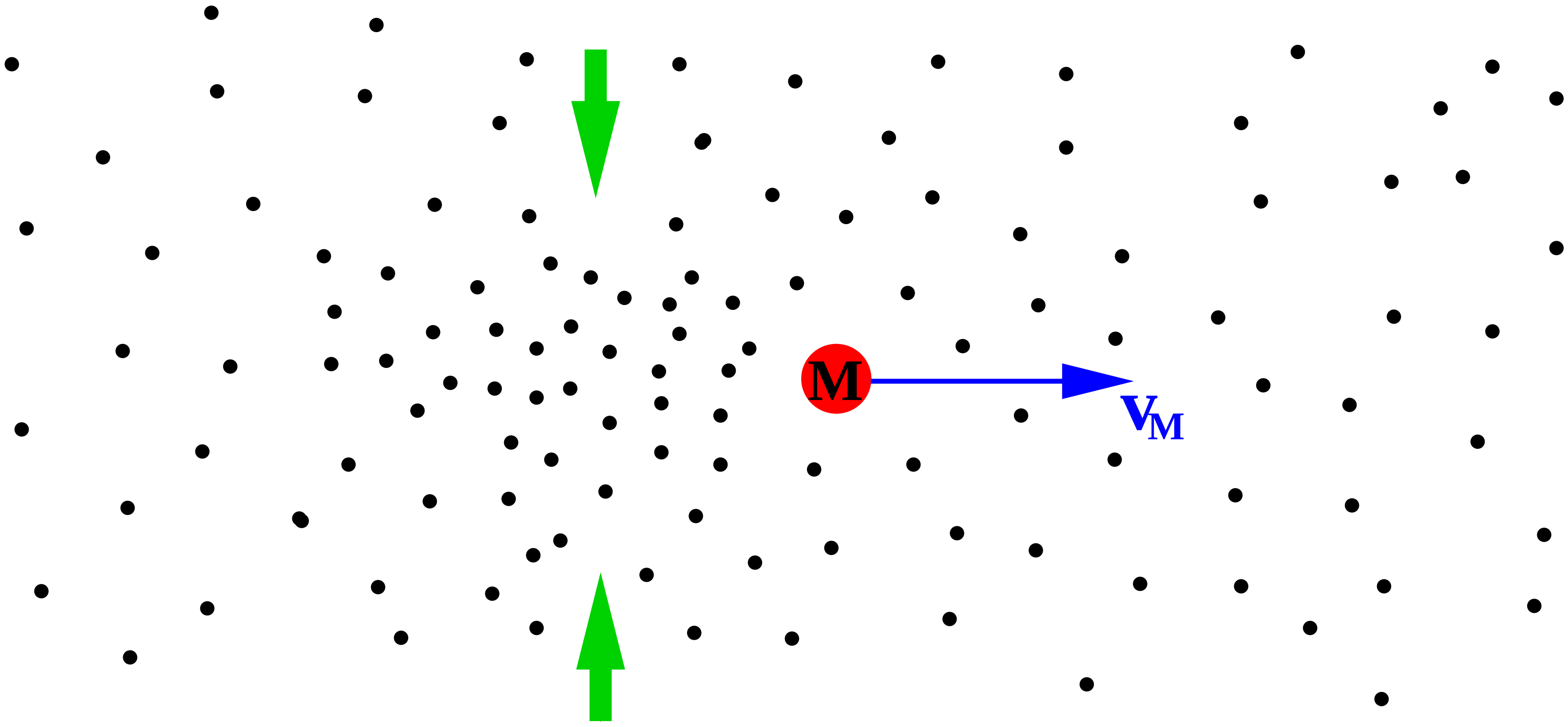
This implies that we can define a critical velocity $v_{\rm crit}$, such that for $v_\infty > v_{\rm crit}$ galaxy A reaches $\vec{r}_0$ with sufficient energy to escape to infinity. If, on the other hand, $v_\infty < v_{\rm crit}$ then systems A and B will merge.

If $v_\infty \gg v_{\rm crit}$ then we can use the impulse approximation to analytically calculate the effect of the encounter.

In most cases of astrophysical interest, however, $v_\infty \lesssim v_{\rm crit}$ and we have to resort to numerical simulations to compute the outcome of the encounter. However, in the special case where $M_A \ll M_B$ or $M_A \gg M_B$ we can describe the evolution with dynamical friction, for which analytical estimates are available.

Dynamical Friction I

Consider the motion of a system with mass M through a medium consisting of many individual 'particles' of mass $m \ll M$. As an example, think of a satellite galaxy moving through the dark matter halo of its parent galaxy.



Due to gravitational focussing M creates an overdensity of particles behind its path (the wake). The backreaction of this wake on M is called dynamical friction and causes M to slow down. Consequently, energy is transferred from the massive to the less massive bodies: dynamical friction is a manifestation of mass segregation.

Dynamical Friction II

Assuming, for simplicity, a uniform density medium with an isotropic velocity distribution $f(v_m)$ of the particles $m \ll M$, then

$$\vec{F}_{\rm df} = M\frac{{\rm d}\vec{v}_M}{{\rm d}t} = -\frac{4\pi G^2 M^2}{v_M^2}\ln\Lambda\,\rho(< v_M)$$

with $\ln\Lambda$ the Coulomb logarithm and

$$\rho(< v_M) = 4\pi \int_0^{v_M} f(v_m) v_m^2 {\rm d}v_m$$

the mass density of background particles with velocities $v_m < v_M$.

