UNder CLose Examination: Semi Analytical Models

Jerusalem Winter School 2003-2004

Frank van den Bosch (ETH Zürich)

Outline

Semi-Analytical Models: Ingredients

- Merger Trees; the skeleton of hierarchical formation
- Cooling, Star Formation & Feedback
- Mergers & Galaxy Morphology
- Chemical Evolution, Stellar Population Synthesis & Dust

Semi-Analytical Models: Results

- The Galaxy Luminosity Function (galaxy abundances)
- The Tully Fisher Relation (mass-to-light ratios)
- The Color-Magnitude Relation (ages, metallicities & extinction)

The Galaxy-Dark Matter Connection

- Large Scale Structure with SAM (physics)
- Halo Occupation Models & Conditional Luminosity Function
- Large Scale Structure with CLF (statistics)

SAM BASICS

- **Hierarchical formation of DM haloes**
- **Baryons get shock heated to halo virial temperature**
- **Hot gas cools and settles in a disk in the center of the potential well.**
- **Cold gas in disk is transformed into stars (star formation)**
- **Energy output from stars (feedback) reheats some of cold gas**
- **After haloes merge, galaxies sink to center by dynamical friction**
- **Galaxies merge, resulting in morphological transformations.**

$$M_{\rm vir} = M_{\rm DM} + M_{\rm hot} + M_{\rm cold} + M_*$$
$$\dot{M}_{\rm hot} = -\dot{M}_{\rm cool} + \dot{M}_{\rm reheat} + f_{\rm bar} \cdot \dot{M}_{\rm vir}$$
$$\dot{M}_{\rm cold} = \dot{M}_{\rm cool} - \dot{M}_* - \dot{M}_{\rm reheat}$$

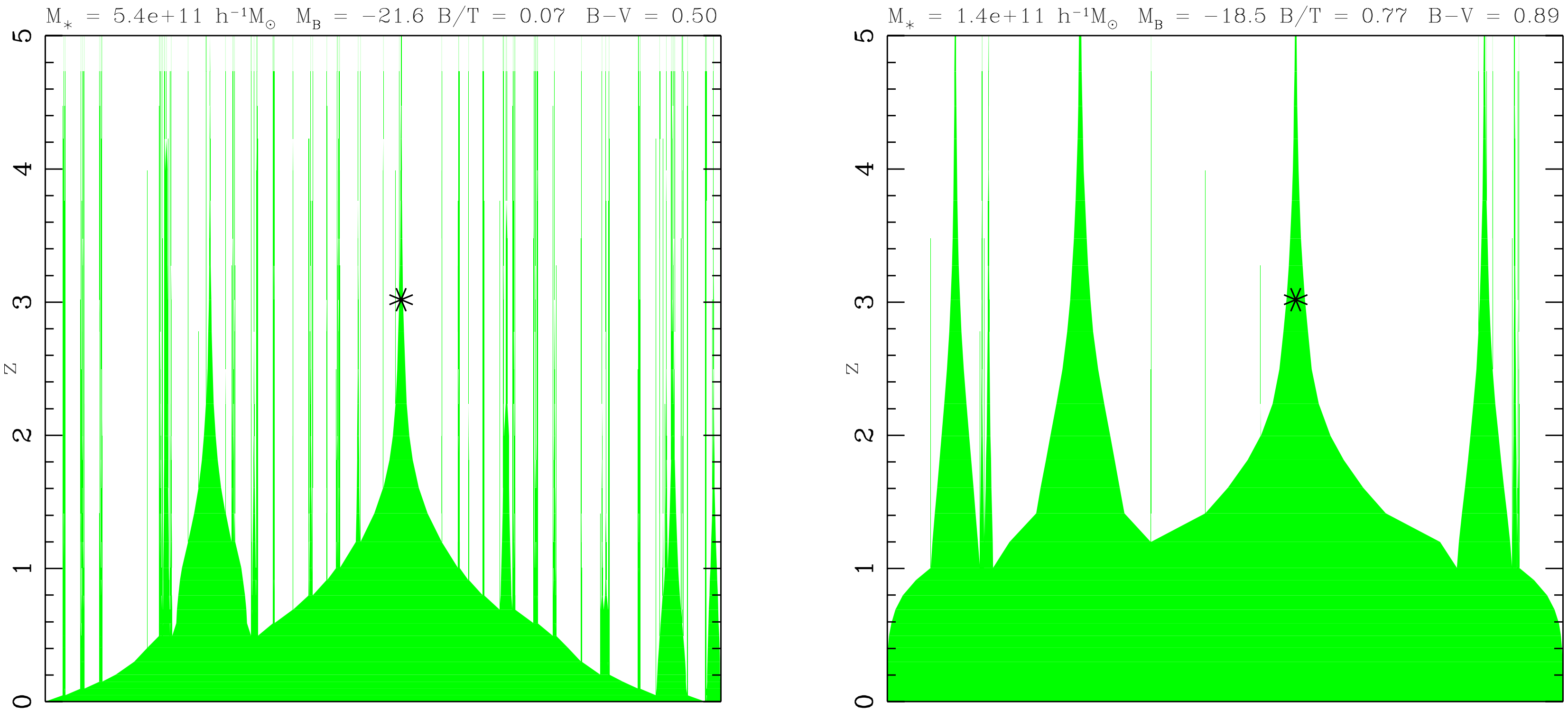
- $\dot{M}_{\rm vir} \iff$ merger history
- $\dot{M}_{\rm cool} \iff$ cooling recipe
- $\dot{M}_* \iff$ star formation recipe
- $\dot{M}_{\rm reheat} \iff$ feedback recipe

Merger Histories

The extended Press-Schechter theory yields the conditional probability $P(M_2, z_2|M_1, z_1)$ that a particle in a halo of mass M_1 at redshift z_1 was embedded in a halo of mass M_2 at redshift $z_2 > z_1$.

Bond et al. 1991; Bower 1991; Lacey & Cole 1993

With this conditional probability one can construct merger histories of dark matter haloes. **Kauffmann & White 1993; Somerville & Kolatt 1999; Sheth & Lemson 1999**



Baugh et al 1998

Gas Cooling I

The **cooling time** is defined as $\tau_{\rm cool} = \frac{U}{|{\rm d}U/{\rm d}t|}$ with $U = \frac{3}{2} N k T$ and ${\rm d}U/{\rm d}t = n_e n_t \Lambda(T,Z)$. Here

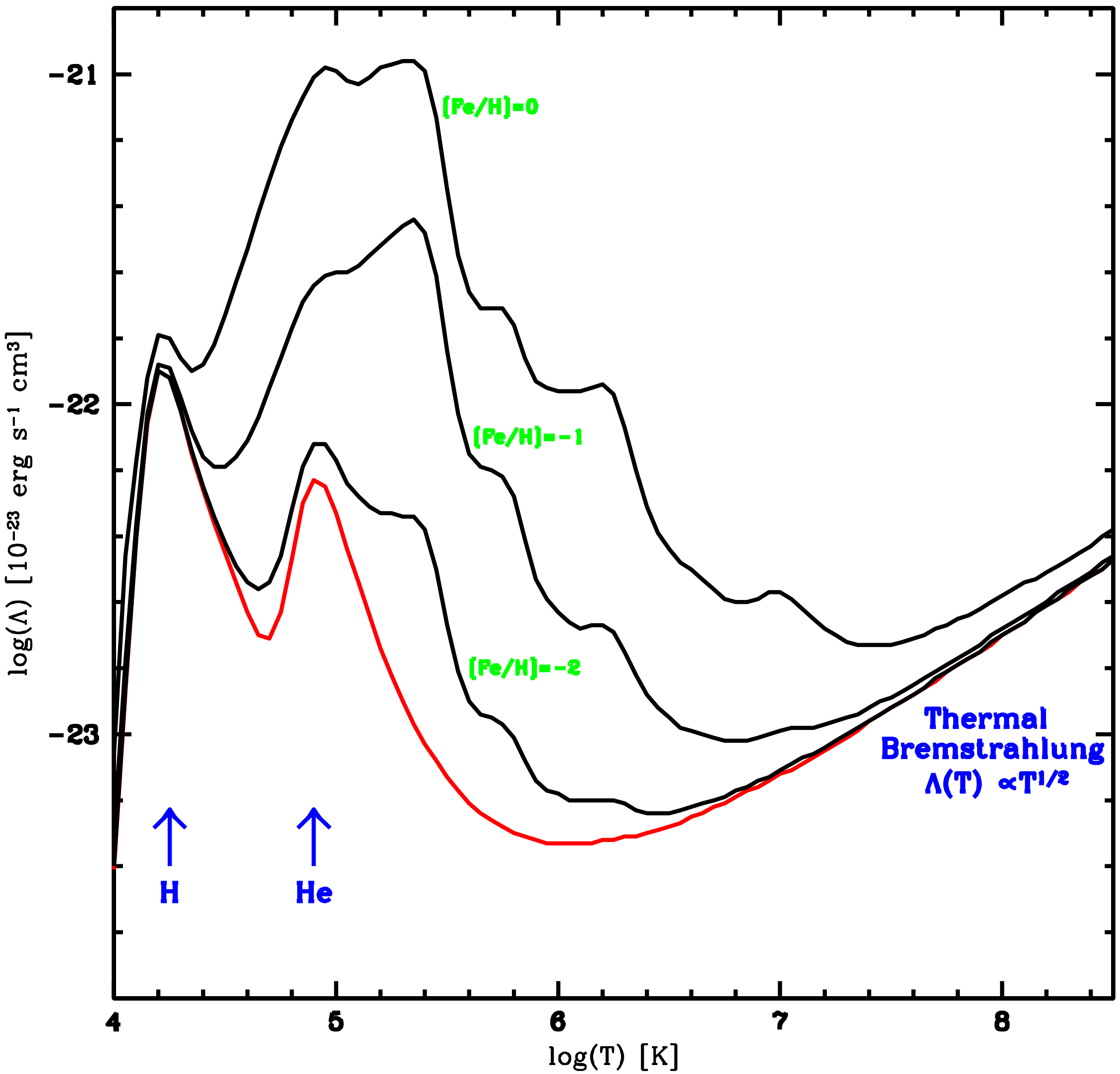
- $N =$ number density of particles $= n_e + n_t$
  $n_e =$ number density of electrons
  $n_t = \sum_i n_i =$ number density of all ions
- $k =$ Boltzman's constant
- $T =$ Temperature
- $\Lambda(T,Z) =$ **net cooling function** (in erg/s/cm^3)
  depends on temperature T and metallicity Z

Defining $\mu_e \equiv \frac{N}{n_e}$ = number of particles per electron, and $\bar{\mu} =$ mean mass per particles, we can write $\rho_{\rm gas} = N\,\bar{\mu} = n_e\,\mu_e\,\bar{\mu}$ and $n_t = (\mu_e - 1) n_e$, so that

$$\tau_{\rm cool} = \frac{3}{2}\,\bar{\mu}\,\frac{kT}{\rho_{\rm gas}\Lambda(T,Z)}\,\frac{\mu_e^2}{\mu_e - 1}$$

NOTE: $\bar{\mu}$ and μ_e depend on both composition and ionization state of gas. For completely ionized, primordial gas (Y=0.25, Z=0) $\bar{\mu} = 0.6 m_p$ and $\mu_e = 27/14$.

Gas Cooling II



Sutherland & Dopita 1993

Net cooling functions as function of temperature and metallicity

Gas Cooling III

In order to compute the cooling time of gas in dark matter haloes we need both the density and the temperature of the gas, both as function of radius r.

Modelling the Gas Density

Truncated Isothermal Sphere: $\rho_{\rm gas} \propto r^{-2}$

The β-model: $\rho_{\rm gas} \propto (r^2 + r_c^2)^{-3\beta/2}$

Simulations indicate that $\beta \simeq 2/3$ for which $\rho_{\rm gas}$ equals an isothermal sphere with a constant density core ($r < r_c$).

Modelling the Gas Temperature

General assumption: when gas enters virial radius, it is shock heated to virial temperature to bring the infalling gas in virial equilibrium with the dark matter;

$$T(r) = T_{\rm vir} = \frac{1}{2}\frac{\bar{\mu}}{k} V_{\rm vir}^2$$

For completely ionized, primordial gas: $T_{\rm vir} = 35.9\mathrm{K} \left(\frac{V_{\rm vir}}{\mathrm{km\,s^{-1}}}\right)^2$

Gas Cooling IV

In SAMs the cooling rates are computed as follows:

Define the cooling radius as $\tau_{\rm cool}(r_{\rm cool}) = t(z) =$ age of Universe.

$$\dot{M}_{\rm cool} = 4\pi\rho_{\rm gas}(r_{\rm cool})r_{\rm cool}^2 \frac{{\rm d}r_{\rm cool}}{{\rm d}t}$$

NOTE: this ignores the fact that gas without pressure support still takes a free-fall time to reach the center

The free-fall time is defined as: $t_{\rm ff} = \sqrt{\frac{3\pi}{32G\bar{\rho}}}$

One can easily take this free-fall time into account by replacing $r_{\rm cool}$ with $r_{\rm cool} = \min[r_{\rm cool}, r_{\rm ff}]$. Here the free-fall radius is defined as the radius where the free-fall time is equal to the age of the Universe: $t_{\rm ff}(r_{\rm ff}) = t(z)$.

For a halo with $\bar{\rho} = \Delta_{\rm crit}\rho_{\rm crit}$ in a ΛCDM 'concordance' cosmology $t_{\rm ff}(z=0) = 1.53h^{-1}$ Gyr and $t_{\rm ff}(z=10) = 0.06h^{-1}$ Gyr

Quiescent Star Formation

Typically, SAMs consider two modes of starformation: (i) quiescent SF in disk and (ii) star bursts during major mergers that produce spheroids

All SAMs use the simple star formation rate: $\dot{M}_* = \frac{M_{\rm cold}}{\tau_*}$

Here the star formation timescale is defined as:

(1) $$\tau_* = \tau_0 \left(\frac{V_{\rm vir}}{V_0}\right)^{\alpha_*}$$

(2) $$\tau_* = \epsilon_*^{-1} \tau_{\rm dyn} \left(\frac{V_{\rm vir}}{V_0}\right)^{\alpha_*} \text{ with } \tau_{\rm dyn} \propto \frac{R_{\rm disk}}{V_{\rm disk}} \propto \frac{R_{\rm vir}}{V_{\rm vir}} = f(z)$$

With (2) the SFR depends on both $V_{\rm vir}$ and z, since $\tau_{\rm dyn}$ decreases with increasing z. With (1), however, the SFR depends only on $V_{\rm vir}$

- **Munich, Santa Cruz & Paris use (2) with $\alpha_* = 0$**
- **Durham used to adopt (1) with $\alpha_* = -1.5$, but changed to (2) in 2000, still using $\alpha_* = -1.5$**

NOTE: α_* and ϵ_* are typically tuned to fit the cold gas mass fraction as function of luminosity.

Feedback

The energy released by stellar winds and SNe can heat (thermal feedback) and/or eject (kinetic feedback) gas from disk and/or halo.

Feedback in SAMs modelled as: $\dot{M}_{\rm reheat} \propto \left(\frac{V_{\rm vir}}{V_0}\right)^{-\alpha_{\rm fb}} \dot{M}_*$

Typically $\alpha_{\rm fb} = 2$, which is "motivated" by a simple wind model:

Consider a galactic wind with mass ejection rate $\dot{M}_{\rm reheat}$ and wind velocity v_w. Energy balance requires

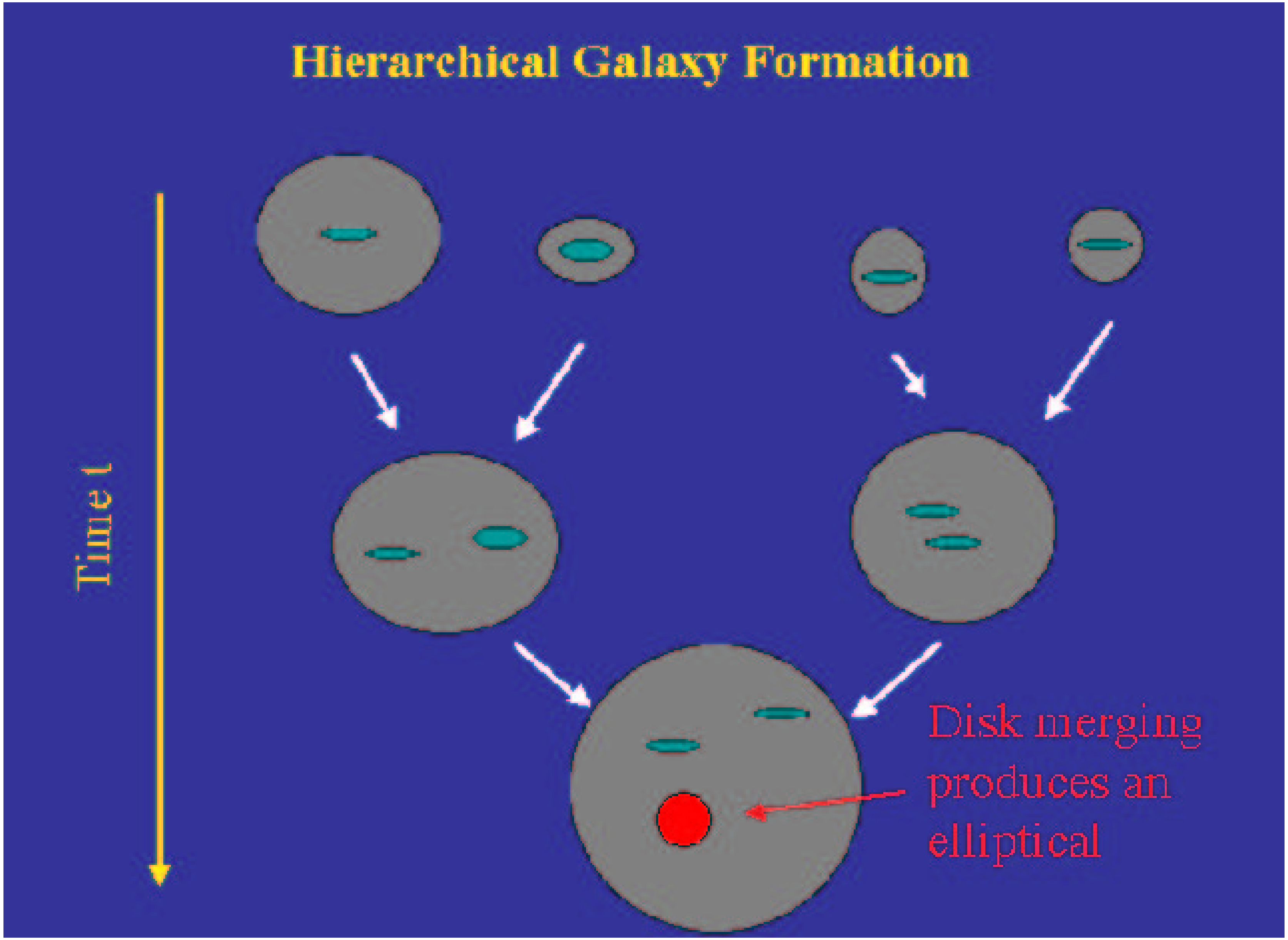
$$\frac{1}{2}\dot{M}_{\rm reheat} v_w^2 = \epsilon_{\rm fb}\eta_{\rm SN}\dot{M}_* E_{\rm SN}$$

Assume $v_w = v_{\rm esc} = \sqrt{2}V_{\rm vir}$ than: $\dot{M}_{\rm reheat} = \frac{\epsilon_{\rm fb}\eta_{\rm SN}E_{\rm SN}}{V_{\rm vir}^2}\dot{M}_*$

- $\epsilon_{\rm fb}$ = fraction of SN energy turned into kinetic wind energy
- $\eta_{\rm SN}$ = number SN per solar mass of stars formed.
  For Scale IMF, $\eta_{\rm SN} \simeq 4 \times 10^{-3}$
- $E_{\rm SN} = 10^{51}$ ergs = energy produced per SN

$M_{\rm reheat}$ is either added to $M_{\rm hot}$ (retention) or ejected from halo (rejection).

