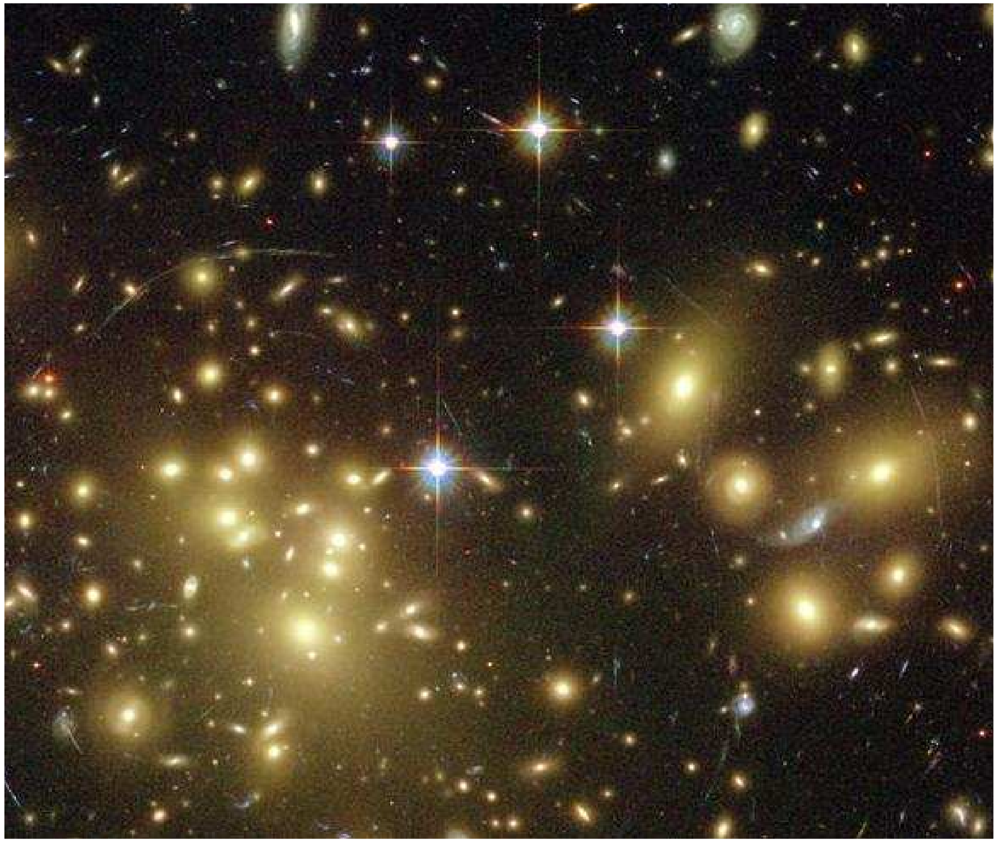
The Galaxy–Dark Matter Connection

cosmology & galaxy formation with the CLF



Frank C. van den Bosch (MPIA)

Outline

- Statistical Description of Large Scale Structure
- Galaxy Bias & The Galaxy-Dark Matter Connection
- The Halo Model, Halo Bias & Halo Occupation Statistics
- The Conditional Luminosity Function (CLF)
- The Universal Relation between Light and Mass
- Constraining Cosmological Parameters with the CLF
- Halo Occupation Statistics from Galaxy Groups
- Constraining Galaxy Formation with Galaxy Ecology
- Conclusions

Correlation Functions

Define the dimensionless density perturbation field: $\delta(\vec{x}) = \frac{\rho(\vec{x}) - \bar{\rho}}{\bar{\rho}}$

For a Gaussian random field, the one-point probability function is:

$$P(\delta)\mathrm{d}\delta = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{\delta^2}{2\sigma^2}\right] \mathrm{d}\delta$$

$$\begin{aligned} \langle\delta\rangle &= \int \delta P(\delta)\mathrm{d}\delta &= 0 \\ \langle\delta^2\rangle &= \int \delta^2 P(\delta)\mathrm{d}\delta &= \sigma^2 \end{aligned}$$

Define n-point probability function: $P_n(\delta_1, \delta_2, \cdots, \delta_n)\, \mathrm{d}\delta_1\, \mathrm{d}\delta_2 \cdots \mathrm{d}\delta_n$

