

THE 30TH JERUSALEM WINTER SCHOOL IN THEORETICAL PHYSICS

Lecture 4

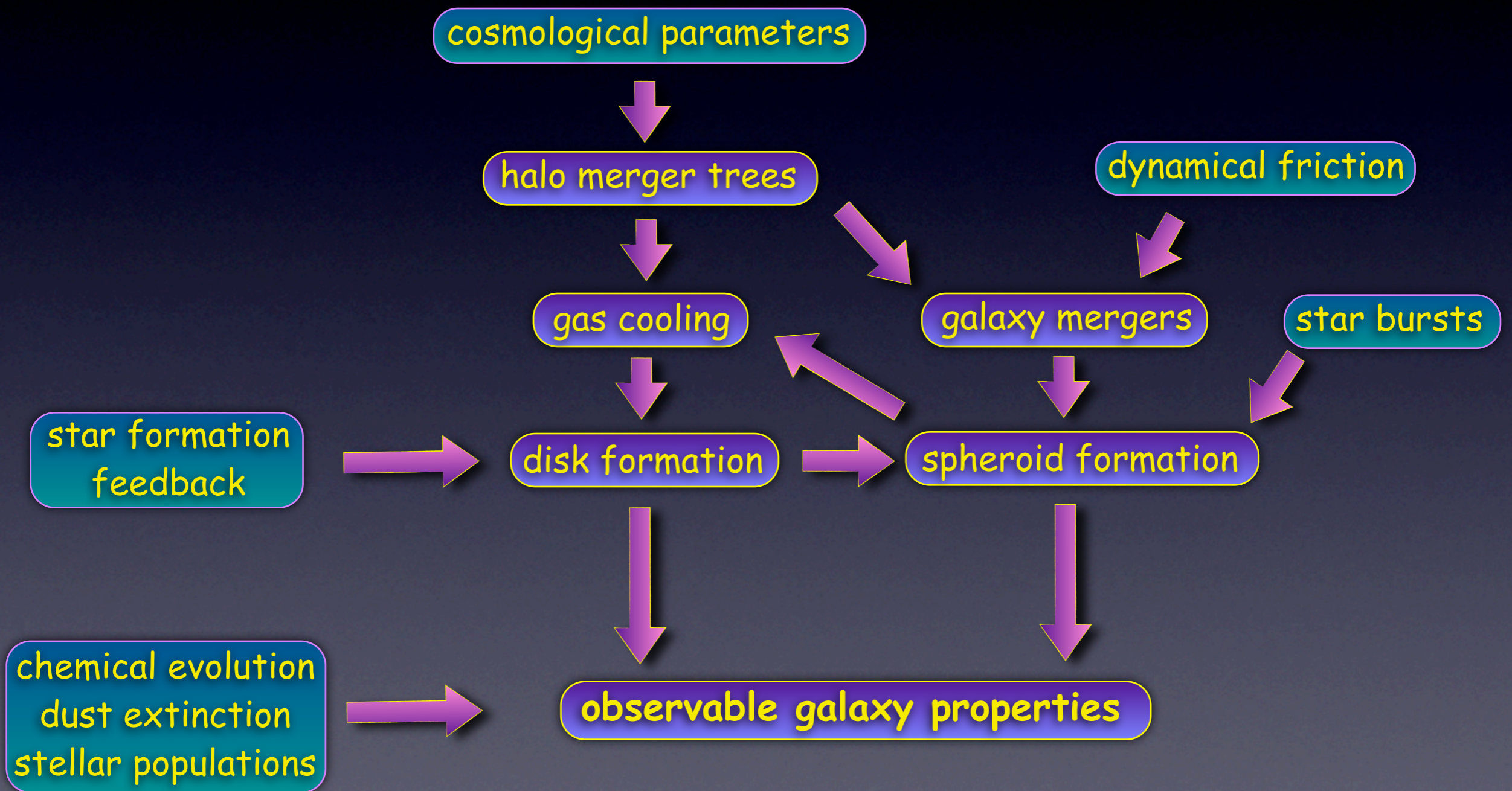
Semi-Analytical Models

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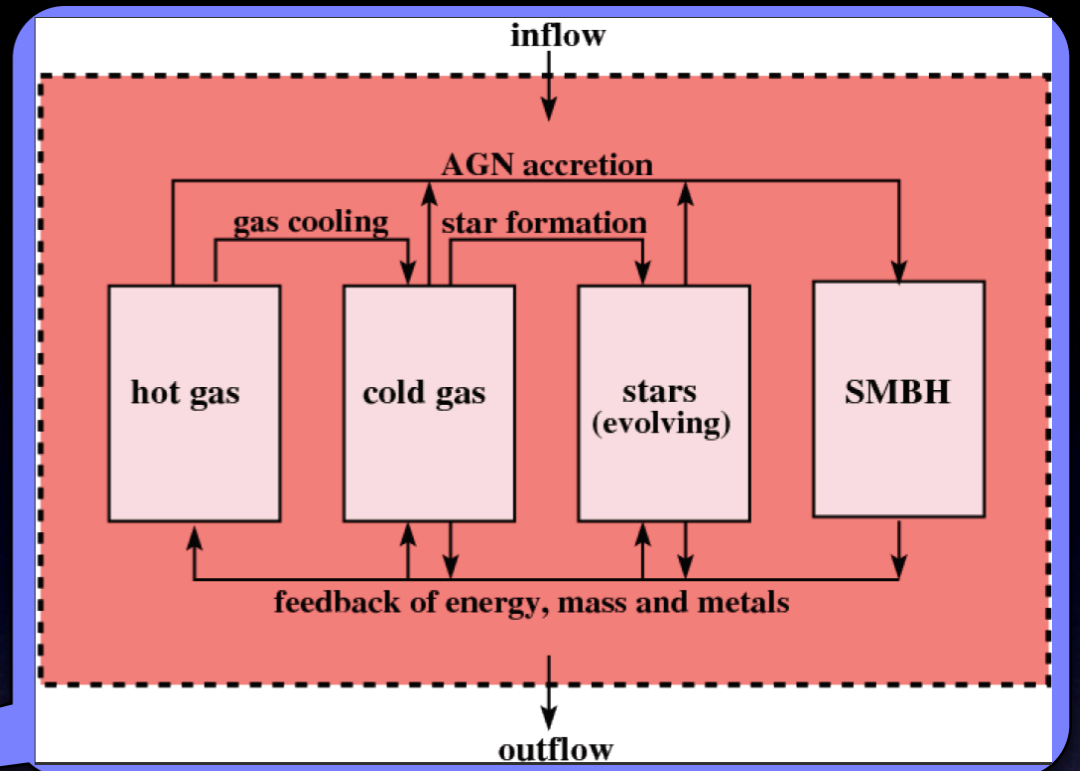
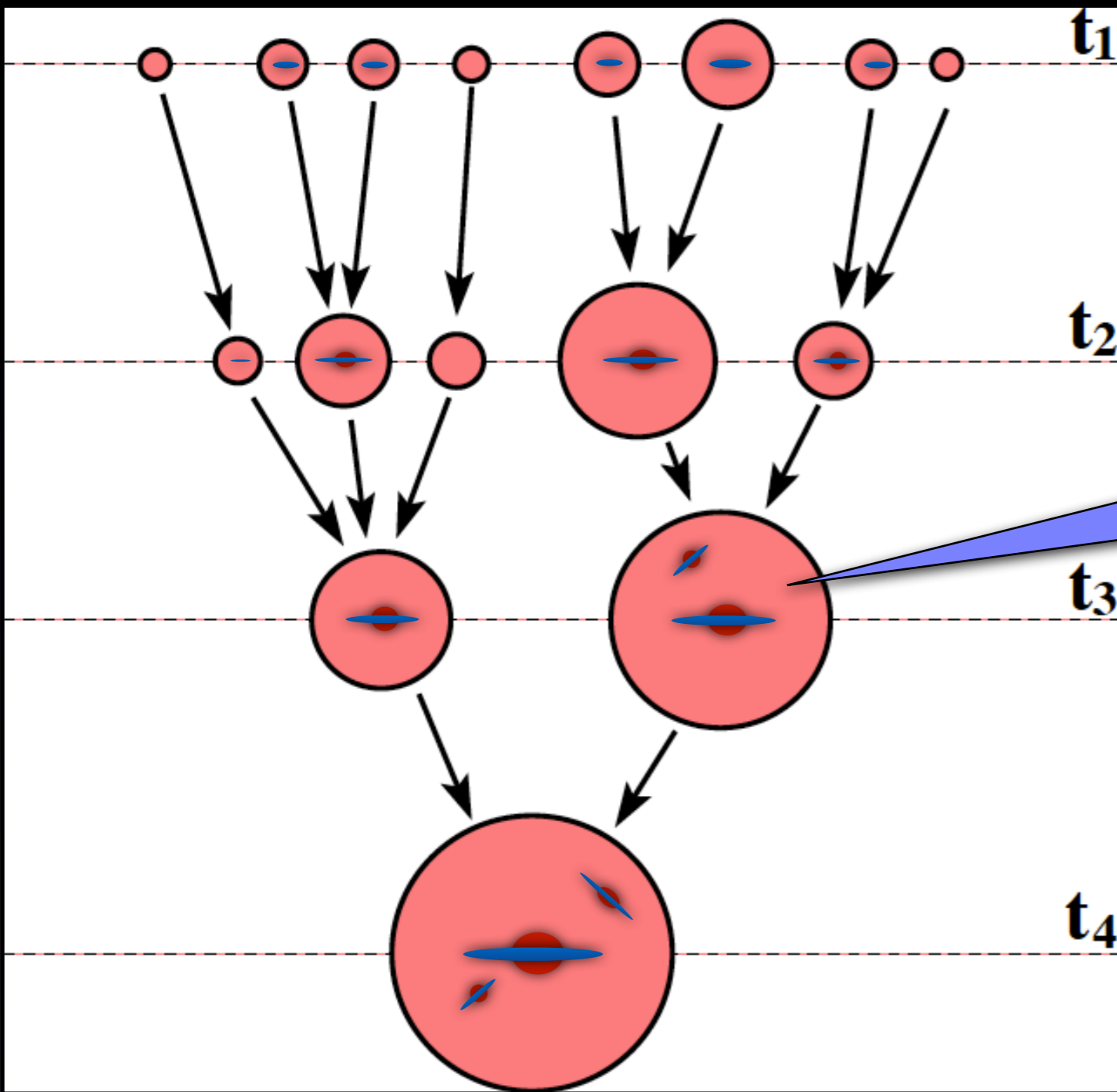


SAM Basics

Semi-Analytical Models (SAMs) for galaxy formation are phenomenological models that use approximate, analytical descriptions to describe the various processes relevant for galaxy formation in order to make predictions that can be compared to observations.



SAM Basics



$$M_{\text{vir}} = M_{\text{DM}} + M_{*} + M_{\text{cold}} + M_{\text{hot}}$$

$$\dot{M}_{\text{hot}} = -\dot{M}_{\text{cool}} + \dot{M}_{\text{reheat}} + f_{\text{bar}} \dot{M}_{\text{vir}}$$

$$\dot{M}_{\text{cold}} = \dot{M}_{\text{cool}} - \dot{M}_{\text{reheat}} - \dot{M}_{*}$$

\dot{M}_{vir} \longleftrightarrow mass accretion

\dot{M}_{cool} \longleftrightarrow cooling

\dot{M}_{*} \longleftrightarrow star formation

\dot{M}_{reheat} \longleftrightarrow feedback

SAM Basics

Currently there are many different **SAMs** available in the literature, all with different treatments of different physical processes....

Feature	Model				
	DURHAM	MUNICH	SANTA-CRUZ	MORGANA	GALICS
Merger Trees					
→ Analytic	Modified ePS	ePS	ePS	PINOCCHIO	×
→ N-body	✓	✓	✓	×	✓
Halo Profiles	Einasto	Isothermal	NFW	NFW	Empirical
Cooling Model					
→ Metal-dependent	✓	✓	✓	✓	✓
Star Formation	✓	✓	✓	✓	✓
Feedbacks					
→ SNe	✓	✓	✓	✓	✓
→ AGN	✓	✓	✓	✓	✓
→ Reionization	✓	×	✓	✓	✓
Merging					
→ Substructure ¹⁶	N-body	N-body	DF	DF	N-body
→ Substructure-Substructure ¹⁹	✓	×	✓	×	✓
Environments					
→ Ram Pressure Stripping	✓	✓	×	×	✓
→ Tidal Stripping	✓	×	✓	✓	✓
→ Harassment	×	×	×	×	×
Disks					
→ Disk Stability	✓	✓	✓	✓	✓
→ Dynamical Friction ²⁷	✓	×	×	×	×
→ Thickness	✓	×	×	×	×
Sizes					
→ Adiabatic contraction	✓	×	✓	✓	×
Chemical Enrichment	✓ [delayed ¹⁰]	✓ [instant ²⁹]	✓ [delayed ³⁰]	✓ [instant]	✓ [delayed ³¹]
Dust	GRASIL	Screen	Slab	GRASIL	Slab

Source: Benson 2010, Physical Reports, 495, 33

SAM Basics

Currently there are many different **SAMs** available in the literature, all with different treatments of different physical processes....

And each `individual' **SAM** is constantly being updated/expanded/modified...

A 1.5hr lecture can't possibly do justice to the immense amount of work done on and with **SAMs**. Furthermore, a comparison among different models is largely meaningless...

Instead, we will focus on the basic structure of **SAMs**, discuss the treatment of some of the main ingredients (cooling, SF, feedback, dynamical friction), and examine the current status of some of the models, focussing only on

- luminosity/stellar mass functions at $z=0$
- galaxy-dark matter connection (clustering & TF relation)
- stellar mass assembly histories

Topics that are important but which I won't be able to cover include

- Stellar Population Synthesis Modeling
- Dust (absorption & emission)
- Chemical Evolution (metallicities & abundances)
- Galaxy Sizes
- BH-Bulge mass relation
- Disk Instabilities

The background of the slide is a complex, fractal-like network of blue and purple filaments, representing the cosmic web of galaxies and dark matter. The filaments are interconnected and form a dense, web-like structure against a dark background.

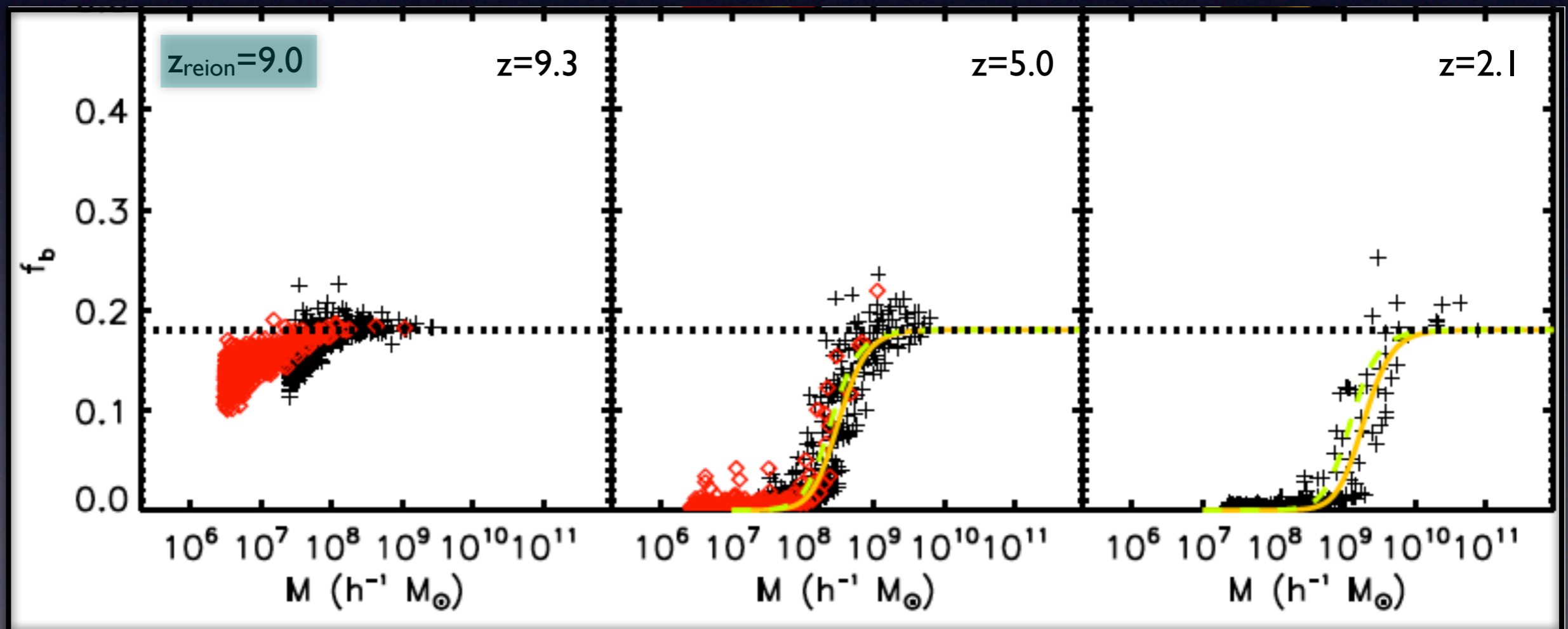
Heating & Cooling

Reionization

The fraction of baryonic matter that is bound to a dark matter halo is regulated by the potential depth and the pressure of the baryons.

When the Universe **reionizes**, the temperature and thus the pressure of the baryons increases. This results in a strongly **reduced baryon fraction** in low mass halos.

Okamoto et al. (2008) used hydro simulations to study the impact of reionization imposing a uniform, ionizing background as computed by Haardt & Madau (2001).



Source: Okamoto et al. 2008, MNRAS, 390. 920.