Mergers & Galaxy Morphologies I



Mergers & Galaxy Morphologies II

- Central galaxy in most massive halo becomes central galaxy in new halo
- All other galaxies become satellite galaxies which sink towards central galaxy due to dynamical friction
- Gas continues to cool onto central galaxy only.
- If galaxies merge, morphological outcome depends on mass ratio; if $M_2/M_1 > f_{\rm ellip}$ stars form ellipsoid & cold gas undergoes starburst.

Dynamical Friction: (Chandrasekhar 1943; White 1976; Tremaine 1980)

Starting from the virial radius, a satellite reaches the center after a time

$$\tau_{\rm df} = f_{\rm df} \frac{f(\eta)}{\ln\Lambda} \frac{V_{\rm vir} r_{\rm vir}^2}{M_{\rm sat}}$$

Here $\ln\Lambda \simeq \ln(1 + M_{\rm vir}/M_{\rm sat})$ is the Coulomb logarithm, $\eta = J_{\rm orbit}/J_{\rm circ}(E)$ is the orbital circularity, and $f_{\rm df}$ is a fudge factor.

The dependence of the dynamical friction time scale on the orbital eccentricity is modelled as $f(\eta) = \eta^a$. Different authors have advocated different values of a: [0.78 (Lacey & Cole 1993), 0.53 (van den Bosch et al. 1999), 0.40 (Colpi, Mayer & Governato 1999)]

Extra Ingredients

Satellite-Satellite Mergers Satellite galaxies can also merge among themselves. Cross sections and merger rates ($\tau_{\rm coll}$) can be obtained from literature (Makino & Hut 1997; Mamon 1992, 2000)

Implementation: each time step Δt the probability for a satellite to merge with another satellite is $P = \Delta t / \tau_{\rm coll}$. The morphological outcome depends on mass ratio of the satellites (Santa Cruz & Galics).

Spheroid Formation from Disk Instabilities Disks that are too compact are unstable. They typically form a bar which may later 'dissolve' into a bulge.

(Combes et al. 1990; Pfenniger & Norman 1990)

Stability Criterion: A disk is unstable if

$$\alpha_c = \frac{V_{\rm disk}(3R_d)}{V_{\rm circ}(3R_d)} \gtrsim 0.6$$

(Efstathiou et al. 1982; Christodoulou et al. 1995)

Thus, if disk contributes more than ~ 60 percent to the circular velocity at $R = 3R_d$, the self-gravity of the disk causes instability.

Implementation: When disk is unstable, transfer as much disk mass to bulge until disk is marginally stable ($\alpha_c = 0.6$).

(van den Bosch 1998; Mao & Mo 1998; Cole et al. 2000; Galics)

Chemical Evolution

Instantaneous Recycling Approximation: Each mass M_* formed in new stars immediately returns a fraction $\mathcal{R}\, M_*$ back to the ISM and produces a mass $Y\, M_*$ of newly synthesized metals. The remaining $(1-\mathcal{R})\, M_*$ is assumed to live as stars forever.

Both the return fraction, $\mathcal{R}$, and the yield, Y, depend on the IMF. Especially the yield is rather uncertain

$$\mathcal{R} \approx 0.3 - 0.4 \qquad\qquad Y \approx 0.01 - 0.03$$

Transporting Metals

$$\dot{M}_*^Z = (1-\mathcal{R}) Z_{\rm cold} \dot{M}_*$$
$$\dot{M}_{\rm hot}^Z = Z_{\rm cold} \dot{M}_{\rm reheat} - Z_{\rm hot} \dot{M}_{\rm cool}$$
$$\dot{M}_{\rm cold}^Z = Z_{\rm hot} \dot{M}_{\rm cool} - Z_{\rm cold} \dot{M}_{\rm reheat} - (1-\mathcal{R}) Z_{\rm cold} \dot{M}_* + Y \dot{M}_*$$

Stellar Population Synthesis

SAMs yield star formation rate, $\psi(t)$, and metallicity of the cold gas, $Z(t)$. One can compute the galactic flux at time t in a waveband λ as:

$$F_\lambda(t) = \int_0^t \mathrm{d}\tau \int_{m_1}^{m_2} \mathrm{d}m \, f_\lambda(m, Z, t-\tau) \, \phi(m) \, \psi(\tau)$$

Here $\phi(m)$ is the IMF and $f_\lambda(m, Z, t)$ is the flux in λ of a star with initial mass m, initial metallicity Z, and age t.

Stellar Population Synthesis Models typically provide a list of $\hat{F}_\lambda(t, Z)$ for an instantaneous burst of SF ($\psi(t) = \delta(t - t_0)$) for given IMF and metallicity Z. (e.g., Guiderdoni & Rocca-Volmerange 1987; Bruzual & Charlot 1993). This allows computation of $F_\lambda(t)$ for any $\psi(t)$ and $Z(t)$.

$$F_\lambda(t) = \int_0^t \hat{F}_\lambda(t-\tau, Z(\tau)) \, \psi(\tau) \, \mathrm{d}\tau$$

Typically, brown dwarfs are not modelled $(m_1 = 0.1\ \mathrm{M}_\odot)$. Their contribution can be included by multiplying all mass-to-light ratios with

$$\Upsilon = \frac{\text{mass in visible stars + brown dwarfs}}{\text{mass in visible stars}}$$

Durham SAMs typically include Υ as a free parameter, which they tune by fitting the LF.

Dust

Model magnitudes need to be corrected for extinction by dust:

$$M_{\lambda,\mathrm{corr}} = M_{\lambda,\mathrm{intr}} + A_\lambda$$

For a disk galaxy, the amount of extinction depends on inclination angle i.
For a standard slab model; a thin disk with stars & dust uniformly mixed together (Tully & Fouqué 1985):

$$A_\lambda = -2.5^{10}\log\left(\frac{1-\mathrm{e}^{-\tau_\lambda/\cos i}}{\tau_\lambda/\cos i}\right)$$

with τ_λ the face-on optical depth.

Santa Cruz & Munich use empirical relation due to Wang & Heckmann 1996.

$$\tau_B = 0.8\,\left(\frac{L}{L^*}\right)^{0.5}$$

Durham uses analytical models of Ferrara et al. 1999.

$$\tau \propto \frac{M_{\mathrm{dust}}}{r^2_{\mathrm{disc}}} \propto \frac{Z_{\mathrm{cold}}\,M_{\mathrm{cold}}}{r^2_{\mathrm{disc}}}$$

More detailed/complicated dust models, taking account of the energy reradiated at far-IR and sub-mm wavelengths, have been develloped by Silva et al. 1998 and Granato et al. 2000. See also GALICS models (Hatton et al. 2003).

Tuning SAM

Free Parameters

- **Cosmology:** $\Omega_m, \Omega_\Lambda, \Omega_b, H_0, \sigma_8, n_{\rm spec}$
- **Cooling:** $\beta, r_{\rm core}$
- **Star formation:** ϵ_*, α_*
- **Feedback:** $\epsilon_{\rm fb}, \alpha_{\rm fb}$
- **Mergers:** $f_{\rm ellip}, f_{\rm df}$
- **IMF:** $\eta_{\rm SN}, \mathcal{R}, Y, \Upsilon$
- **Dust:** $\tau_\Lambda/\tau_B, \tau_B$

Normalization

- **TF zero-point & cold gas mass fraction of MW** — Munich & Santa Cruz
- **Luminosity Function in B and/or K-band** — Durham & Paris

Challenging Uncle Sam

SUCCES & FAILURE IN A DECADE OF SAM

The LF–TF challenge

The shapes of the galaxy luminosity function, $\Phi(L)$, and the halo mass function, $n(M)$, are very different.

- At low mass end $n(M) \propto M^{-2}$, whereas at faint end $\Phi(L) \propto L^{-1.2}$. Therefore, galaxy formation has to become extremely inefficient at low M: Feedback? Reionization?
- If $M/L = \mathrm{cst}$, then $n(M) > \Phi(L)$ at high mass end. Therefore, galaxy formation has to become less efficient at high M: Cooling? Merging? Conduction? Superwinds?

$\frac{M}{L}(M)$ has to vary dramatically with M with a characteristic shape

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The Tully-Fisher relation has the form $L_{\rm gal} \propto V_{\rm obs}^3$ in the I-band.

For a virialized haloes: $M_{\rm vir} \propto V_{\rm vir}^3$ so that $L_{\rm gal} \propto \left(\frac{M}{L}\right)^{-1} V_{\rm vir}^3$. If $V_{\rm obs} \propto V_{\rm vir}$ the slope of the TF relation implies $M/L = \mathrm{cst}$.

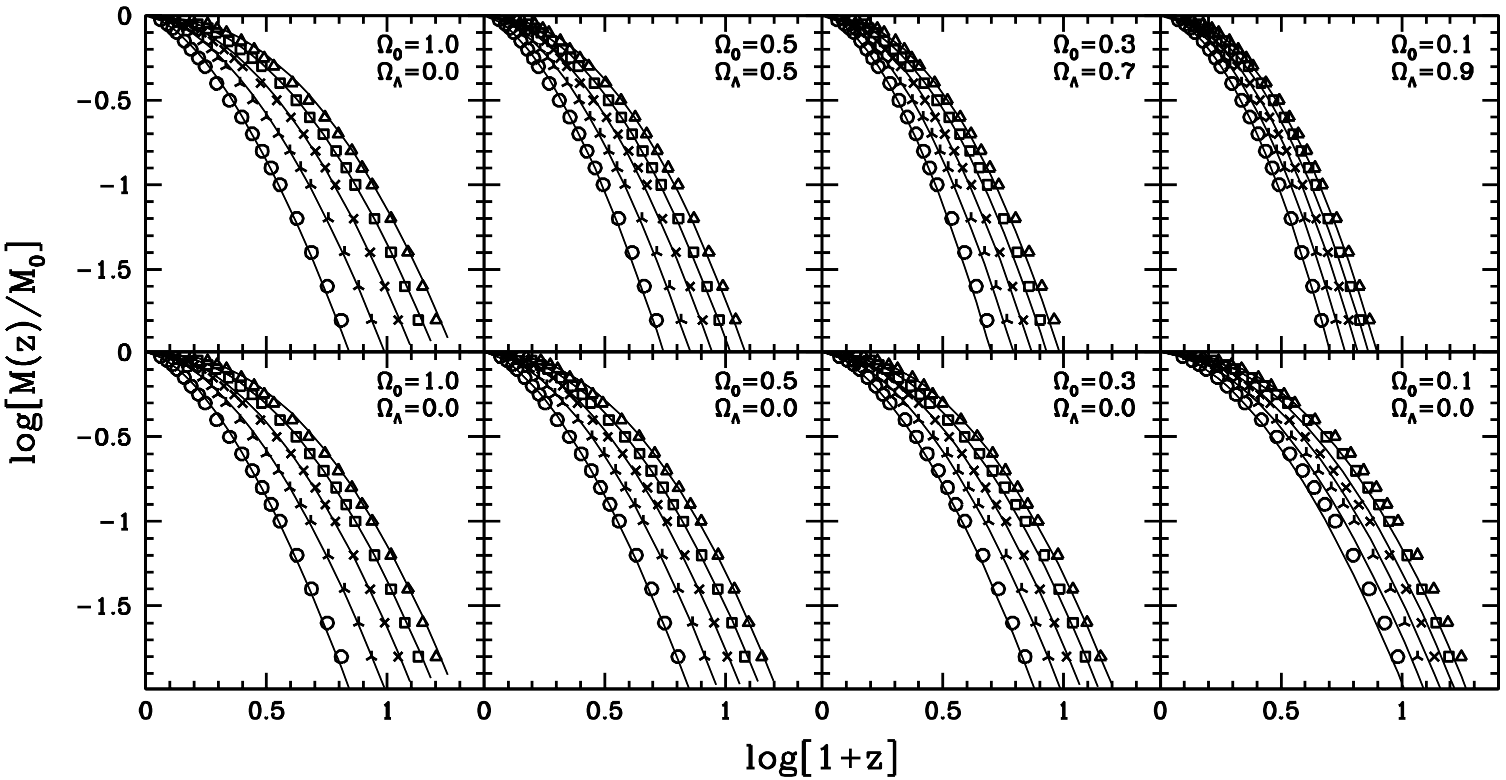
This is inconsistent with what is required to match the LF!!!

More general; matching the slope of the TF relation requires $\frac{V_{\rm obs}}{V_{\rm vir}} \propto \left(\frac{M}{L}\right)^{-1/3}$

Colors & Morphologies

Galaxies reveal color-magnitude , and morphology-density relations.

In CDM cosmologies, small mass haloes form earlier than more massive haloes. One therefore naively expects more massive haloes to be bluer.



Average mass accretion histories for varies cosmologies (Ω_m and Ω_Λ as indicated; all with $\sigma_8 = 1.0$ and $h = 0.65$). Results are shown for five masses: $5.0 \times 10^9 h^{-1}$ $M_\odot$ (open triangles), $5.0 \times 10^{10} h^{-1}$ $M_\odot$ (open squares), $5.0 \times 10^{11} h^{-1}$ $M_\odot$ (crosses), $5.0 \times 10^{12} h^{-1}$ $M_\odot$ (tripods), $5.0 \times 10^{13} h^{-1}$ $M_\odot$ (open circles). (van den Bosch 2002).

SAM history I

1991 White & Frenk — Cole — Lacey & Silk

First, highly oversimplified SAMs

- First indications of overcooling problem; too many galaxies

$\Rightarrow$ Feedback must play important role

1993 Kauffmann, White & Guiderdoni

SCDM / No dust / No chem. evol. / Improved Merger Trees

- Faint-end slope of LF too steep despite feedback
- Can't match LF and TF simultaneously
- color-magnitude relation inverted

1994 Cole, Aragon-Salamanca, Frenk, Navarro & Zepf

SCDM / No dust / No chem. evol. / Extreme Feedback ($\alpha_{\rm fb} = 5.5$)

- Faint-end slope of LF still too steep
- TF zero-point two magn too faint
- Flat color-magnitude relation; celebrated as big success

SAM history II

1995 Heyl, Cole, Frenk & Navarro

Application of Durham models to low-Ω_m cosmologies

- Lowering Ω_m results in earlier formation; cooling continues relatively longer; makes galaxies brighter & bluer $\Rightarrow$ Problems at bright end of LF
- Lowering Ω_m results in fewer halos; fitting LF yields lower mass-to-light ratios $\Rightarrow$ Better match to TF zero-point.

1996 Kauffmann — Baugh, Cole & Frenk

More detailed investigation of ages & colors.

- Reasonable match to color-morphology relation
- Models match scatter but not slope of col-mag rel.

