The Physics of Disk Formation

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The Standard Picture

Disks galaxies are rotation supported systems in centrifugal equilibrium

Structure of disks is governed by angular momentum content

In standard picture angular momentum originates from cosmological torques

(Hoyle 1953; Peebles 1969; Doroshkevich 1970; White 1984)

Torques work in linear regime and are of gravitational origin. $\Rightarrow$ baryons & dark matter are expected to acquire identical specific angular momentum.

After being shock heated to $T_{\mathbf{vir}}$ the gas cools and is assumed to conserve its specific angular momentum **(Fall & Efstathiou 1980)**

- **Gas settles in disk in centrifugal equilibrium**
- **Density distribution of disk is direct reflection of angular momentum distribution (AMD) of baryons before cooling.**

Numerous models, of ever increasing complexity, have been constructed based on this general framework:

Fall & Efstathiou 1980; Faber 1982; Dalcanton, Spergel & Summers 1997; Mo, Mao White 1998

Kauffmann 1996; Jimenez et al. 1998; Buchalter, Jimenez & Kamionkowski 2001;

Avila-Reese & Firmani 2000; Firmani & Avila-Reese 2000; van den Bosch 1998, 2000, 2001, 2002

Halo Virial Properties

Define the **virial radius**, $r_{\rm vir}$, as the radius inside of which the average density is equal to $\Delta_{\rm vir}\,\rho_{\rm crit}$

$$\bar{\rho} = \frac{3\,M_{\rm vir}}{4\,\pi\,r_{\rm vir}^3} = \Delta_{\rm vir}\,\frac{3\,H^2(z)}{8\,\pi\,G}$$

For a ΛCDM concordance cosmology ($\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$) at redshift $z = 0$ one has that $\Delta_{\rm crit} = 101$ (Bryan & Norman 1998)

Substituting some characteristic values then yields

$$\boxed{r_{\rm vir} = 282 h^{-1}\ {\rm kpc}\ \left(\frac{V_{\rm vir}}{200\ {\rm km\,s^{-1}}}\right)\ \left(\frac{\Delta_{\rm vir}}{101}\right)^{-1/2}\ \left(\frac{H(z)}{H_0}\right)^{-1}}$$

and using the definition of **virial velocity**, $V_{\rm vir} = \sqrt{G\,M_{\rm vir}/r_{\rm vir}}$ one obtains that

$$\boxed{M_{\rm vir} = 2.7 \times 10^{12} h^{-1}\ {\rm M}_\odot\ \left(\frac{V_{\rm vir}}{200\ {\rm km\,s^{-1}}}\right)^3\ \left(\frac{\Delta_{\rm vir}}{101}\right)^{-1/2}\ \left(\frac{H(z)}{H_0}\right)^{-1}}$$

The Spin Parameter

Standard Definition: $\lambda = \frac{J\,|E|^{1/2}}{G\,M^{5/2}}$ (Peebles 1969)

More Convenient Definition: $\lambda' = \frac{J}{\sqrt{2}\,M_{\rm vir}\,r_{\rm vir}\,V_{\rm vir}}$ (Bullock et al. 2001)

For a halo in virial equilibrium $K + W/2 = 0$. Therefore, the total energy $E \equiv K + W = -K$. Thus, $|E| = |K|$ which is easily obtained when considering all particles on circular orbits:

$$\boxed{|E| = 2\pi \int_0^{r_{\rm vir}} \rho(r)\, V_c^2(r)\, r^2\, {\rm d}r \equiv \tfrac{1}{2}\, f\, M\, V_{\rm vir}^2}$$

Thus, $\lambda/\lambda' = \sqrt{f}$. Note that for a singular isothermal sphere $f = 1$.

Numerical simulations have shown that the distribution of λ' for CDM haloes is log-normal

$$\boxed{p(\lambda)\,{\rm d}\lambda = \frac{1}{\sqrt{2\pi}\sigma_\lambda} \exp\left[-\frac{\ln^2(\lambda/\bar{\lambda})}{2\sigma_\lambda^2}\right] \frac{{\rm d}\lambda}{\lambda}}$$

with $\bar{\lambda} \simeq 0.04$ and $\sigma_\lambda \simeq 0.5$

Barnes & Efstathiou 1987; Ryden 1988; Warren et al. 1992; Bullock et al. 2001

Disk Scale Lengths

Consider a disk with mass M_d that formed inside a halo of mass $M_{\rm vir}$. If the disk has an exponential mass density then

$$\Sigma(R) = \Sigma_0\,{\rm e}^{-R/R_d} \qquad \text{with} \qquad M_d = 2\,\pi\,\Sigma_0\,R_d^2$$

The angular momentum of the disk is given by

$$\begin{aligned} J_d &= 2\,\pi\,\int_0^\infty \Sigma(R)\,R\,V_c(R)\,R{\rm d}R \\ &= 2\,\pi\,\Sigma_0\,R_d^3\,V_{\rm vir}\,\int_0^\infty x^2\,{\rm e}^{-x}\,\frac{V_c(x\,R_d)}{V_{\rm vir}}\,{\rm d}x \\ &= M_d\,R_d\,V_{\rm vir}\,f_R \end{aligned}$$

f_R is weighted measure of $V_c(R)/V_{\rm vir}$. For singular isothermal sphere and $M_d/M_{\rm vir} \rightarrow 0$ one obtains $f_R = 1$.

Let specific angular momentum of disk be a fraction f_j of that of halo:

$$j_d = R_d\,V_{\rm vir}\,f_R = f_j\,\sqrt{2}\,\lambda\,R_{\rm vir}\,V_{\rm vir}$$

and thus: $\boxed{R_d = \sqrt{2}\,\left(\frac{f_j}{f_R}\right)\,\lambda\,R_{\rm vir}}$

(Mo, Mao & White 1998)

Substituting typical values yields:

(In standard model $f_j = 1$)

$$\boxed{R_d = 8h^{-1}\,{\rm kpc}\,\left(\frac{f_j}{f_R}\right)\,\left(\frac{\lambda}{0.04}\right)\,\left(\frac{V_{\rm vir}}{200\,{\rm km\,s^{-1}}}\right)\,\left(\frac{\Delta_{\rm vir}}{101}\right)^{-1/2}\,\left(\frac{H(z)}{H_0}\right)^{-1}}$$

Disk Scale Lengths II

f_R depends on $\frac{M_{\rm disk}}{M_{\rm vir}}$, λ, and concentration c. Typically $f_R > 1$

NFW halo:

$$\boxed{\frac{V_c(r)}{V_{\rm vir}} = \frac{1}{x}\,\frac{\ln(1+cx) - cx/(1+cx)}{\ln(1+c) - c/(1+c)}}$$

with $x = r/r_{\rm vir}$. The circular velocity $V_c(r)$ reaches a maximum $V_{\rm max}$ at $r_{\rm max} = 2.163 r_s = 2.163 r_{\rm vir}/c$.

$$\boxed{\frac{V_{\rm max}}{V_{\rm vir}} \simeq 0.465 \sqrt{\frac{c}{\ln(1+c) - c/(1+c)}}}$$