Gravity induces correlations between δ_i so that

$$P_n(\delta_1, \delta_2, \cdots, \delta_n) \neq \prod_{i=1}^{n} P(\delta_i)$$

Correlations are specified via n-point correlation function:

$$\langle \delta_1 \delta_2 \cdots \delta_n \rangle = \int \delta_1 \delta_2 \cdots \delta_n P_n(\delta_1, \delta_2, \cdots, \delta_n)\, \mathrm{d}\delta_1\, \mathrm{d}\delta_2 \cdots \mathrm{d}\delta_n$$

In particular, we will often use the two-point correlation function

$$\xi(x) = \langle \delta_1 \delta_2 \rangle \quad \text{with } x = |\vec{x}_1 - \vec{x}_2|$$

Galaxy Bias

Consider the distribution of matter and galaxies, smoothed on some scale R

$$\delta(\vec{x}) = \frac{\rho(\vec{x}) - \bar{\rho}}{\bar{\rho}} \qquad \delta_{\rm gal}(\vec{x}) = \frac{n_{\rm gal}(\vec{x}) - \bar{n}_{\rm gal}}{\bar{n}_{\rm gal}}$$

Mass distribution **Galaxy distribution**

$$\xi(r) = \langle \delta(\vec{x}_1)\delta(\vec{x}_2)\rangle \qquad \xi_{\rm gal}(r) = \langle \delta_{\rm gal}(\vec{x}_1)\delta_{\rm gal}(\vec{x}_2)\rangle$$

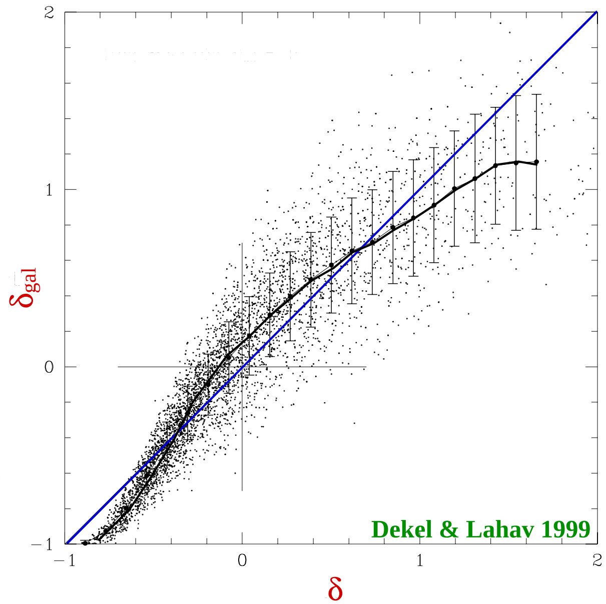
- **There is no good reason why galaxies should trace mass.**
- **Ratio is galaxy bias:** $b(\vec{x}) = \delta_{\rm gal}(\vec{x})/\delta(\vec{x})$
- **One can distinguish various types of bias:**

  linear, deterministic: $\delta_{\rm gal} = b\,\delta$

  non-linear, deterministic: $\delta_{\rm gal} = b(\delta)\,\delta$

  stochastic: $\delta_{\rm gal} \neq \langle \delta_{\rm gal}|\delta\rangle$

- **Real bias probably non-linear and stochastic**
- **Bias also depends on smoothing scale R**
- **Since $\delta_{\rm gal} = \delta_{\rm gal}(L, M_*, \ldots)$ bias also depends on galaxy properties**



Handling Bias

- **Bias is an imprint of galaxy formation, which is poorly understood**
- **Consequently, little progress constraining cosmology with LSS**

Q: How can we constrain and quantify galaxy bias in a convenient way?

Handling Bias

- Bias is an imprint of galaxy formation, which is poorly understood
- Consequently, little progress constraining cosmology with LSS

Q: How can we constrain and quantify galaxy bias in a convenient way?

A: With Halo Model plus Halo Occupation Statistics!