$\Rightarrow$ Col-Mag-Rel is metallicity rather than age effect

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1998 Kauffmann & Charlot

SCDM — chem. evol. — new stel. pop. models — No dust

- Models can be found that match col-mag relation but only with extreme feedback and unrealistically high yield

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SAM history III

1999 Somerville & Primack

Various cosmologies — dust — retention/rejection feedback

- Dust allows for good fit to bright end of B-band LF, but not of K-band LF
- Faint-end slope of LF well-fit for ΛCDM$_{0.3}$
- TF & LF fit simultaneously for ΛCDM$_{0.3}$ if $V_{\rm rot} = V_{\rm vir}$
- No (detailed) discussion on color-magnitude relation

2000 Cole, Lacey, Baugh & Frenk

ΛCDM$_{0.3}$ with $\Omega_b = 0.02$ — dust — chem. evol. — β-profile

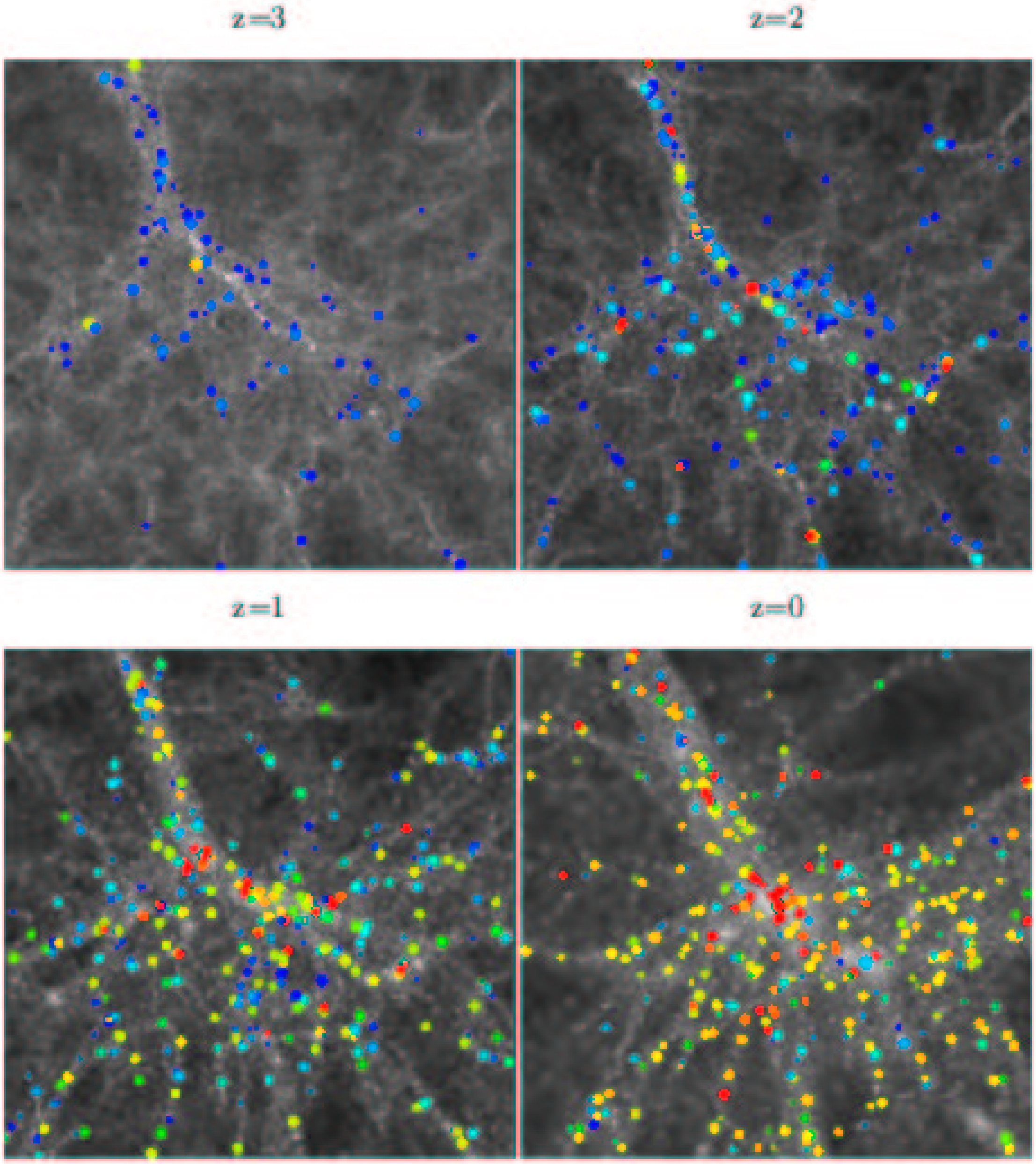
- Good fit to entire LFs in B and K (dust $+$ β-profile $+$ $\Omega_b = 0.02$)
- TF zero-point still no match ($V_{\rm rot} \neq V_{\rm vir}$ too large)
- Col-Mag relation poor; bright galaxies too blue

2003 Benson, Bower, Frenk, Lacey, Baugh & Cole

ΛCDM$_{0.3}$ with $\Omega_b = 0.04$ — dust — chem. evol. — β-profile

- Bright-end of LF can no longer be fit; overcooling is back
  Good fit requires (unrealistic amount of) conduction and/or superwinds
- TF zero-point still too faint by ~ 0.5 mag

The Galaxy-Dark Matter Connection



from Kauffmann, Colberg, Diaferio & White 1999

Large Scale Structure: Theory

Observations yield $\xi(r_p, \pi)$ with r_p and π the pair separations perpendicular and parallel to the line-of-sight.

redshift space CF: $\xi(s)$ with $s = \sqrt{r_p^2 + \pi^2}$.

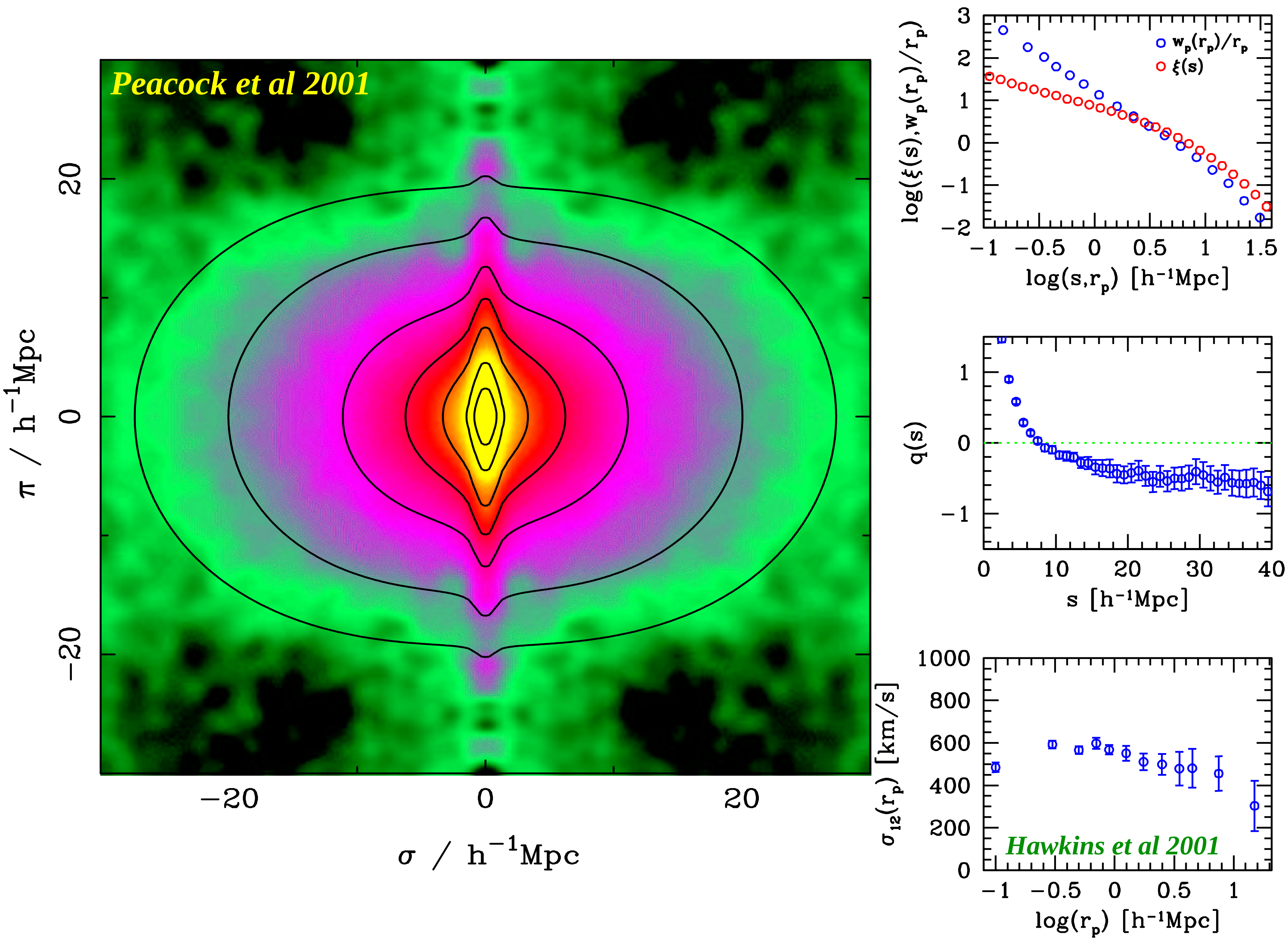
projected CF: $w_p(r_p) = \int_{-\infty}^{\infty} \xi(r_p, \pi) \mathrm{d}\pi = 2 \int_{r_p}^{\infty} \xi(r) \frac{r \, \mathrm{d}r}{\sqrt{r^2 - r_p^2}}$

Peculiar velocities cause anisotropy of $\xi(r_p, \pi)$ and differences between $\xi(s)$ and $\xi(r)$.

Anisotropy of $\xi(r_p, \pi)$ is quantified by quadrupole-to-monopole ratio denoted by $q(s)$.

- **Large Scales: Infall (“Kaiser Effect”); boosts $\xi(s)$ w.r.t. $\xi(r)$. $q(s)$ is negative and a measure of $\beta \equiv \Omega_m^{0.6}/b$.**
- **Small Scales: Virialized motion (“Finger-of-God”); suppresses $\xi(s)$ w.r.t. $\xi(r)$. $q(s)$ is positive and a measure for the pairwise velocity dispersions (PVDs) denoted by σ_{12}.**

Large Scale Structure: The 2dFGRS

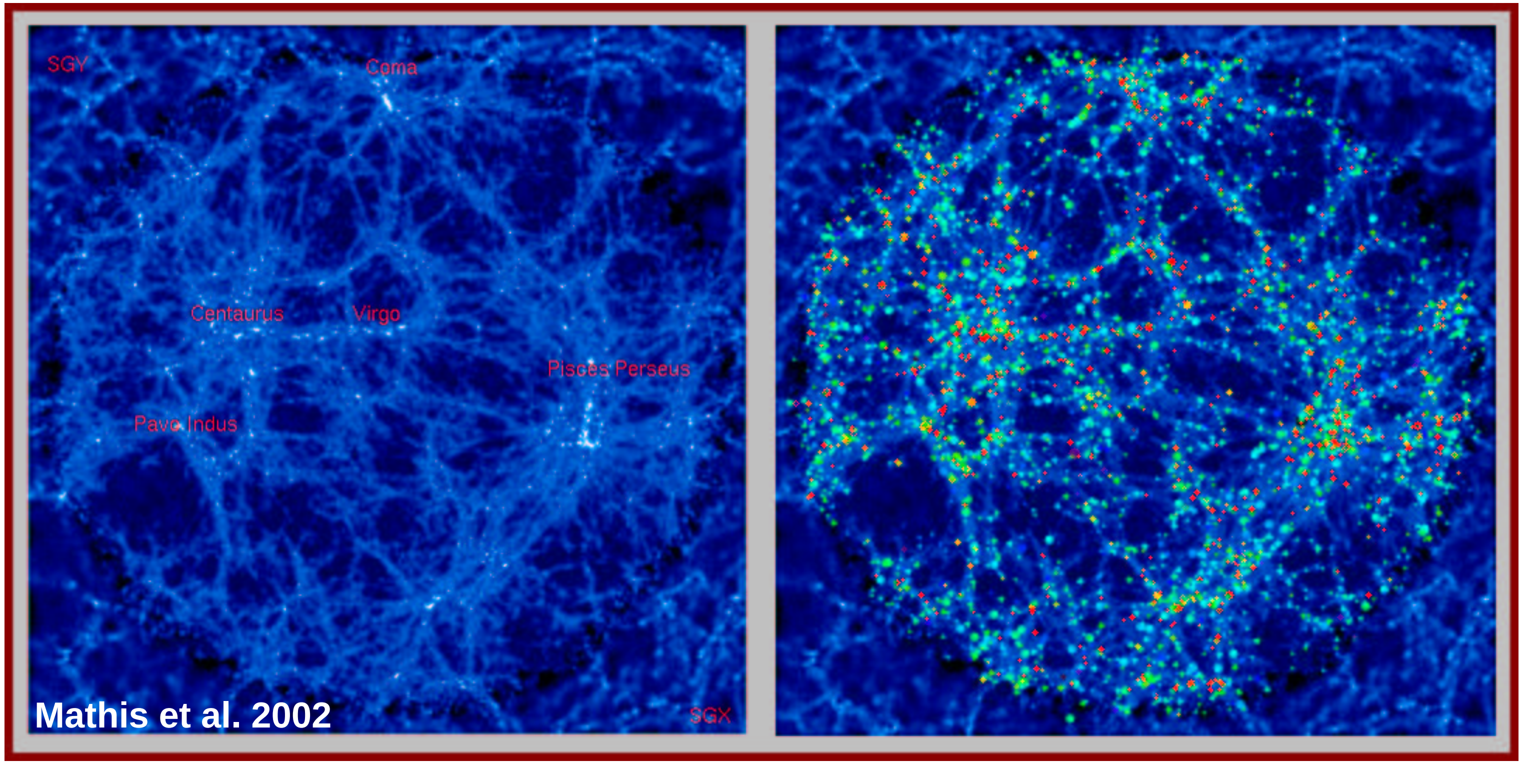


Hybrid SAM/N-BODY Models

Recently, SAMs have been combined with N-body simulations. This allows LSS studies (Kauffmann et al. 1999; Benson et al. 2000a,b), and allows investigations of the spatial distribution of galaxies in clusters (Springel et al. 2001) and in the local environment (Mathis et al. 2002).

Either merger histories are obtained directly from the N-body simulation (Munich & Paris), or they are computed from EPS theory and the simulation is only used to locate the galaxies (Durham).

(Trade-off between Self-consistency & Resolution Effects)



SAM history IV

1999 Kauffmann, Colberg, Diaferio & White

ΛCDM$_{0.3}$ with $\Omega_b = 0.05$ — N-body merger histories

- Extreme Resolution Effects: only $L > L^*$.
- Very bad fit to LF; too many bright galaxies, too few L^* galaxies
- $\xi(r)$ poorly fit & PVD of galaxies only marginally smaller than for DM particles

2000 Benson, Cole, Frenk, Baugh & Lacey

ΛCDM$_{0.3}$ with $\Omega_b = 0.02$ — EPS merger histories

- Same $L_{\rm box} = 141 h^{-1}\,{\rm Mpc}$ simulation as Kauffmann et al. (1999)
  But better resolution: $L \gtrsim 0.1 L^*$
- Good fits to LFs & TF zero-point
- Good fits to $\xi(r)$ and PVDs
- Baryonic mass fraction too low!!

SAM's Level of Success for matching LSS observations remains unclear

Halo Occupation Numbers

PARADIGM: **Galaxies live in extended Cold Dark Matter Haloes.**

QUESTION: **What Galaxy lives in What Halo?**

- **How many galaxies, on average, per halo?**
- **How does $\langle N \rangle$ depend on M?**
- **How does $\langle N \rangle$ depend on L?**
- **What is $\langle L \rangle (M)$?**

The answers to these questions hold important information regarding

- **Galaxy Formation (cooling/starformation/feedback)**
- **Large Scale Structure (galaxy bias)**
- **Cosmology (Halo mass function/CDM distribution)**

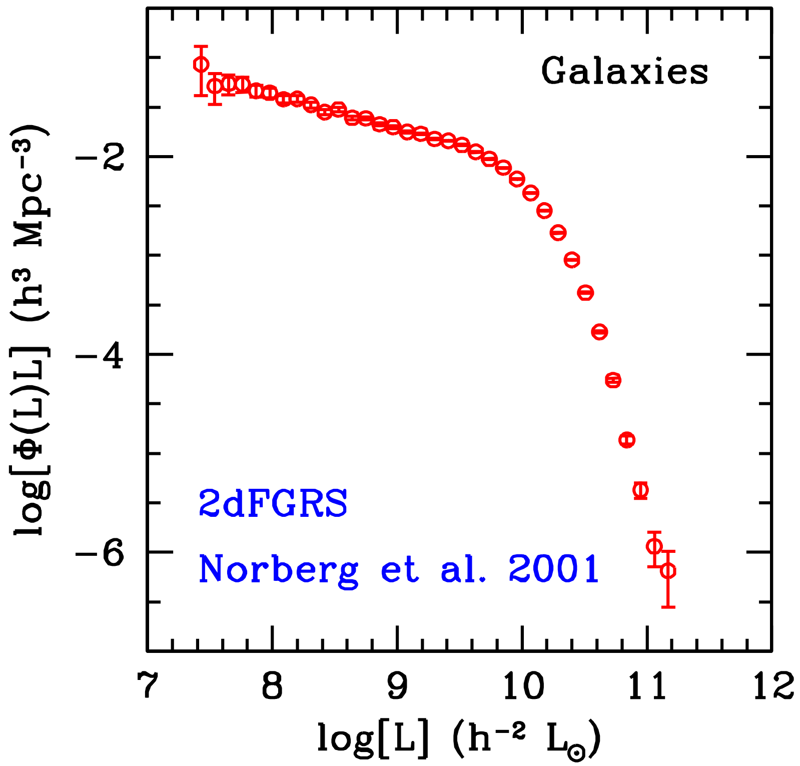
The galaxy-dark matter connection can be studied

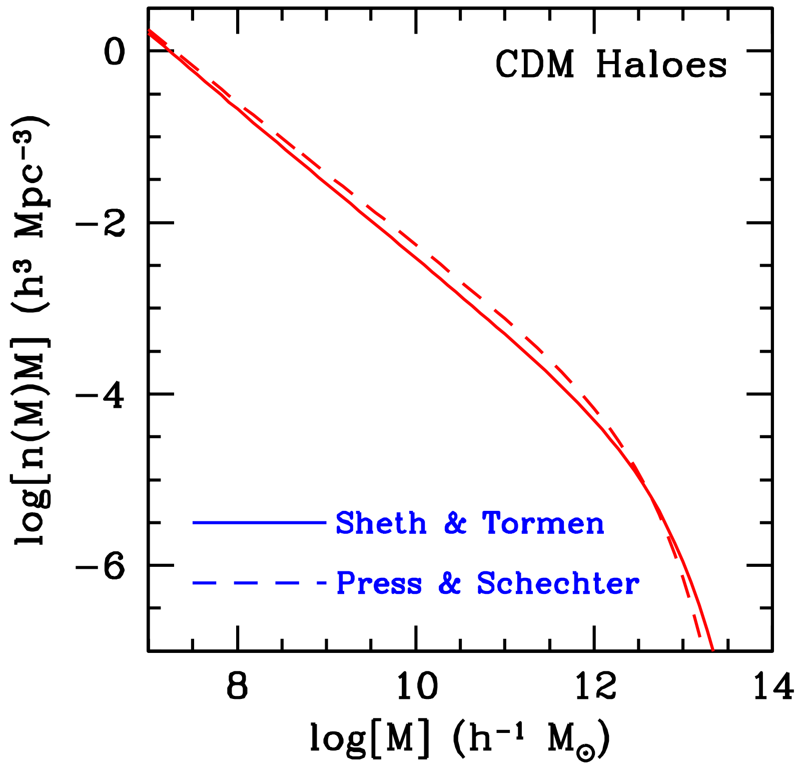
Physically: Ab initio galaxy formation models (SAMs)

Statistically: The Conditional Luminosity Function (CLF)

Lighting-Up the Dark Matter

$\Phi(L)\mathrm{d}L =$ comoving number density of galaxies with luminosities in the range L, $L + \mathrm{d}L$.