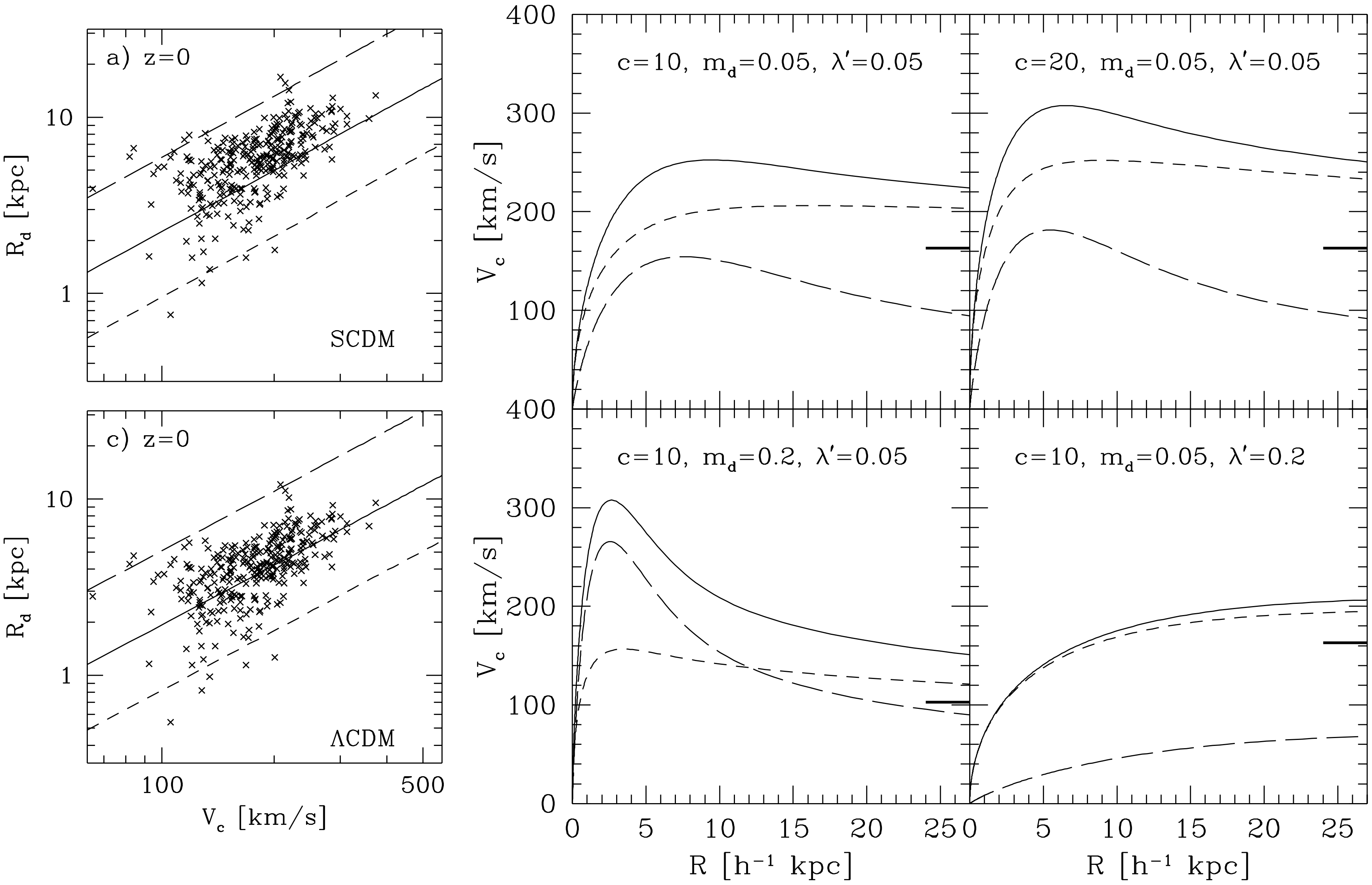
which is larger than unity for all realistic values of c (1.2 for $c = 10$)

Disk contribution : disk adds mass, therefore increases $V_c(r)$ and thus f_R.

Adiabatic Contraction: when disk formation is slow compared to dynamical time the halo responds adiabatically to the formation of the disk (actions are adiabatic invariants): halo becomes more concentrated, increasing f_R

Adiabatic contraction is typically taken into account by considering the approximate adiabatic invariant $r\,M(r)$; which is only exact for circular orbits in a spherical potential. Nevertheless, tests have shown this approximation to be sufficiently accurate (Barnes & White 1984; Blumenthal et al. 1986; Jesseit, Naab & Burkert 2000)

Disk Scale Lengths III



Mo, Mao & White 1998

Cooking Up a Disk Galaxy

In the MMW picture (i) disk formation is instantaneous, (ii) disks are assumed to be exponential, and (iii) rotation curves can be unrealistic.

Towards More Realism

- **Mass Accretion History (MAH):** $M_{\rm vir}(r,\phi,\theta,t|M_0)$
- **Angular Momentum Distribution (AMD):** $J_{\rm vir}(r,\phi,\theta,t|\lambda_0)$
- **Cooling model:** $t_{\rm form} = \max[t_{\rm cool}(Z/Z_{\odot}), t_{\rm ff}]$
- **Bulge Formation:** $\alpha_c = V_{\rm disk}(3R_d)/V_{\rm circ}(3R_d) = 0.6$

After a time $t_{\rm form}$ mass element $m(r,\phi,\theta,t)$ ends up in the disk at a radius R given by $j(r,\phi,\theta,t) = R \cdot V_{\rm circ}(R, t + t_{\rm form})$.

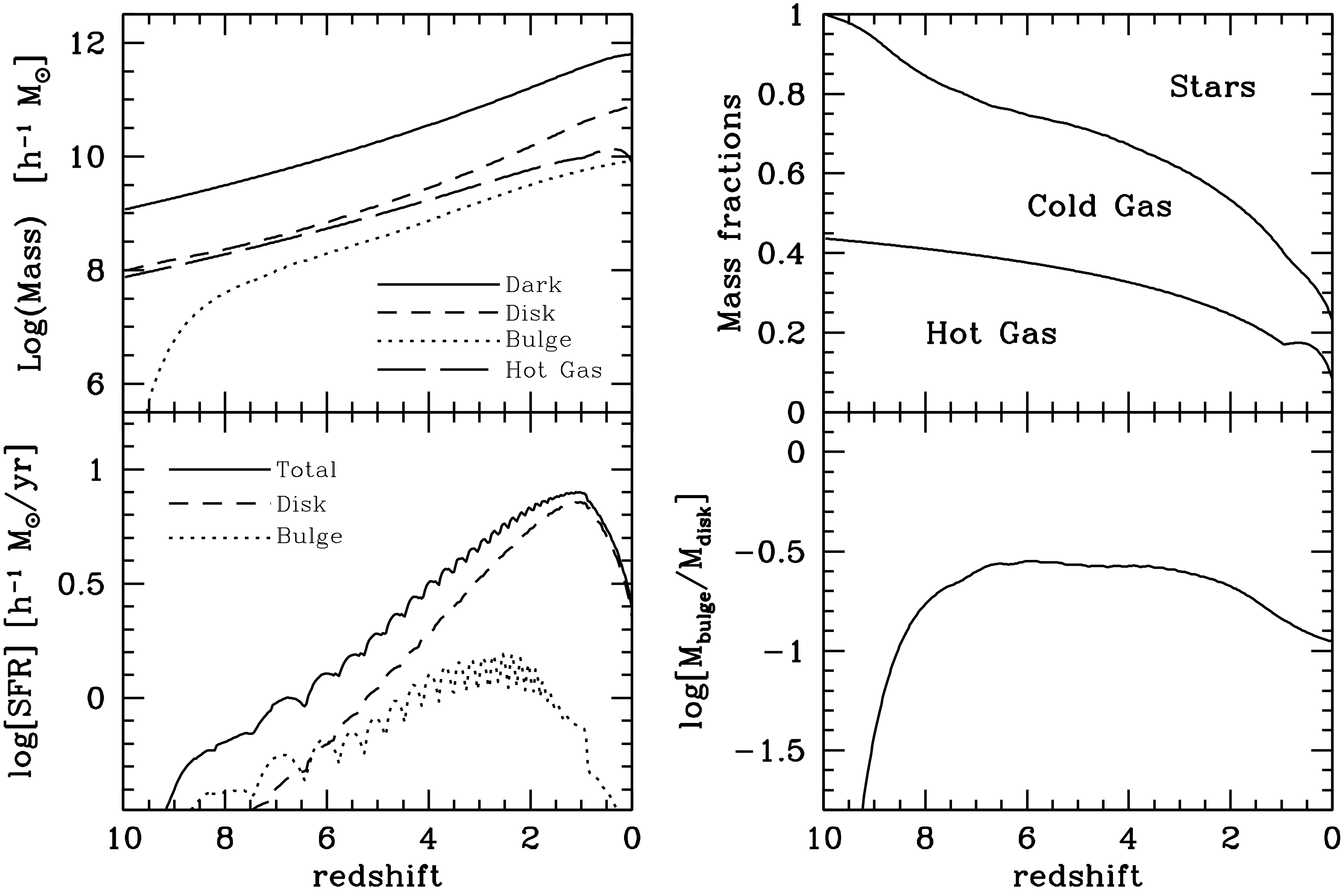
$$\boxed{\text{MAH + AMD} \rightarrow j(r,\phi,\theta,t) \rightarrow M_{\rm disk}(R,t)}$$

Additional model ingredients: star formation, feedback, stellar population models, chemical evolution all à la SAMs