The derivation of this equation (see B&T Sect. 7.1) is due to Chandrasekhar (1943), and one therefore often speaks of Chandrasekhar dynamical friction.

Note that $\vec{F}_{\rm df} \propto M^2$: the amount of material that is deflected (i.e., the 'mass' of the wake) is proportional to M and the gravitational force that this wake exerts on M is proportional to M times its own mass.

Note that $\vec{F}_{\rm df} \propto v_M^{-2}$ in the limit of large v_M, but $\vec{F}_{\rm df} \propto v_M$ in the limit of small v_M [i.e., for sufficiently small v_M one may replace $f(v_m)$ with $f(0)$].

Note that $\vec{F}_{\rm df}$ is independent of m!

The Coulomb Logarithm

One has that $\Lambda = b_{\rm max}/b_{\rm min}$ with $b_{\rm min}$ and $b_{\rm max}$ the minimum and maximum impact parameters for which encounters can be considered effective:

Encounters with $b > b_{\rm max}$ don't cause a significant deflection, and these therefore do not contribute significantly to the wake. Encounters with $b < b_{\rm min}$ cause a very strong deflection so that these also do not contribute to the wake.

We can estimate $b_{\rm min}$ as the impact parameter that corresponds to a close encounter (see first lecture), and thus $b_{\rm min} \simeq \frac{GM}{\langle v^2 \rangle}$ with $\langle v^2 \rangle^{1/2}$ the rms velocity of the background particles.

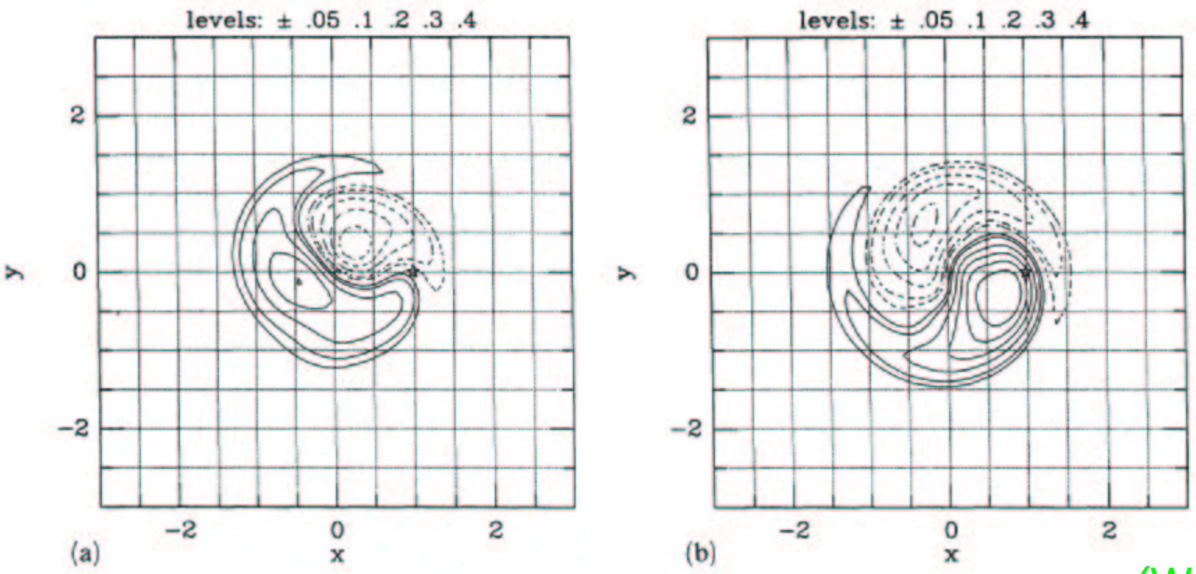
The maximum impact parameter, $b_{\rm max}$, is much harder to estimate (see White 1976), and one typically simply takes $b_{\rm max} \simeq L$ with L the size of the system.

Typical values that one encounters for the Coulomb Logarithm are $3 \lesssim \ln\Lambda \lesssim 30$.

Dynamical Friction: Local vs. Global

Note that Chandrasekhar Dynamical Friction is a purely local phenomenon:
The dynamical friction force $\vec{F}_{\rm df}$ depends only on the local density $\rho(< v)$, and the backreaction owes to a local phenomenon, namely wake-creation due to gravitational focussing.

However, a system A can also experience dynamical friction due to a system B when it is located outside of B (Lin & Tremaine 1983). Clearly, this friction can not arise from a wake. Instead, it arises from torques between A and stars/particles in B that are in resonance with A (Tremaine & Weinberg 1984).



(Weinberg 1989)

The extent to which dynamical friction is a local (wake) versus a global (resonant-coupling) effect is still being debated.