We use the Conditional Luminosity Function to link the distributions of galaxies and CDM haloes

$\Phi(L|M)\mathrm{d}L =$ average number of galaxies with luminosities in the range L, $L + \mathrm{d}L$ that 'live' in haloes of mass M.

The Conditional Luminosity Function

The luminosity function:

$$\Phi(L) = \int_0^\infty \Phi(L|M)\, n(M)\, \mathrm{d}M$$

The average luminosity in a halo of mass M:

$$\langle L \rangle(M) = \int_0^\infty \Phi(L|M)\, L\, \mathrm{d}L$$

The average number of galaxies in a halo of mass M with $L > L_1$:

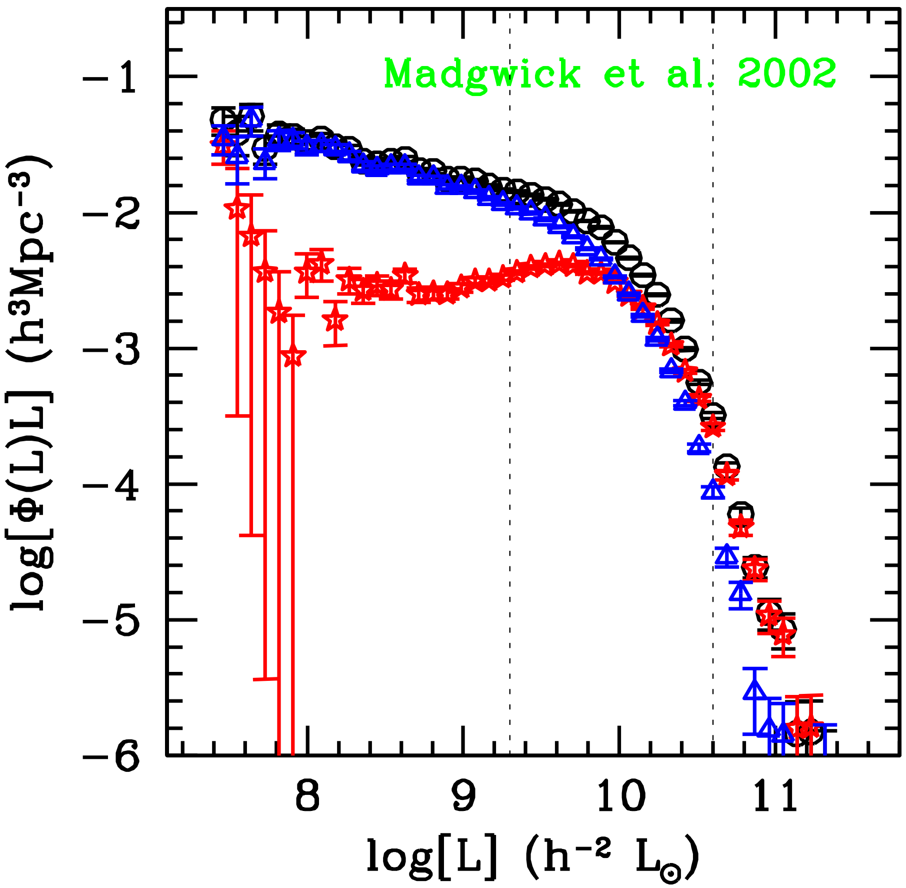
$$N_M(L > L_1) = \int_{L_1}^\infty \Phi(L|M)\, \mathrm{d}L$$

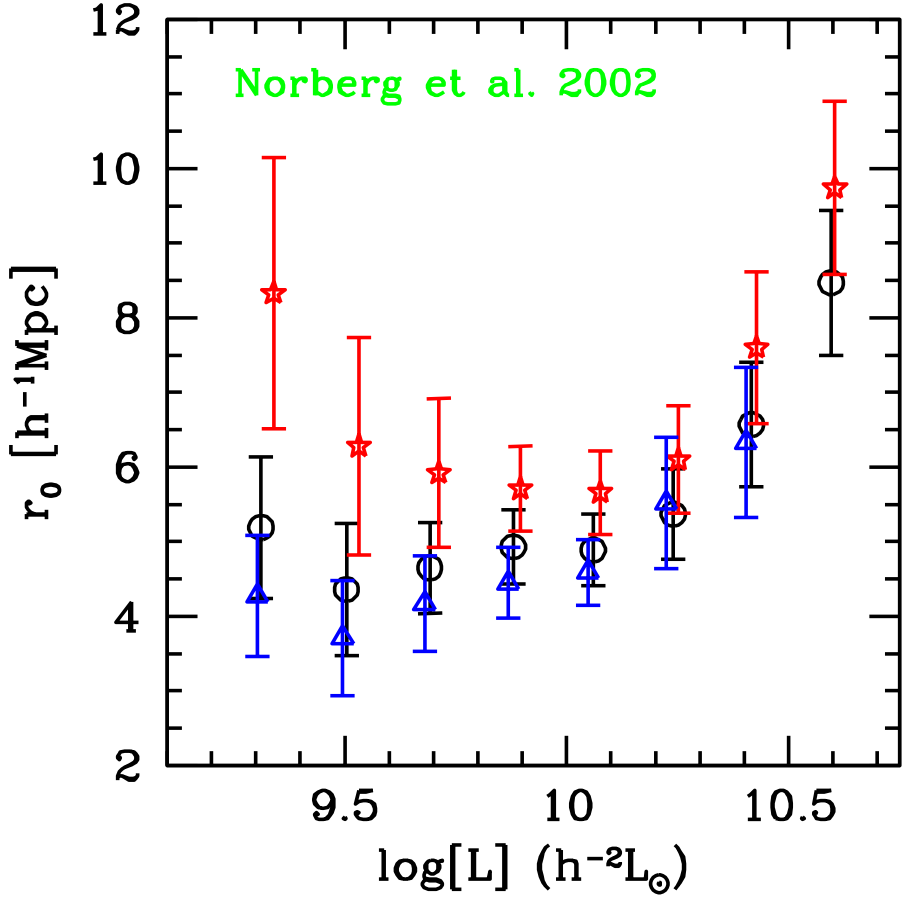
The conditional LFs of late- and early-type galaxies:

$$\Phi_l(L|M) = f_l(L, M)\, \Phi(L|M)$$
$$\Phi_e(L|M) = [1 - f_l(L, M)]\, \Phi(L|M)$$

The conditional LF is the ideal statistical ‘tool’ to link the distributions of dark matter haloes and galaxies.

Luminosity & Correlation Functions





- On average, early-type galaxies are more luminous and more strongly clustered than late-type galaxies.
- In general, more luminous galaxies are more strongly clustered.

REMINDER: Correlation length r_0 defined by $\xi(r_0) = 1$

The Galaxy Correlation Function

The two-point galaxy-galaxy correlation function can be split in a 1-halo and a 2-halo term

$$\xi_{\rm gg}(r) = \xi^{\rm 1h}_{\rm gg}(r) + \xi^{2h}_{\rm gg}(r) = \xi^{\rm 1h}_{\rm gg}(r) + \bar{b}^2 \xi^{\rm 2h}_{\rm hh}(r)$$

Here $\bar{b}$ and $\xi^{2h}_{\rm hh}(r)$ are computed as follows:

- $\bar{b} = \frac{1}{\bar{n}_g} \int_0^\infty n(M)\, \langle N(M) \rangle\, b(M)\, {\rm d}M$ (Berlind & Weinberg 2002)
- $\langle N(M) \rangle = \int_{L_1}^{L_2} \Phi(L|M)\, {\rm d}L$
- $\xi^{2h}_{\rm hh}(r; M_1, M_2) = b(M_1)\, b(M_2)\, \xi^{\rm 2h}_{\rm dm}(r)$ (Mo & White 1996)
- $\xi^{\rm 2h}_{\rm dm}(r) = \xi_{\rm dm}(r) - \xi^{1h}_{\rm dm}(r)$ (Ma & Fry 2000)
- $\xi_{\rm dm}(r) = \int_0^\infty \Delta^2_{\rm nl}(k)\, \frac{\sin(kr)}{kr}\, \frac{{\rm d}k}{k}$

NOTE: $\xi^{\rm 1h}_{\rm gg}(r)$ can be ignored at large r

The Model

- The LFs of clusters are well fit by a Schechter function
- The LF of all field galaxies has a Schechter form
- The halo mass function has a Press-Schechter form

We therefore assume that the CLF also has the Schechter form:

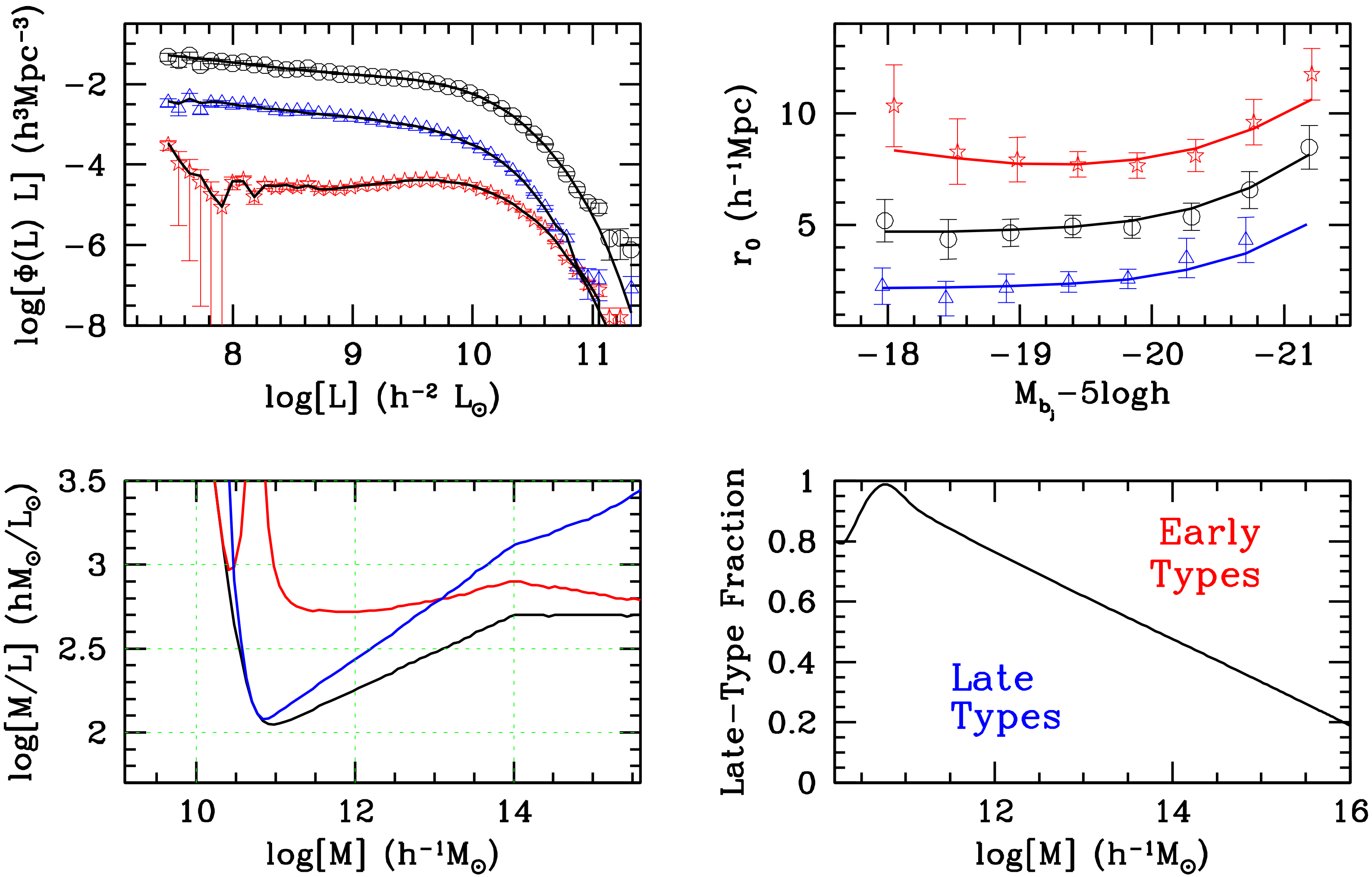
$$\Phi(L|M)\mathrm{d}L = \frac{\tilde{\Phi}^*}{\tilde{L}^*} \left(\frac{L}{\tilde{L}^*}\right)^{\tilde{\alpha}} \exp(-L/\tilde{L}^*)\,\mathrm{d}L$$

Here $\tilde{\Phi}^*$, $\tilde{L}^*$ and $\tilde{\alpha}$ all depend on M.

- Parameterize $\tilde{\Phi}^*$, $\tilde{L}^*$ and $\tilde{\alpha}$
- Find parameters that best fit $\Phi(L)$ and $r_0(L)$
- Let $f_l(L,M) = g(L)h(M)$ and constrain $g(L)$ such that the model reproduces $\Phi_l(L)/\Phi(L)$.
- Find the $h(M)$ that best reproduces the $r_0(L)$ of late- and early-type galaxies.