van den Bosch 1998, 2000, 2001, 2002

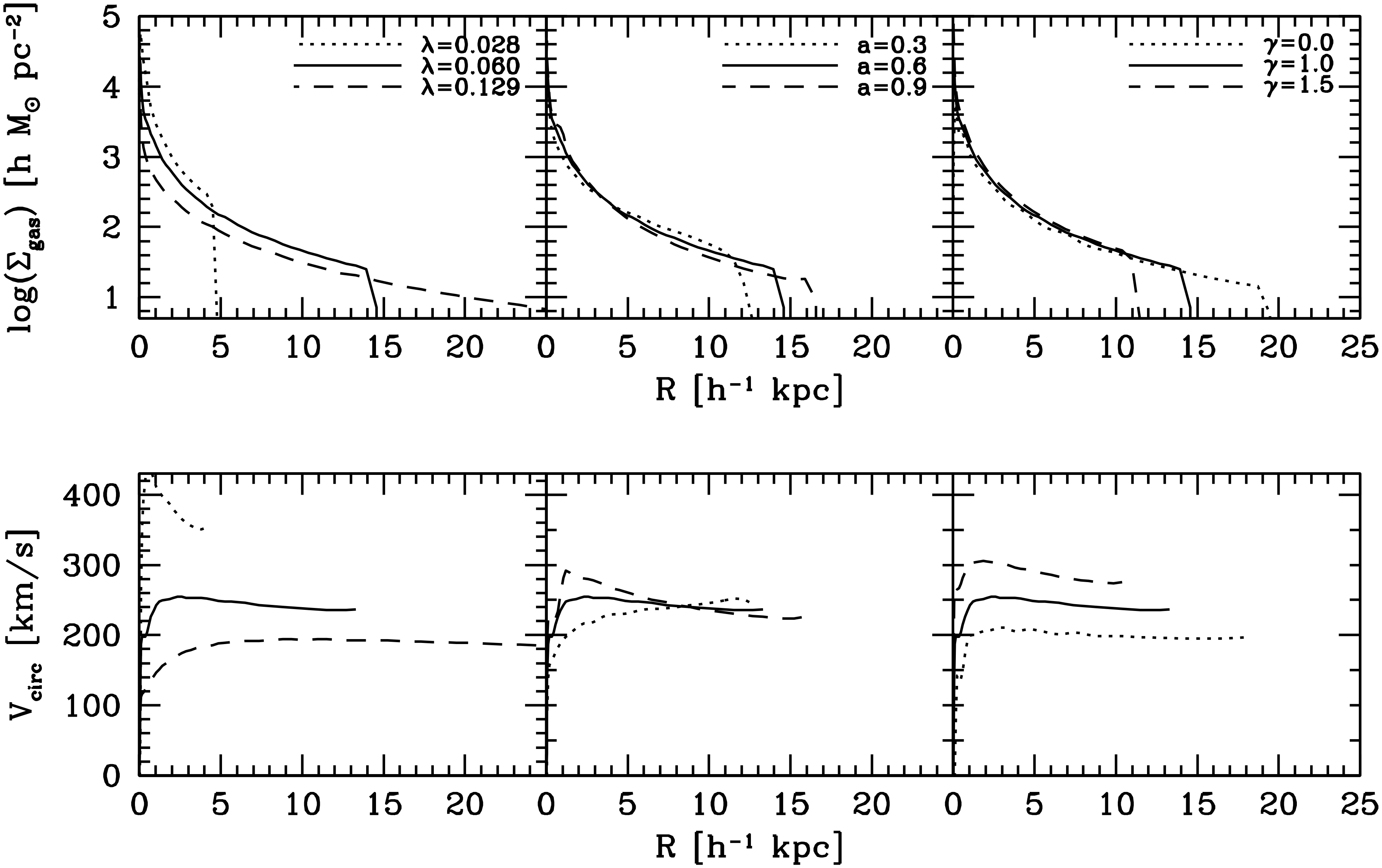
Avila-Reese & Firmani 2000; Firmani & Avila-Reese 2000

An Example



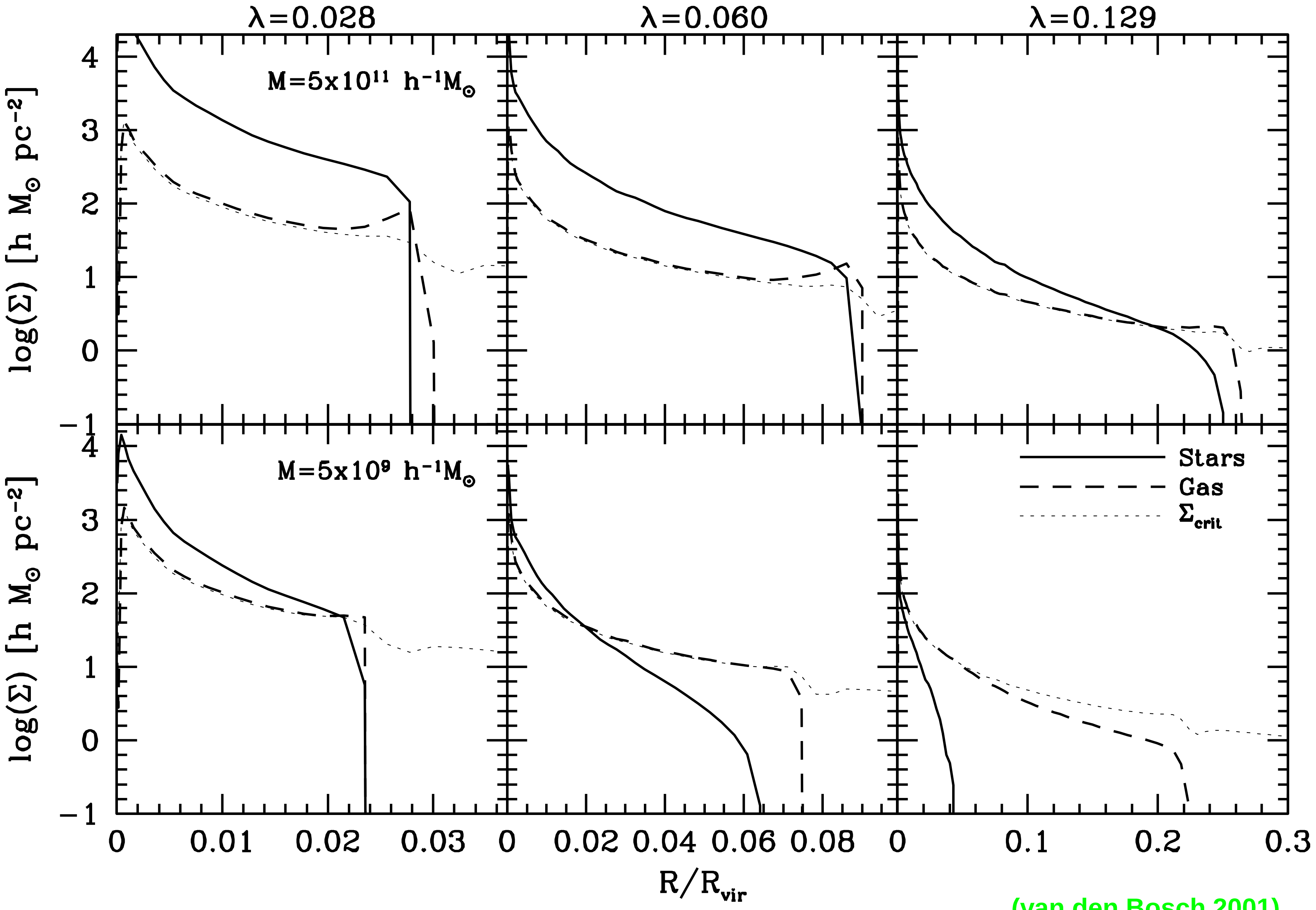
$M_{\rm vir} = 5 \times 10^{11} h^{-1}\,{\rm M}_\odot$, $\lambda = 0.06$, average MAH

Cooling Only



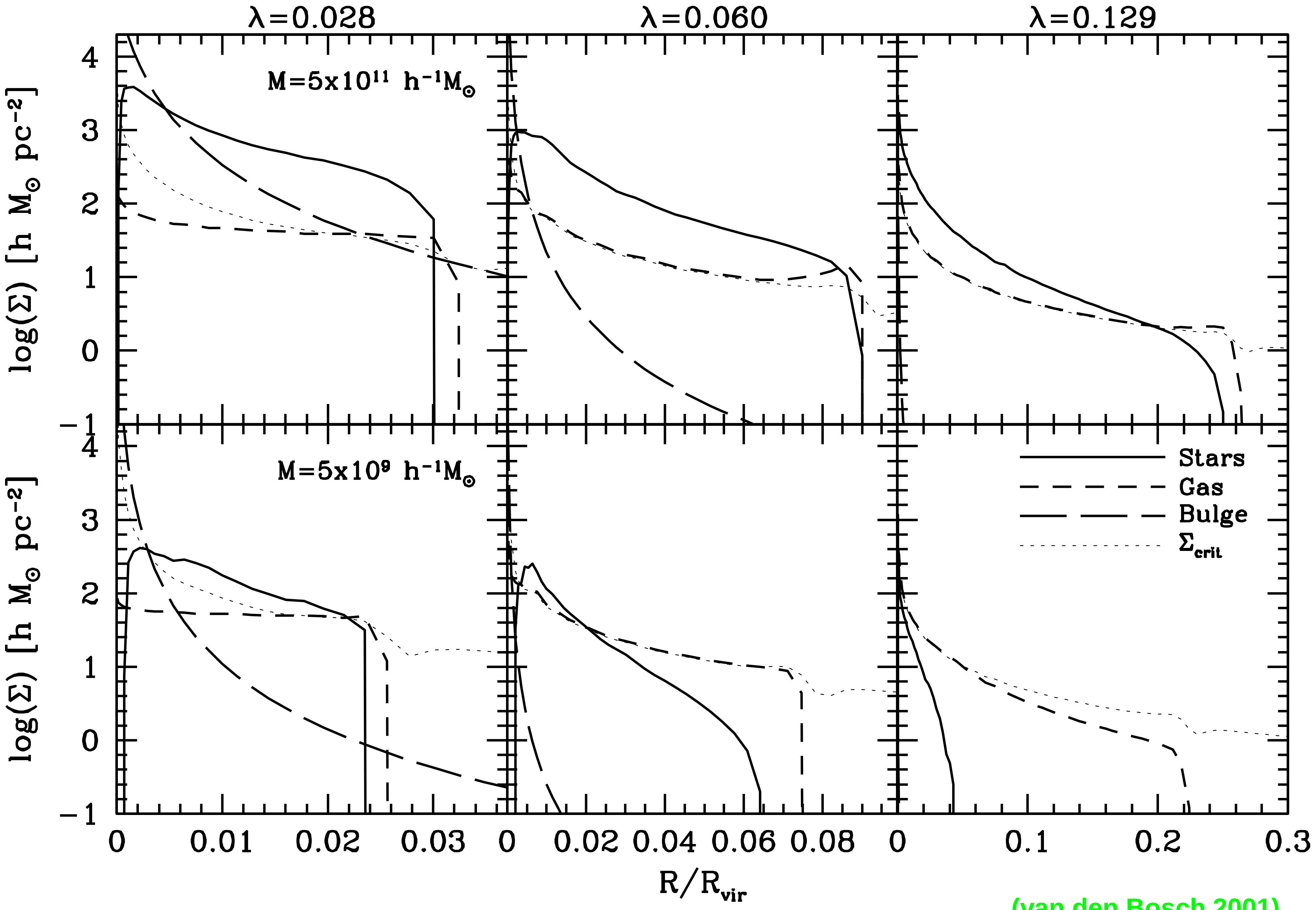
(van den Bosch 2001)