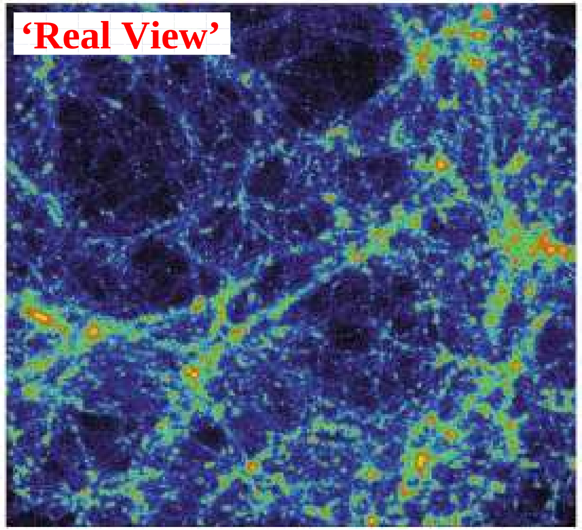
The Halo Model describes CDM distribution in terms of halo building blocks, under assumption that every CDM particle resides in virialized halo

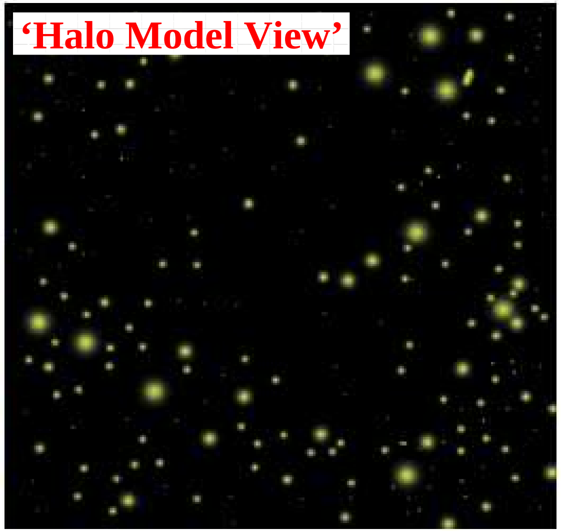
- On small scales: $\delta(\vec{x})$ reflects density distribution of haloes (NFW profiles)
- On large scales: $\delta(\vec{x})$ reflects spatial distribution of haloes (halo bias)

PARADIGM: All galaxies live in Cold Dark Matter Haloes.

galaxy bias = halo bias + halo occupation statistics

Halo Model Ingredients





Cooray & Sheth (2002)

Halo Density Distributions: (Navarro, Frenk & White 1997)

$$\rho(r) = \frac{\rho_s}{(r/r_s)(1+r/r_s)^2}$$

Halo Mass Function: (Press & Schechter 1974)

$$n(m) = \sqrt{\frac{2}{\pi}}\frac{\bar{\rho}}{m^2}\left|\frac{\mathrm{d}\ln\sigma}{\mathrm{d}\ln m}\right|\sqrt{\nu}\mathrm{e}^{-\nu/2}$$