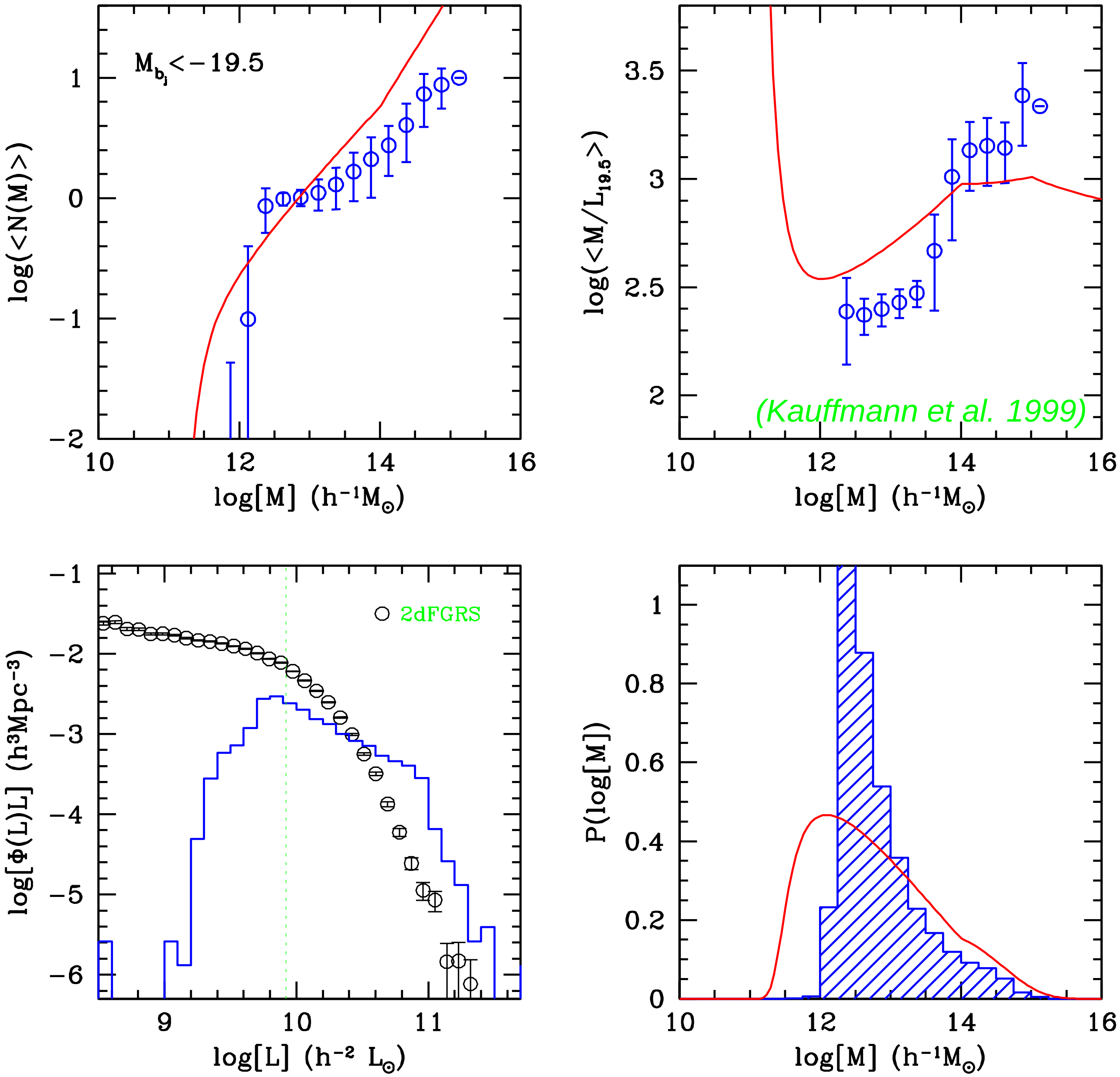
The Concordance Cosmology

$\Omega_m = 0.3; \Omega_\Lambda = 0.7, h = 0.7, \sigma_8 = 0.9, n = 1.0$



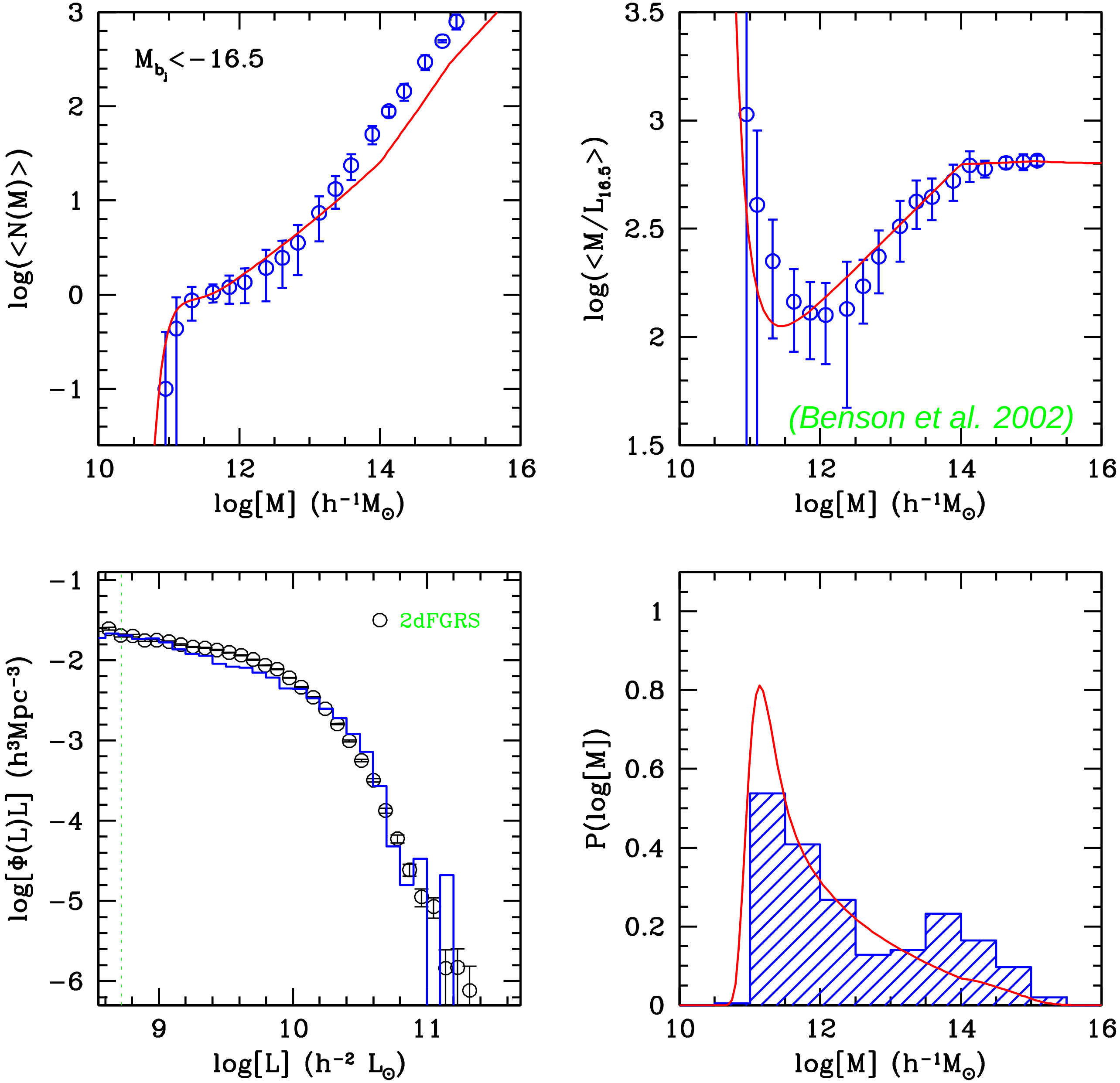
Concordance model fits $\Phi(L)$ and $r_0(L)$ of both early- and late-type galaxies.

Semi-Analytical Models I



Poor agreement with CLF; but SAM doesn't fit LF

Semi-Analytical Models II



Good agreement with SAMs that fit LF

The Issue of Galaxy Bias

CDM distribution	Galaxy distribution
$\delta_{\rm DM}(\vec{x}) = \frac{\rho(\vec{x}) - \bar{\rho}}{\bar{\rho}}$	$\delta_{\rm gal}(\vec{x}) = \frac{n_{\rm gal}(\vec{x}) - \bar{n}_{\rm gal}}{\bar{n}_{\rm gal}}$
(cosmology dependent)	(observable)

Unknown bias $b(\vec{x}) = \delta_{\rm gal}(\vec{x})/\delta_{\rm DM}(\vec{x})$, which is an imprint of various galaxy formation processes, leads to

- **Degeneracy between β and Ω_m.**
- **Degeneracy between r_0 and σ_8.**

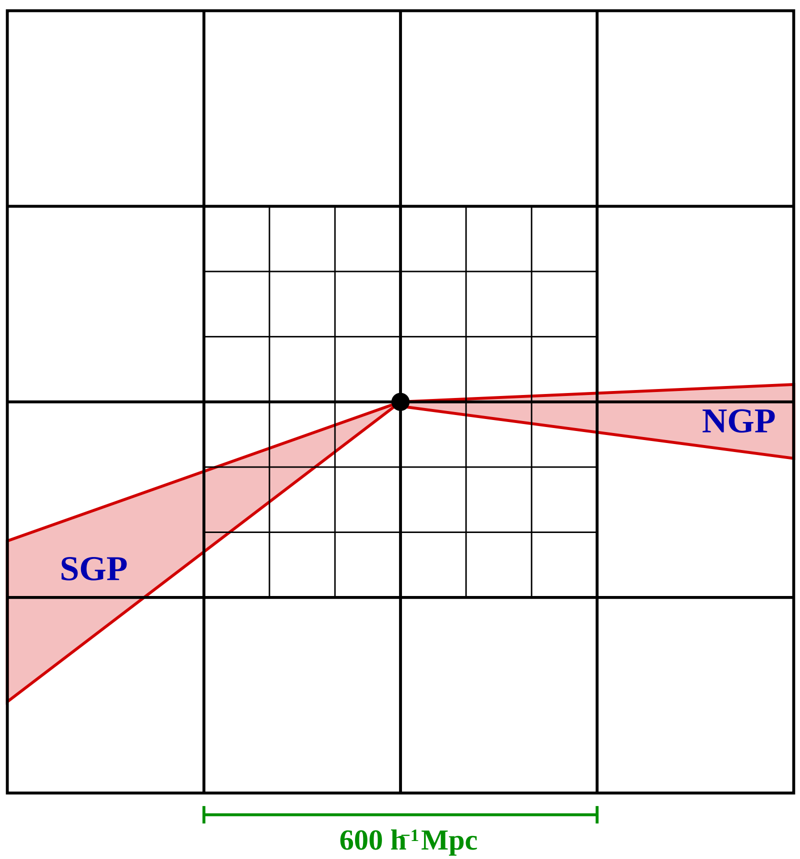
Conditional Luminosity Function provides a statistical description of the bias as function of both luminosity and type.

$$\bar{b}(L) = \frac{1}{\bar{n}_{\rm gal}} \int_0^\infty \Phi(L|M)\, b(M)\, n(M)\, {\rm d}M$$

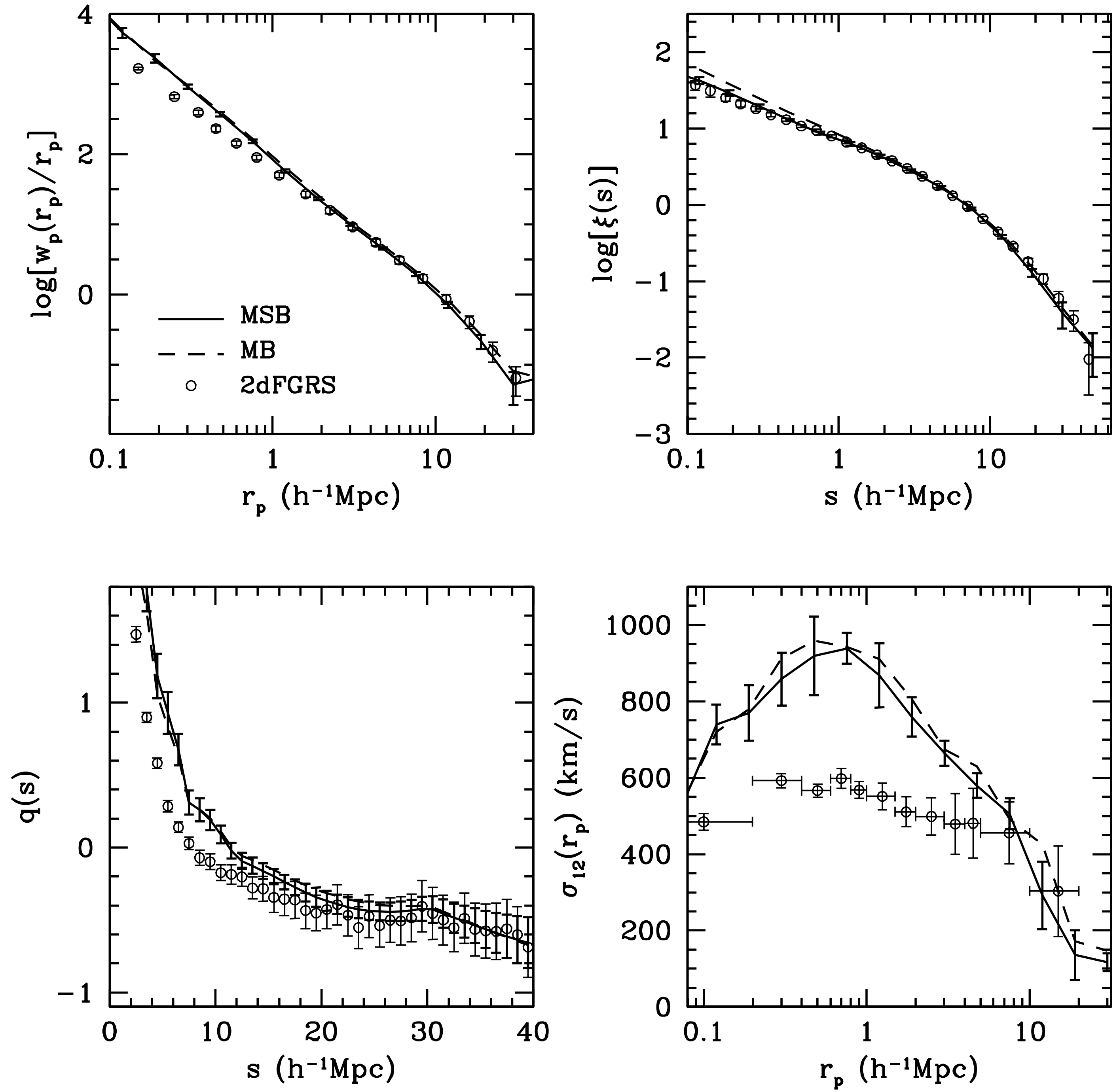
Therefore, the CLF is the ideal tool to study LSS.

Constructing Mock Surveys

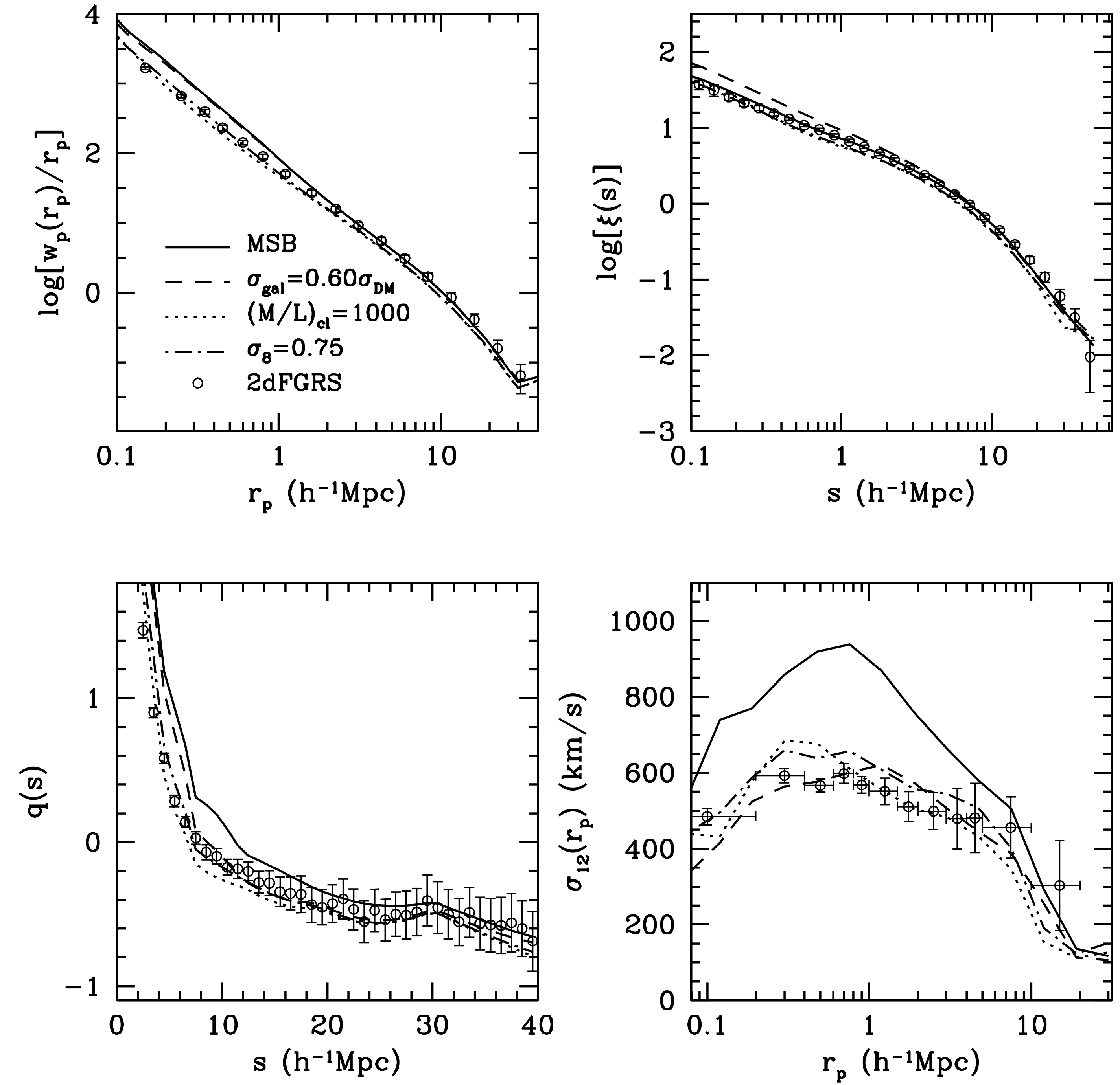
- **Run numerical simulations: ΛCDM concordance cosmology; $L_{\rm box} = 100h^{-1}$ Mpc and $L_{\rm box} = 300h^{-1}$ Mpc with 512^3 CDM particles each.**
- **Identify dark matter haloes (FOF algorithm, $b = 0.2$).**
- **Populate haloes with galaxies using CLF.**
- **Stack boxes to create virtual universe and mimick observations (magnitude limit, completeness, geometry)**