Reionization

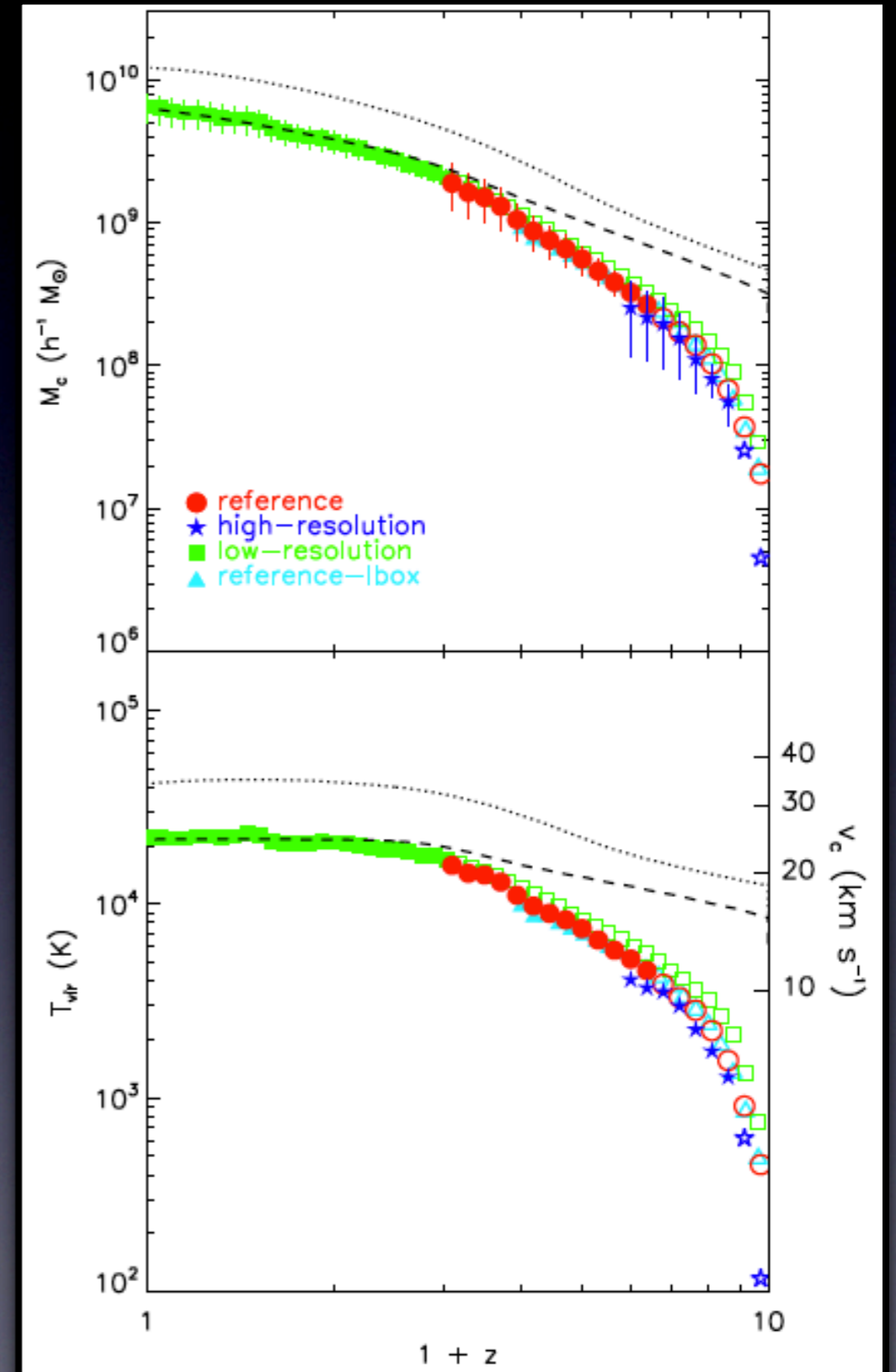
The fraction of baryons that remains bound to their halos is well fit by the following functional form, first introduced by Gnedin (2000):

$$f_b(M, z) = \frac{\Omega_b}{\Omega_m} \left[1 + (2^{2/3} - 1) \left(\frac{M}{M_c(z)} \right)^{-2} \right]^{-3/2}$$

Here $M_c(z)$ is the (redshift-dependent), **characteristic mass** at which half the baryons are photo-evaporated from their host halo..

In the simulations of Okamoto et al. (2008), in which reionization occurs at $z=9$, the characteristic mass increases from $\sim 10^8 M_\odot$ shortly after reionization, to $\sim 7 \times 10^9 M_\odot$ at $z=0$.

Latter corresponds to a virial velocity of $\sim 25 \text{ km/s}$



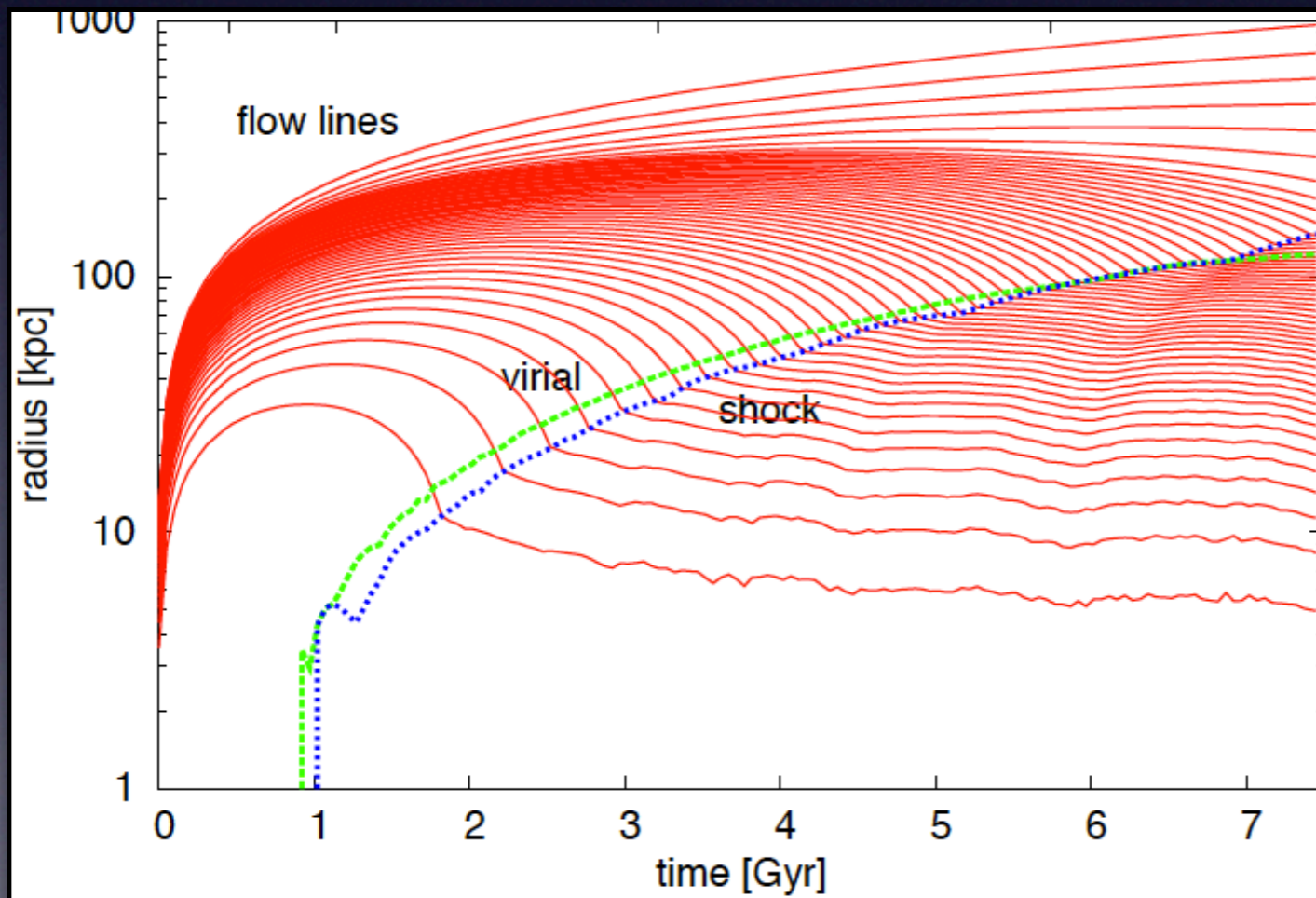
Source: Okamoto et al. 2008, MNRAS, 390, 920,

Accretion Shocks & Virial Temperature

When infalling gas passes the halo's **accretion shock**, its kinetic energy is thermalized, and therefore heated to of order the **virial temperature**

$$T_{\text{vir}} = \frac{\mu m_p}{2 k_B} V_{\text{vir}}^2 \simeq 3.6 \times 10^5 \text{ K} \left(\frac{V_{\text{vir}}}{100 \text{ km/s}} \right)^2$$

where we have assumed that dark matter halos are singular isothermal spheres, and we have adopted that $\mu = 0.59$, appropriate for a primordial gas $(X, Y, Z) = (\frac{3}{4}, \frac{1}{4}, 0)$



CAUTION: in general, gas inside halo has T -profile, and cannot be described by single T . Nevertheless, concept of T_{vir} is useful for order of magnitude estimates, and is therefore frequently used.

The build-up of a virial shock (discontinuity in velocity) at around the virial radius in a collapsing structure. Based on 1D calculations in an expanding Universe...

Radiative Cooling

The primary **cooling processes** relevant for galaxy formation are two-body radiative processes in which gas loses energy through the emission of photons as a consequence of two-body interactions. The main processes are:

	type	reaction	name
1	free-free	$e^- + X^+ \rightarrow e^- + X^+ + \gamma$	bremsstrahlung
2	free-bound	$e^- + X^+ \rightarrow X + \gamma$	recombination
3	bound-free	$e^- + X \rightarrow X^+ + 2e^-$	collisional ionization
4	bound-bound	$e^- + X \rightarrow e^- + X'$ $\rightarrow e^- + X + \gamma$	collisional excitation

In order to compute the cooling rate, one needs to know the above reaction rates, as well as the number densities of the various ionic species. In the case of a pure H/He mixture (the simplest, relevant case), these are $n_e, n_{H_0}, n_{H^+}, n_{He_0}, n_{He^+}, n_{He^{++}}$

One typically makes the following two assumptions:

- **gas is optically thin** (i.e., Case A recombination; all photons generated escape)
- **collisional ionization equilibrium (CIE)**; number densities are in equilibrium)

Cooling Time

It is useful to define the **cooling function**:

$$\Lambda(T, Z) \equiv \frac{\mathcal{C}}{n_{\text{H}}^2}$$

$$[\Lambda] = \text{erg s}^{-1} \text{cm}^3$$

which depends on the temperature, T , and composition (metallicity Z) of the gas, but not on its density, and where \mathcal{C} is the volumetric cooling rate (in erg/s/cm^3)

The **cooling time**, the time it takes the gas to radiate away its internal energy, is given by

$$t_{\text{cool}} \equiv \frac{\rho \varepsilon}{\mathcal{C}} = \frac{3n k_{\text{B}} T}{2 n_{\text{H}}^2 \Lambda(T)}$$



denser gas cools faster...

To assess impact of cooling, compare **cooling time** to two other timescales:

- the age of the Universe, which is roughly the Hubble time

$$t_{\text{H}} = \frac{1}{H(z)} \propto \frac{1}{(G\bar{\rho})^{1/2}} \quad \bar{\rho} = \Omega_{\text{m}}\rho_{\text{crit}}$$

- the dynamical time (or 'free-fall time') of the system

$$t_{\text{ff}} = \left(\frac{3\pi}{32G\bar{\rho}_{\text{sys}}} \right)^{1/2} \propto \frac{1}{(G\bar{\rho}_{\text{sys}})^{1/2}} \quad \bar{\rho}_{\text{sys}} = \bar{\rho}_{\text{gas}} + \bar{\rho}_{\text{DM}}$$

Cooling Time

$$\bar{\rho}_{\text{sys}} \sim 200\bar{\rho} \rightarrow t_{\text{ff}} \sim t_{\text{H}}/10$$

We distinguish three regimes:

$$t_{\text{cool}} > t_{\text{H}}$$

Cooling is not important. Gas is in **hydrostatic equilibrium**, unless it was recently disturbed

$$t_{\text{ff}} < t_{\text{cool}} < t_{\text{H}}$$

System is in **quasi-hydrostatic equilibrium**. It evolves on cooling time scale. Gas contracts and cools, but system has sufficient time to contract and re-establish hydrostatic equilibrium.

hot mode

$$t_{\text{cool}} < t_{\text{ff}}$$

Cooling is **catastrophic**. Gas cannot respond fast enough to loss of pressure. Since cooling time decreases with increasing density, cooling proceeds faster than free-fall time (=catastrophic). Gas falls to center of dynamic system on free-fall time...

cold mode

$$t_{\text{cool}} \propto \rho_{\text{gas}}^{-1} \propto (1+z)^{-3}$$

$$t_{\text{ff}} \propto \rho^{-1/2} \propto (1+z)^{-3/2}$$

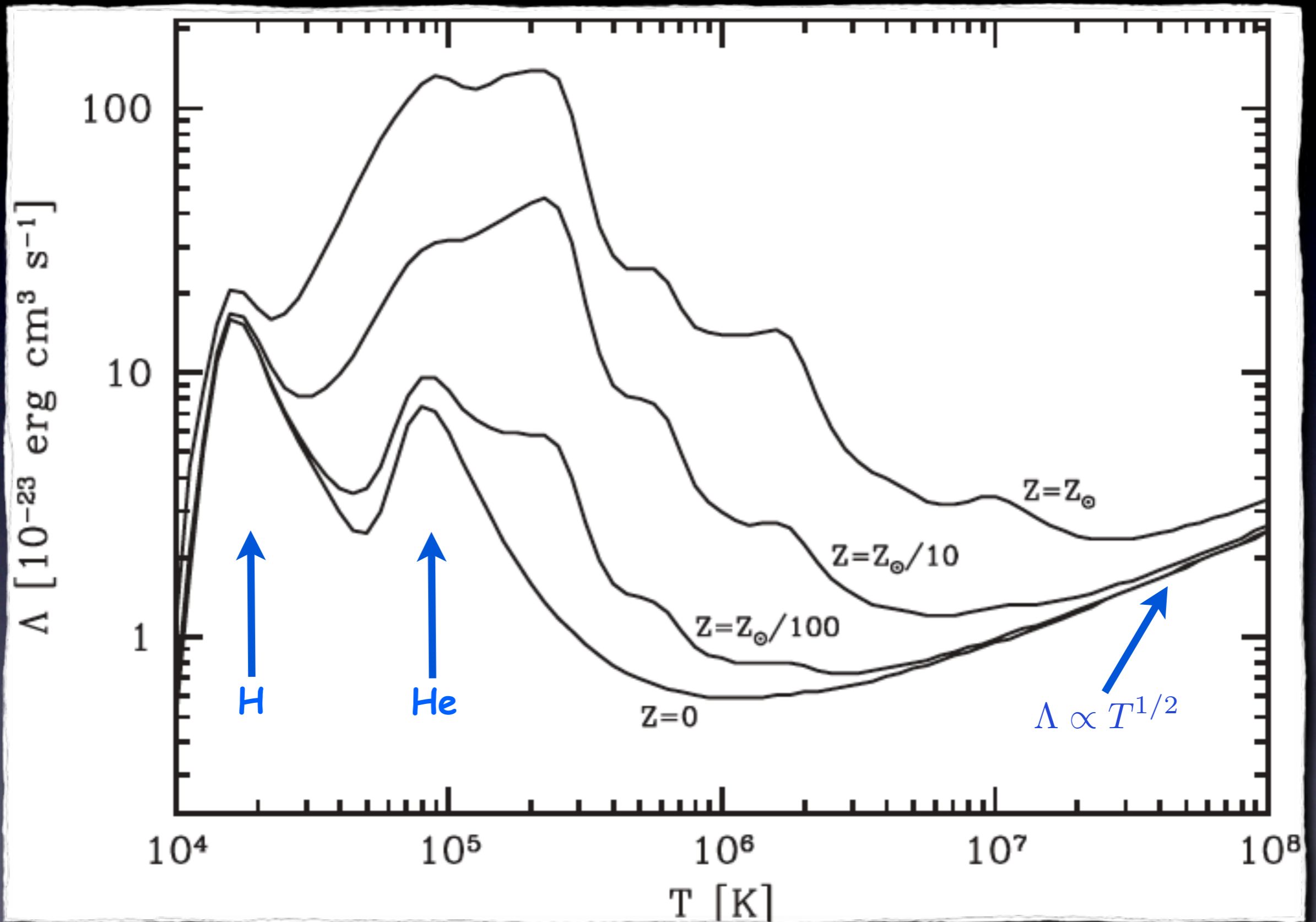


cooling is more efficient at higher redshifts



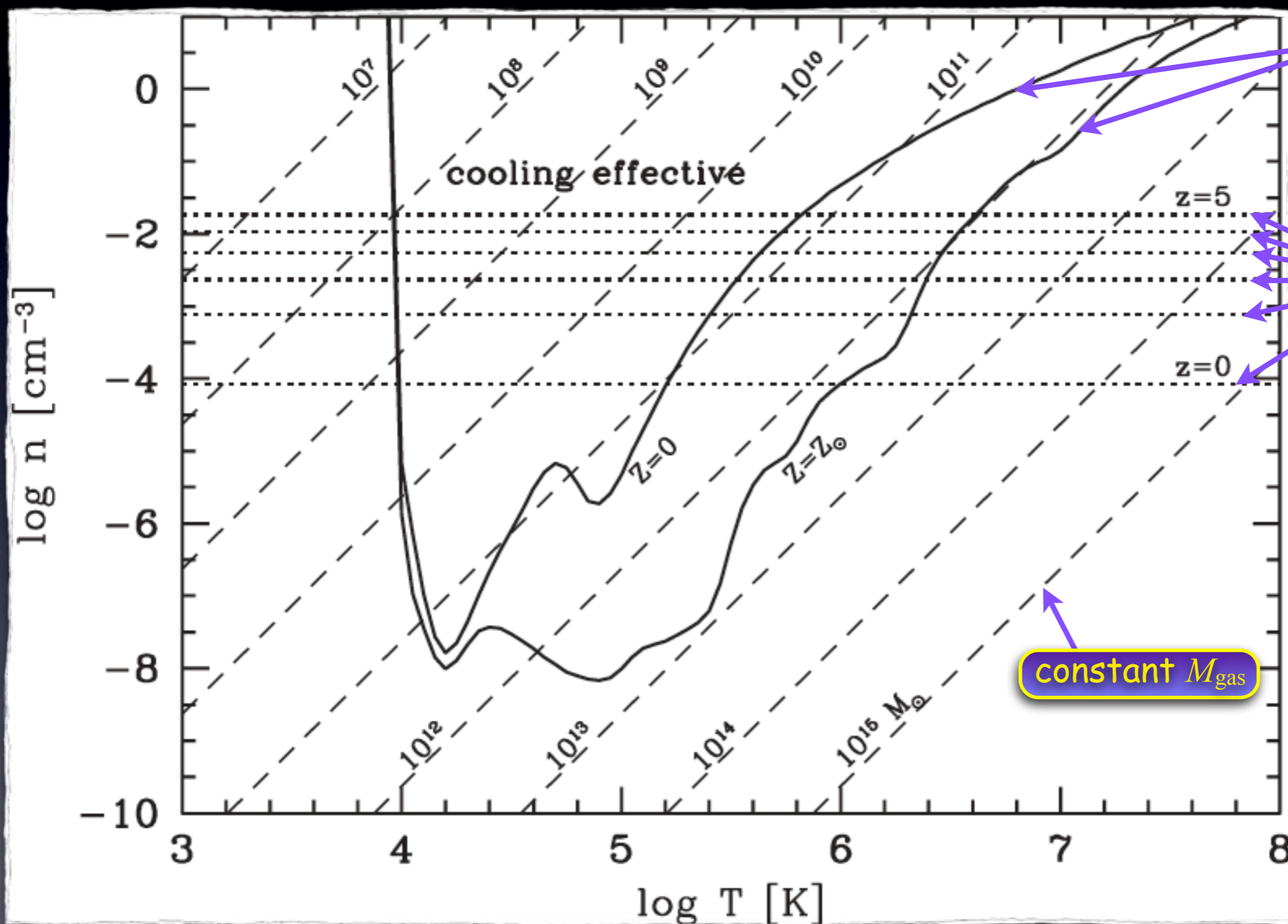
overcooling problem

The CIE Cooling Function



Cooling & Galaxy Formation

Under the assumption of CIE, we can compute $\frac{t_{\text{cool}}}{t_{\text{ff}}} = \frac{t_{\text{cool}}}{t_{\text{ff}}}(n, T, Z)$

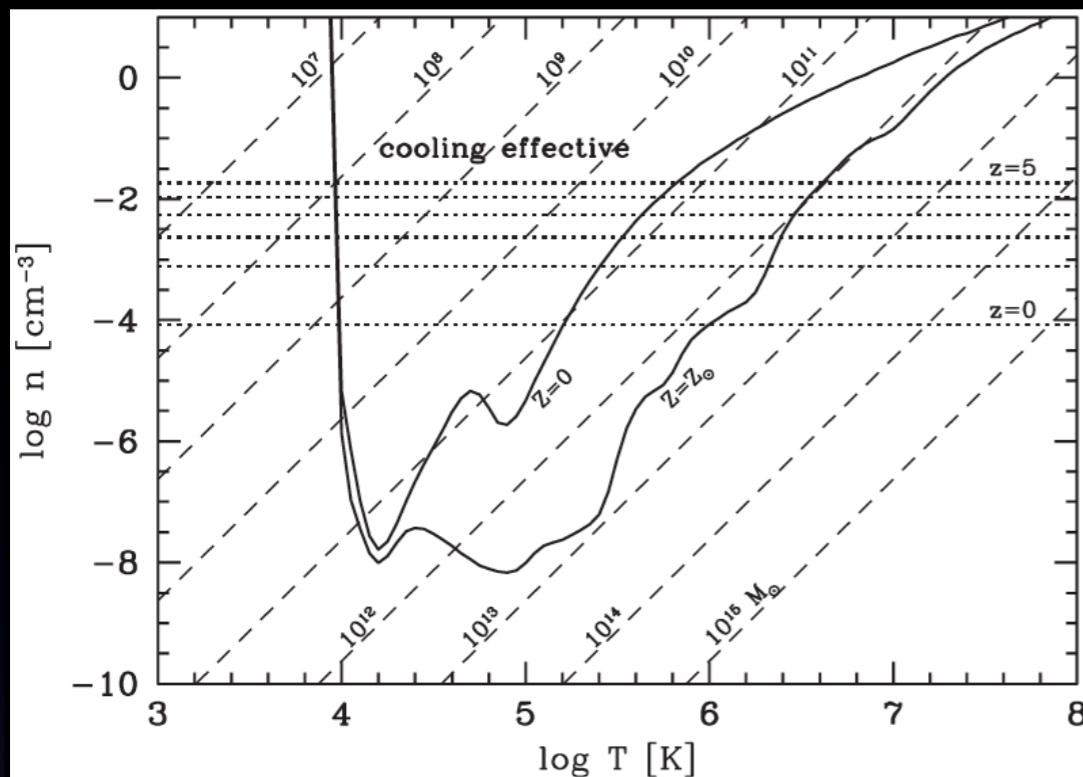


$t_{\text{cool}} = t_{\text{ff}}$

$\delta = 200$

constant M_{gas}

Cooling & Galaxy Formation



$$M_{\text{gas}} = 0.15 M_{\text{vir}} \quad \Rightarrow \quad M_{\text{vir}} \simeq 6.6 M_{\text{gas}}$$

- Haloes with $M_{\text{vir}} < 10^7 M_{\odot}$ ($V_{\text{vir}} < 20 \text{ km/s}$) can't cool their gas (except by molecular cooling...)
- Haloes with $M_{\text{vir}} > 10^{12} M_{\odot}$ ($Z = 0$)
 $M_{\text{vir}} > 10^{13} M_{\odot}$ ($Z = Z_{\odot}$)
 can't cool their gas efficiently either...