Orbital Decay I

Consider a singular, isothermal sphere with density and potential given by

$$\rho(r) = \frac{V_c^2}{4\pi G r^2} \qquad\qquad \Phi(r) = V_c^2 \ln r$$

If we further assume that this sphere has, at each point, an isotropic and Maxwellian velocity distribution, then

$$f(v_m) = \frac{\rho(r)}{(2\pi\sigma^2)^{3/2}} \exp\left[-\frac{v_m^2}{2\sigma^2}\right]$$

with $\sigma = V_c/\sqrt{2}$. Now consider a test-particle of mass M moving on a circular orbit (i.e., $v_M = V_c$) through this sphere. The Chandrasekhar dynamical friction that this particle experiences is

$$F_{\rm df} = -\frac{4\pi \ln\Lambda G^2 M^2 \rho(r)}{V_c^2}\left[\mathrm{erf}(1) - \frac{2}{\sqrt{\pi}}\mathrm{e}^{-1}\right] \simeq -0.428 \ln\Lambda \frac{GM^2}{r^2}$$

The test-particle has an angular momentum $L = r v_M$, which it looses, due to dynamical friction, at a rate

$$\frac{\mathrm{d}L}{\mathrm{d}t} = r\frac{\partial v_M}{\partial t} = r\frac{F_{\rm df}}{M} = -0.428 \ln\Lambda \frac{GM}{r}$$

Due to this angular momentum loss the test-particle moves to a smaller radius, while it continues on circular orbits with $v_M = V_C$.

Orbital Decay II

The rate at which the radius changes follows from

$$V_c \frac{dr}{dt} = -0.428 \ln\Lambda \frac{GM}{r}$$

Solving this differential equation subject to the initial condition $r(0) = r_i$ one finds that the test-particle reaches the center after a time

$$\boxed{t_{df} = \frac{1.17}{\ln\Lambda} \frac{r_i^2 V_c}{GM}}$$

As an example, consider the LMC. Assume for simplicity that the LMC moves on a circular orbit at $r_i = 50$ kpc, that the mass of the LMC is $M = 2 \times 10^{10}\,M_\odot$, and that the MW can be approximated as a singular isothermal sphere with $V_c = 220\,\mathrm{km\,s^{-1}}$ and with a radius of $r = 200$ kpc.

We then find that the LMC will reach the center of the MW halo after a time $t_{df} \simeq \frac{7.26}{\ln\Lambda}$ Gyr. Using the approximation for Λ discussed before we find that $\ln\Lambda \simeq 6$, and thus $t_{df} \simeq 1.2$ Gyr.

Orbital Decay III

The derivation on the previous pages was for a circular orbit. We now focus on the orbital decay of an eccentric orbit, whose eccentricity is defined as

$$e = \frac{r_+ - r_-}{r_+ + r_-}$$

with r_+ and r_- the apo- and pericenter, respectively.

For simplicity, we once again focus on a singular isothermal sphere, for which the radius of a circular orbit with energy E is given by

$$r_c(E) = \frac{1}{\sqrt{e}} \exp\left(\frac{E}{V_c^2}\right)$$

We can express the angular momentum of an eccentric orbit in terms of the orbit's circularity

$$\eta \equiv \frac{L}{L_c(E)} = \frac{L}{r_c(E) V_c}$$

The circularity η is uniquely related to the orbital eccentricity e, with $\mathrm{d}e/\mathrm{d}\eta < 0$:

Circular orbit: $\eta = 1$ and $e = 0$

Radial orbit: $\eta = 0$ and $e = 1$

We now investigate how dynamical friction influences the orbit's evolution.

Orbital Decay IV

Dynamical friction transfers both energy and angular momentum from the test-particle to the particle's that make up the halo. Let's examin how this influences the orbit's eccentricity

$$\frac{\mathrm{d}e}{\mathrm{d}t} = \frac{\mathrm{d}e}{\mathrm{d}\eta}\frac{\mathrm{d}\eta}{\mathrm{d}t}$$

Using the definition of the orbital circularity we obtain

$$\frac{\mathrm{d}\eta}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{L}{L_c(E)}\right) = \frac{1}{L_c(E)}\frac{\mathrm{d}L}{\mathrm{d}t} - \frac{L}{L_c^2(E)}\frac{\partial L_c(E)}{\partial E}\frac{\mathrm{d}E}{\mathrm{d}t} = \eta\left[\frac{1}{L}\frac{\mathrm{d}L}{\mathrm{d}t} - \frac{1}{V_c^2}\frac{\mathrm{d}E}{\mathrm{d}t}\right]$$

where we have used that $L_c(E) = r_c(E)V_c$. Using that $L = rv_\perp$, with $v_\perp$ the velocity in the direction perpendicular to the radial vector, we find that

$$\frac{\mathrm{d}E}{\mathrm{d}t} = v\frac{\mathrm{d}v}{\mathrm{d}t} \qquad\qquad \frac{\mathrm{d}L}{\mathrm{d}t} = r\frac{\mathrm{d}v_\perp}{\mathrm{d}t} = \frac{L}{v}\frac{\mathrm{d}v}{\mathrm{d}t}$$

Combining all the above we finally find that

$$\frac{\mathrm{d}e}{\mathrm{d}t} = \frac{\eta}{v}\frac{\mathrm{d}e}{\mathrm{d}\eta}\left[1 - \left(\frac{v}{V_c}\right)^2\right]\frac{\mathrm{d}v}{\mathrm{d}t}$$

where $\mathrm{d}v/\mathrm{d}t = F_{\mathrm{df}}/M < 0$ (see van den Bosch et al. 1999).

Orbital Decay V

At pericenter we have that $v > V_c$. Since $\eta > 0$, $\frac{de}{d\eta} < 0$, and $\frac{dv}{dt} < 0$ we thus have that $\frac{de}{dt} < 0$; the eccentricity decreases and the orbit becomes more circular.