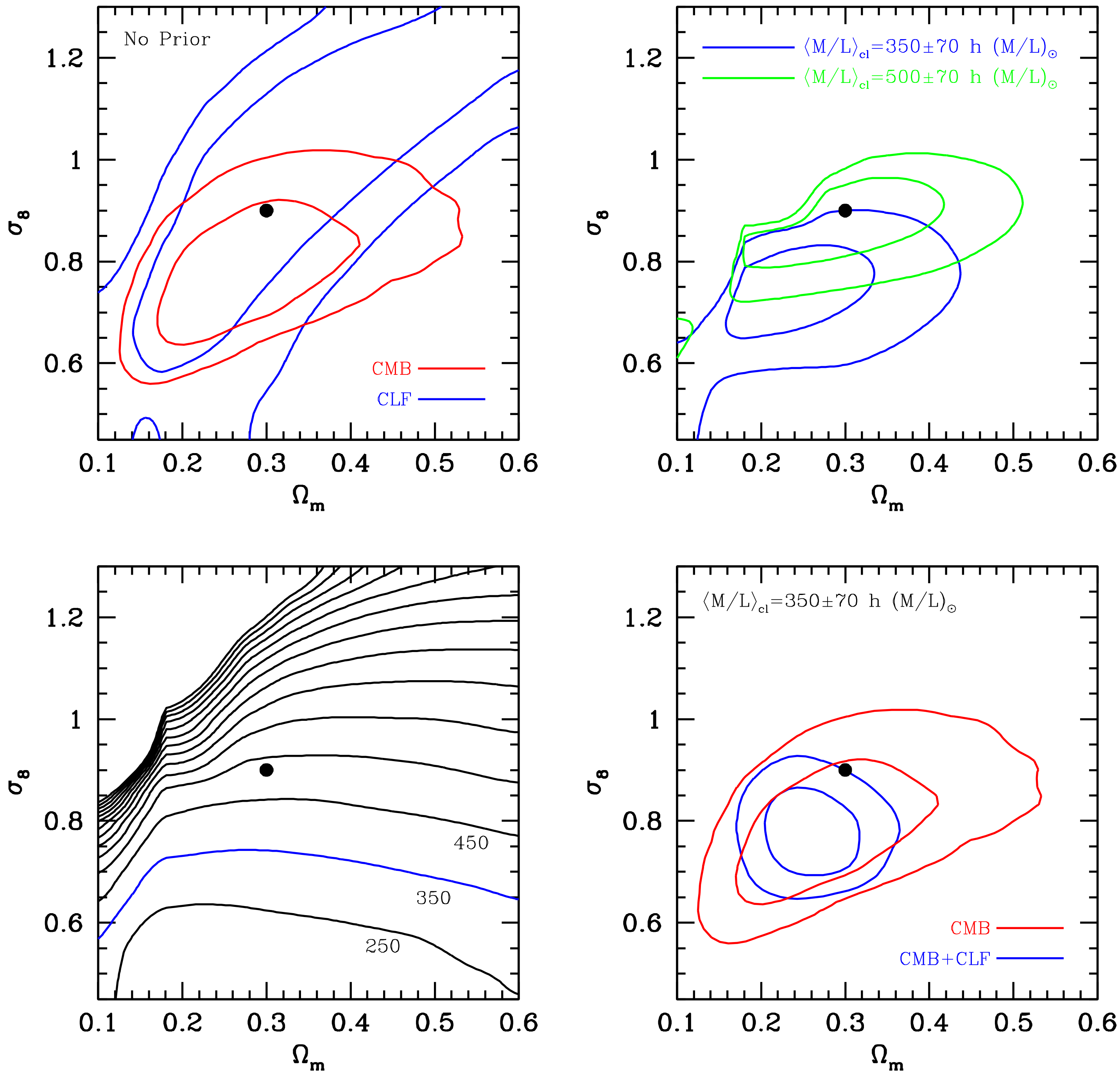
Mock versus 2dFGRS: round 1



Mock versus 2dFGRS: round 2

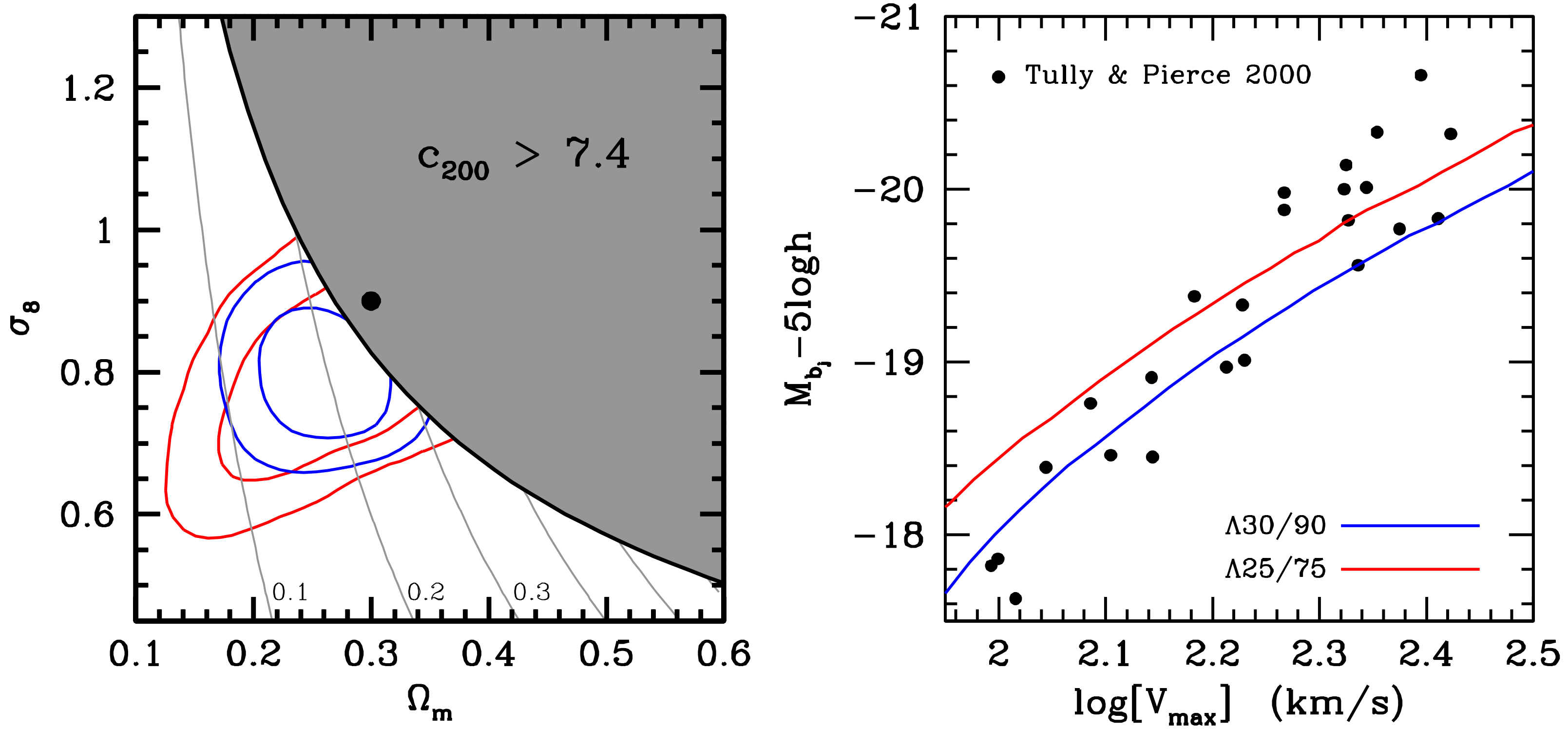


Constraints on Ω_m and σ_8



Concordance on Galactic Scales?

$$\Omega_m = 0.25 \pm 0.04 \qquad \sigma_8 = 0.78 \pm 0.06$$



Cosmologies with lower Ω_m and lower σ_8 yield dark matter haloes that are significantly less concentrated. This

- **Alleviates problem with rotation curves of dwarf and LSB galaxies.**
- **Results in a TF zero-point that is ~ 0.5 magnitudes brighter.**

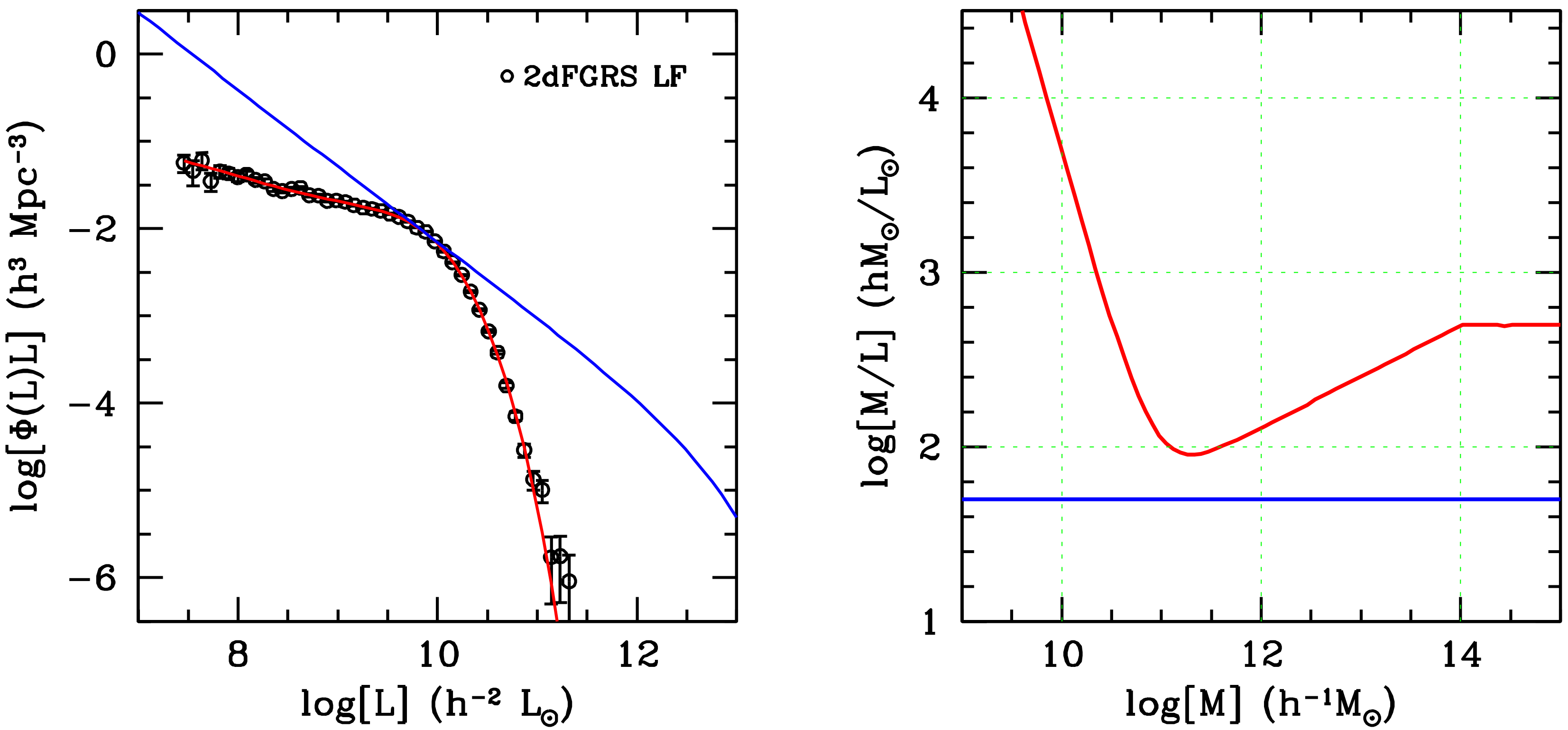
THE END

For questions/comments: contact me at vdbosch@phys.ethz.ch

The LF Challenge

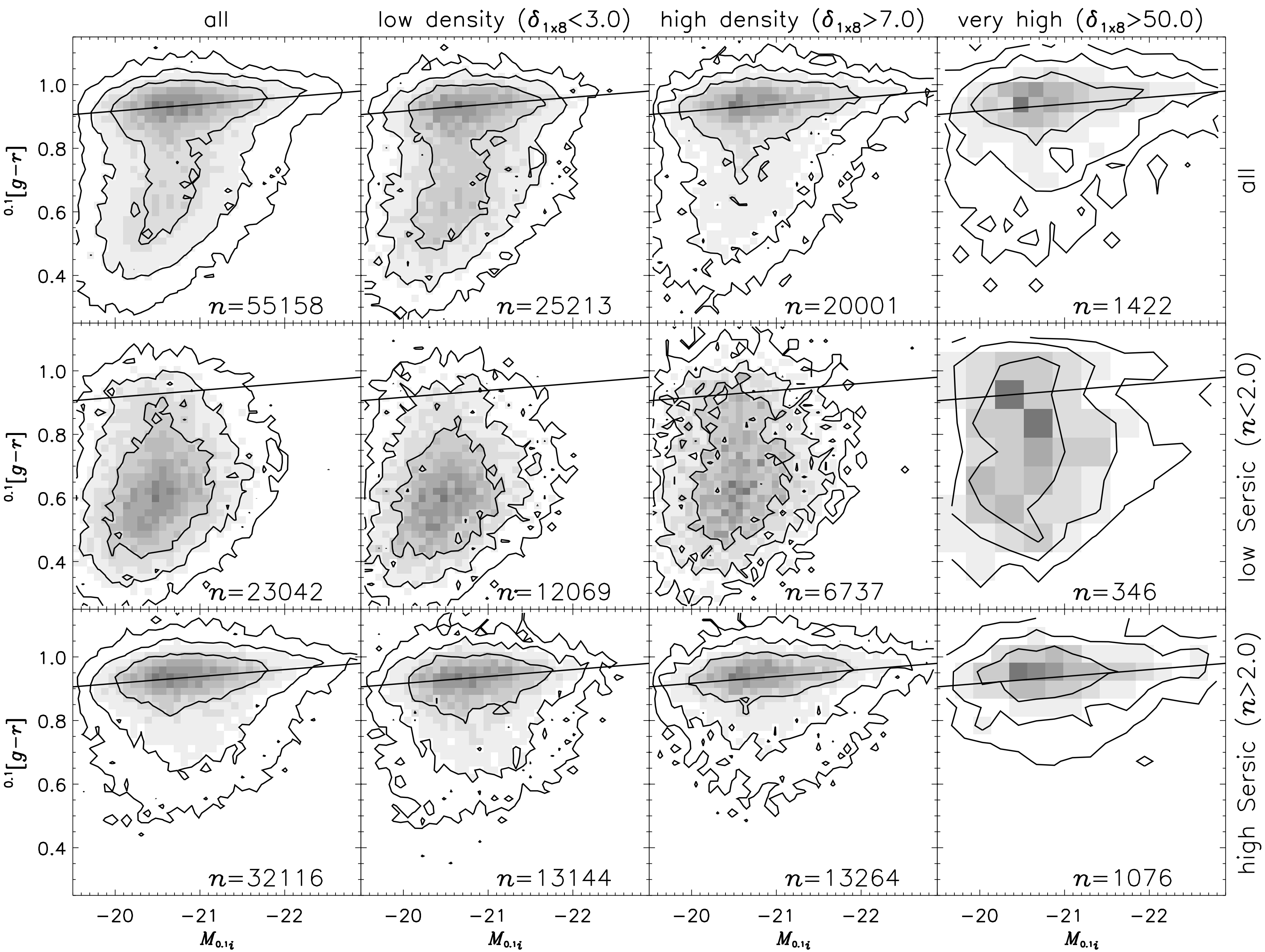
$\Phi(L)\mathrm{d}L =$ comoving number density of galaxies with luminosities in the range L, $L + \mathrm{d}L$.

$n(M)\mathrm{d}M =$ comoving number density of dark matter haloes with masses in the range M, $M + \mathrm{d}M$.



back

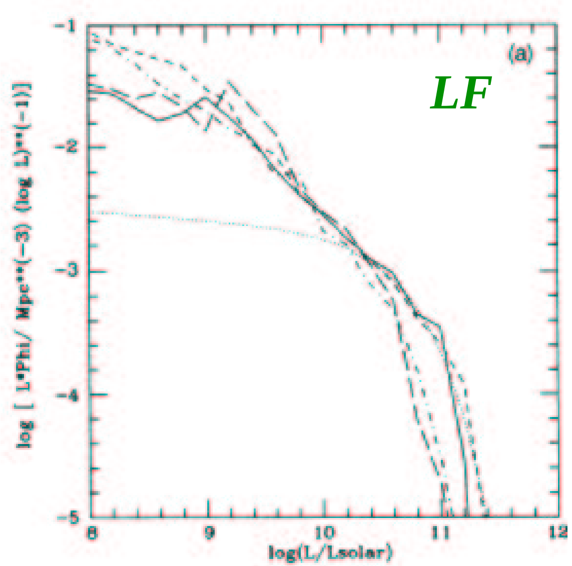
Color-Magnitude Relations from SDSS



Hogg et al. 2003

back

SAM history Ib



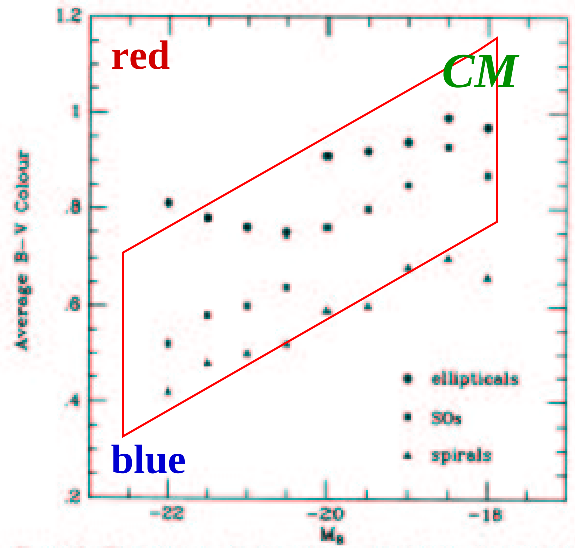


Figure 5. The average $B-V$ colours of galaxies of different morphological type in a dark matter halo of circular velocity 1000 km s^{-1} for the same model as in Fig. 4.

Uncle Sam in Trouble...