Cooling + Starformation



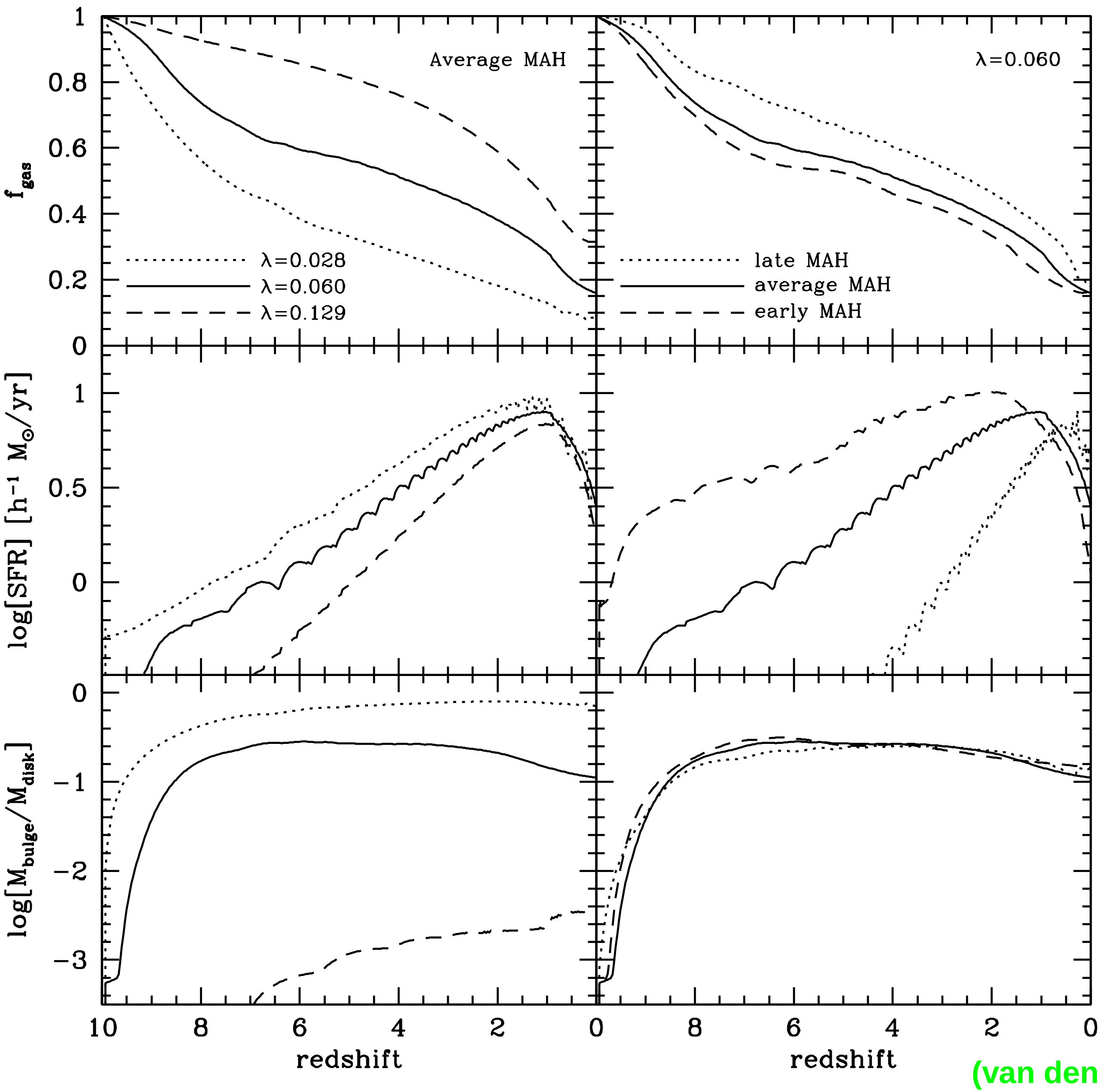
(van den Bosch 2001)

With Bulge Formation



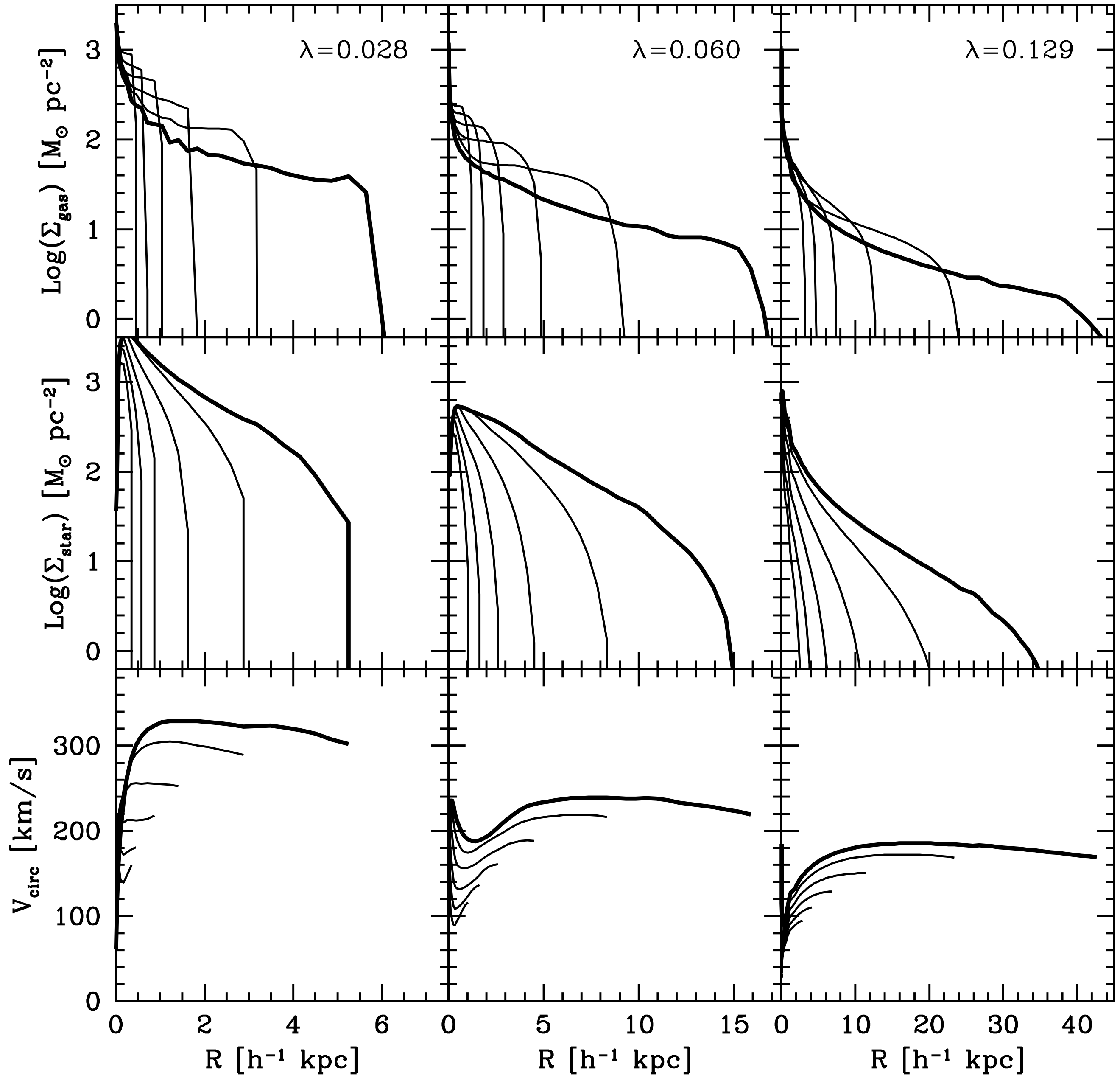
(van den Bosch 2001)