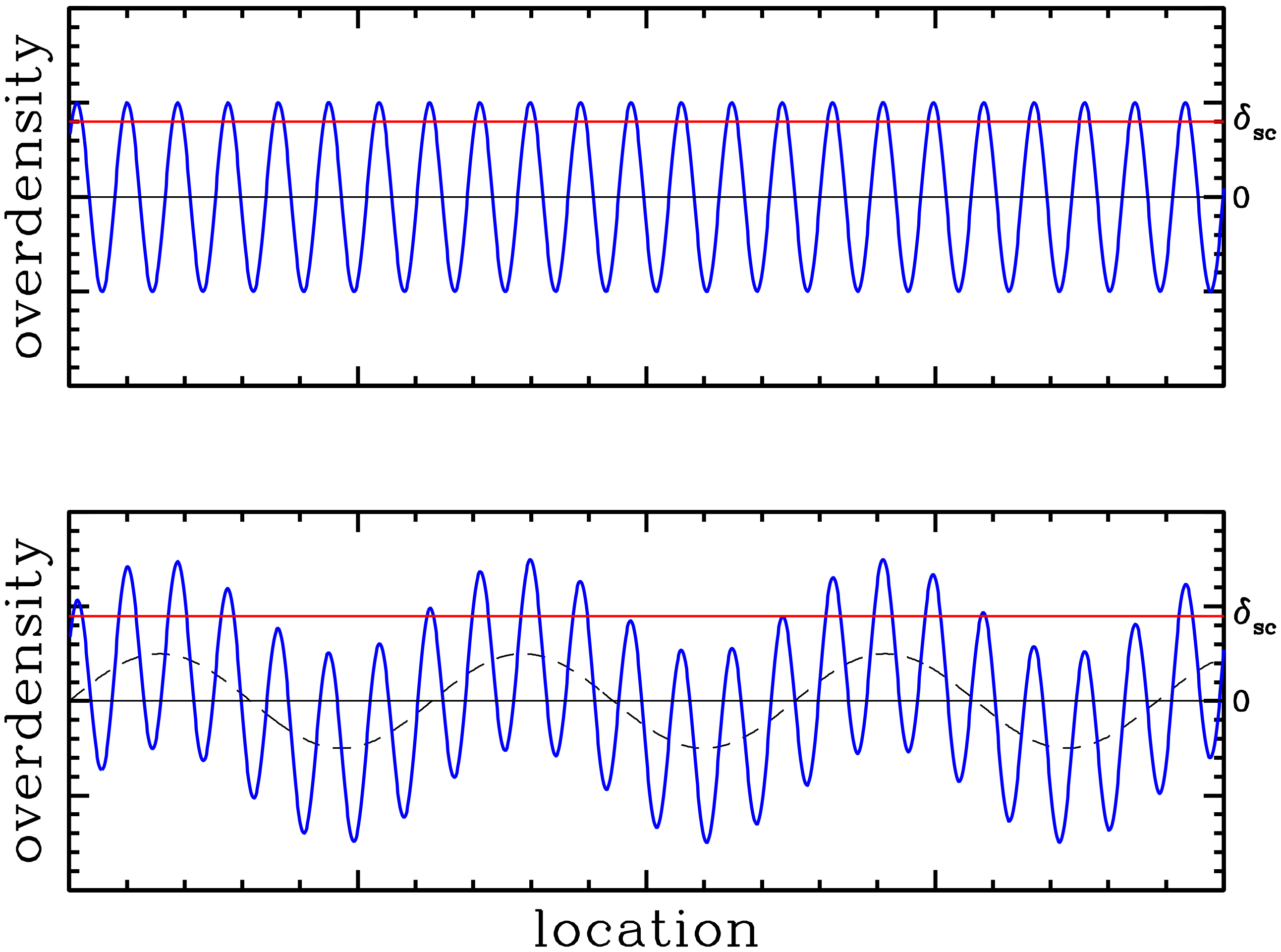
Halo Bias Function: (Kaiser 1994; Mo & White 1996)

$$b(m) \equiv \frac{\delta_h(m)}{\delta} = \frac{n(m|\delta)-n(m)}{n(m)\,\delta} = 1 + \frac{\nu-1}{\delta_{\rm sc}}$$

$\delta_{\rm sc}$ is critical spherical collapse overdensity, $\sigma^2(m)$ is mass variance, and $\nu = \delta_{\rm sc}^2/\sigma^2(m)$