First results from Kauffmann, White & Guiderdoni 1993

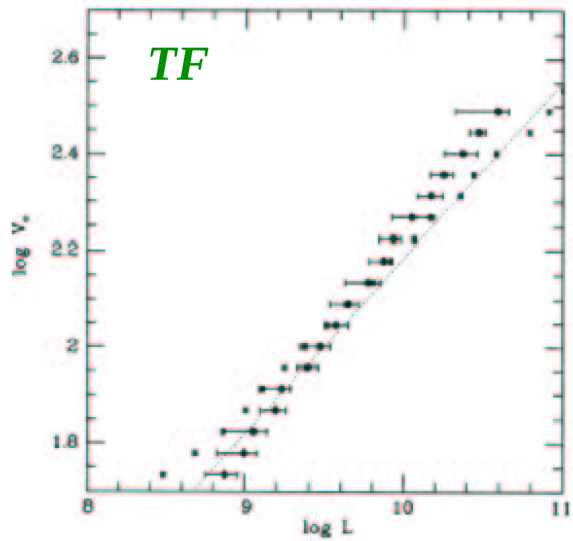
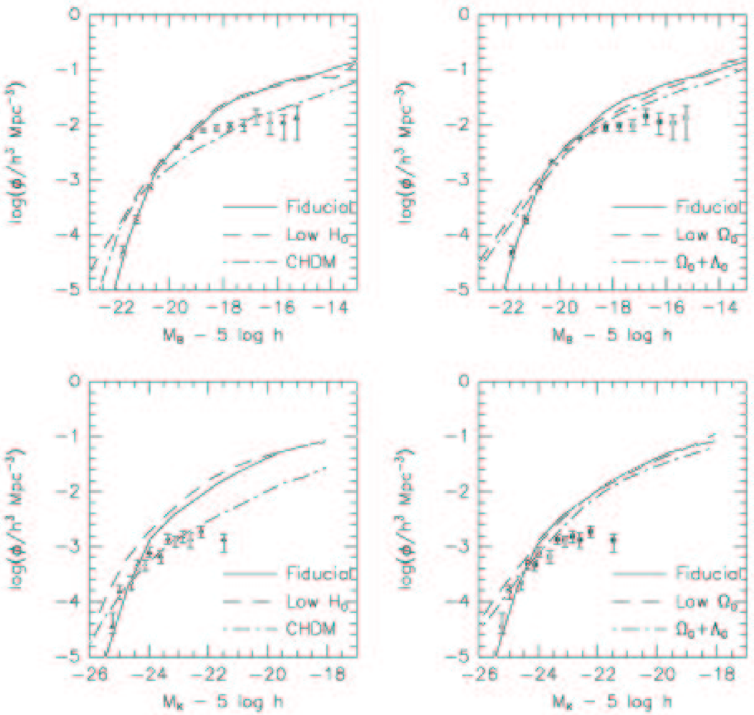
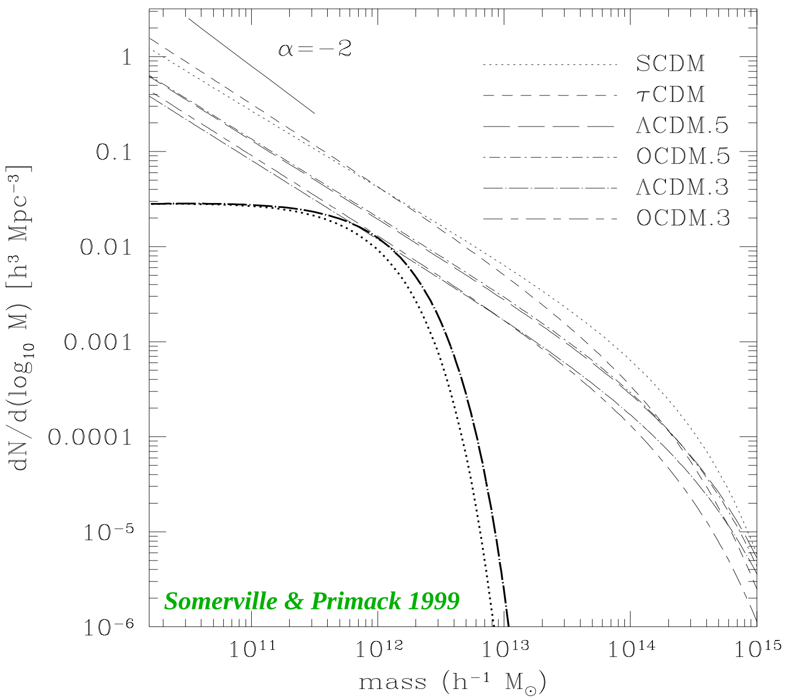


Figure 7. The circular velocities of the haloes of present-day central galaxies versus their B-band luminosities for models with $\Omega_b=0.1$, $f_{merg}=1$ and dwarf suppression (filled circles) and $\Omega_b=0.2$, $f_{merg}=1$ and dwarf suppression (squares). Horizontal error bars show the scatter obtained in 20 realizations of the merging history for haloes of each circular velocity. The dotted line shows the observed 'Tully–Fisher' relation measured by Pierce & Tully (1988) after correction to $H_0=50$ km s^{-1} Mpc^{-1}.

back

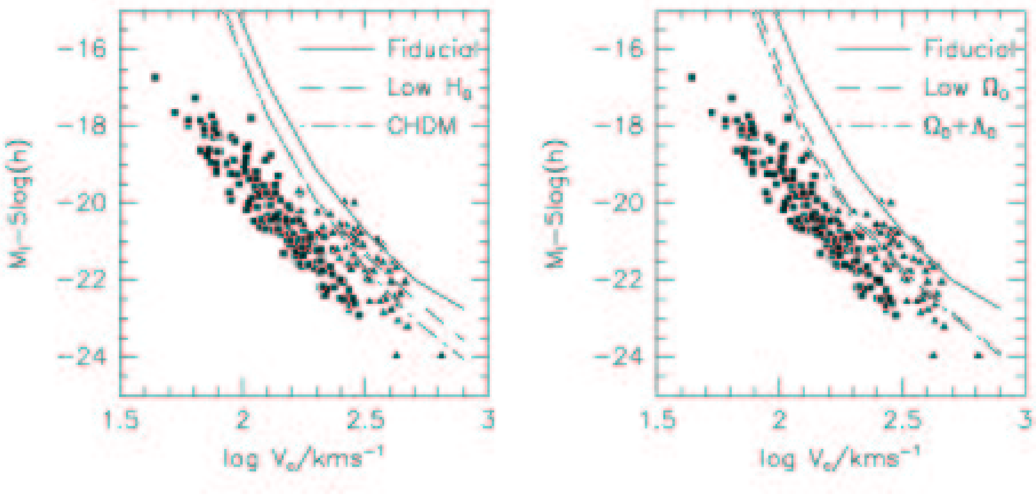
SAM history IIb





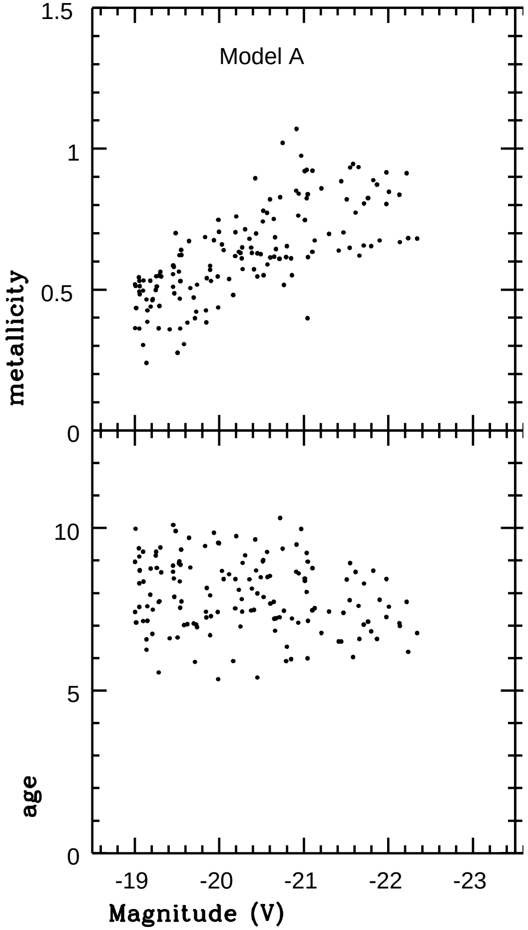
Somerville & Primack 1999

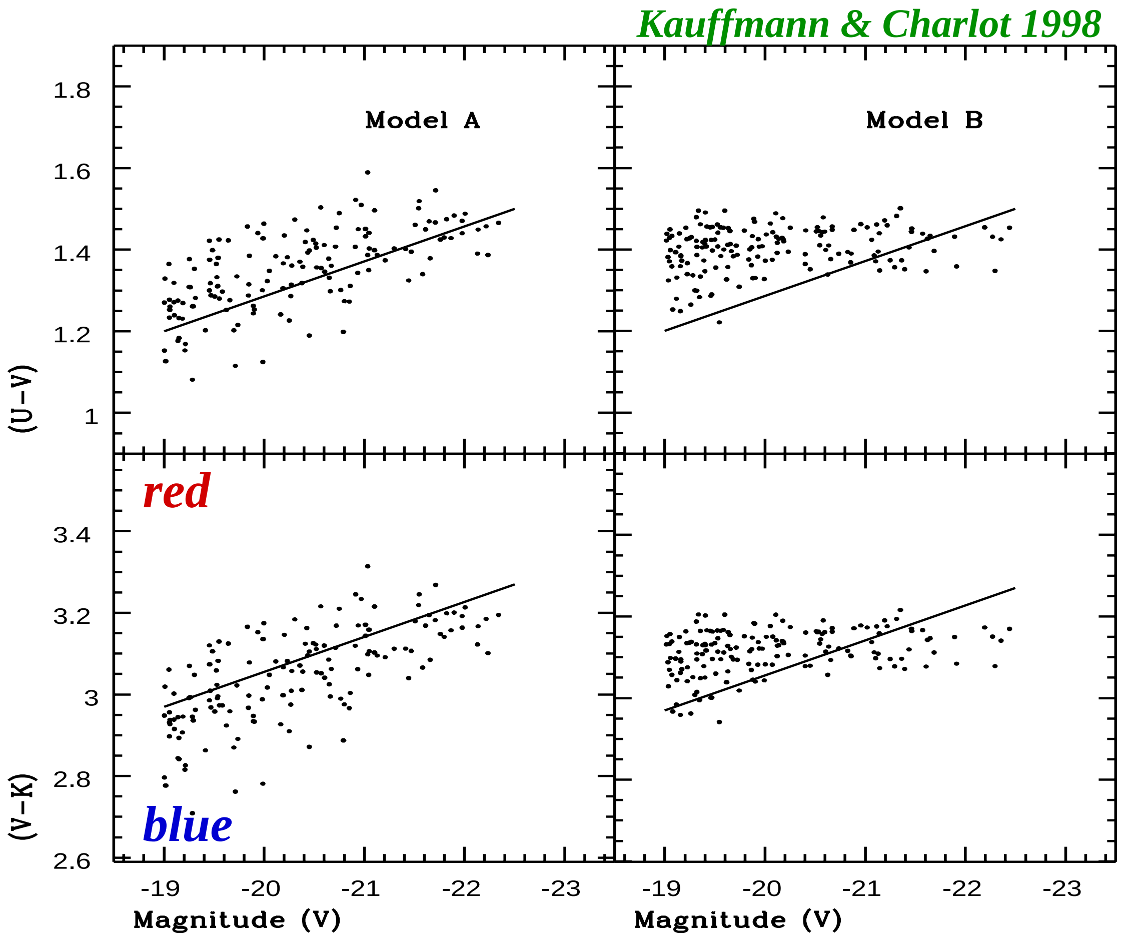
Heyl et al 1995



back

SAM history IIc

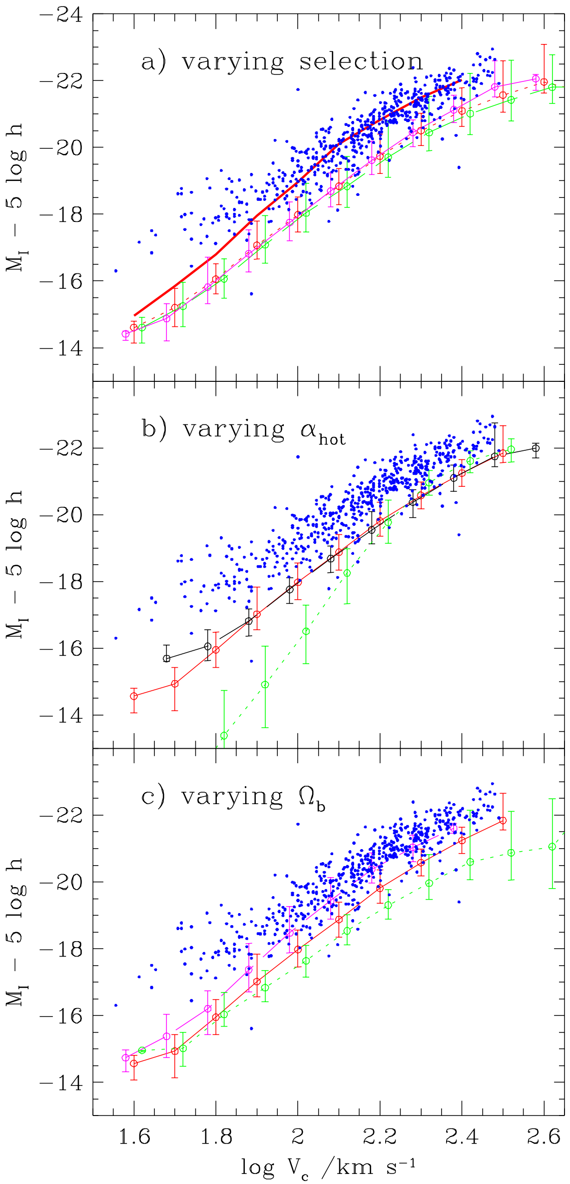




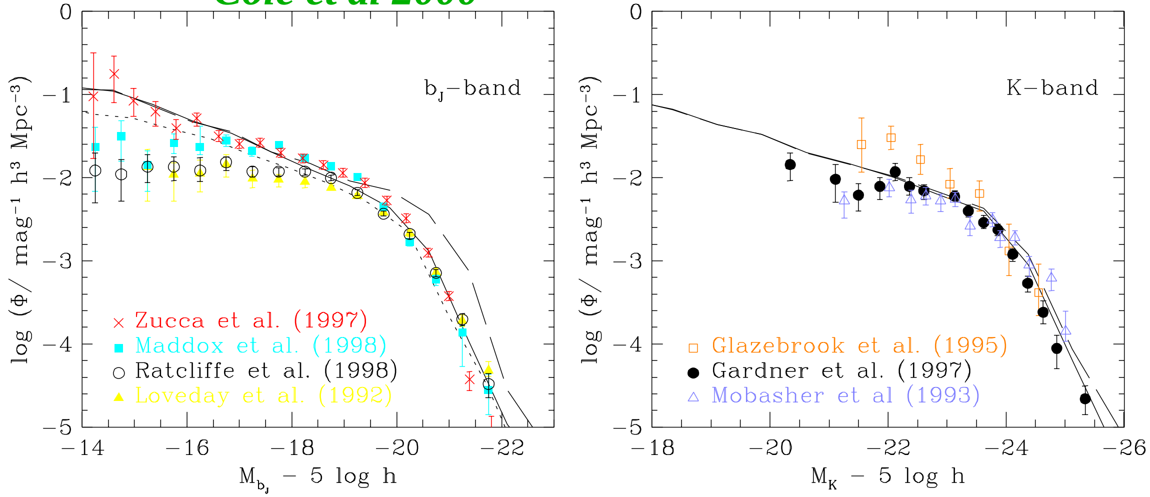
Color−Magnitude relation is a metallicity effect...