However, at apocenter $v < V_c$ and therefore $\frac{de}{dt} > 0$: the orbit becomes more eccentric during an apocentric passage.

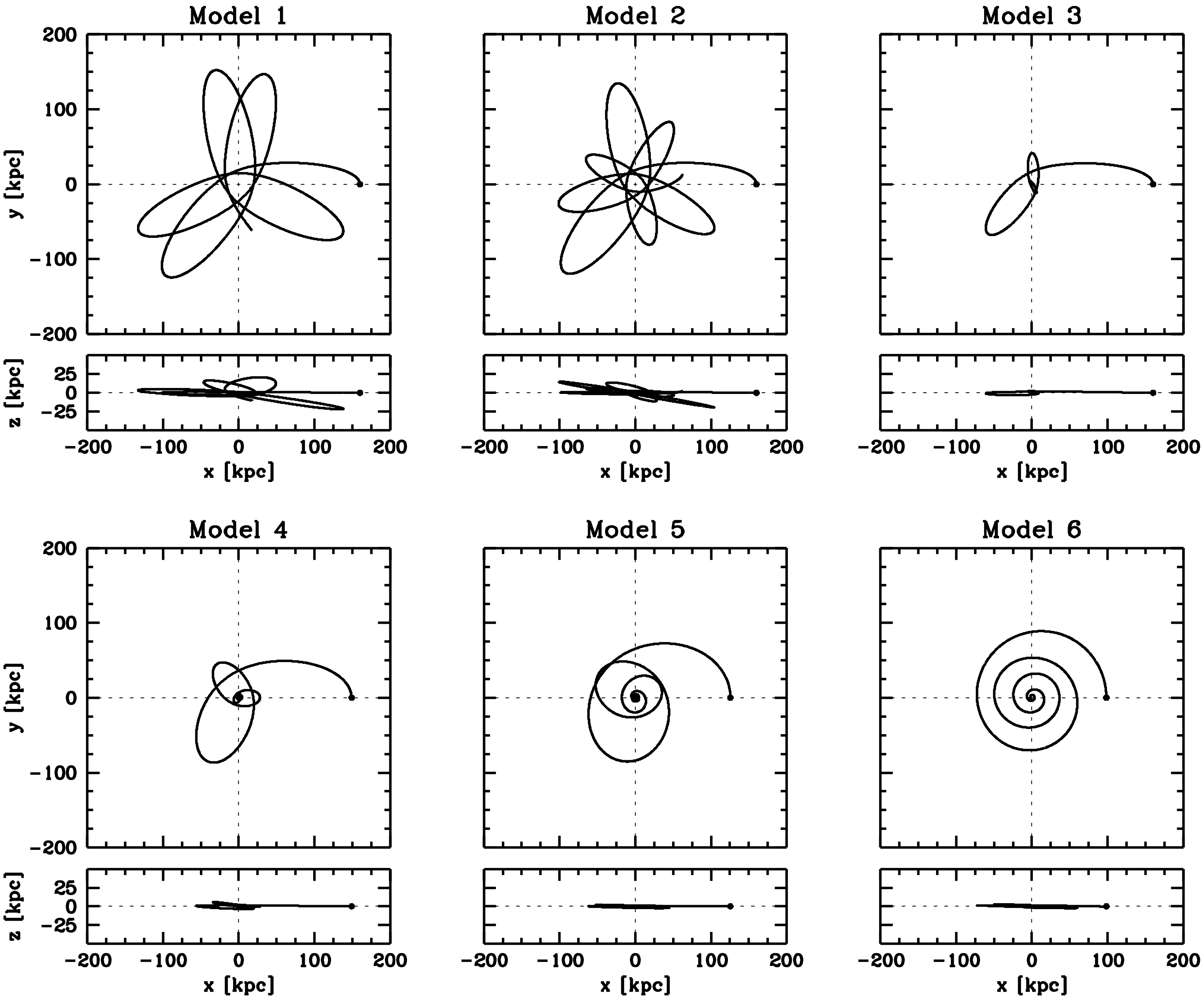
The overall effect of dynamical friction on the orbit's eccentricity, integrated over an entire orbit, can not be obtained from inspection: numerical simulations are required.

For realistic density distributions one finds that $\frac{de}{dt} \sim 0$: contrary to what is often claimed in the literature, dynamical friction does (in general) not lead to circularization of the orbit (see van den Bosch et al. 1999).

As an example of an orbit that circularizes, consider a space-ship on an eccentric orbit around the Earth. It only experiences a friction, due to the Earth's atmosphere, during a pericentric passage, and this causes the 'grazing' orbit of the space-ship to circularize.