The Origin of Halo Bias

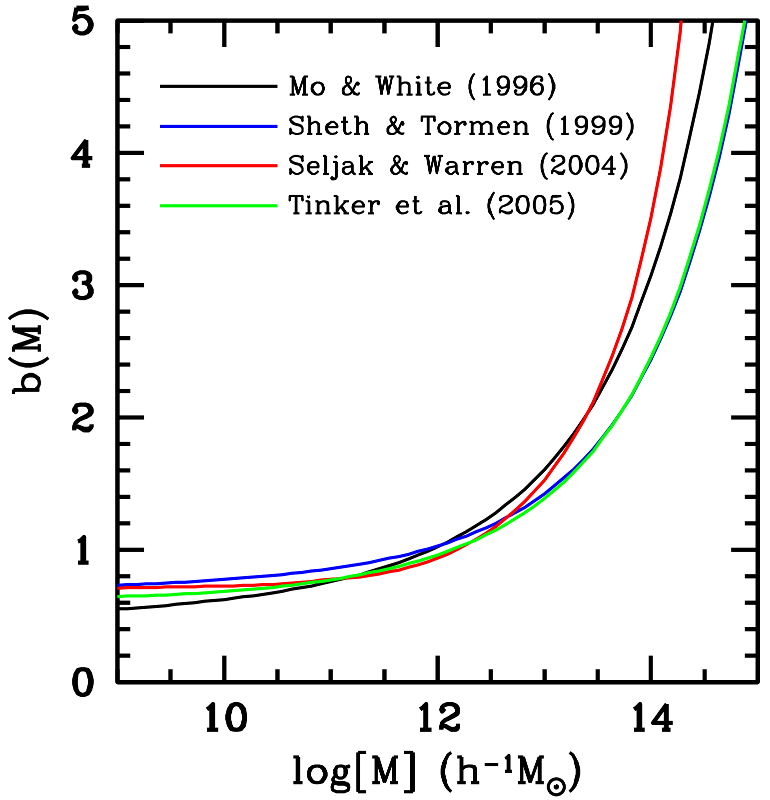
Dark Matter Haloes are a biased tracer of the dark matter mass distribution!



Modulation causes statistical bias of peaks (haloes)

Modulation growth causes dynamical enhancement of bias

Analytical Description of Halo Bias



Define **halo bias** as $b(m) = \delta_h(m)/\delta$

$b(m) > 1$ if $m > m^*$ (biased)

$b(m) = 1$ if $m = m^*$ (unbiased)

$b(m) < 1$ if $m = m^*$ (unbiased)

Halo bias has absolute minimim:

$$b > 1 - \frac{1}{\delta_{\rm sc}} \simeq 0.41$$

Halo-Halo correlation function: for haloes of mass m

$$\xi_{\rm hh}(r) \equiv \langle \delta_{h_1}\delta_{h_2}\rangle = b(m)^2 \langle \delta_1 \delta_2 \rangle = b(m)^2 \xi(r)$$

Halo Occupation Statistics

How many galaxies, on average, per halo?

Halo Occupation Distribution: The HOD $P(N|M)$ specifies the probability that a halo of mass M contains N galaxies.

Of particular importance: first moment $\langle N \rangle_M = \sum_{N=0}^{\infty} N\, P(N|M)$

How are galaxies distributed (spatially & kinematically) within halo?

Central Galaxy : located at center of dark matter halo.

Satellite Galaxies: $n_{\rm sat}(r) \propto \rho_{\rm dm}(r) \iff \sigma_{\rm sat}(r) = \sigma_{\rm dm}(r)$

Supported by distribution of sub-haloes in N-body simulations

What are physical properties of galaxies (luminosity, color, morphology)

One needs separate HOD for each sub-class of galaxies...

Introduce Conditional Luminosity Function, $\Phi(L|M)$, which expresses average number of galaxies with luminosity L that reside in halo of mass M

The Conditional Luminosity Function

The CLF $\Phi(L|M)$ is the direct link between halo mass function $n(M)$ and the galaxy luminosity function $\Phi(L)$:

$$\Phi(L) = \int_0^\infty \Phi(L|M)\, n(M)\, \mathrm{d}M$$

The CLF contains a lot of important information, such as:

- halo occupation numbers as function of luminosity:

$$N_M(L > L_1) = \int_{L_1}^\infty \Phi(L|M)\, \mathrm{d}L$$

- The average relation between light and mass:

$$\langle L \rangle(M) = \int_0^\infty \Phi(L|M)\, L\, \mathrm{d}L$$

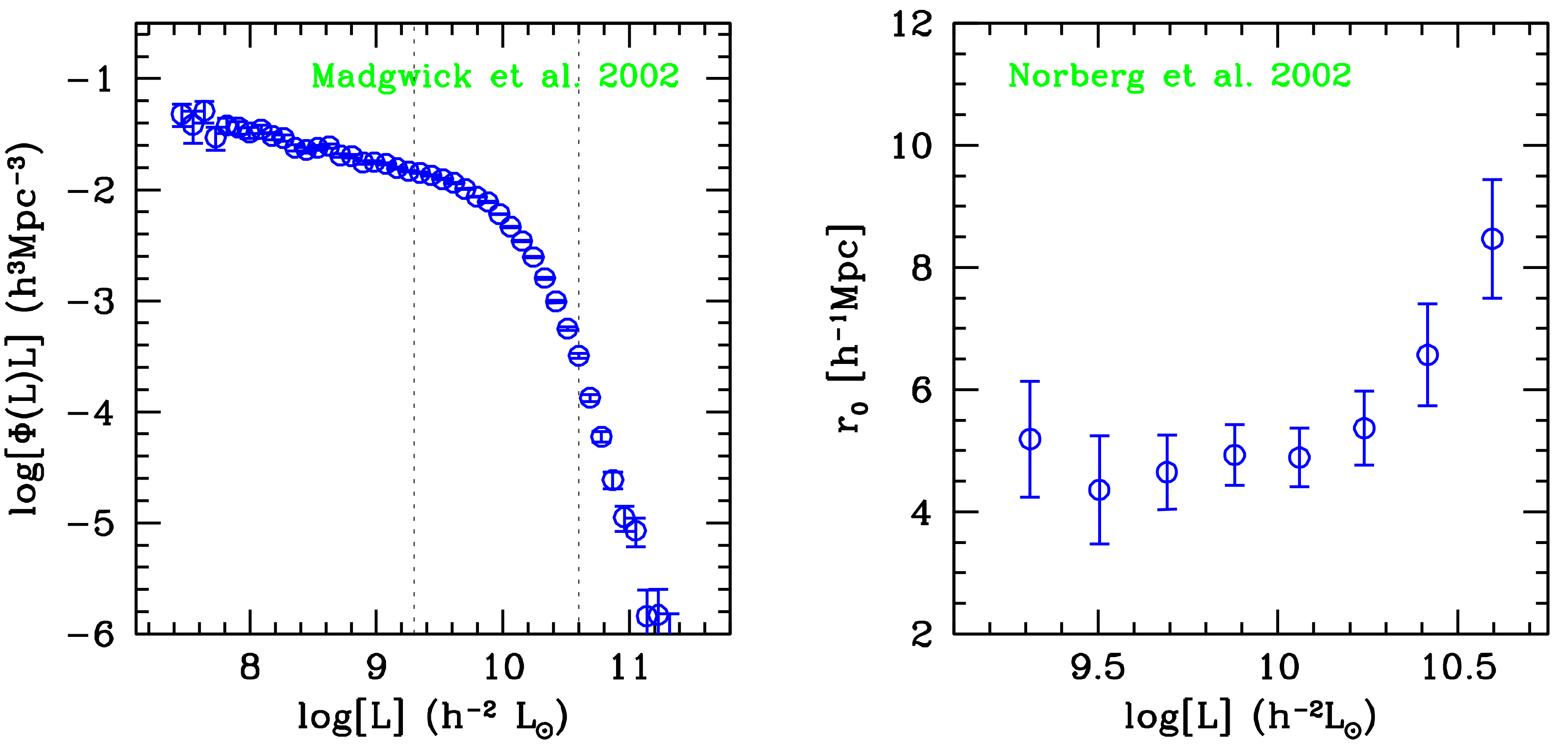
- The bias of galaxies as function of luminosity:

$$\xi_{\mathrm{gg}}(r|L) = b^2(L)\, \xi_{\mathrm{dm}}(r)$$

$$b(L) = \frac{1}{\Phi(L)} \int_0^\infty \Phi(L|M)\, b(M)\, n(M)\, \mathrm{d}M$$

CLF is ideal statistical ‘tool’ to investigate Galaxy-Dark Matter Connection

Luminosity & Correlation Functions



- **2dFGRS:** More luminous galaxies are more strongly clustered.
- **ΛCDM:** More massive haloes are more strongly clustered.