In early papers (and textbooks) on galaxy formation, this mass scale of $10^{12} - 10^{13} M_{\odot}$ was invoked to explain the exponential cut-off in the luminosity/stellar mass function of galaxies; more massive galaxies can't form because they can't efficiently cool their gas...
 (e.g., Binney, 1977, ApJ, 215, 483; Silk, 1977, ApJ, 211, 683; Ostriker & Rees, 1977, MNRAS, 179, 541)

However, this argument is seriously flawed for two reasons:

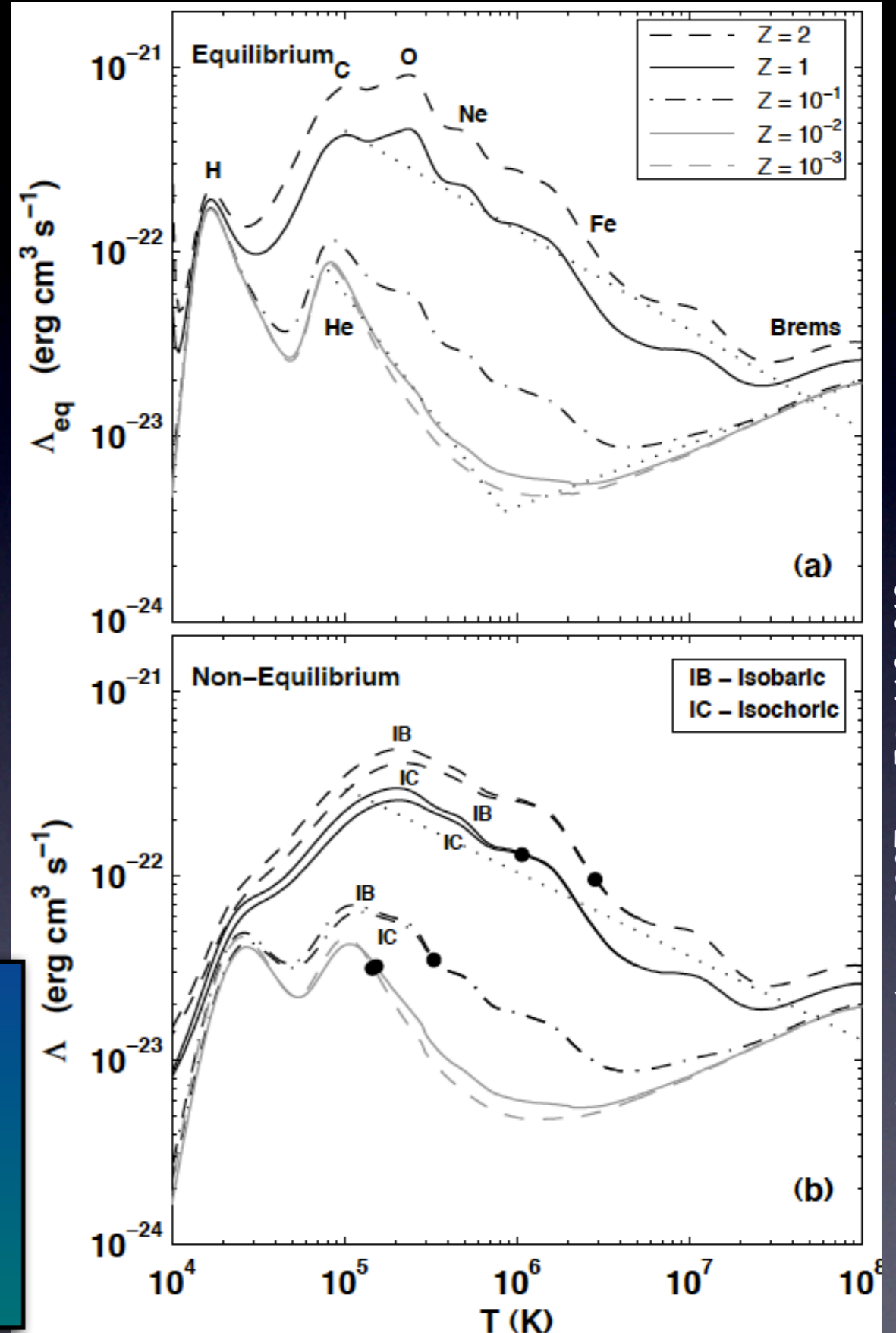
- Haloes and galaxies form hierarchically \Rightarrow the progenitors of massive haloes can cool, especially at higher redshifts...
- The curve $t_{\text{cool}} = t_{\text{ff}}$ is calculated for an overdensity $\delta = 200$. The gas in a halo typically has a density profile, and can have $\delta \gg 200$ near the center.
 \Rightarrow at least some fraction of the gas should have cooled...

Deviations from CIE

In particular for low T gas, the recombination time-scale can exceed the cooling time scale of the gas, so that, while it cools, it starts to drift away from CIE. This can result in significantly different cooling times (by up to factors of two to three).

To my knowledge this is not accounted for in any SAM...

Comparison of cooling functions for gas in CIE (upper panel) with the effective cooling functions for gas in which time-dependent ionization states are computed throughout the cooling (lower panel). IB and IC correspond to cases in which gas is assumed to be cooling isobarically or isochorically...



Source: Gnat & Sternberg, 2007, ApJS, 168, 213

Cooling in SAMs

Almost all **SAMs** treat cooling by defining a cooling radius, r_{cool} , as the radius where

$$t_{\text{cool}}(r) = \frac{3 n(r) k_B T(r)}{2 n_{\text{H}}^2(r) \Lambda(T)} = \tau_{\text{cool}}$$

Here τ_{cool} is some 'characteristic' time, which varies from **SAM** to **SAM**.

Some **SAMs** set it equal to the Hubble time, others to the time since the last major merger, or the free-fall time....Computing this cooling radius also requires assumptions about the density and temperature profiles of the hot gas, which also differ from **SAM** to **SAM**....

Once the **cooling radius** has been computed, the **cooling rate** is computed as follows:

$$\dot{M}_{\text{cool}} = \begin{cases} \frac{M_{\text{hot}}}{t_{\text{ff}}} & \text{if } r_{\text{cool}} > r_{\text{vir}} \\ \frac{M_{\text{hot}}(<r_{\text{cool}})}{\tau_{\text{cool}}} & \text{if } r_{\text{cool}} \leq r_{\text{vir}} \end{cases}$$

Typically cooling rates differ by factors two to three from **SAM** to **SAM**, but no **SAM** includes free parameter(s) to scale their cooling rates...





Star Formation

Star Formation in SAMs

Lacking a detailed theory for **star formation**, the treatment of **SF** in **SAMs** has thus far been very pragmatic:

Old SAMs: no internal structure of galaxies

$$\dot{M}_* = \varepsilon_{\text{SF}} \frac{M_{\text{cold}}}{\tau_{\text{SF}}} \quad \tau_{\text{SF}} = t_{\text{dyn}} \left(\frac{V_{\text{vir}}}{200 \text{ km/s}} \right)^{\alpha_{\text{SF}}}$$

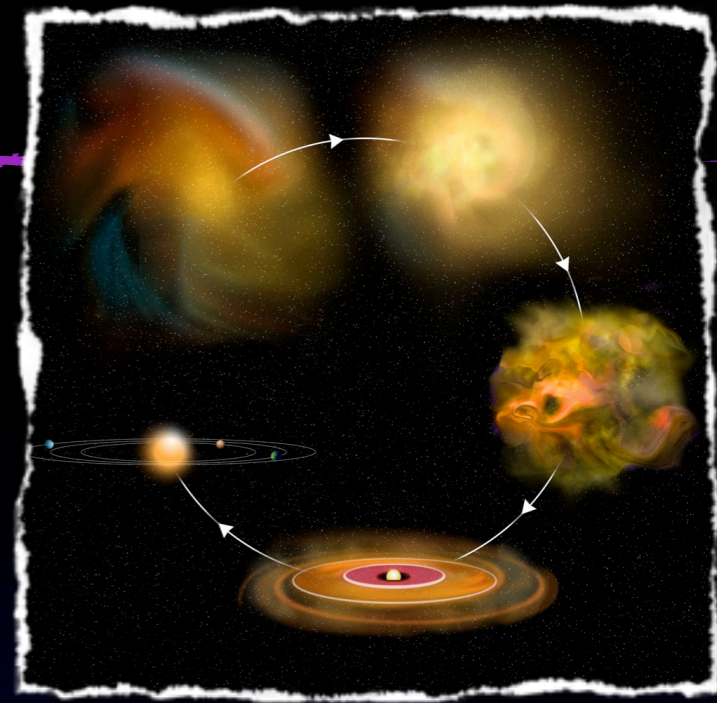
free parameters ($\varepsilon_{\text{SF}}, \alpha_{\text{SF}}$) vary from **SAM** to **SAM** with $-2.5 \leq \alpha_{\text{SF}} \leq 0$

Modern SAMs: disk galaxies are assumed to be exponential

$$\Sigma_{\text{cold}}(R) = \left(\frac{M_{\text{cold}}}{2\pi R_d^2} \right) e^{-R/R_d} \quad R_d \propto \lambda_{\text{DM}} r_{\text{vir}} \quad (\text{Mo, Mao \& White 1998})$$

$$\dot{M}_* = 2\pi \int \dot{\Sigma}_*(R) R dR \quad \dot{\Sigma}_*(R) = \begin{cases} \varepsilon_{\text{SF}} \Sigma_{\text{cold}}^{1.4}(R) & \text{if } \Sigma_{\text{cold}} > \Sigma_{\text{crit}} \\ 0 & \text{otherwise} \end{cases}$$

Hence, the **SFR** follows the Kennicutt-Schmidt law (with 'free' parameter for normalization), and takes account of a **critical surface density** for star formation in agreement with empirical findings (e.g., Kennicutt 1998)



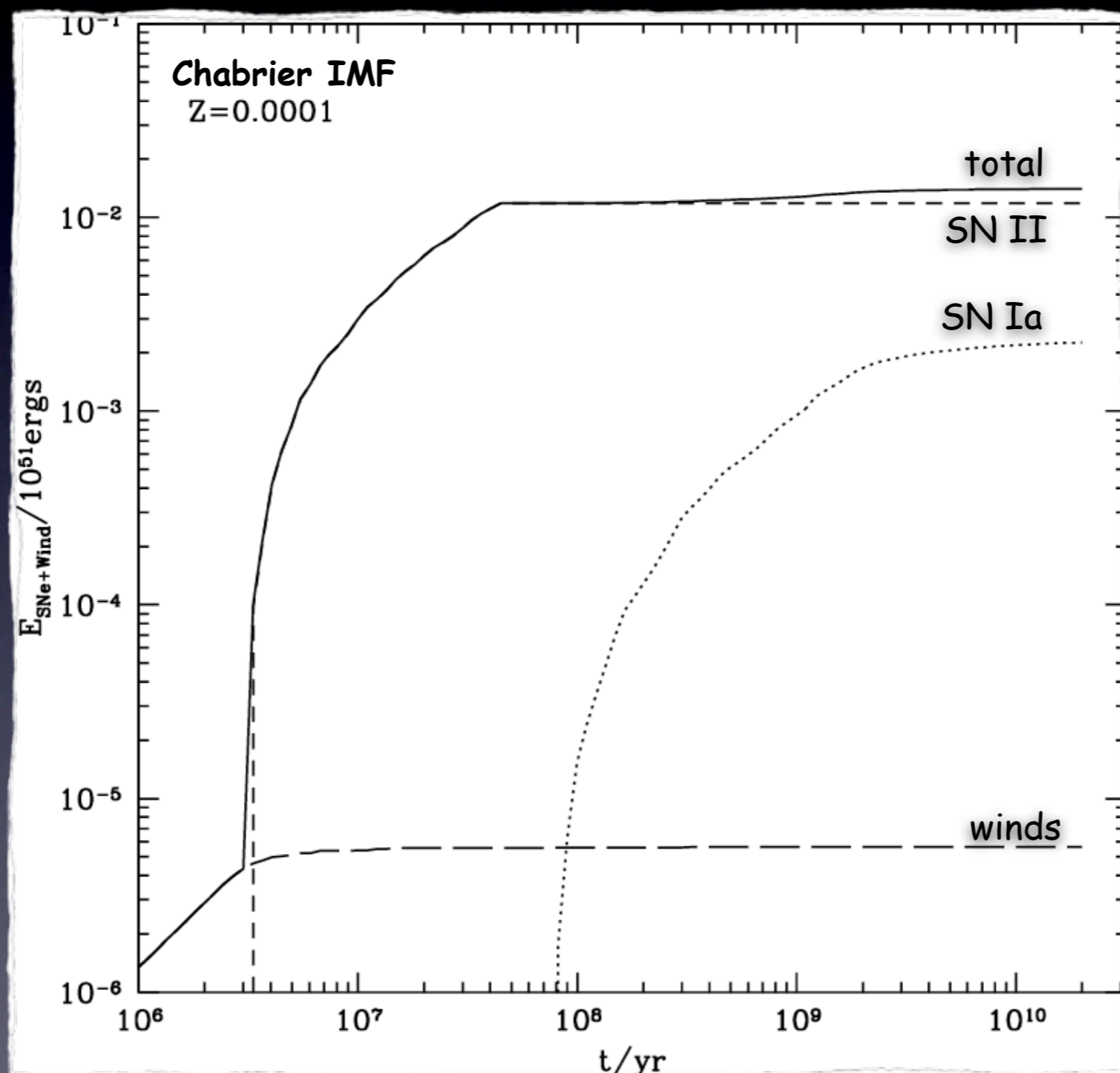


Some Feedback Please...

Stellar Feedback

Stellar evolution injects energy & momentum back into ICM, mainly in form of stellar winds, supernovae and radiation pressure. For a Chabrier IMF, the effective number of SN ($E_{\text{inject}}/10^{51}$ erg) per solar mass of stars formed is ~ 0.01 and dominated by SNII

Effective number of SN per solar mass of stars formed



Source: Benson, 2010, Phys. Rep. 495, 33

SuperNova Feedback (ejection)

To get a feel for whether the energy input from SN can be relevant for galaxy formation, imagine ejecting a mass M_{ej} from the center of a NFW dark matter halo.

This requires an energy injection of $E_{\text{ej}} = \frac{1}{2} M_{\text{ej}} V_{\text{esc}}^2$. Using that, to a good approximation, the escape velocity from the center of a NFW halo is $V_{\text{esc}} \simeq \sqrt{2} c V_{\text{vir}}$ where c is the halo concentration parameter, we have that $E_{\text{ej}} \simeq c M_{\text{ej}} V_{\text{vir}}^2$

The energy available from SN is $E_{\text{fb}} = \epsilon_{\text{SN}} \zeta M_* E_{\text{SN}}$

$\epsilon_{\text{SN}} \leq 1$ = fraction of SN energy available for feedback (not just radiated away)

$\zeta \simeq 0.01 M_{\odot}^{-1}$ = number of SN produced per Solar mass of stars formed (IMF dependent)

$E_{\text{SN}} \simeq 10^{51}$ erg = energy supplied per SN

Equating E_{fb} to E_{ej} we obtain that

$$\frac{M_{\text{ej}}}{M_*} \simeq \epsilon_{\text{SN}} \left(\frac{c}{12} \right)^{-1} \left(\frac{V_{\text{vir}}}{200 \text{ km/s}} \right)^{-2}$$



Hence, even if 100% of SN energy can be converted into kinetic energy of a galactic wind, SN can only eject about the equivalent of the stellar mass from a MW-sized halo.

This efficiency increases with decreasing halo mass; for $V_{\text{vir}} = 50 \text{ km/s}$ we have that $M_{\text{ej}} \leq 10 M_*$.

SuperNova Feedback (reheating)

Rather than ejecting gas from the halo, SN energy can also be used to reheat gas.

The **internal energy** of gas is $E_{\text{int}} = \frac{3}{2} M_{\text{gas}} \frac{k_B T}{\mu m_p}$

Imagine we want to reheat this gas from an initial temperature of $T_{\text{init}} = 10^4 \text{ K}$ to the virial temperature of the halo, $T_{\text{vir}} = \frac{\mu m_p}{2 k_B} V_{\text{vir}}^2$

This requires an energy $E_{\text{reheat}} = \frac{3}{2} M_{\text{gas}} \frac{k_B (T_{\text{vir}} - T_{\text{init}})}{\mu m_p} = \frac{3}{4} M_{\text{gas}} V_{\text{vir}}^2 \left(1 - \frac{T_{\text{init}}}{T_{\text{vir}}} \right)$

Equating E_{reheat} to E_{ej} yields $\frac{M_{\text{gas}}}{M_*} \simeq 17 \epsilon_{\text{SN}} \left(\frac{V_{\text{vir}}}{200 \text{ km/s}} \right)^{-2} \left(1 - \frac{T_{\text{init}}}{T_{\text{vir}}} \right)^{-1}$

Hence, in a MW halo, SN can reheat up to $17 M_{\odot}$ for every Solar mass of stars formed.