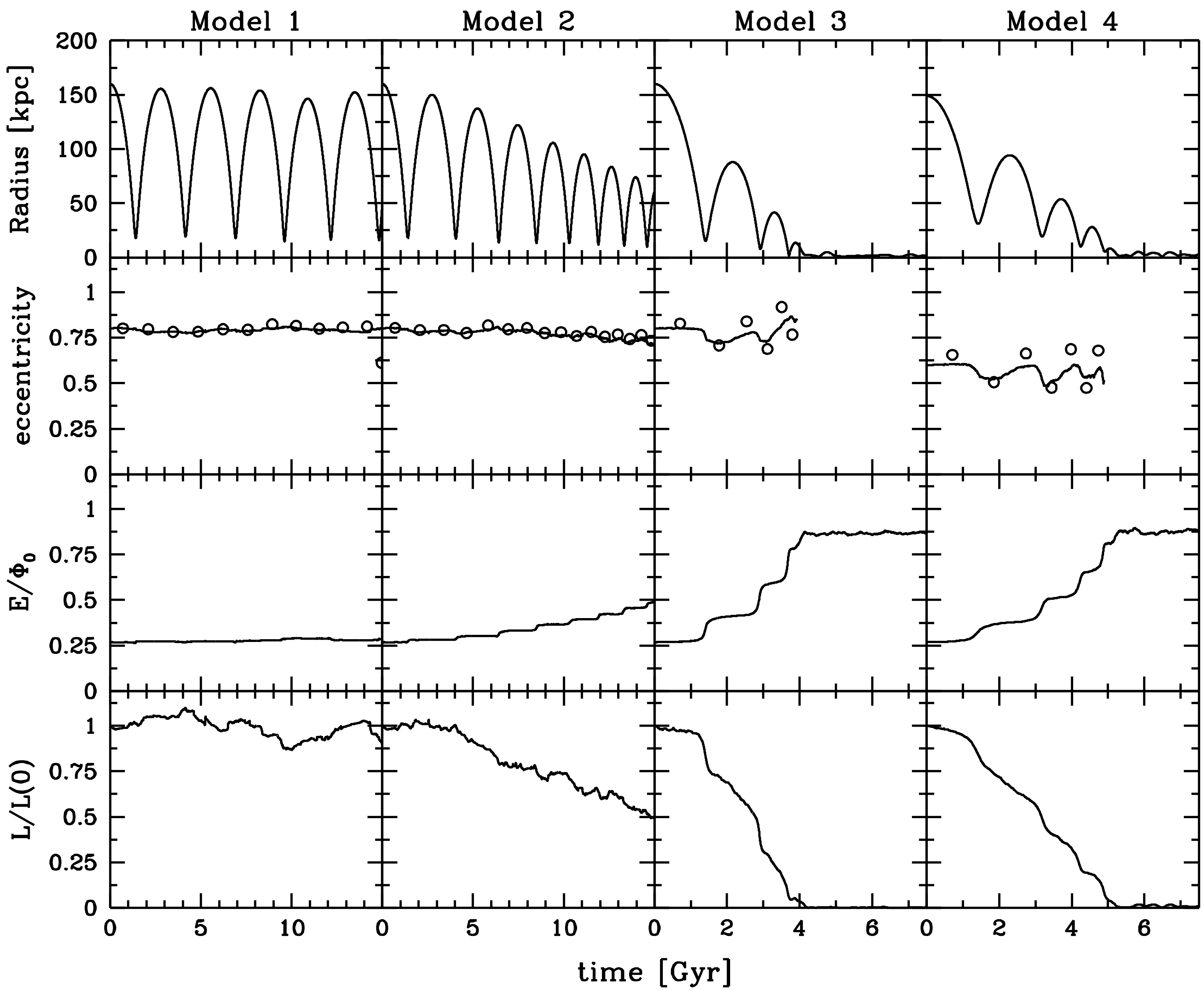
Numerical simulations have shown that $t_{\rm df} \propto \eta^{0.53}$.

Orbital Decay VI



van den Bosch et al. (1999)

Orbital Decay VII



van den Bosch et al. (1999)

The Impulse Approximation I

There are two kinds of encounters between collisionless systems that can be treated analytically:

- Encounters of very unequal mass $\triangleright$ Dynamical Friction
- Encounters of very high speed $\triangleright$ Impulse Approximation

As we have seen before, when v_∞ becomes larger, the effect of the encounter diminishes. Therefore, for sufficiently large v_∞ we can treat the encounter as a perturbation.