Parameter Dependencies



(van den Bosch 2002)

The Inside-Out Formation of Disks



(van den Bosch 2002)

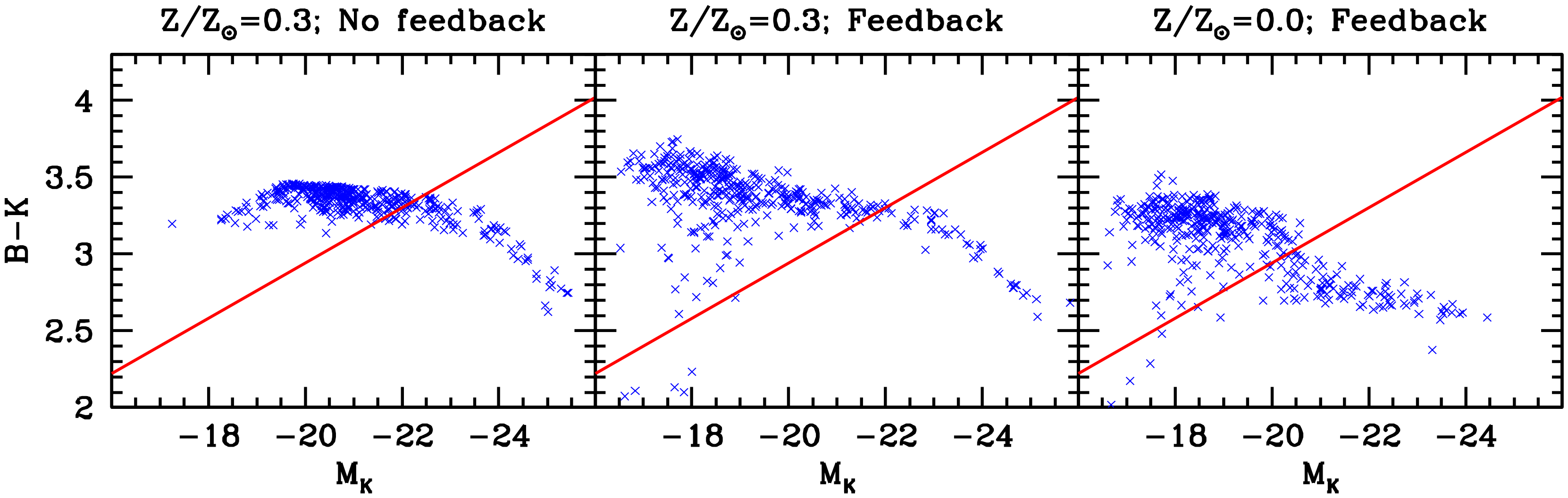
Success & Failure

Successes

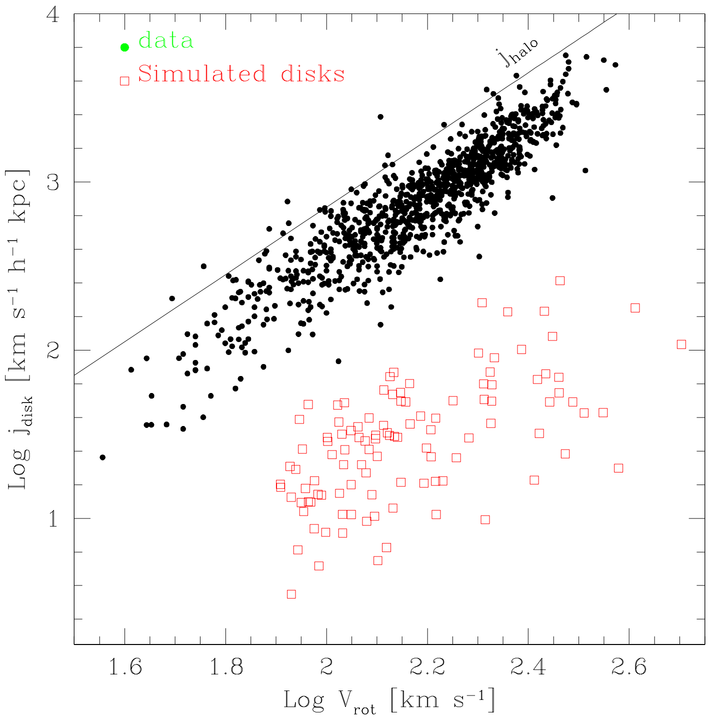
- **Distribution of disk scale lengths matches data** (de Jong & Lacey 2000)
- **With bulge, exponential surface brightness profile** (van den Bosch 2001)
- **Radial color-gradients** (Firmani & Avila-Reese 2000)

Problems

- **Failure to form bulge-less exponential disks á la M33** (van den Bosch 2001)
- **Inverted color-magnitude relation** (van den Bosch 2002; Bell et al. 2003)



The Angular Momentum Catastrophe



Steinmetz & Navarro 1999

- **Disks that form in simulations are an order of magnitude too small**
- **Gas looses large fraction of specific angular momentum to dark matter**
- **Hierarchical formation & “over-cooling” are to blaim**

White & Navarro 1993; Navarro & Steinmetz 1999

SOLUTIONS

(1) Prevent Cooling: feedback, preheating **(Weil et al. 1998; Sommer-Larsen et al. 1999)**

(2) Modify Power Spectrum: WDM, BSI, RSI... **(Sommer-Larsen & Dolgov 2001)**

Disk Scaling Relations I

Observations:

- $M_{\rm disk} = 3.1 \times 10^9 \, h^{-2} \, {\rm M}_\odot \left(\frac{V_{\rm rot}}{100 \; {\rm km \, s^{-1}}}\right)^{3.5}$ (Bell & de Jong 2001)
- $j_{\rm disk} = 3.3 \times 10^2 \; {\rm km \, s^{-1}} h^{-1} \; {\rm kpc} \left(\frac{V_{\rm rot}}{100 \; {\rm km \, s^{-1}}}\right)^2$ (Navarro 1998)

Theoretical Predictions:

- $M_{\rm disk} = f_m \left(\frac{\Omega_b}{\Omega_m}\right) M_{\rm vir}$
- $j_{\rm disk} = \sqrt{2} \, f_j \, \lambda' \, R_{\rm vir} V_{\rm vir}$
- $M_{\rm vir} \propto V_{\rm vir}^3 \qquad R_{\rm vir} \propto V_{\rm vir}$

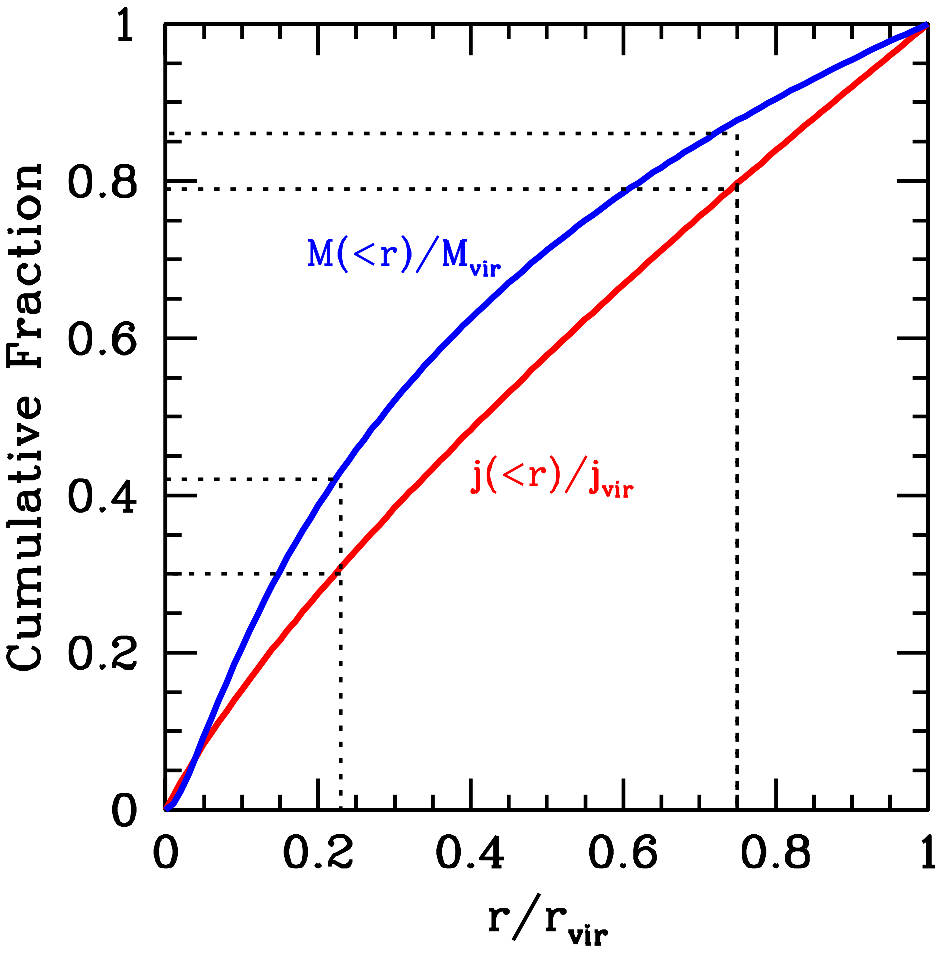
Example: $\Omega_m = 0.3 \quad h = 0.7 \quad \lambda = 0.04 \quad V_{\rm rot}/V_{\rm vir} = 1.4$

$$\boxed{f_m = 0.42 \left(\frac{V_{\rm vir}}{200 \; {\rm km \, s^{-1}}}\right)^{1/2} \qquad f_j = 0.79}$$