Reheating is more efficient than ejecting gas, by roughly factor $(V_{\text{esc}}/V_{\text{vir}})^2 \simeq 2c$

The all important question for gauging the potential impact of SN feedback is

what is the SN feedback efficiency parameter ϵ_{SN}

Depending on the ISM and SF conditions, one can have $0.01 < \epsilon_{\text{SN}} \leq 1$

AGN Feedback



What about feedback from AGN?

The energy output from an AGN (over its lifetime) is

$$E_{\text{AGN}} \sim 0.1 M_{\text{BH}} c^2$$

where we have assumed that roughly 10% of the rest-mass energy is radiated away. If we assume that a fraction ϵ_{AGN} of this radiation is used to reheat gas or to eject it from the halo, and we use that $M_{\text{BH}} \simeq 0.002 M_{\text{bulge}}$ we obtain that

$$\frac{E_{\text{AGN}}}{E_{\text{SN}}} \sim 36 \frac{\epsilon_{\text{AGN}}}{\epsilon_{\text{SN}}} \left(\frac{M_{\text{bulge}}}{M_*} \right)$$

If $\epsilon_{\text{AGN}} \sim \epsilon_{\text{SN}}$ AGN feedback can be order of magnitude more efficient than SN feedback

Key question: what is a realistic value for ϵ_{AGN}

NOTE: fact that we can see AGN implies that ϵ_{AGN} has to be significantly smaller than unity

SN Feedback in SAMs

- **SAMs** use the **SN** energy to either eject or reheat cold gas from the disk.
- Fraction of **SN** energy used for ejection vs. reheating varies from **SAM** to **SAM**.
- Ejected gas is often stored in 'reservoir' for later re-accretion into the halo.

Majority of **SAMs** adopt

$$\dot{M}_{\text{fb}} \equiv \dot{M}_{\text{eject}} + \dot{M}_{\text{reheat}} = \varepsilon_{\text{fb}} \dot{M}_* \quad \text{with} \quad \varepsilon_{\text{fb}} = \varepsilon_{\text{fb},0} \left(\frac{V_{\text{vir}}}{200 \text{ km/s}} \right)^{-\alpha_{\text{fb}}}$$

Energy driven winds have $\alpha_{\text{fb}} = 2$ (e.g., Dekel & Silk 1986)

Momentum driven winds have $\alpha_{\text{fb}} = 1$ (e.g., Murray et al. 2005)

SAMs typically 'require' $\alpha_{\text{fb}} > 2$ in order to match faint-end slope of LF.

Extreme example: $\alpha_{\text{fb}} = 5.5$ (Cole et al. 1994)

The recent 'Munich' **SAM** of Guo et al. (2011) adopt $\alpha_{\text{fb}} = 3.5$, but with a modified functional form:

for reheating

$$\varepsilon_{\text{fb}} = \varepsilon_{\text{fb},0} \left[0.5 + \left(\frac{V_{\text{vir}}}{70 \text{ km/s}} \right)^{-3.5} \right]$$

'arbitrary' characteristic scale...

AGN Feedback in SAMs

- AGN feedback is `required' in order to **quench** star formation in massive haloes
- Two different modes of **AGN feedback** are included: Quasar mode & Radio mode
- **AGN feedback** has become a `standard ingredient' in **SAMs** since 2006

"Quasar Mode"

associated with mergers or disk instabilities when large amounts of gas are funneled to galaxy's center. Main channel of BH growth

$$\dot{M}_{\text{BH}} = M_{\text{BH},\text{min}} + f(M_{\text{cold},1}, M_{\text{cold},2}, M_{\text{vir},1}, M_{\text{vir},2})$$

Typically the function f is tuned to match the empirical $M_{\text{BH}}-M_{\text{bulge}}$ relation.

Treatment of feedback from **Quasar Mode** varies strongly from **SAM** to **SAM**

'Santa Cruz' SAMs: momentum driven outflow $\dot{M}_{\text{out}} = \epsilon_{\text{quasar}} \dot{M}_{\text{BH}} c/V_{\text{esc}}$

'Munich' SAMs: no explicit treatment of quasar mode feedback, but starburst associated with merger typically ejects all cold gas via `regular' SN feedback

AGN Feedback in SAMs

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"Radio Mode"

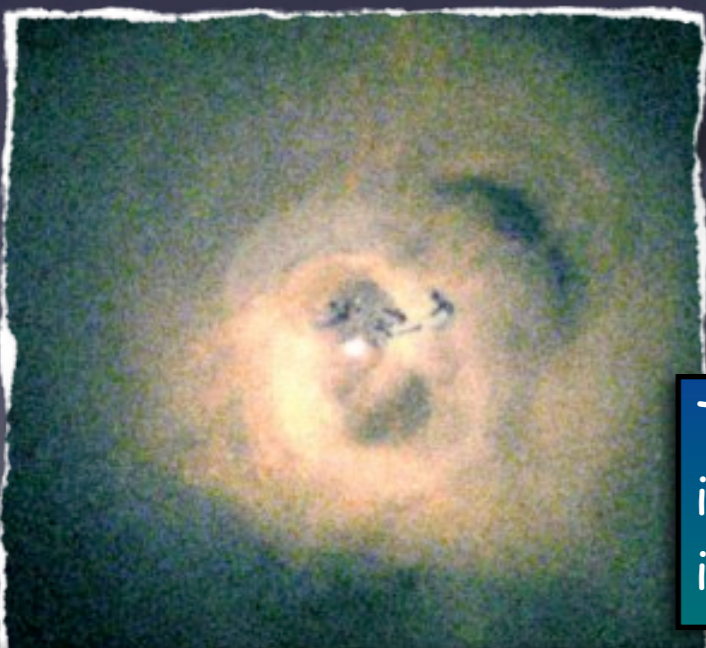
associated with low-Eddington Bondi accretion of hot gas in massive haloes. Little **BH** growth, but main mode to offset cooling

$$\dot{M}_{\text{BH}} = \epsilon_{\text{radio}} \left(\frac{M_{\text{BH}}}{10^8 M_{\odot}} \right) f(M_{\text{hot}}, T_{\text{hot}})$$

Motivation is that low-Eddington ratio accretion results in powerful jets. Most **SAMs** assume that this energy output heats the hot gas, thereby `quenching' star formation

$$\dot{M}_{\text{cool,eff}} = \dot{M}_{\text{cool}} - \frac{0.1 \dot{M}_{\text{BH}} c^2}{V_{\text{vir}}^2}$$

There is ample observational evidence to suggest that radio-mode AGN impact the ICM of clusters, such as here in the Perseus cluster. However, it is unclear whether the AGN energy is actually thermalized...



A visualization of the cosmic web, showing a complex network of blue and red filaments and nodes against a black background. The filaments represent the large-scale structure of the universe, with nodes indicating regions of high density.

**Dynamical Friction,
Mergers &
Morphological
Transformations**

Galaxy Mergers in SAMs

- When two haloes **merge**, most massive galaxy becomes central galaxy in new halo.
- All other galaxies become satellites that orbit central.
- Satellites experience **dynamical friction**, causing them to "sink" to center of halo.
- Satellites will **merge** with central after **dynamical friction** time, t_{df} (cannibalism)
- Some **SAMs** also allow for **mergers** between satellite galaxies...

When two galaxies merge:

$M_1/M_2 \leq \mathcal{R}_{maj}$ \Rightarrow major merger \Rightarrow disks are transformed into spheroid
 $M_1/M_2 > \mathcal{R}_{maj}$ \Rightarrow minor merger \Rightarrow disk masses added together

\mathcal{R}_{maj} is free parameter, tuned to match observed morphologies (typically $\mathcal{R}_{maj} \sim 3 - 4$)

Dynamical friction time is based off Chandrasekhar's derivation

$$t_{df} = \epsilon_{df} t_{dyn} \left(\frac{M_{vir}}{M_{sat}} \right) \frac{f_{orbit}}{\ln \Lambda}$$

free fudge parameter $\rightarrow \epsilon_{df}$
 'Coulomb' logarithm $\rightarrow \ln \Lambda$
 function describing orbital dependence $\rightarrow f_{orbit}$

Numerical simulations and analytical calculations suggest $\epsilon_{df} \sim 3$ (see MBW §12.3)

Starbursts



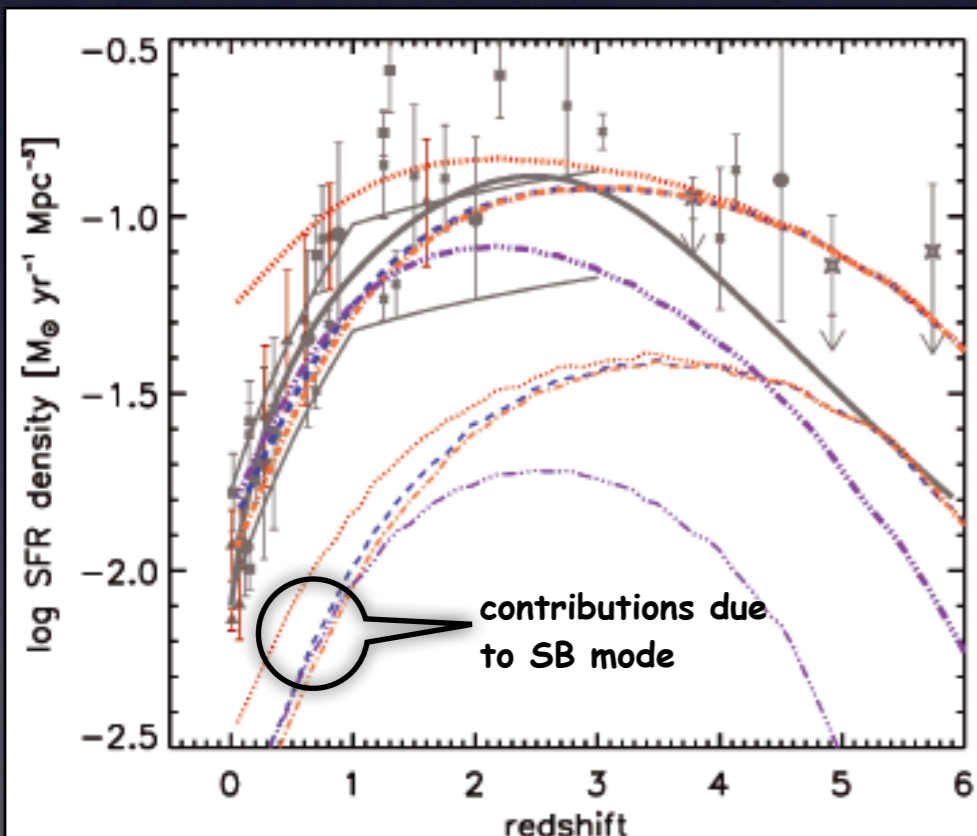
Most **SAMs** assume that during (major) mergers a (large) fraction of the cold gas is consumed in a **burst** of star formation:

$$\dot{M}_{*,\text{burst}} = \frac{e_{\text{SB}} M_{\text{cold}}}{\tau_{\text{SB}}} \quad e_{\text{SB}} = \epsilon_{\text{SB}} \left(\frac{M_{\text{min}}}{M_{\text{maj}}} \right)^{\gamma_{\text{SB}}}$$

'Santa Cruz' SAMs: $(\epsilon_{\text{SB}}, \gamma_{\text{SB}}, \tau_{\text{SB}})$ calibrated against hydro sims (Cox et al. 2008)

'Munich' SAMs: $\epsilon_{\text{SB}} = 0.56$, $\gamma_{\text{SB}} = 0.7$ and $\tau_{\text{SB}} = 0$ (major+minor mergers)

'Durham' SAMs: $\epsilon_{\text{SB}} = 1$, $\gamma_{\text{SB}} = 0$ and $\tau_{\text{SB}} = t_{\text{ff},\text{sph}}$ (major mergers only)



Cosmic star formation histories in the SAMs of Somerville et al. (2008). Different colors correspond to different models. The thin lines show the contributions due to starbursts, which typically are less than 10 percent...

A visualization of the cosmic web, showing a complex network of blue and orange filaments and nodes against a black background. The filaments represent the large-scale structure of the universe, with nodes indicating regions of high density where galaxies are likely to form.

Satellite Galaxies

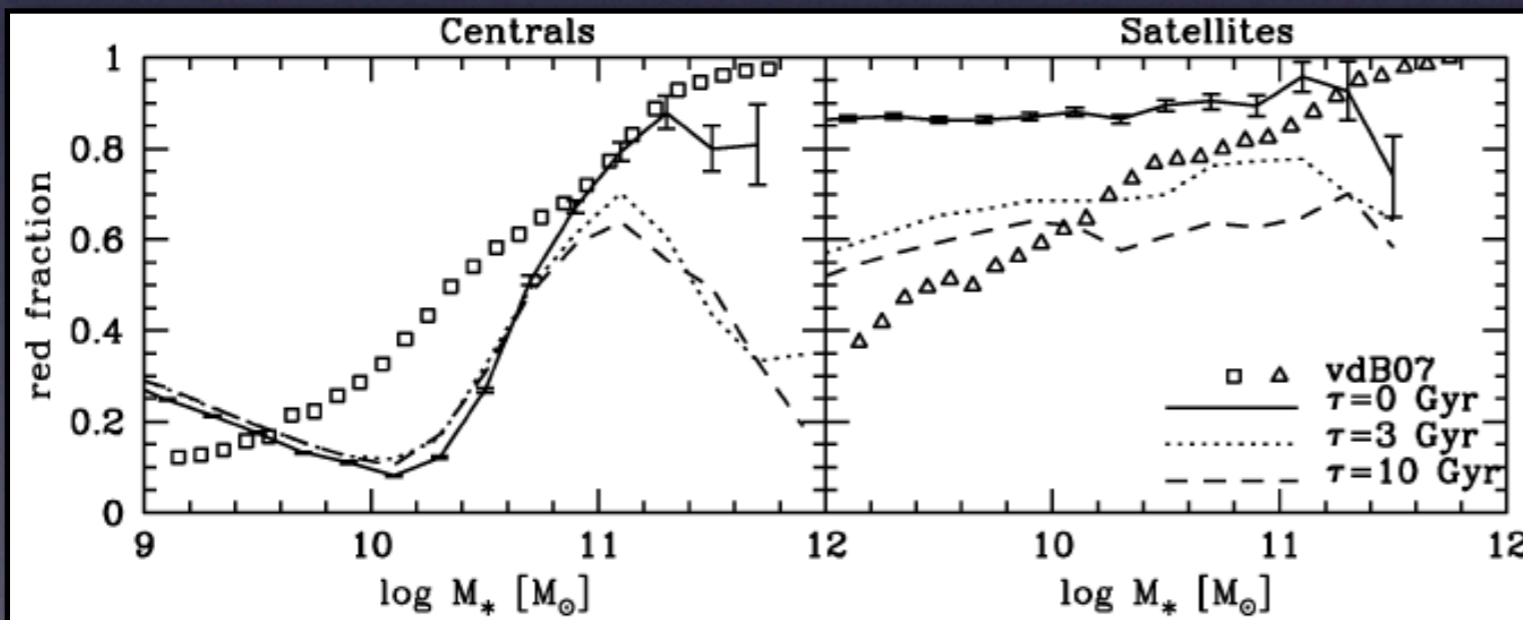
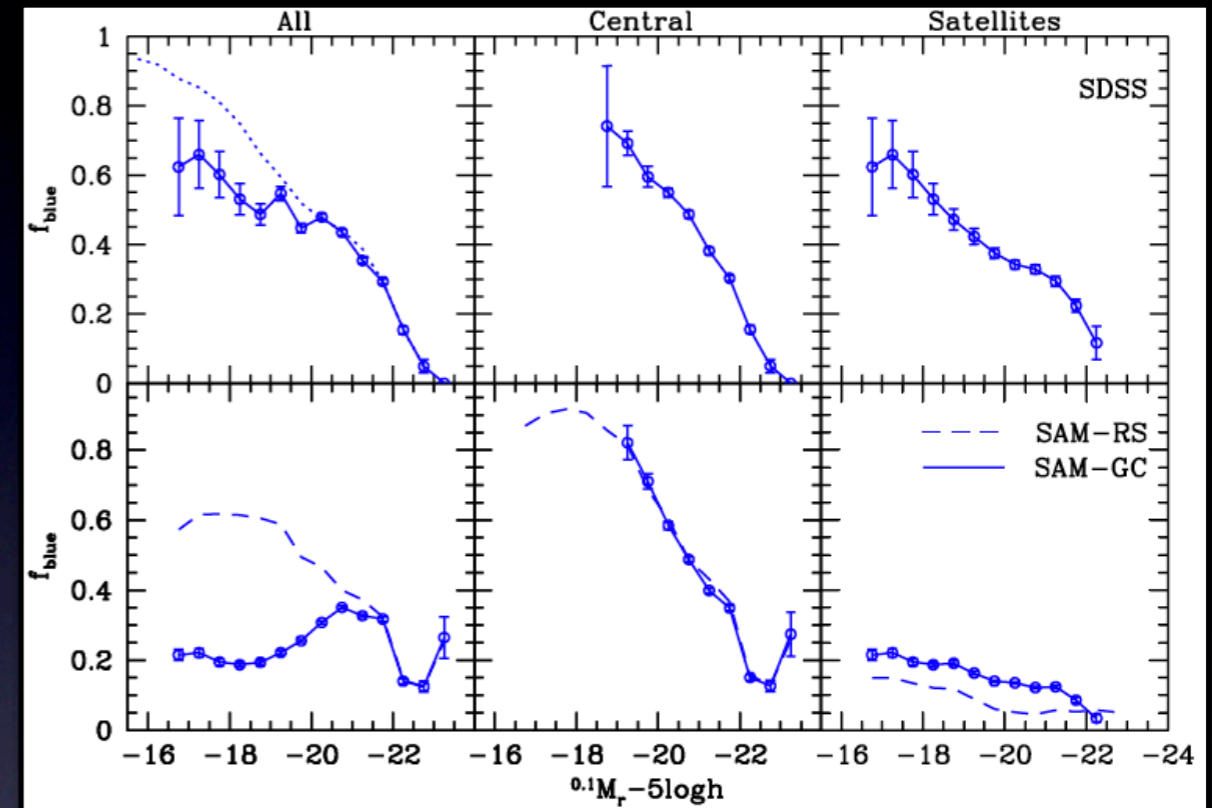
Satellite Quenching

Prior to ~2005 most SAMs instantaneously stripped satellite galaxies of their hot gas reservoir at infall, resulting in rapid **quenching** of their star formation (**strangulation**)

However, as first shown in Weinmann et al. (2006), this results in **blue fractions** of satellite galaxies that are much too low compared to observations.

➔ Instantaneous strangulation is too rapid.

As shown by Kang & vdB (2008), simply slowing down rate at which hot gas is stripped results in problems with the centrals: they become **blue** (despite **AGN feedback**) because they accrete **blue satellites**...