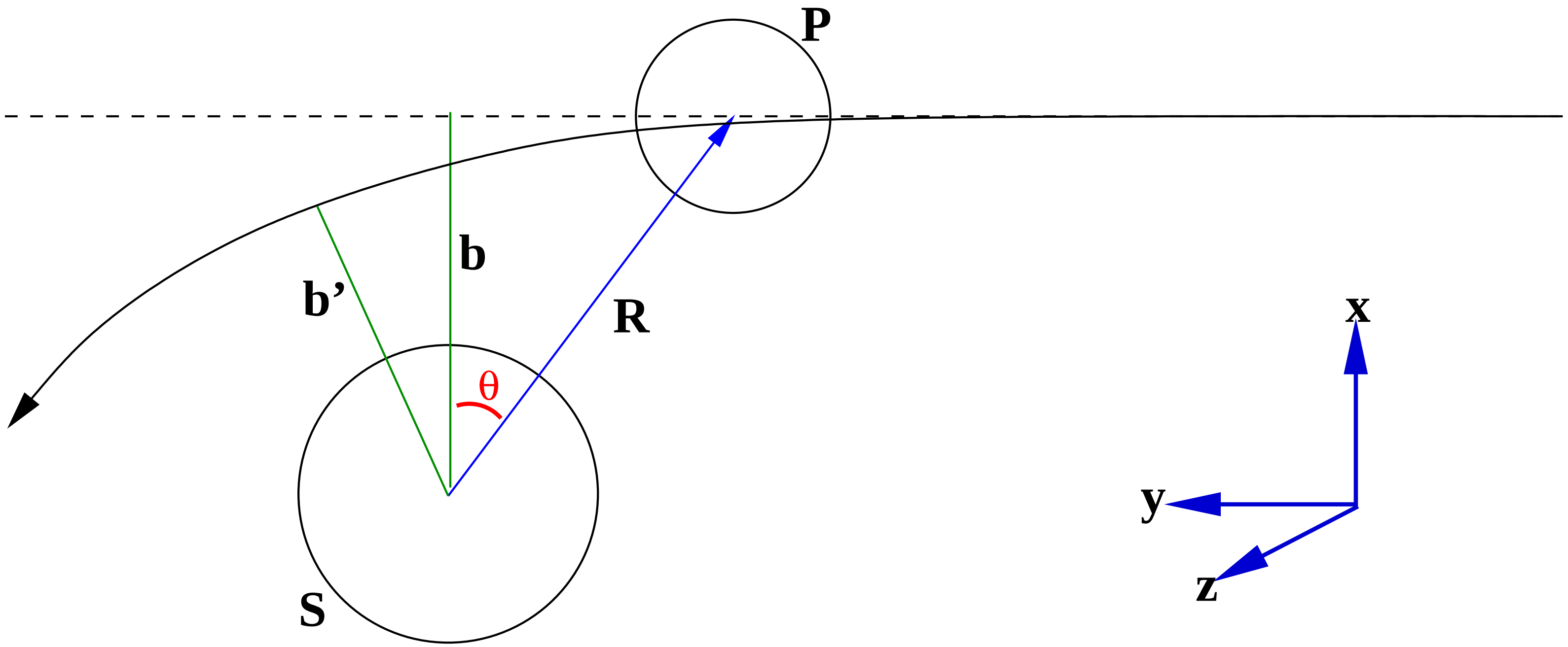
The crucial assumption of the impulse approximation is that the tidal forces due to the perturber act on a timescale $\ll$ orbital time scale of the perturbed stars, so that we may consider the star stationary during the encounter.

- $\triangleright$ No resonant effects
- $\triangleright$ Instantaneous change in velocity of each star
- $\triangleright$ Magnitude of $\Delta\vec{v}$ depends on location of star but not on its velocity
- $\triangleright$ If the encounter speed is sufficiently large then perturber moves in straight line with $v_p(t) = v_\infty \vec{e}_y \equiv v_p \vec{e}_y$ and $\vec{R}(t) = (b, v_p t, 0)$.

Note that the equations for $v_p(t)$ and $\vec{R}(t)$ define the coordinate system that we will adopt in what follows.

The Impulse Approximation II

Consider a system P, which we call the perturber, encountering another system S with an impact parameter b and an initial velocity v_∞. Let $\vec{R}(t)$ be the position vector of P from S and $v_p(t)$ the velocity of P wrt S.



In the large-v_∞ limit we have the $b' \simeq b$ and $v_p(t) \simeq v_\infty \vec{e}_y \equiv v_p \vec{e}_y$ so that $\vec{R}(t) = (b, v_p t, 0)$.

A star in S experiences a gravitational force due to P given by

$$\vec{a}_*(t) = \frac{G M_p f(R) \vec{R}}{R^3}$$

with $f(R)$ the fraction of P's mass that falls within R.

The Impulse Approximation III

We consider the case with $b > \max[R_p, R_s]$ with R_p and R_s the sizes of P and S, respectively.

In this distant encounter approximation we have that $f(R) = 1$, and

$$
\begin{aligned}
\Delta\vec{v}_* &= \int_{-\infty}^{\infty} \vec{a}(t)\mathrm{d}t \\
&= GM_p \int_{-\infty}^{\infty} \frac{(b, v_p t, 0)\mathrm{d}t}{(b^2+v_p^2 t^2)^{3/2}} \\
&= \frac{GM_p}{v_p}\left(\int_{-\infty}^{\infty} \frac{b\ \mathrm{d}s}{(s^2+b^2)^{3/2}}, \int_{-\infty}^{\infty} \frac{s\ \mathrm{d}s}{(s^2+b^2)^{3/2}}, , 0\right) \\
&= \frac{GM_p}{v_p}\left(\frac{2}{b}, 0, 0\right) \\
&= \frac{2GM_p}{v_p b}\vec{e}_x
\end{aligned}
$$

The ratio M_p/v_p is called the collision strength. In impulse approximation the mass and velocity of the perturber only enter through this ratio.

We can split this $\Delta\vec{v}_*$ in two components: the component $\Delta\vec{v}_S$ which describes change in center of mass velocity of S, and the component $\Delta\vec{v}$ wrt the systematic velocity of S.

The Impulse Approximation IV

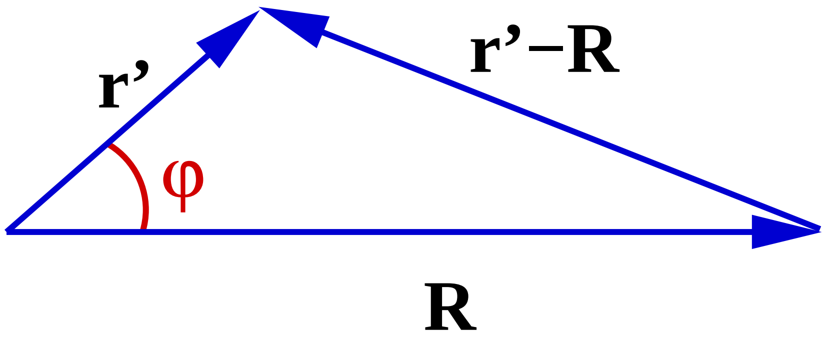
Since we are interested in how P modifies the internal structure of S, we are only interested in $\Delta\vec{v}$.