(see also Navarro & Steinmetz 2000)

Disk Scaling Relations II

$$f_m = 0.42 \left(\frac{V_{\rm vir}}{200\,{\rm km\,s^{-1}}}\right)^{1/2} \qquad f_j = 0.79 \left(\frac{\lambda'}{0.04}\right)^{-1}$$



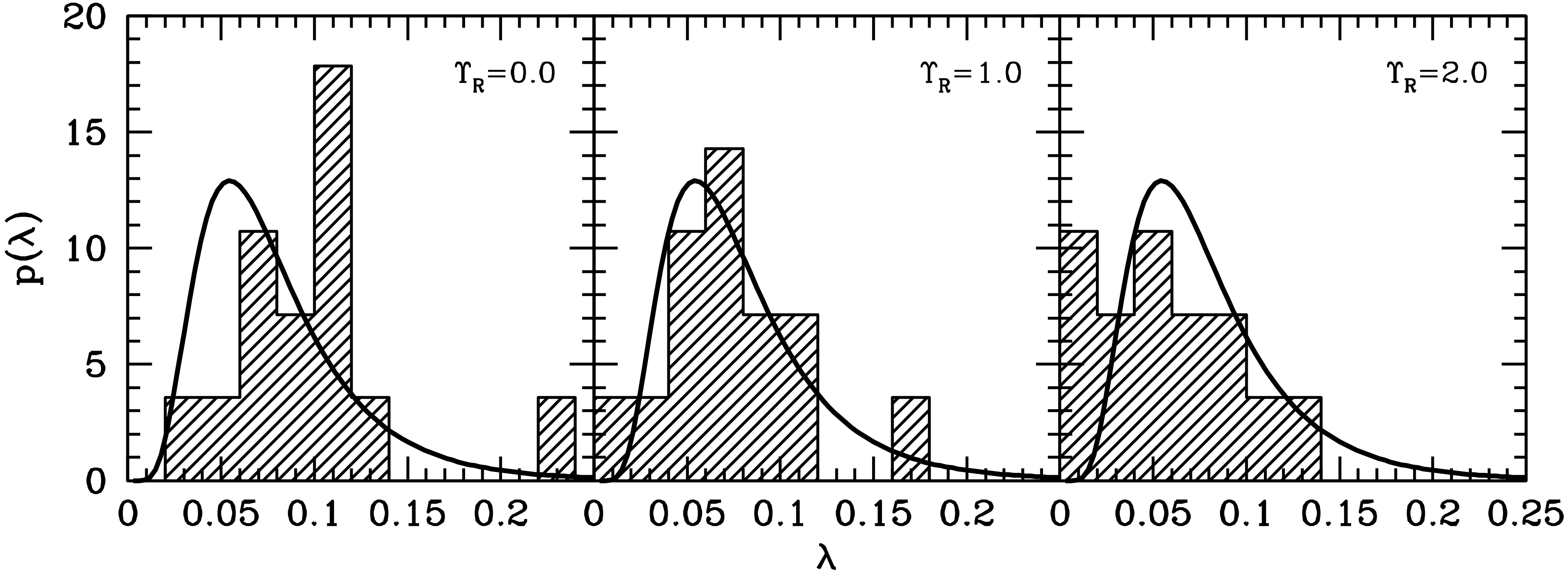
- $M(r)$ from NFW profile with $c = 20$ **(Navarro, Frenk & White 1997)**
- $j(r) \propto r$ from N-body simulations **(Bullock et al. 2001)**

Testing the Paradigm

TEST: **Compare angular momentum distributions of disks and CDM haloes. If standard paradigm is correct, these should be identical.**

DATA: **14 dwarf galaxies whose rotation curves are in good agreement with CDM haloes (van den Bosch & Swaters 2001).**

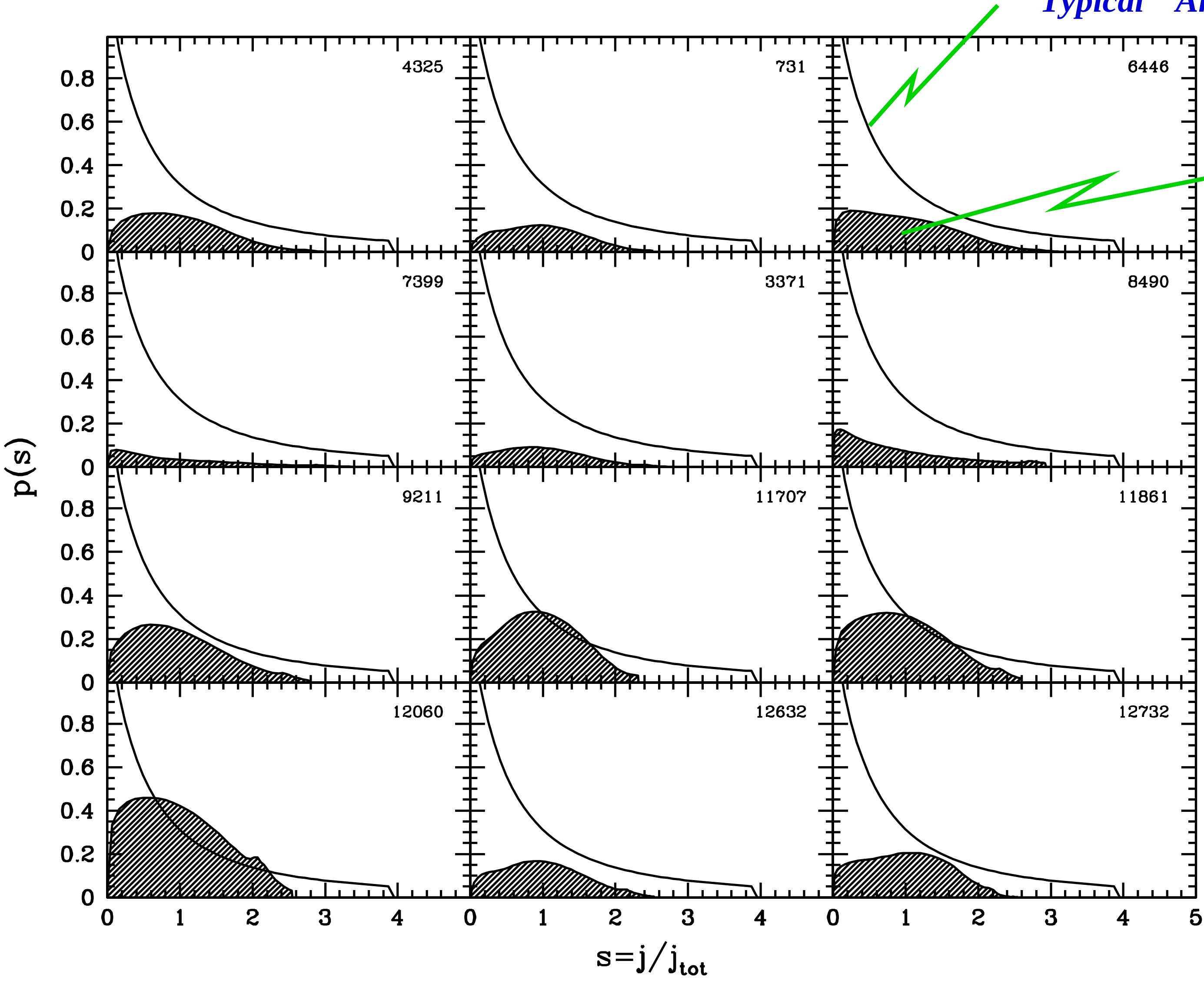
$$M(<j) = 2\pi \int_0^{R_j} \Sigma_{\rm disk}(R)\, R\, {\rm d}R \quad \text{with} \quad j = R_j V_{\rm circ}(R_j)$$



Disks and CDM haloes have same $p(\lambda)$.

van den Bosch, Burkert & Swaters 2001

Angular Momentum Distributions



‘‘Typical’’ Angular momentum distribution of dark matter halo.