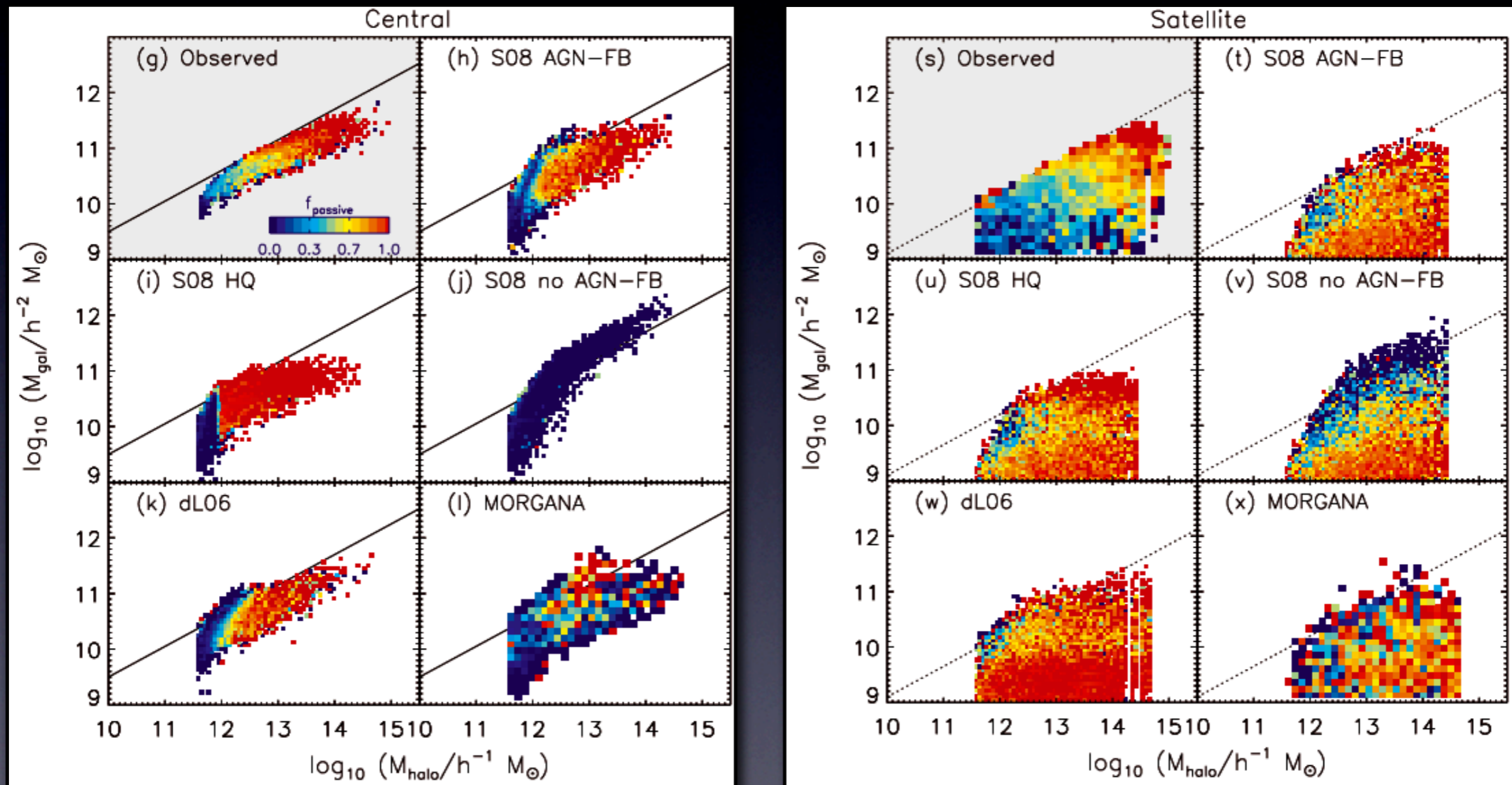
Satellite galaxies mainly need to be disrupted, rather than cannibalized



IntraClusterLight & Stellar Halos

Quenching in SAMs

Kimm et al. (2009) performed a detailed comparison of the **passive fractions** of central and satellite galaxies in **SDSS** (based on galaxy group catalog of Yang et al. 2005) with those in different **SAMs** reveals severe problems for the latter...



Source: Kimm et al. 2009, MNRAS, 394, 1131



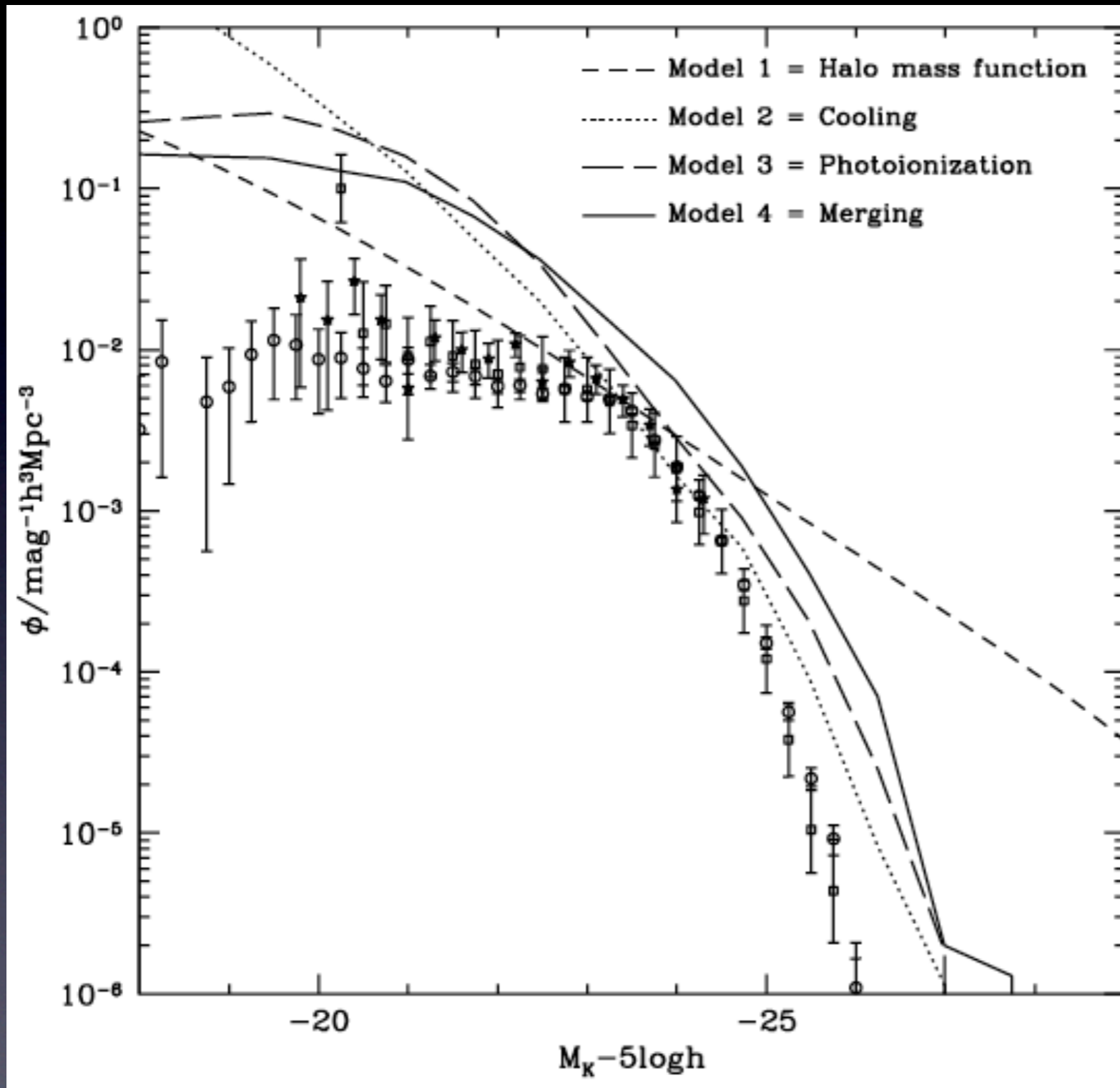
Quenching is not yet treated properly in SAMs



A visualization of the cosmic web, showing a complex network of blue and orange filaments and nodes against a black background. The filaments represent the large-scale structure of the universe, with nodes indicating regions of high density where galaxies are more likely to form.

Galaxy Abundances

Fitting the Galaxy Luminosity Function



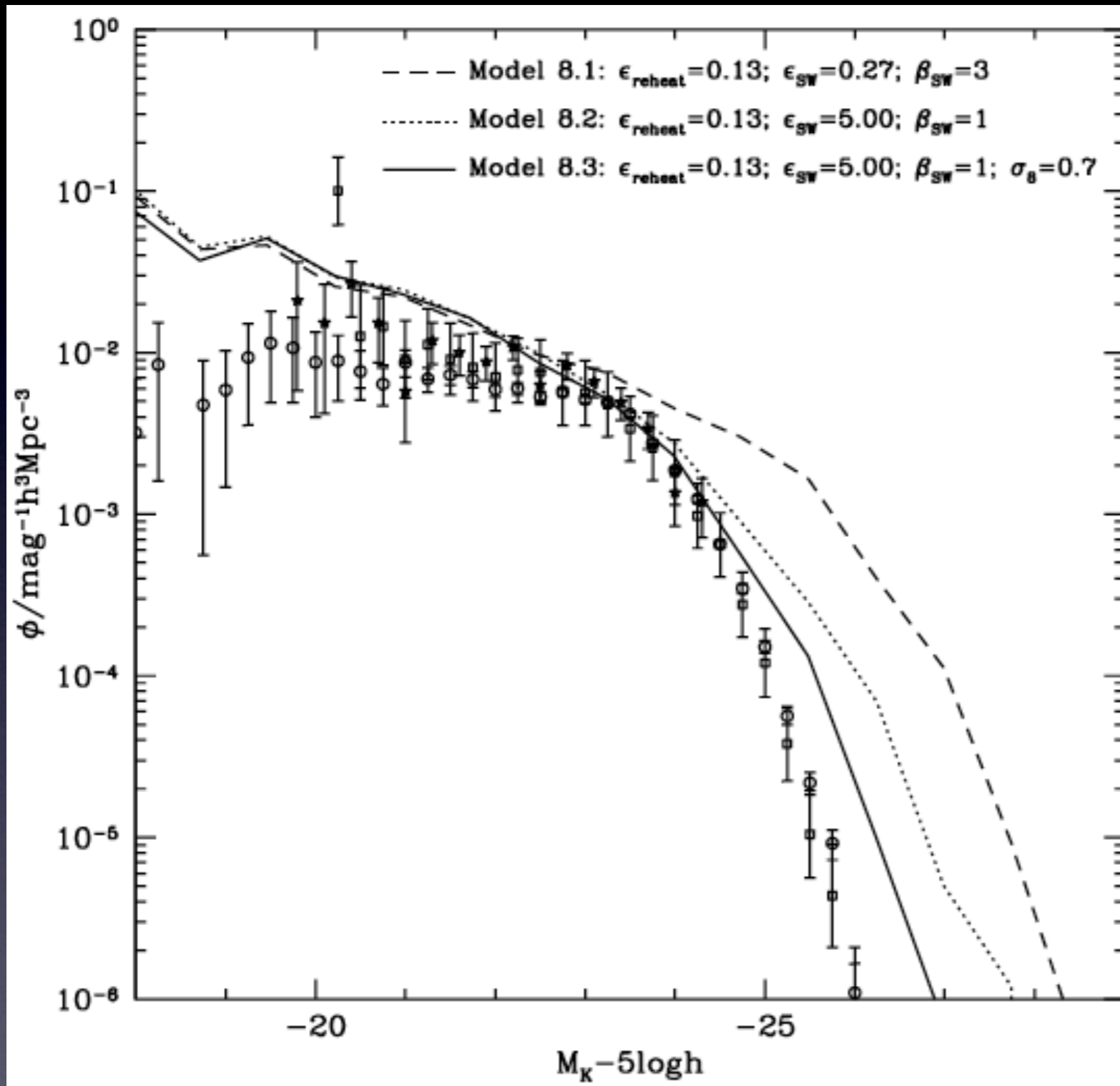
If galaxy formation would be equally efficient in halos of all masses, the shape of the LF would be very different from what is observed.

Since **cooling** is less efficient in more massive haloes, this was originally believed to explain the bright end of the LF.

However, **hierarchical formation** results in too much cooling at early times...

Photoionization by UV background is not sufficient to solve overcooling problem

Fitting the Galaxy Luminosity Function

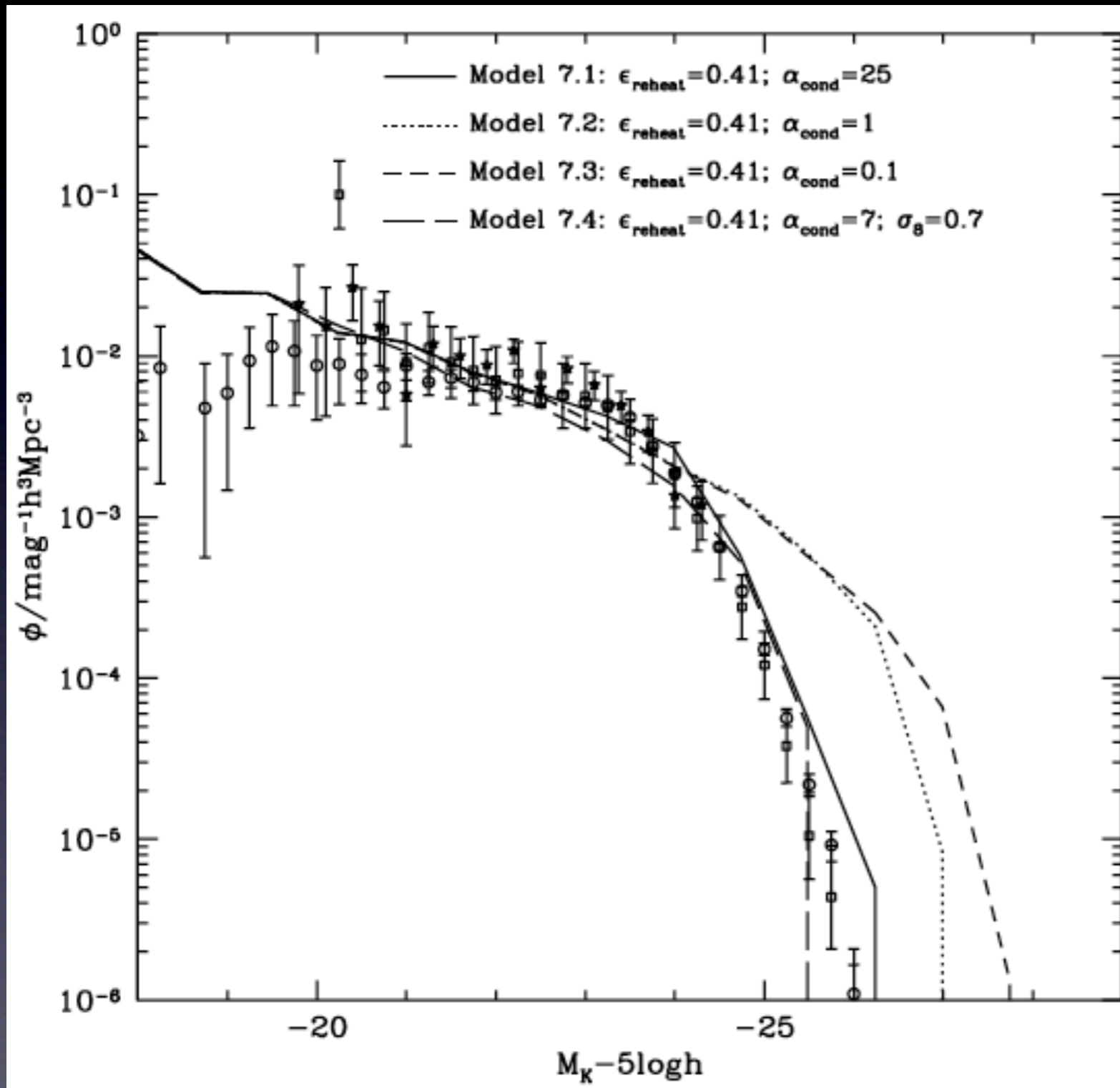


Adding **SN feedback** helps in suppressing galaxy formation in low mass haloes, resulting in a reduced abundance of faint galaxies...

Matching the **bright-end** of the LF requires at least **5x** as much energy as what is available from stellar evolution...

SN feedback by itself cannot solve the overcooling problem

Fitting the Galaxy Luminosity Function

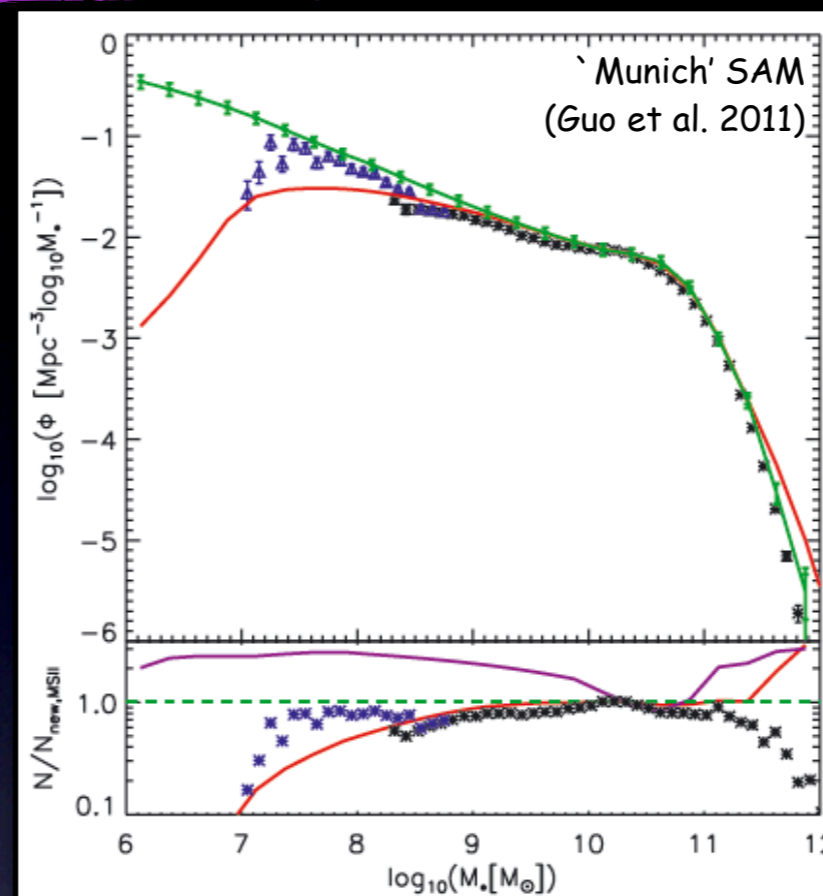
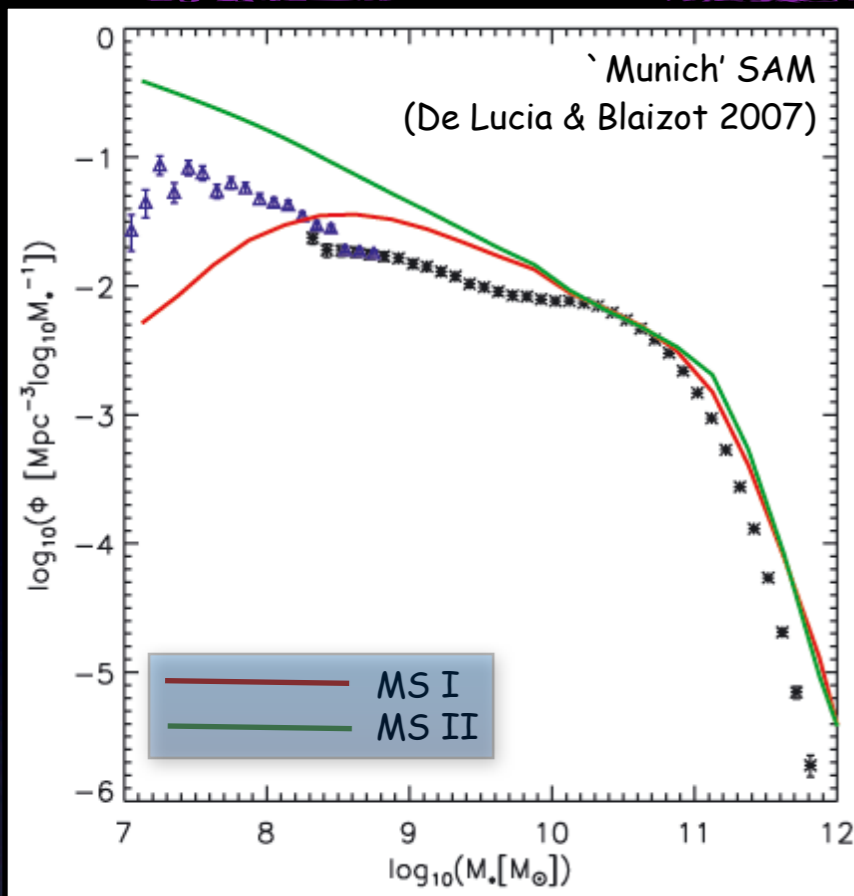


Trying to solve the problems at the bright end of the LF with **conductivity** requires efficiencies that are **25x** the maximum allowed Spitzer conductivity.