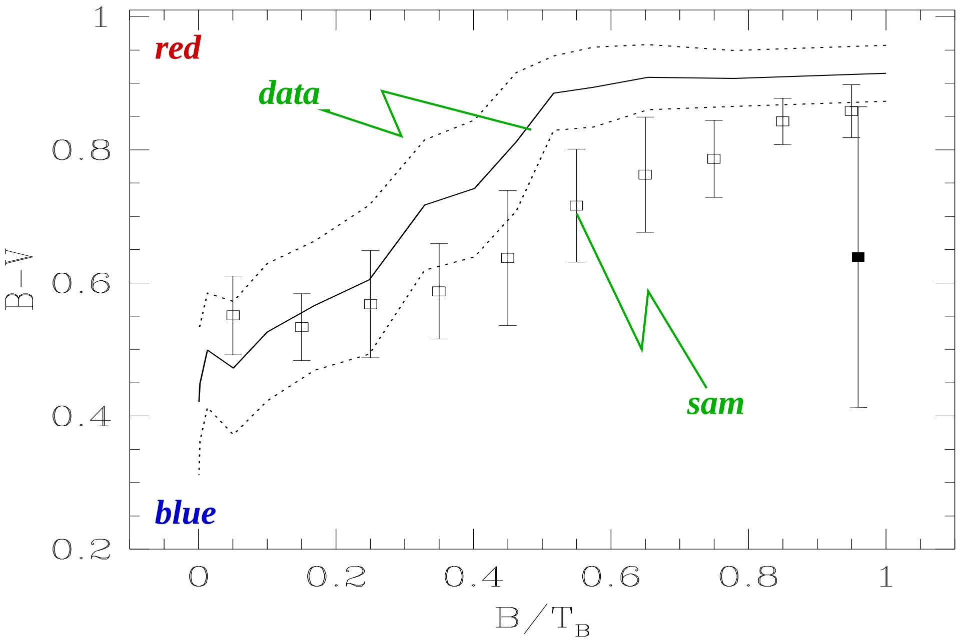
back

SAM history IIIb



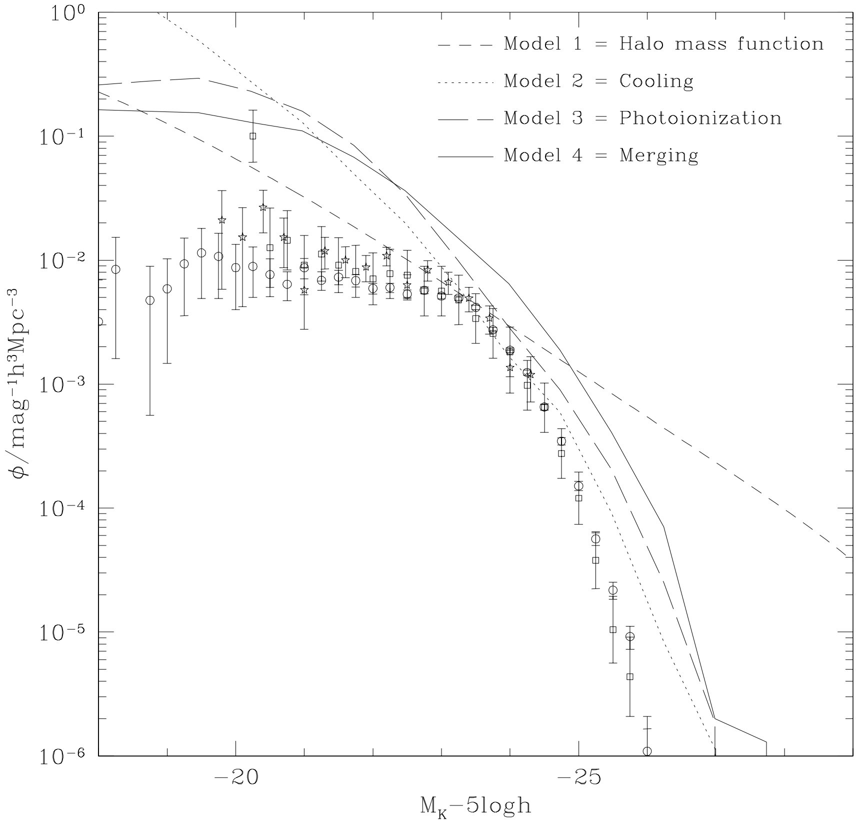
Cole et al 2000

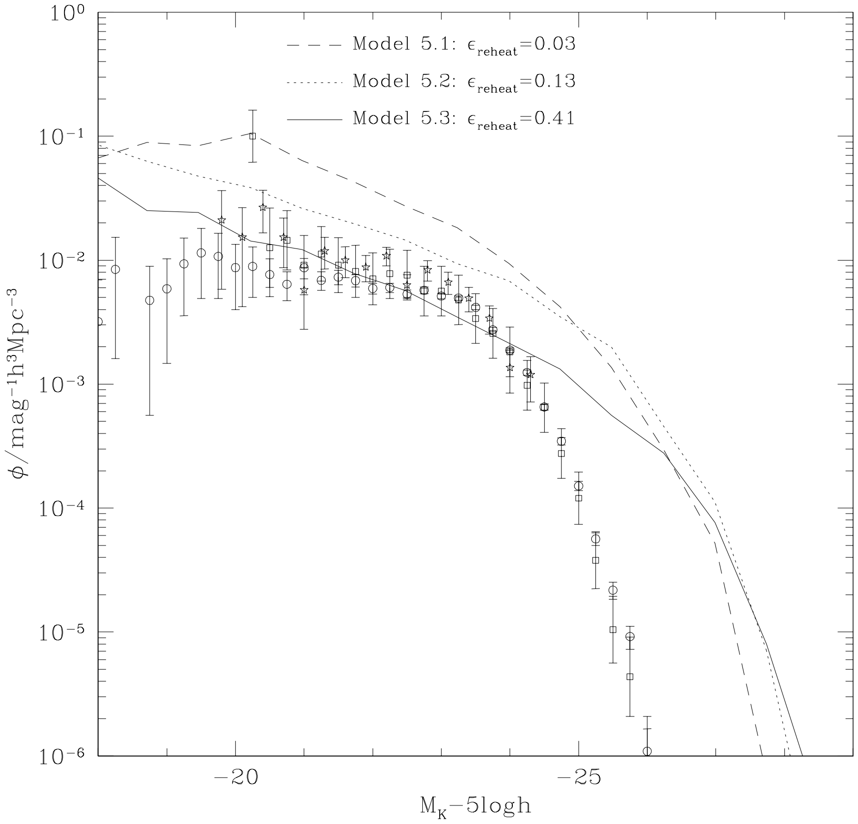




back

Current Status of SAM LFs I





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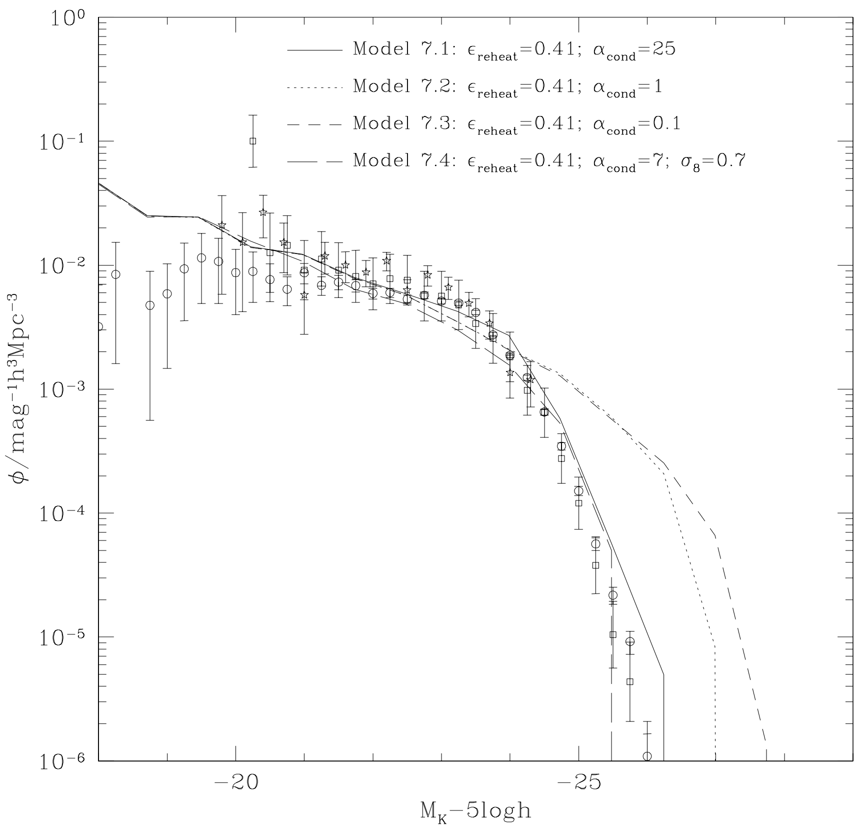
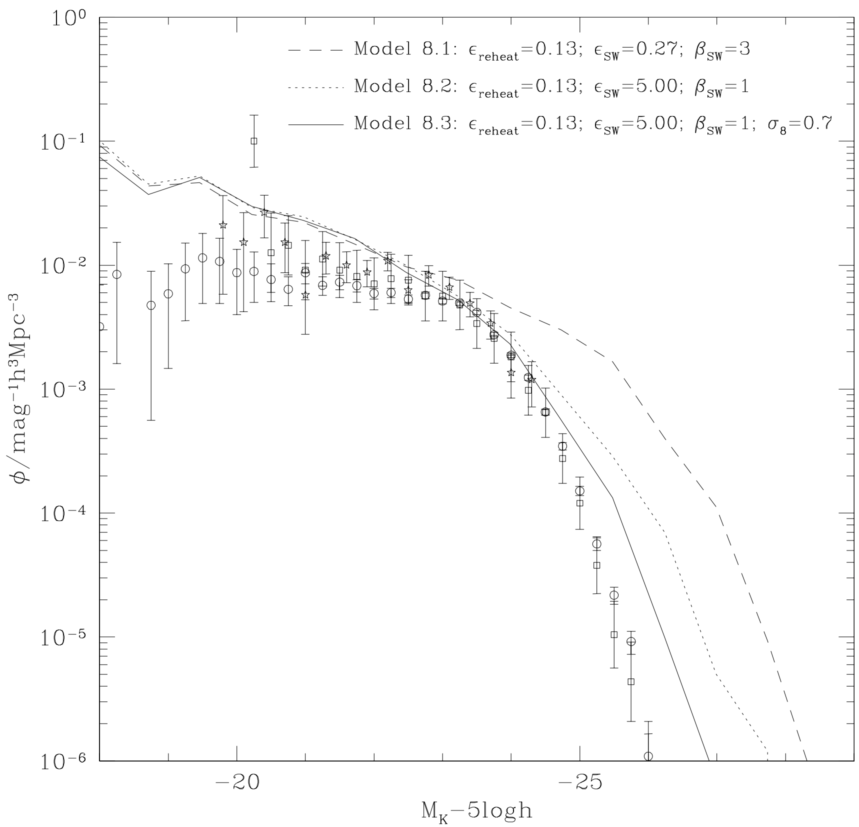
Current Status of SAM LFs II





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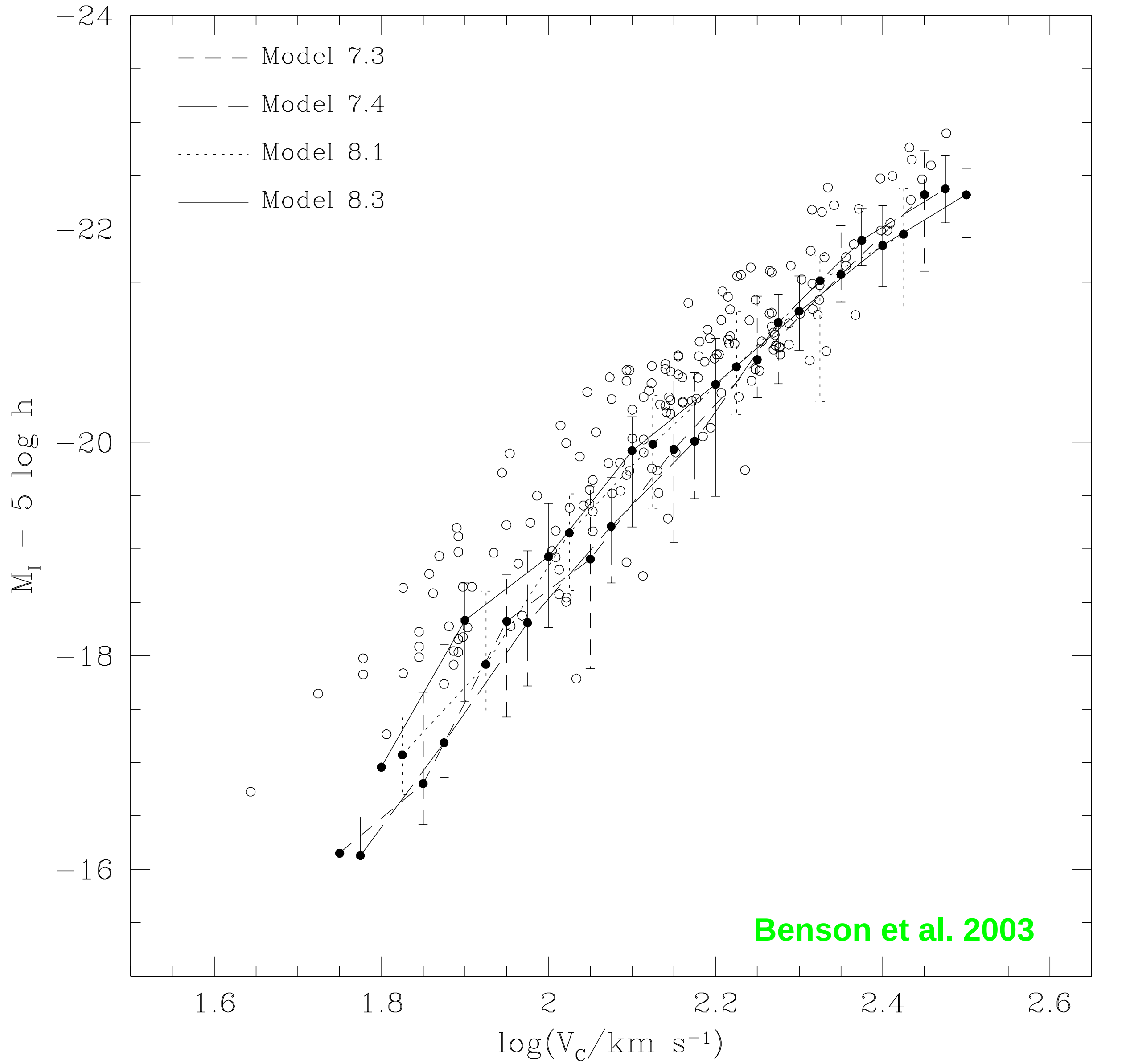
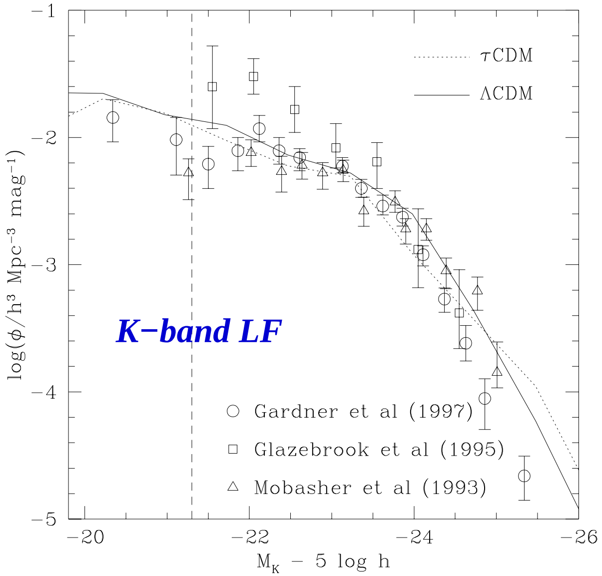
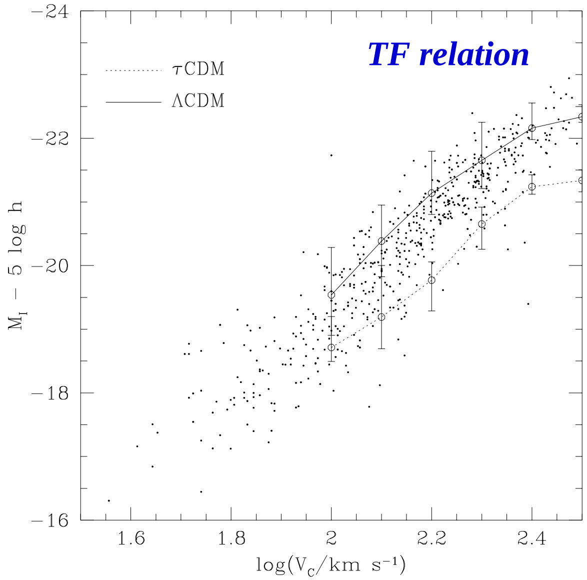
Current Status of SAM's TF

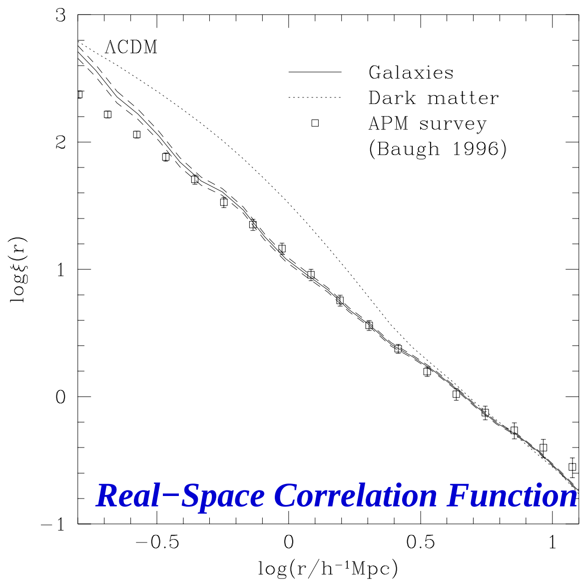


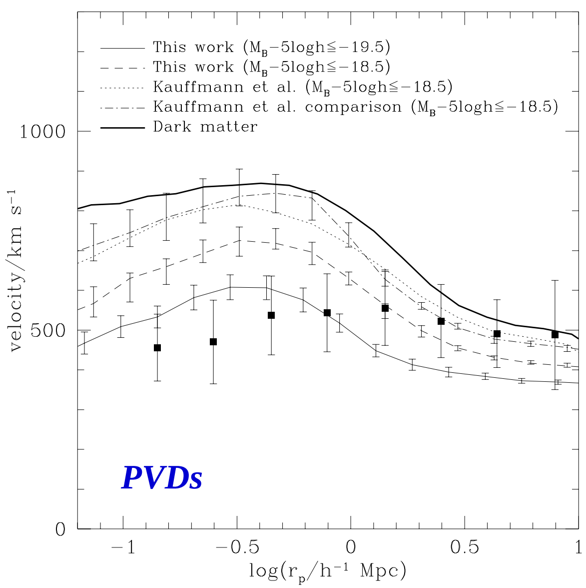
back

SAM's LSS









Benson et al., 2000a,b

back