Note that $\Delta\vec{v}$ arises due to the tidal forces on S, which arise from the fact that the gravitational attraction of P is not uniform over S.

We define a rotating coordinate frame (x', y', z') centered on the center of S, and with the x'-axis pointing towards the instantaneous location of P.

Let $\vec{r}'$ be the position vector of a star in S, and $\vec{R} = R\vec{e}_{x'}$ the position vector of P.

The potential at $\vec{r}'$ due to P is $\Phi(\vec{r}') = -\frac{GM_p}{|\vec{r}' - \vec{R}|}$.



From the above figure one can see that

$$|\vec{r}' - \vec{R}|^2 = (R - r'\cos\phi)^2 + r'^2\sin^2\phi = R^2 - 2rR\cos\phi + r'^2$$

The Impulse Approximation V

Using the series expansion $\frac{1}{\sqrt{1+x}} = 1 - \frac{1}{2}x + \frac{1\cdot3}{2\cdot4}x^2 - \frac{1\cdot3\cdot5}{2\cdot4\cdot6}x^3 + \ldots$ this yields

$$\frac{1}{|\vec{r}' - \vec{R}|} = \frac{1}{R}\left[1 - \frac{1}{2}\left(-2\frac{r'}{R}\cos\phi + \frac{r'^2}{R^2}\right) + \frac{3}{8}\left(-2\frac{r'}{R}\cos\phi + \frac{r'^2}{R^2}\right)^2 + \ldots\right]$$

which allows us to write

$$\Phi(\vec{r}') = -\frac{GM_p}{R} - \frac{GM_p r'}{R^2}\cos\phi - \frac{GM_p r'^2}{R^3}\left(\frac{3}{2}\cos^2\phi - \frac{1}{2}\right) - \ldots$$

The first term is a constant and foes not yield any force.

The second term yields a uniform acceleration $\frac{GM_p}{R^2}\vec{e}_{x'}$ directed towards P. This is the term that causes the center of mass of S to change its velocity, and is not of interest to us.

In the tidal approximation one considers the third term:

$$\Phi_3(\vec{r}') = -\frac{GM_p}{R^3}\left(\frac{3}{2}r'^2\cos^2\phi - \frac{1}{2}r'^2\right)$$

Using that $r'\cos\phi = x'$ and that $r'^2 = x'^2 + y'^2 + z'^2$ we obtain

$$\Phi_3(x', y', z') = -\frac{GM_p}{2R^3}\left(2x'^2 - y'^2 - z'^2\right)$$

The Impulse Approximation VI

The above allows us to write the tidal forces on S as

$$F_{x'} = \frac{2GM_p x'}{R^3} \qquad F_{y'} = -\frac{GM_p y'}{R^3} \qquad F_{z'} = -\frac{GM_p z'}{R^3}$$

These are related to the tidal forces in the (x, y, z) coordinate system:

$$\begin{aligned} F_x &= F_{x'} \cos\theta - F_{y'} \sin\theta \\ F_y &= F_{x'} \sin\theta + F_{y'} \cos\theta \\ F_z &= F_{z'} \end{aligned}$$

while (x', y', z') are related to (x, y, z) according to

$$\begin{aligned} x' &= x\cos\theta - y\sin\theta \\ y' &= -x\sin\theta + y\cos\theta \\ z' &= z \end{aligned}$$

so that we obtain, after some algebra

$$\begin{aligned} F_x = \frac{\mathrm{d}v_x}{\mathrm{d}t} &= \frac{GM_p}{R^3}\left[x\,(2 - 3\sin^2\theta) + 3\,y\,\sin\theta\cos\theta\right] \\ F_y = \frac{\mathrm{d}v_y}{\mathrm{d}t} &= \frac{GM_p}{R^3}\left[y\,(2 - 3\cos^2\theta) + 3\,x\,\sin\theta\cos\theta\right] \\ F_z = \frac{\mathrm{d}v_z}{\mathrm{d}t} &= -\frac{GM_p z}{R^3} \end{aligned}$$

The Impulse Approximation VII

Integrating these equations over time yields the cumulative velocity changes wrt the center of S. Using that $\vec{R}(t) = (b, v_p t, 0)$, and thus $\cos\theta = b/R$ and $\sin\theta = v_p t/R$ we obtain

$$\Delta v_x = \frac{2GM_p x}{v_p b^2} \qquad \Delta v_y = 0 \qquad \Delta v_z = -\frac{2GM_p z}{v_p b^2}$$

We thus have that $\Delta\vec{v} = \frac{2GM_p}{v_p b^2}(x, 0, -z)$, and

$$\Delta E = \tfrac{1}{2}\left(\vec{v} + \Delta\vec{v}\right)^2 + \Phi(\vec{r}') - \tfrac{1}{2}\vec{v}^2 - \Phi(\vec{r}') = \vec{v}\cdot\Delta\vec{v} + \tfrac{1}{2}(\Delta v)^2$$

Note that, in the impulse approximation, the potential energy does not change during the encounter.