In presence of magnetic fields, conductivity is expected to be significantly less than Spitzer...

Conductivity is not a solution to the overcooling problem

The Current State-of-Affair

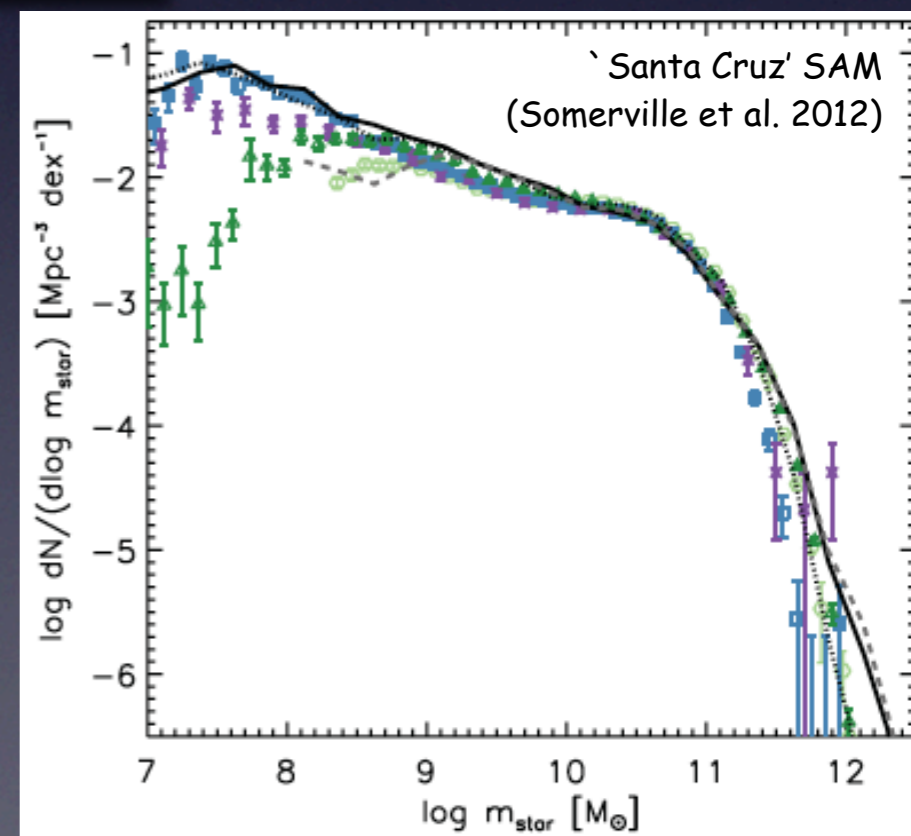


Main difference is in treatment of **SN feedback**; the Guo et al. model has introduced additional degrees of freedom.

This results in a much improved fit of **stellar mass function** at low-mass end.

With **AGN feedback (radio mode)** SAMs nowadays have little difficulty matching high-mass end of stellar mass function.

The latest 'Santa Cruz' SAM of Somerville et al. (2012) also matches low-mass end of stellar mass function, but using virtually same **SN feedback** model as De Lucia & Blaizot (2007). They do include **Quasar Mode Feedback** though....



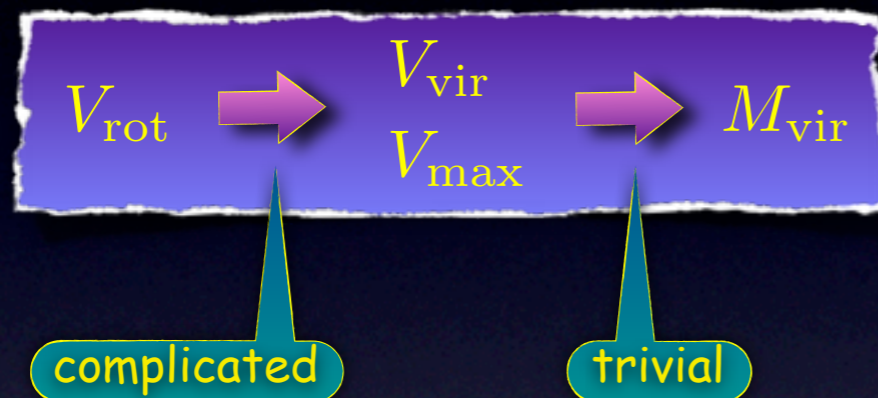
A visualization of the cosmic web, showing a complex network of blue and orange filaments and nodes against a black background. The filaments represent dark matter structures, and the nodes represent galaxy clusters.

The Galaxy-Dark Matter Connection

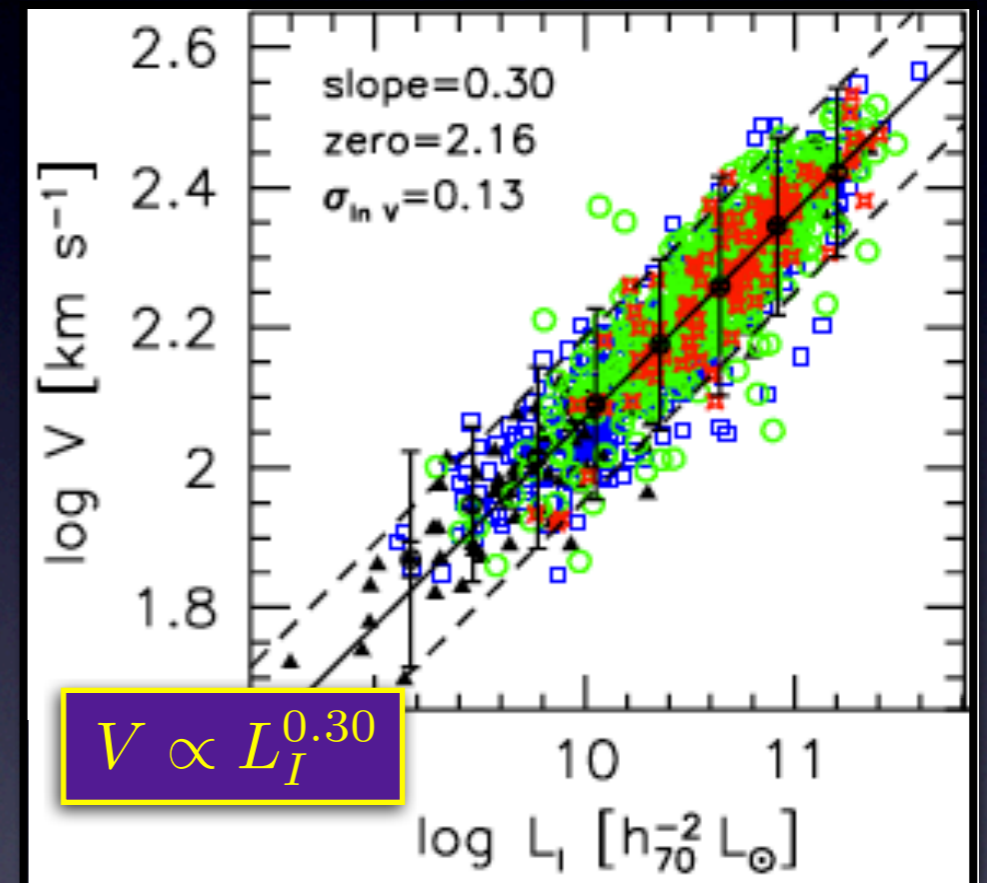
The Tully-Fisher Zero-Point

Reproducing the galaxy luminosity or stellar mass function(s) does not yet guarantee that these galaxies reside in the correct dark matter haloes.

Historically, the main 'test' that was used to check this was the **Tully-Fisher relation**.



Prior to ~2005 SAMS failed to match zero-point of TF relation, overpredicting V_{rot} at fixed L . Only when **SAMs** started to adopt Λ CDM concordance cosmology did problem 'disappear', but ONLY under the 'naive' assumption that $V_{\text{rot}} = V_{\text{max}}$ or $V_{\text{rot}} = V_{\text{vir}}$



Source: Courteau et al. 2007, ApJ, 671, 203

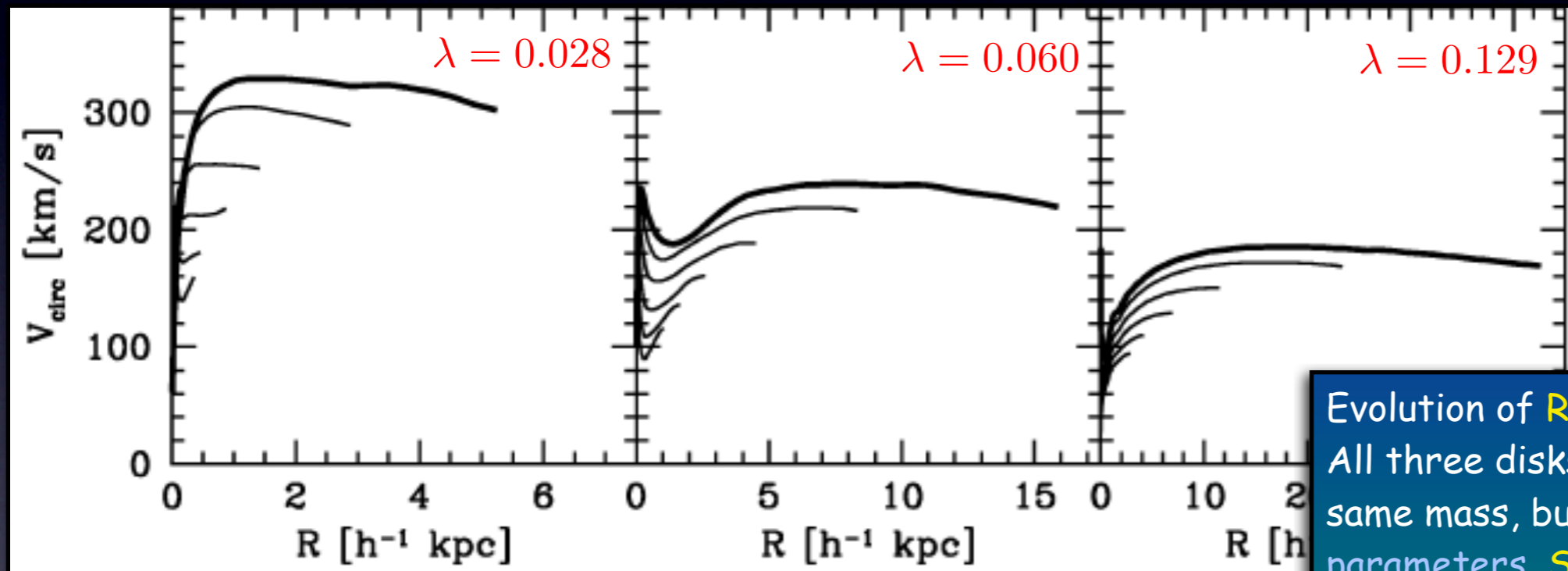
That assumption, however, ignores

- self-gravity of the disk galaxy
- adiabatic contraction of the dark matter halo

Adiabatic Contraction

Adiabatic Contraction: contraction of the dark matter halo as a gravitational response to baryons accumulating at center of potential well.

Following Blumenthal et al. (1986), adiabatic contraction is typically modeled assuming that $r M(r)$ is an **adiabatic invariant** during the formation of a disk galaxy.



Evolution of **RCs** in disk formation models. All three disks reside in haloes of the same mass, but with different spin parameters. **Self-gravity** of disk and **AC** are both taken into account

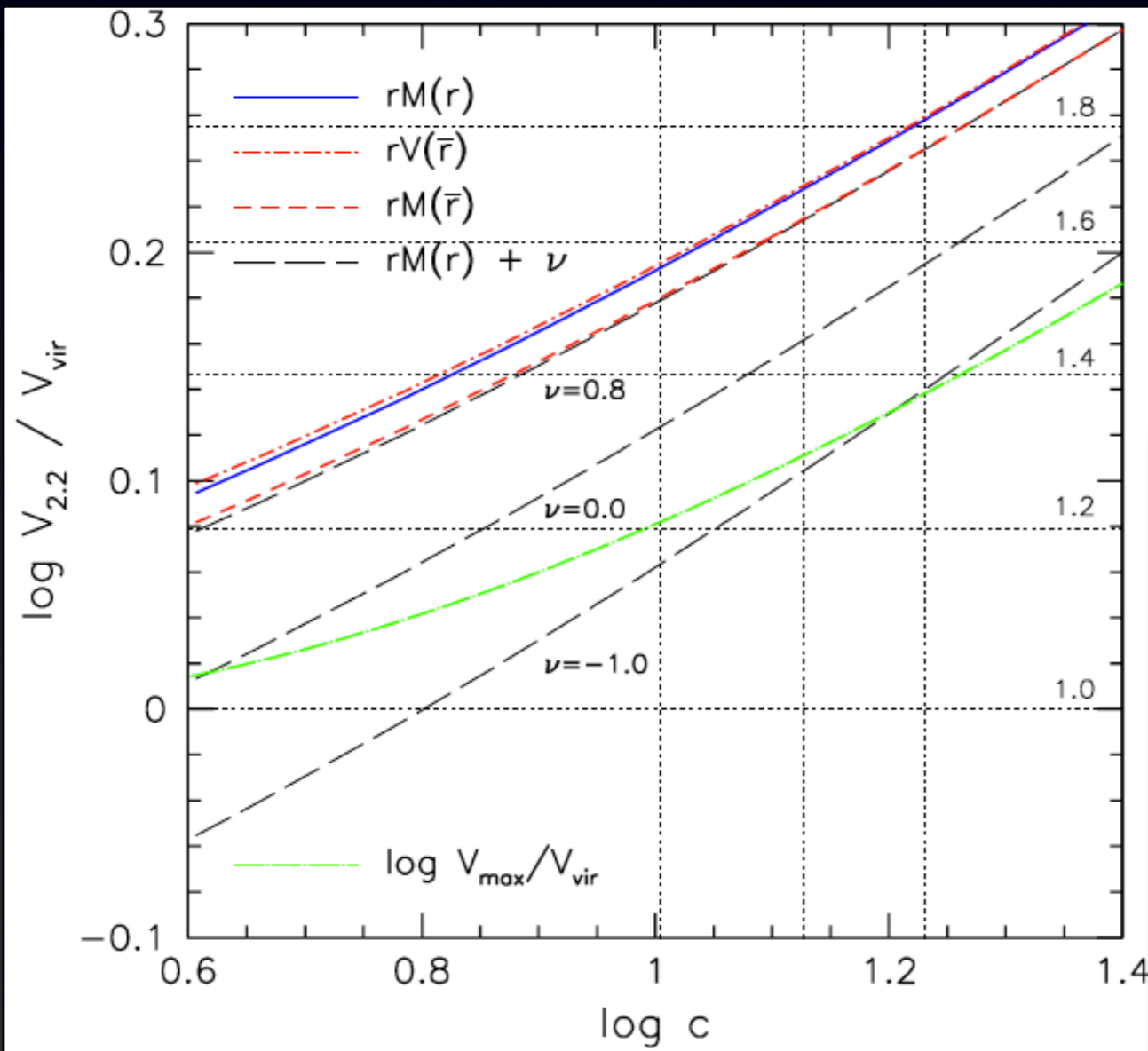
Realistic, detailed models of disk formation that include **self-gravity** of disk and **adiabatic contraction** of halo typically predict that $V_{\text{rot}}/V_{\text{vir}} \sim 1.4-1.8$, whereas **NFW** haloes typically have $V_{\text{max}}/V_{\text{vir}} \sim 1.2...$

Adiabatic Contraction

Note: $r M(r)$ is only an adiabatic invariant under oversimplified assumptions. Under realistic conditions, halo contraction can be modelled using $r_f = \Gamma^\nu r_i$, where $\Gamma = r_f/r_i$ is the simplified **AC** model of Blumenthal et al. (1986) and ν is a 'free parameter'.

Simulations suggest that $\nu \simeq 0.8$

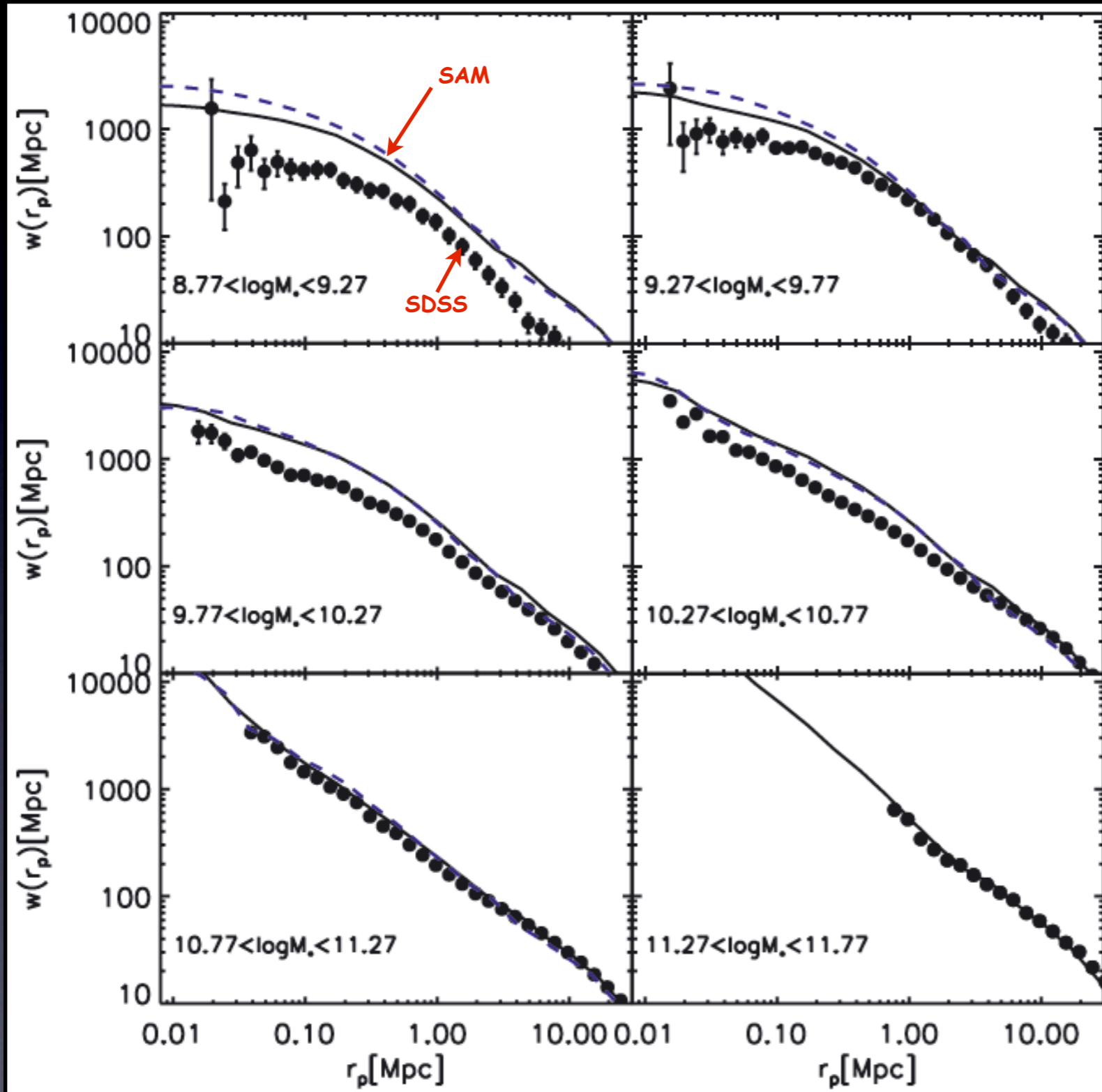
- $\nu = 1$ standard AC
- $\nu = 0$ no adiabatic contraction
- $\nu < 0$ halo expansion



Note how even in the case without **AC** ($\nu = 0$) $V_{2.2} > V_{\max}$, which is simply due to the self-gravity of the disk

Model predictions for the ratio $V_{2.2}/V_{\text{vir}}$ for disk galaxies embedded in **NFW** haloes. Results are shown as function of the halo concentration parameter, c . Here $V_{2.2}$ is the rotation velocity of the disk measured at 2.2 disk scale-lengths. Results are shown for different 'forms' of adiabatic contraction (different values of ν). For comparison, the **green curve** shows the ratio V_{\max}/V_{vir} .