Angular momentum distribution of disk.

van den Bosch, Burkert & Swaters 2001

Disks (of dwarf galaxies) have angular momentum distributions that are clearly different than those of cold dark matter haloes!!!

Gas in Proto-Galaxies

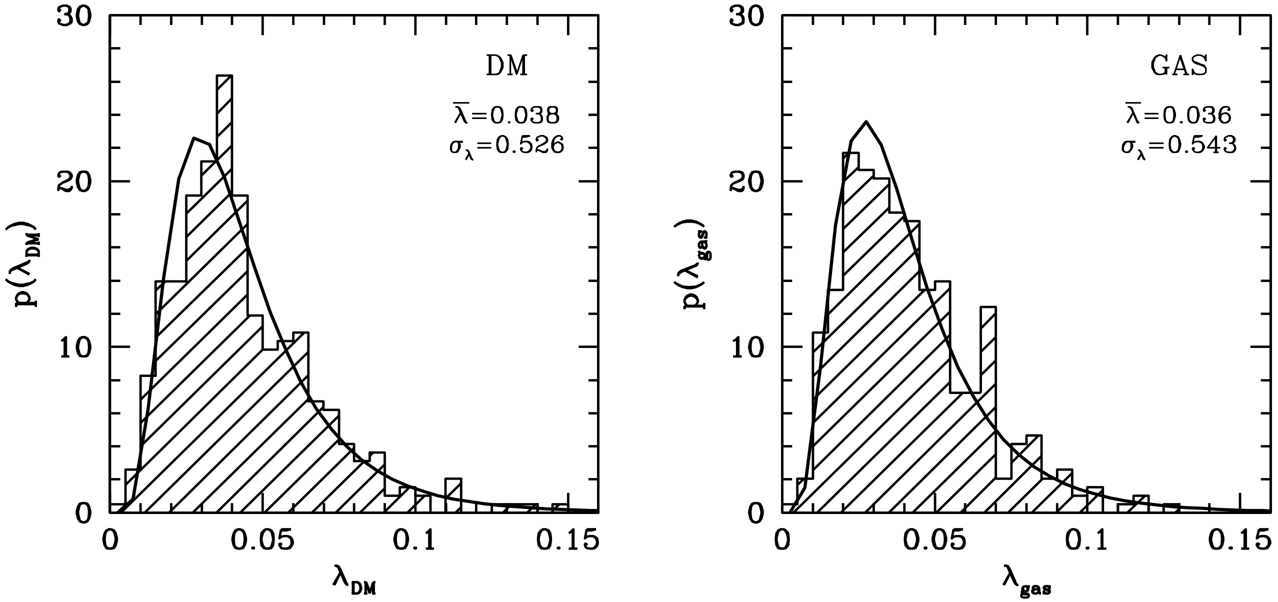
TEST: **Do the gas and dark matter have the same angular momentum distributions before cooling?** **gas can shock...**

TOOL: **Numerical N-body/SPH simulation of ΛCDM cosmology with non-radiative gas; Analyze individual haloes.**

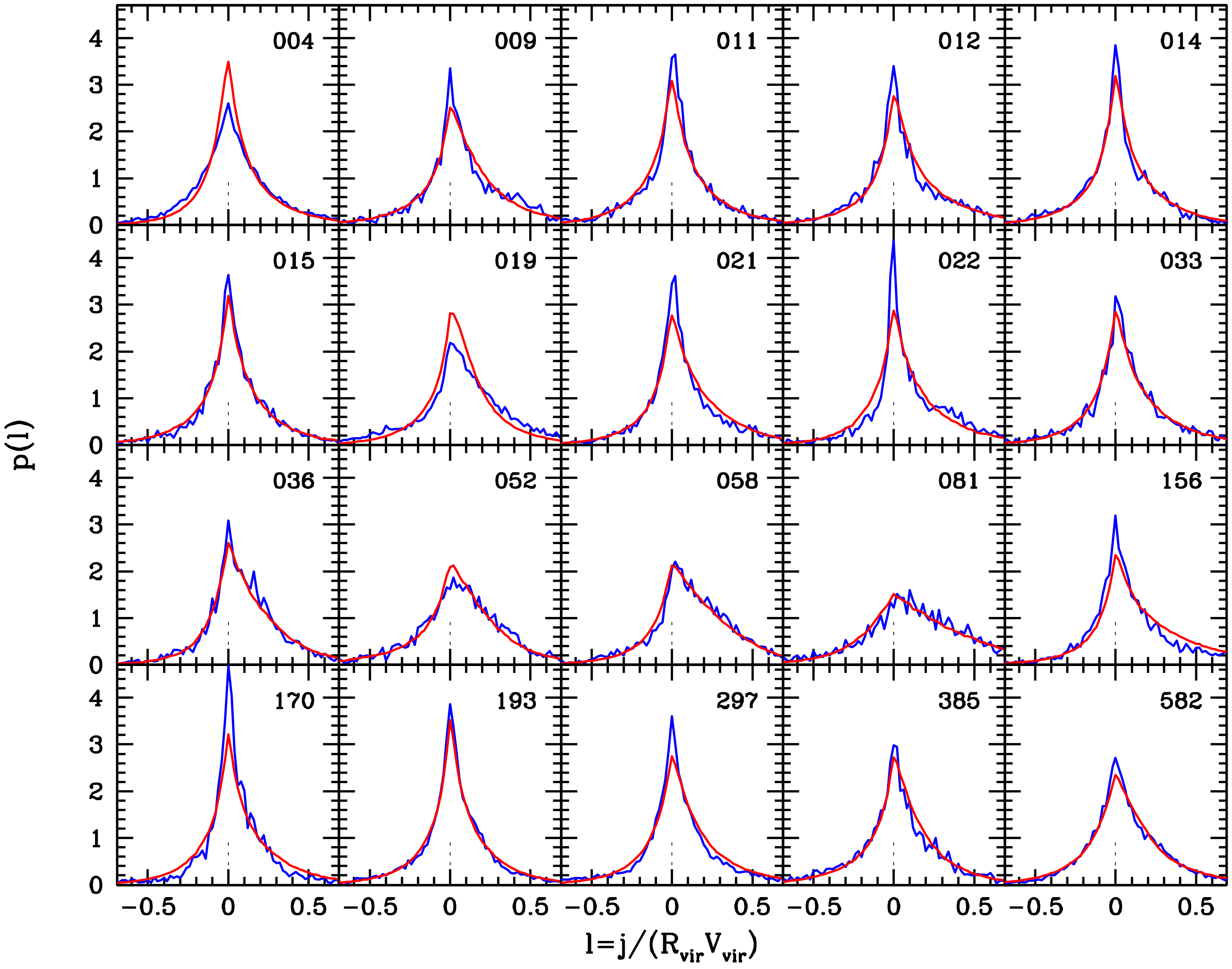
Gas and dark matter are fluids for which $\boxed{\vec{v} = \vec{u} + \vec{w}}$

- $\vec{v}$ =**microscopic velocity (DM particles in simulation)**
- $\vec{u}$ =**streaming motions (SPH particles in simulation)**
- $\vec{w}$ =**random motions (related to temperature of gas particles)**

THERMAL BROADENING: **Add random velocities to SPH particles with dispersion given by particle's temperature.**



A more detailed comparison...