We are interested in computing $\Delta E_{\rm tot}$ which is obtained by integrating ΔE over the entire system S.

First we note that the integral of the first term of ΔE typically is equal to zero, by symmetry. Therefore

$$\begin{aligned}
\Delta E_{\rm tot} &= \tfrac{1}{2}\int \rho(\vec{r}')|\Delta\vec{v}|^2 {\rm d}^3\vec{r}' \\
&= \frac{2G^2M_p^2}{v_p^2 b^4}\int \rho(\vec{r}')(x^2+z^2){\rm d}^3\vec{r}' \\
&= \frac{2G^2M_p^2}{v_p^2 b^4} M_s \langle x^2 + z^2\rangle
\end{aligned}$$

The Impulse Approximation VIII

Assuming spherical symmetry for S, so that $\langle x^2 + z^2 \rangle = \frac{2}{3}\langle x^2 + y^2 + z^2 \rangle = \frac{2}{3}\langle r^2 \rangle$ we finally obtain

$$\Delta E_{\rm tot} = \frac{4}{3} G^2 M_s \left(\frac{M_p}{v_p} \right)^2 \frac{\langle r^2 \rangle}{b^4}$$

As shown by Aguilar & White (1985), this derivation, which is originally due to Spitzer (1958), is surprisingly accurate for encounters with $b \gtrsim 5 \max[r_p, r_s]$ (with r_p and r_s the median radii of P and S), even for relatively slow collisions with $v_\infty \simeq \langle v_s^2 \rangle^{1/2}$.

The above shows that fast encounters pump energy into the systems involved. This energy originates from the kinetic energy associated with the orbit of P wrt S. Note that $\Delta E_{\rm tot} \propto b^{-4}$, so that close encounters are far more important that distant encounters.

As soon as the amount of energy pumped into S becomes comparable to its binding energy, the system S will become tidally disrupted.

Some stars can be accelerated to velocities that exceed the local escape velocity $\triangleright$ encounters, even those that do not lead to tidal disruption, may cause mass loss of S. In this case, the first terms of ΔE is not zero, and the above impulse approximation has to be handled with care.

Return to Equilibrium

As we have seen, a fast encounter transfers orbital energy to the two systems involved in the encounter, whose kinetic energy has subsequently increased.

After the encounter the systems are therefore no longer in virial equilibrium. The systems now need to readjust themselves to find a new virial equilibrium. Interestingly, this process changes the internal kinetic energy more than did the encounter itself.

Let the initial kinetic and total energies of a system be T_0 and E_0, respectively. According to the virial theorem we have that $E_0 = -T_0$.

Due to the encounter $T_0 \rightarrow T_0 + \delta T$, and thus also $E_0 \rightarrow E_0 + \delta T$.

Applying the virial theorem we obtain that after the relaxation the new kinetic energy is

$$T_1 = -E_1 = -(E_0 + \delta T) = T_0 - \delta T$$

Thus, the relaxation process decreases the kinetic energy by $2\delta T$ from $T_0 + \delta T$ to $T_0 - \delta T_0$.

Similarly, the gravitational energy becomes less negative:

$$W_1 = 2E_1 = 2E_0 + 2\delta T = W_0 + 2\delta T$$

Since the gravitational radius $r_g = GM^2/|W|$ the system will expand!

Heat Capacity of Gravitating Systems

As we have seen on the previous page, by pumping energy ('heat') into the system, it has actually grown 'colder'. This is a consequence of the negative heat capacity of gravitational systems.

By analogy with an ideal gas we defined the temperature of a self-gravitating system as

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_B T$$

Unlike an isothermal gas, the temperature in a self-gravitating system is typically a function of position. Therefore, we define the mean temperature as

$$\langle T \rangle \equiv \frac{1}{M} \int \rho(\vec{x}) T(\vec{x}) \mathrm{d}^3\vec{x}$$

and the total kinetic energy of a system of N particles is then $K = \frac{3}{2} N k_B \langle T \rangle$. According to the virial theorem we thus have that $E = -\frac{3}{2} N k_B \langle T \rangle$.

This allows us to define the heat capacity of the system as

$$C \equiv \frac{\mathrm{d}E}{\mathrm{d}\langle T \rangle} = -\frac{3}{2} N k_B$$

which is thus negative: by losing energy the system becomes hotter!

Heat Capacity of Gravitating Systems

Note that all systems in which the dominant forces are gravitational have a negative heat capacity. This includes the Sun, where the stability of nuclear burning is a consequence of $C < 0$: If the reaction rates become 'too high', the excess energy input into the core makes the core expand and cool. This makes the reaction rates drop, bringing the system back to equilibrium.

The negative specific heat also results in fascinating phenomena in stellar-dynamical systems.

Consider a central density cusp. If the cusp is sufficiently steep one has that $\sigma(r)$ increases with radius: the center is colder than its surroundings.

Two-body interactions tend towards thermal equilibrium, which means that they transport heat from outside to inside.

Since $C < 0 \quad \Rightarrow \quad \sigma_0 \downarrow$, i.e., the center becomes colder!

As a consequence $\vec{\nabla} T \uparrow$, and the heat flow becomes larger.

This leads to run-away instability, known as Gravothermal Catastrophe.

Thus, if radial temperature gradient exists, and two-body relaxation time is sufficiently short (e.g., in globular clusters), the system can undergo core collapse.