Galaxy Clustering




CDM predicts that more massive haloes are more strongly clustered. Hence, only if galaxies reside in the correct dark matter haloes will the models match clustering data.

"Munich" **SAM** of Guo et al. (2011) fails in that respect, despite good match to stellar mass function...

Part of problem is likely to be that **Millennium Simulation** adopts wrong cosmology....

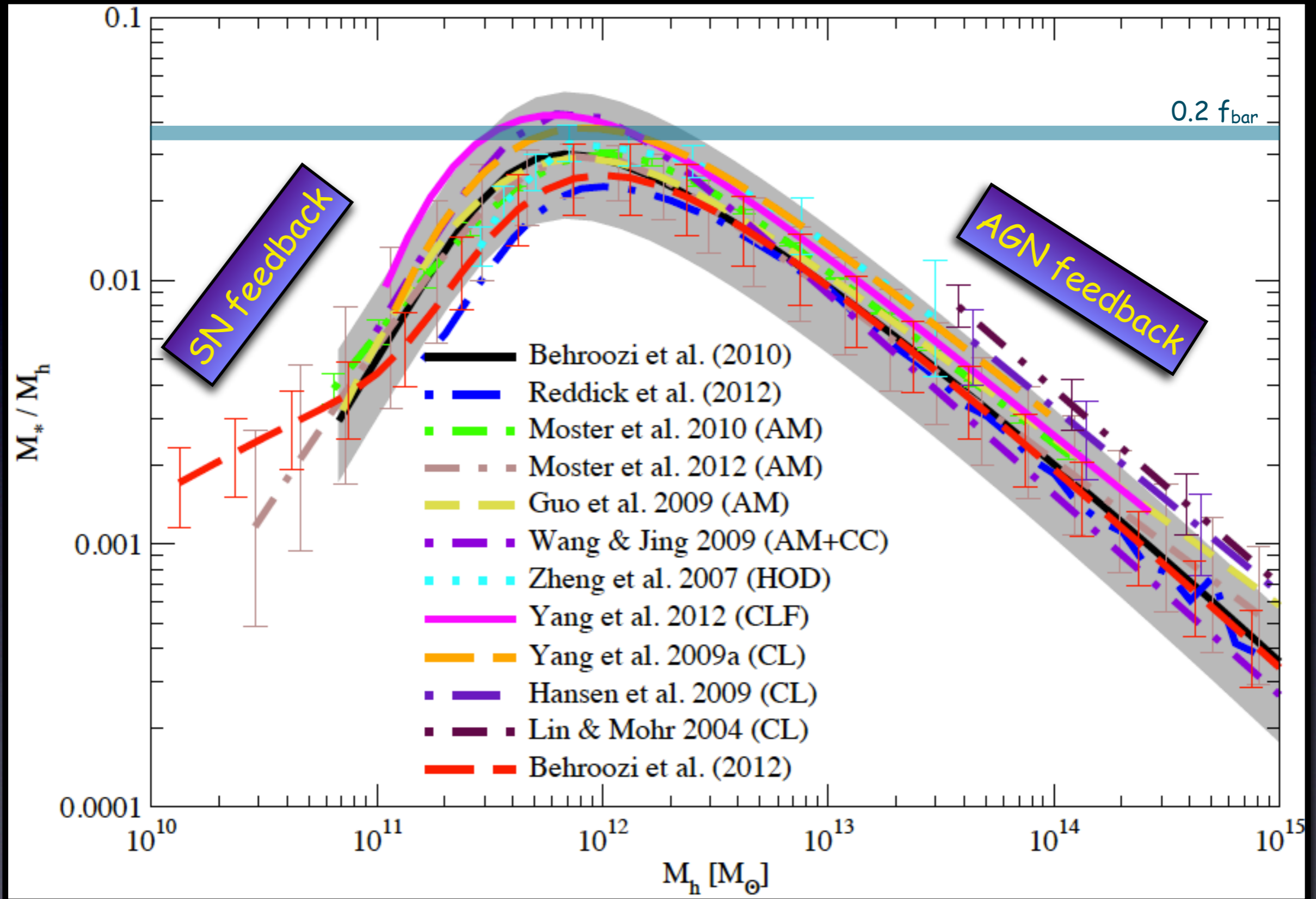
Several studies have shown that the **WMAP1 cosmology** cannot simultaneously match galaxy abundances & galaxy clustering.

(e.g., van den Bosch et al 2004; Cacciato et al. 2009, 2012)

A visualization of the cosmic web, showing a complex network of blue and orange filaments and nodes against a black background. The filaments represent the large-scale structure of the universe, with nodes indicating regions of high density.

Halo Occupation Modeling

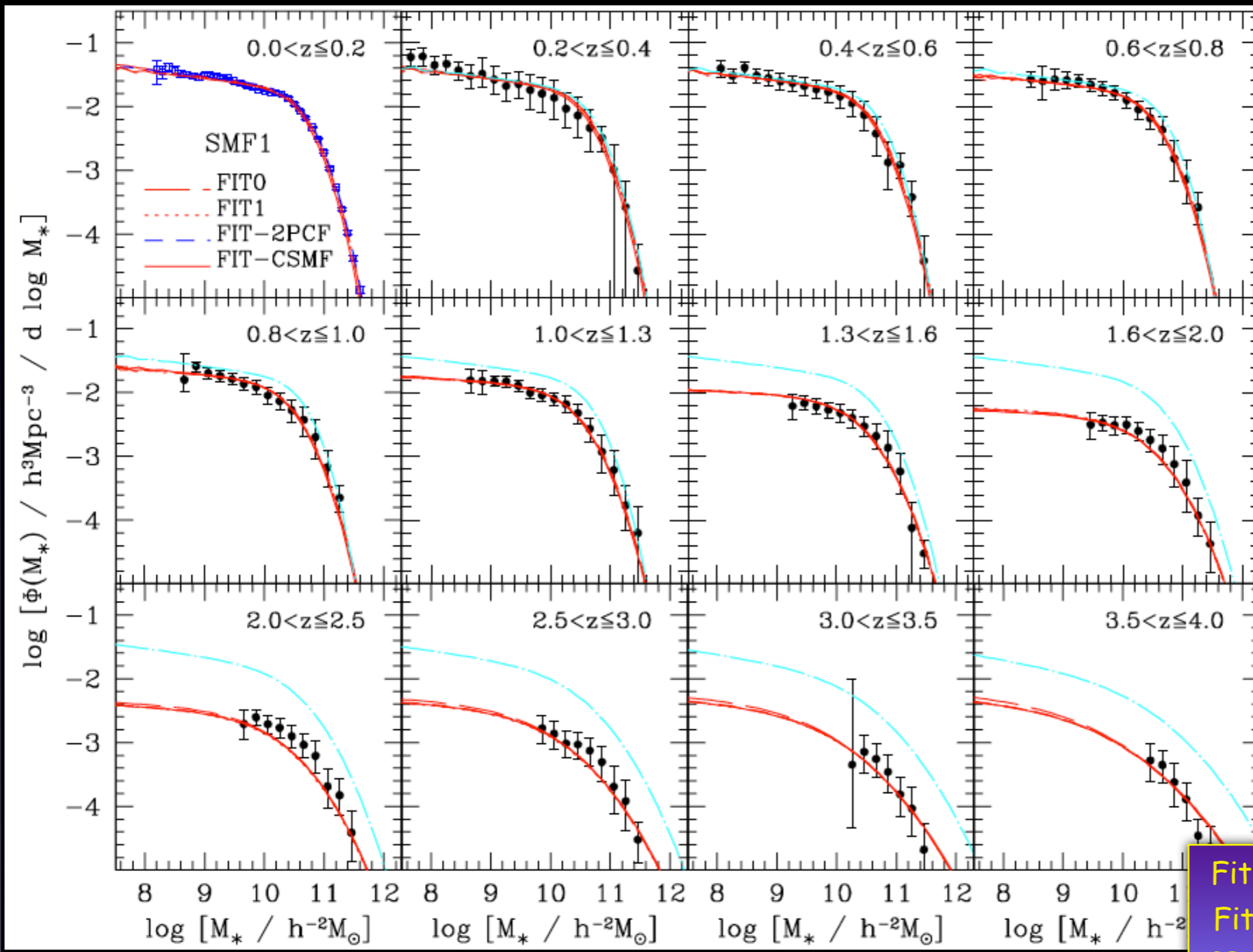
The Galaxy - Dark Matter Connection



Source: Behroozi, Wechsler & Conroy, 2012, arXiv:1207.6105

The Galaxy - Dark Matter Connection

Data: Yang et al (2009; $z \sim 0.1$)
Perez-Gonzales et al. (2008)

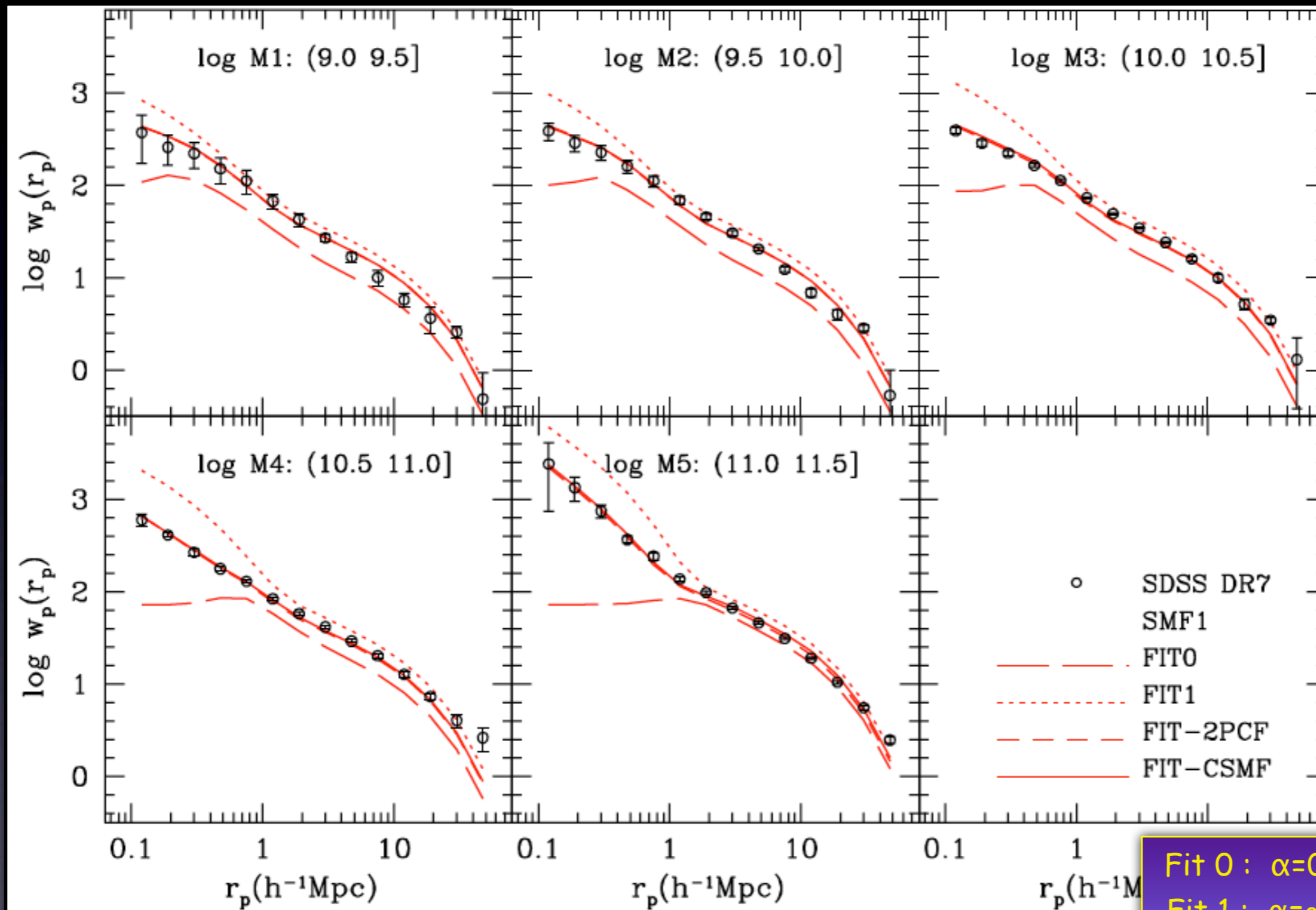


Fit 0 : $\alpha=0 \rightarrow$ no sats
 Fit 1 : $\alpha=\infty \rightarrow$ no evolution
 2PCF : fit to $\Phi(M_*) + 2PCF$
 CSMF : fit to $\Phi(M_*) + \Phi(M_*|M, z=0)$

Source: Yang et al. 2012, ApJ, 752, 41

The Galaxy - Dark Matter Connection

Data: SDSS DR7
(Yang et al. 2012)



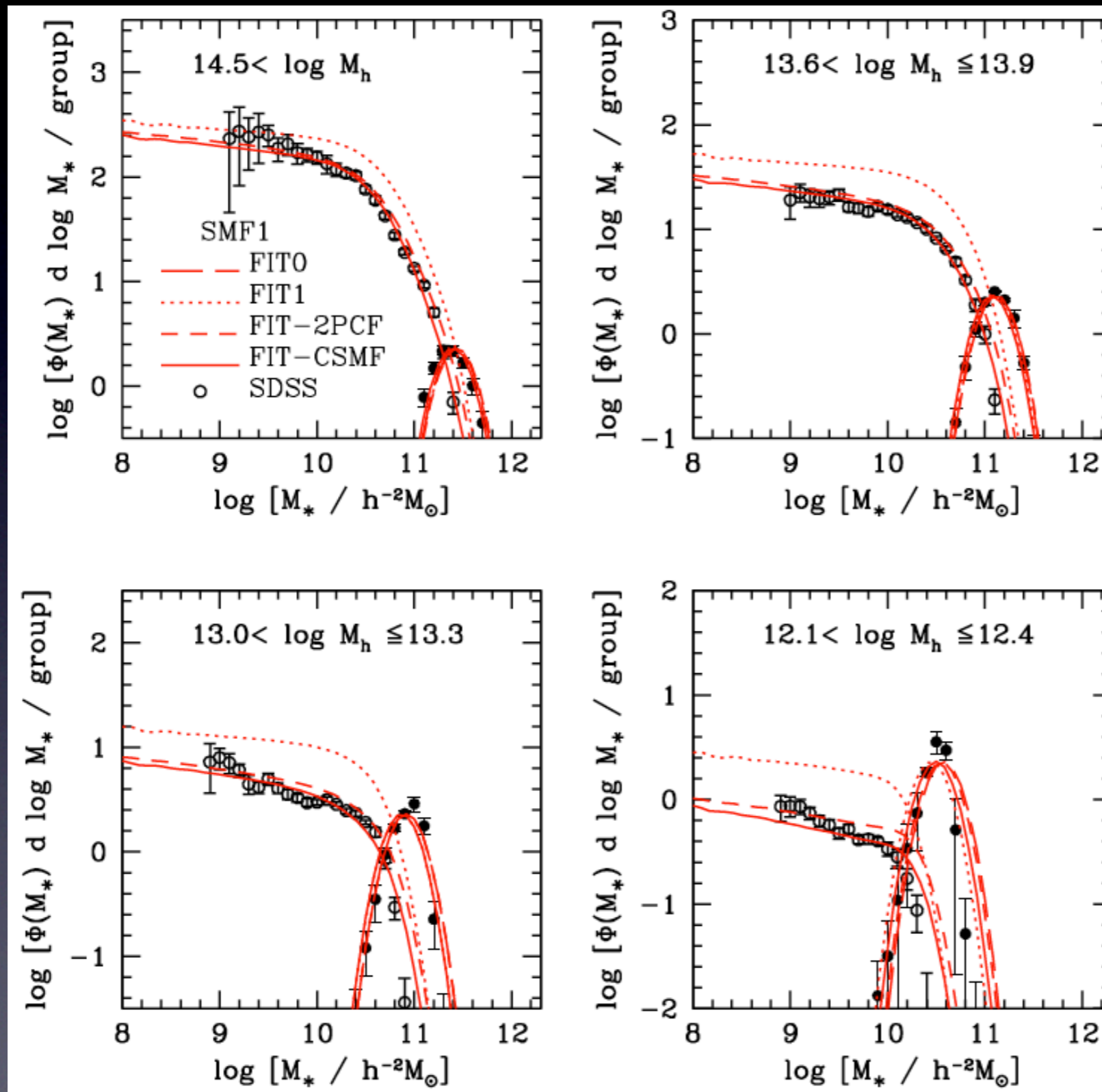
○ SDSS DR7
 SMF1
 — FIT0
 ··· FIT1
 - - - FIT-2PCF
 - - - FIT-CSMF

Fit 0 : $\alpha=0$ --> no sats
 Fit 1 : $\alpha=\infty$ --> no evolution
 2PCF : fit to $\Phi(M_*) + 2PCF$
 CSMF : fit to $\Phi(M_*) + \Phi(M_*|M, z=0)$

Source: Yang et al. 2012, ApJ, 752, 41

The Galaxy - Dark Matter Connection

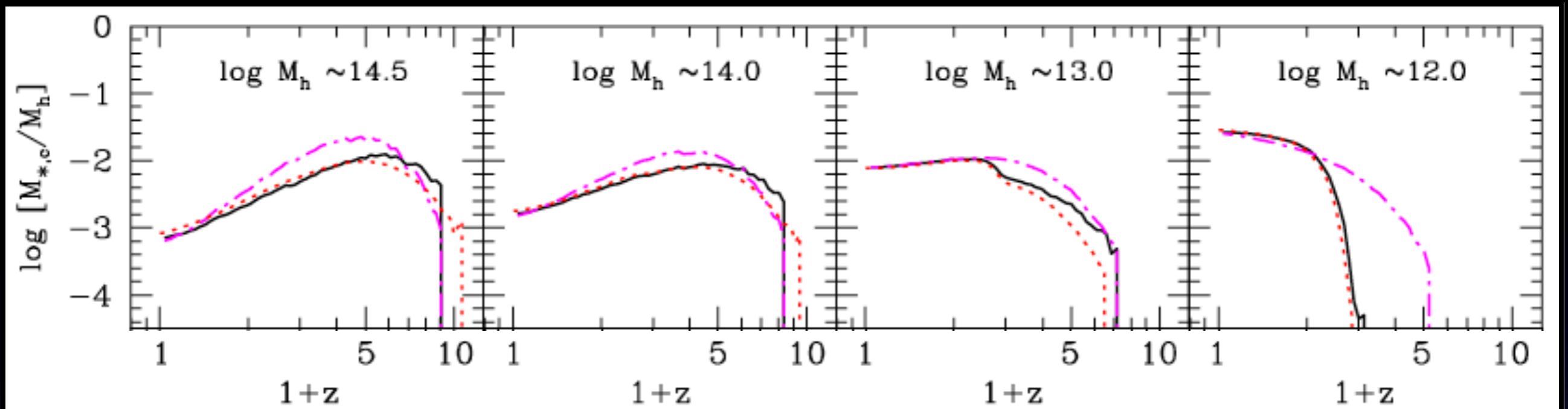
Data: SDSS Galaxy Group Catalogues
(Yang et al. 2009)