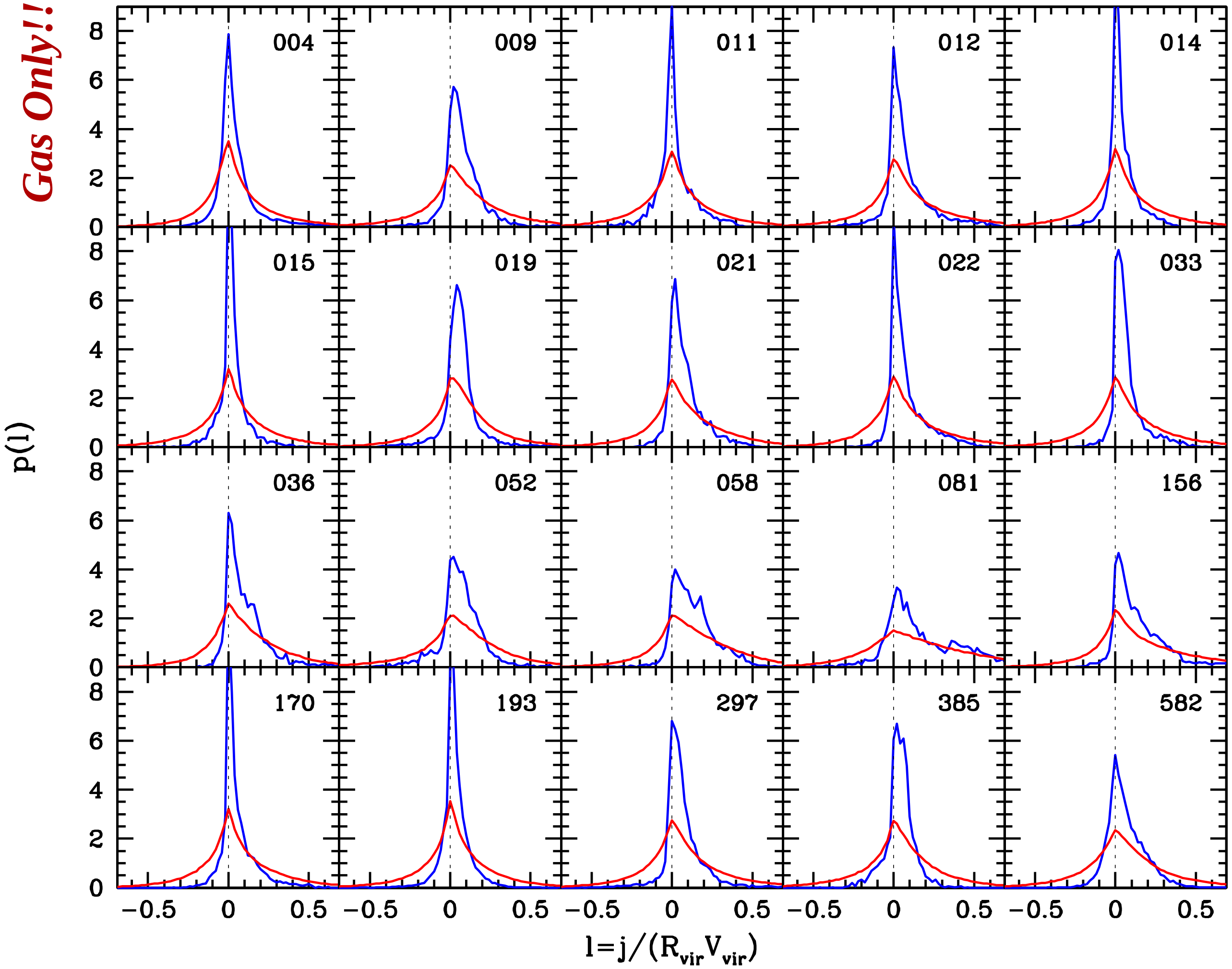
Blue lines:
Dark Matter

Red lines:
Gas

van den Bosch et al 2002

- **AMDs of gas and dark matter are virtually identical**
- **Virialization shocks do not affect AMD of gas**
- **Apparently, the standard assumption is correct**

and what it means for disk formation



Red lines:
(j = r x v)

Blue lines:
(j = r x u)

van den Bosch et al 2002

Between 10 & 40 percent of gas has negative specific angular momentum!!!

A new problem?

Bulge Formation?

- **Disks do not contain counter-rotating material...**
- **About 40% of haloes forms Early-Type galaxies**
- **Virtually no bulge-less systems can form**

CONCLUSIONS

- ★ **small mass haloes form before big mass haloes**
- ★ **cooling very efficient in low mass haloes at high z**

⟹ **Angular Momentum Catastrophe & Inverted Color-Magnitude Relation**

- ★ **haloes have too much low angular momentum material**

⟹ **Morphology Problem! Too much bulge, too little disk**

- ★ **haloes have too much negative angular momentum material**

⟹ **No detailed conservation of specific angular momentum possible**

Standard Model for Disk Formation is Incomplete and/or Incorrect

Future Prospectives

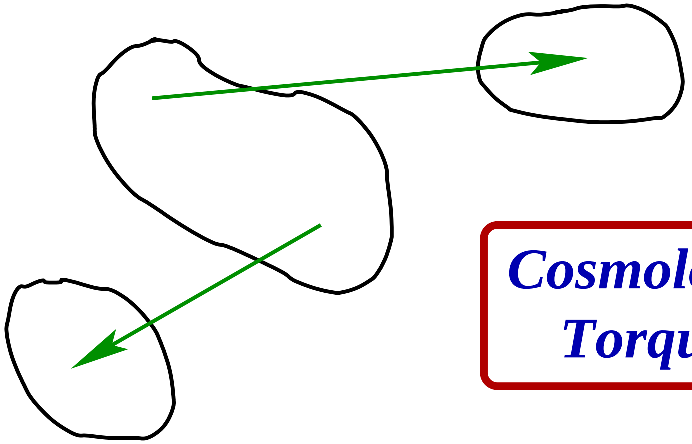
(1) More detailed modelling of feedback & reionization

(2) Cold Accretion *vs* Hot Accretion **Katz et al. 2003; Birnboim & Dekel 2003**

(3) Satellite accretion & streamers

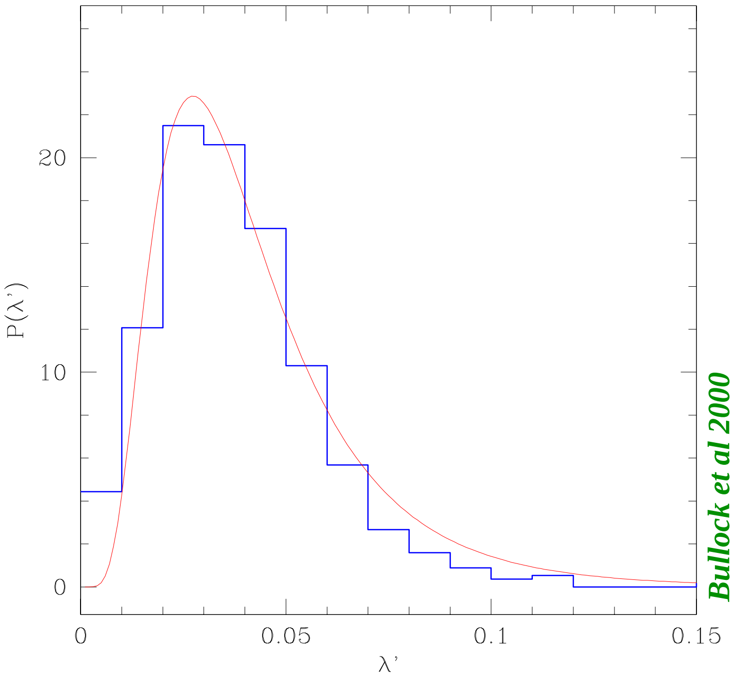
(4) Cosmology...

The Origin of Angular Momentum

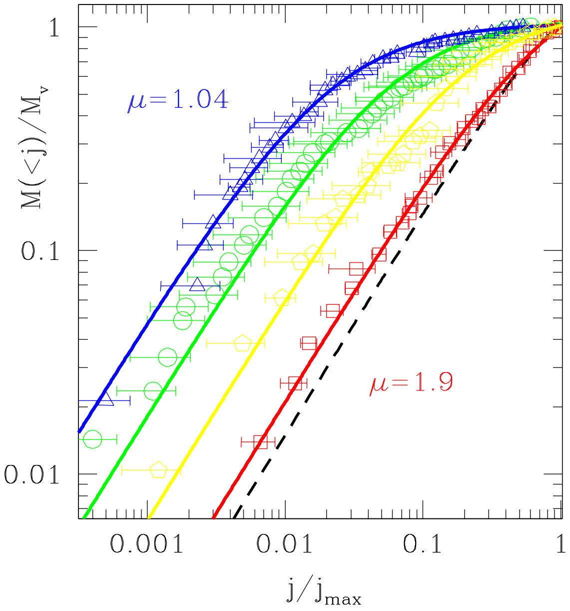


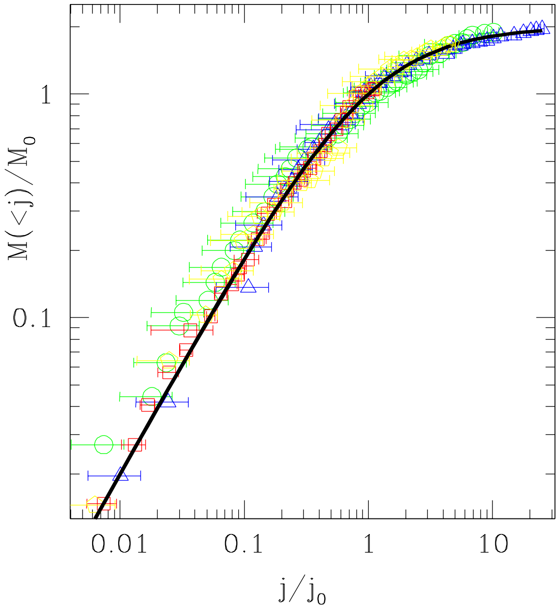
Cosmological Torques

$$\boldsymbol{\lambda = \frac{J\,|E|^{1/2}}{GM^{5/2}} \propto \frac{j_{\rm tot}}{R_{\rm vir}V_{\rm vir}}}$$



Bullock et al 2000





Cold Dark Matter haloes have a log−normal distribution of halo spin parameters...

Cold Dark Matter haloes have a Universal Angular Momentum distribution...

Bullock et al 2000

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