Source: Yang et al. 2012, ApJ, 752, 41

Fit 0 : $\alpha=0 \rightarrow$ no sats
 Fit 1 : $\alpha=\infty \rightarrow$ no evolution
 2PCF : fit to $\Phi(M_*)$ + 2PCF
 CSMF : fit to $\Phi(M_*)$ + $\Phi(M_*|M, z=0)$

The Galaxy - Dark Matter Connection



Source: Yang et al. 2012, ApJ, 752, 41

Stellar mass assembly history of galaxies is completely decoupled from mass assembly histories of their dark matter haloes



Virtually all galaxy formation models try to establish such a 'decoupling' with feedback from supernovae and/or AGN:

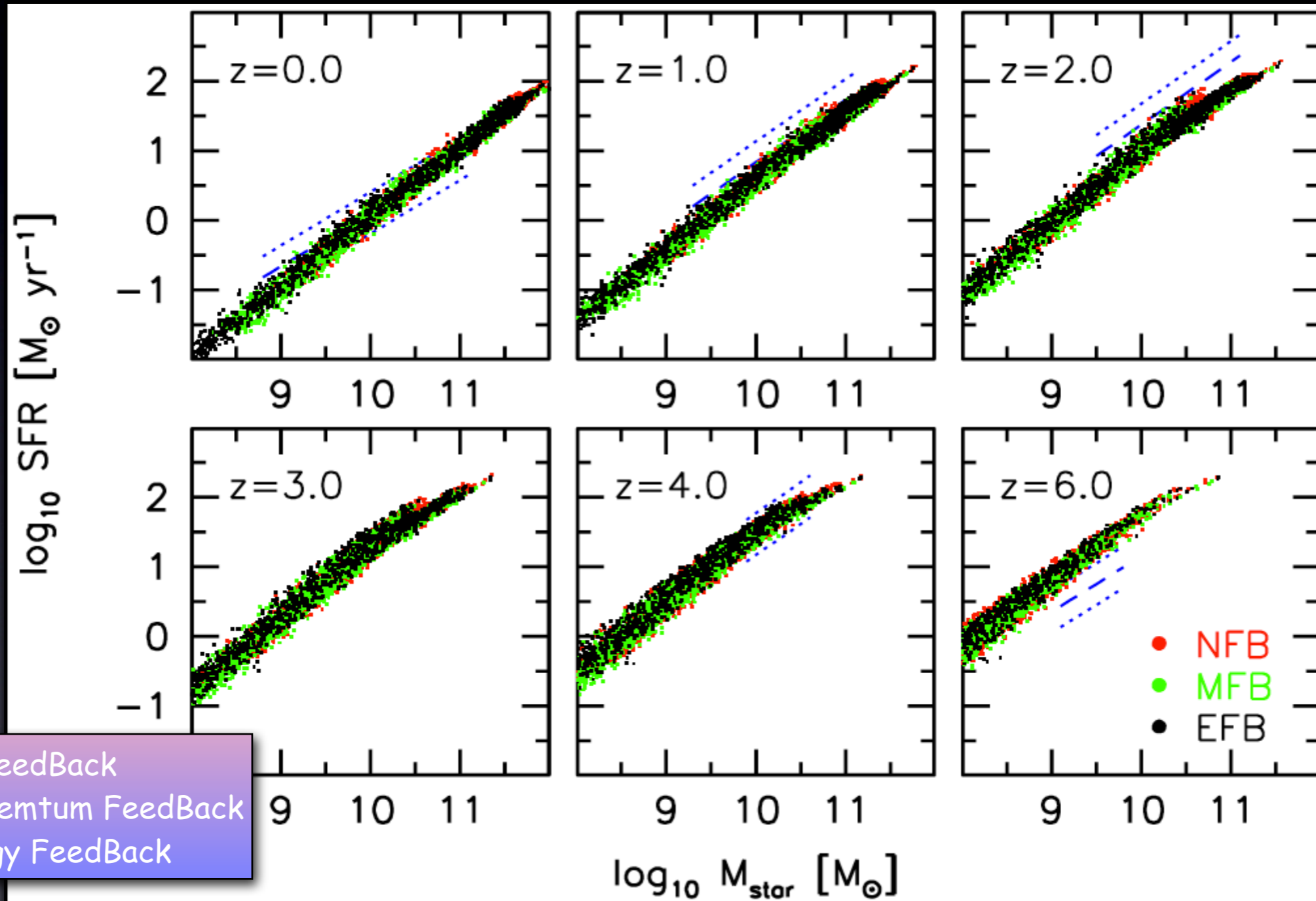
- **AGN feedback** is invoked to quench centrals in massive haloes
- **SN feedback** is invoked to suppress SF in low mass haloes.

The **MAIN** problem with current **SAMs** (and hydro-sims) is that the latter doesn't decouple stellar mass growth from halo mass growth, as I now demonstrate..

Origin of the Galaxy-SFR Sequence

Dutton et al. (2010) used detailed, spatially resolved **SAM** of (disk) galaxy formation, and showed that model naturally reproduces **SFR- M^*** relation, independent of (SN) feedback. This is consequence of **self-regulation** that drives system to **steady-state**!

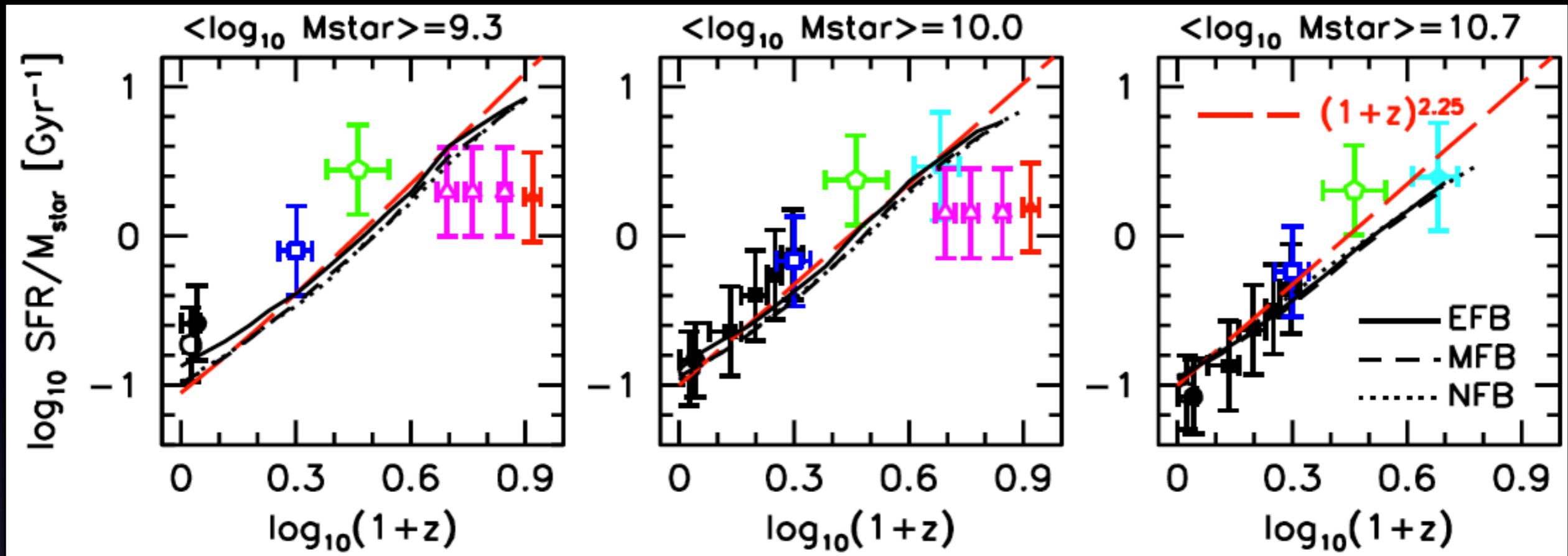
[cf. Bouché et al. (2010), Davé et al. (2010) as well as the lectures by Davé and Lilly]



NFB = No FeedBack
MFB = Momemtum FeedBack
EFB = Energy FeedBack

Source: Dutton, vdB & Dekel, 2010 MNRAS, 405, 1690

Origin of the Galaxy-SFR Sequence

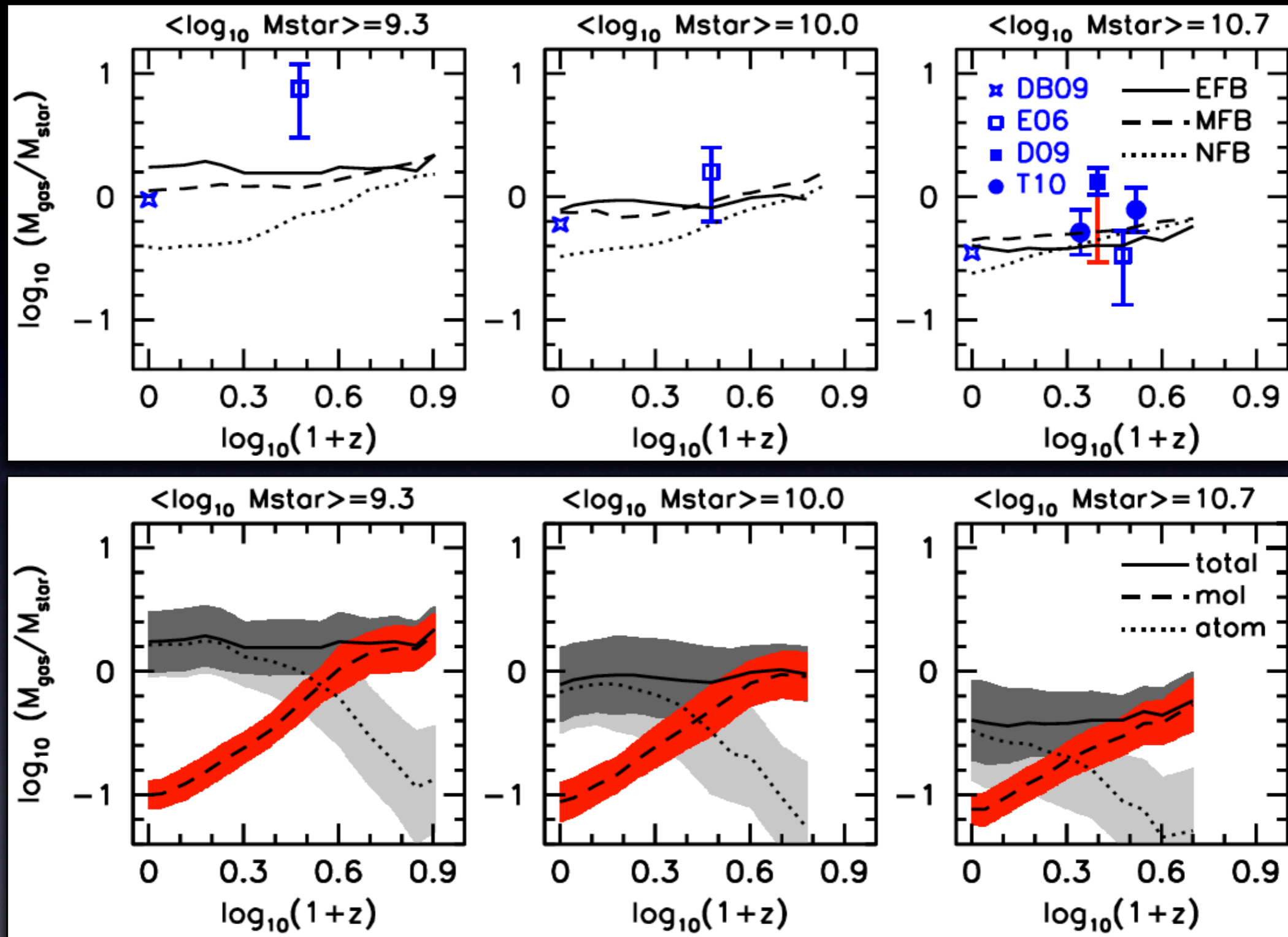


Source: Dutton, vdB & Dekel, 2010 MNRAS, 405, 1690

The stellar mass growth closely follows the cosmological accretion rate of dark matter haloes, independent of SN feedback; Hence, SN feedback cannot decouple halo growth from galaxy growth..



Origin of the Galaxy-SFR Sequence

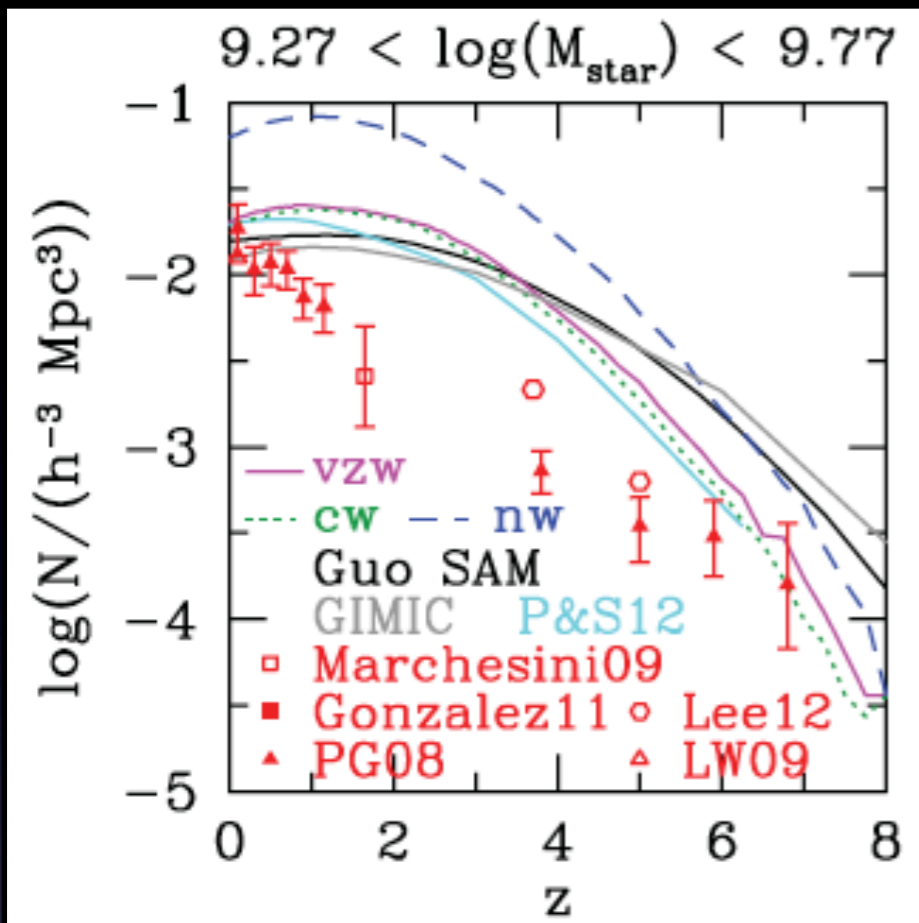


Source: Dutton, vdB & Dekel, 2010 MNRAS, 405, 1690

Model predicts little to no evolution in gas mass fractions at fixed stellar mass, but strong evolution in ratio of molecular-to-atomic (consequence of density evolution).

Outstanding Problems

Source: Weinmann et al. 2012, MNRAS, 426, 2797



In my opinion, the most daunting problem for galaxy formation is understanding why stellar mass assembly is so detached from halo mass assembly.

Most SAMs try to accomplish this using a combination of **reionization** and **feedback** from SN & AGN

The problem with **SN feedback** is that you need star formation to prevent star formation...

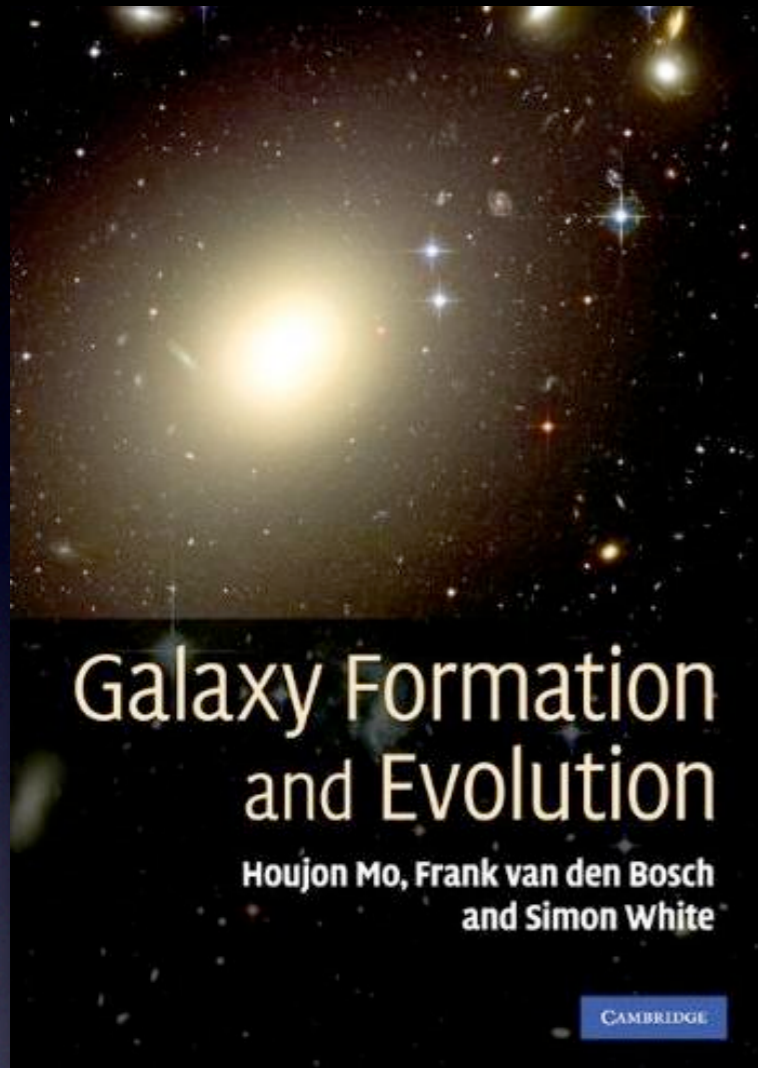
The problem with **AGN feedback** is that currently it is nothing but **wishful thinking**...



Galaxy Formation is far from a solved problem



For Further Reading...



A detailed treatment of the physical processes relevant for galaxy formation, and included in **SAMs**, can be found in the textbook **Galaxy Formation and Evolution**.

An excellent and up-to-date review of galaxy formation is

○ **Benson, 2010, Physical Reports, 495, 33**

A detailed review of Semi-Analytical Models is

○ **Baugh, 2006, Reports on Progress in Physics, 69, 3101**