More luminous galaxies reside in more massive haloes

REMINDER: Correlation length r_0 defined by $\xi(r_0) = 1$

The CLF Model

- The LFs of clusters are well fit by a Schechter function
- The LF of all field galaxies has a Schechter form
- The halo mass function has a Press-Schechter form

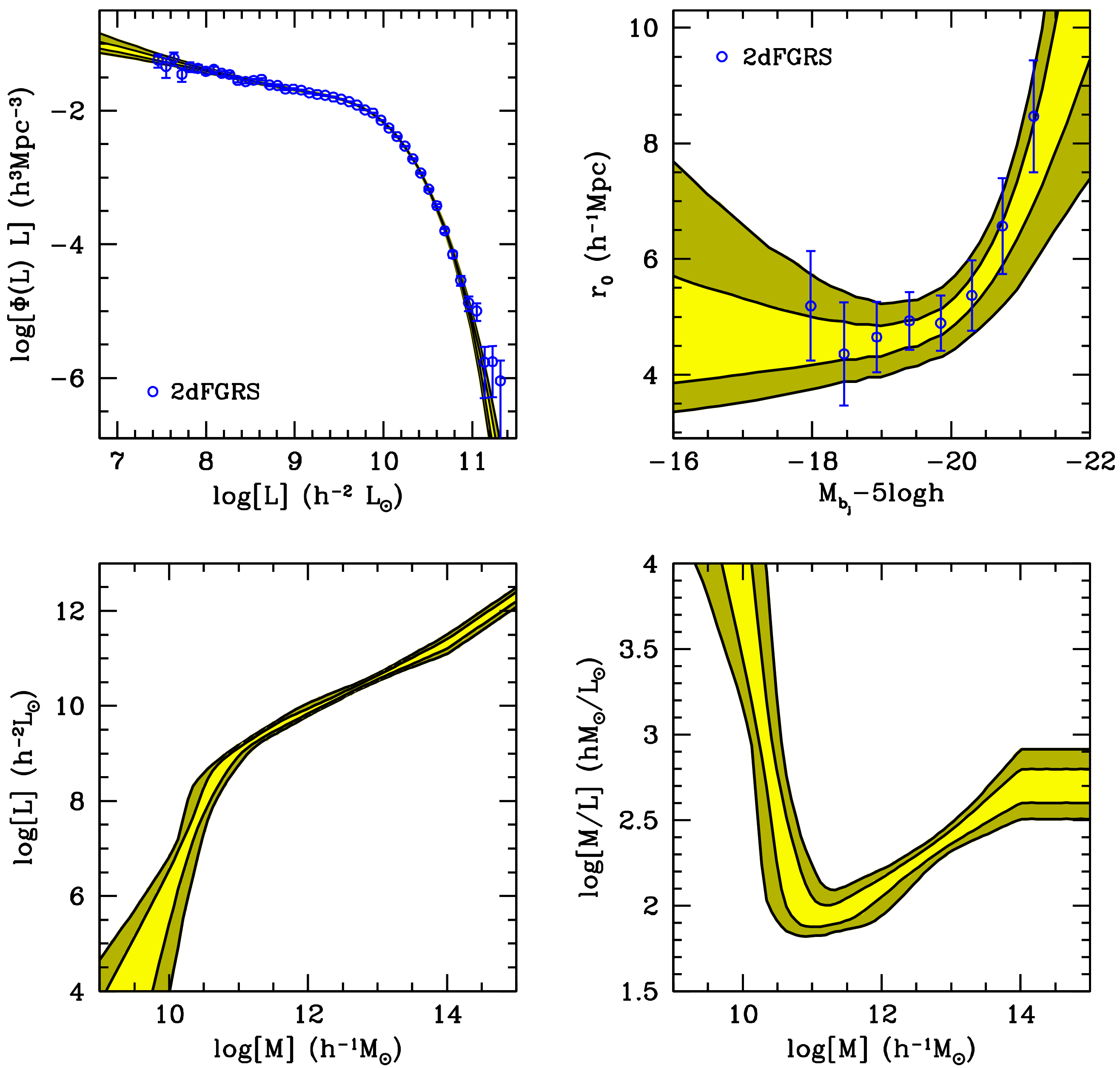
We therefore assume that the CLF also has the Schechter form:

$$\Phi(L|M)\mathrm{d}L = \frac{\tilde{\Phi}^*}{\tilde{L}^*} \left(\frac{L}{\tilde{L}^*}\right)^{\tilde{\alpha}} \exp(-L/\tilde{L}^*)\,\mathrm{d}L$$

Here $\tilde{\Phi}^*$, $\tilde{L}^*$ and $\tilde{\alpha}$ all depend on M.

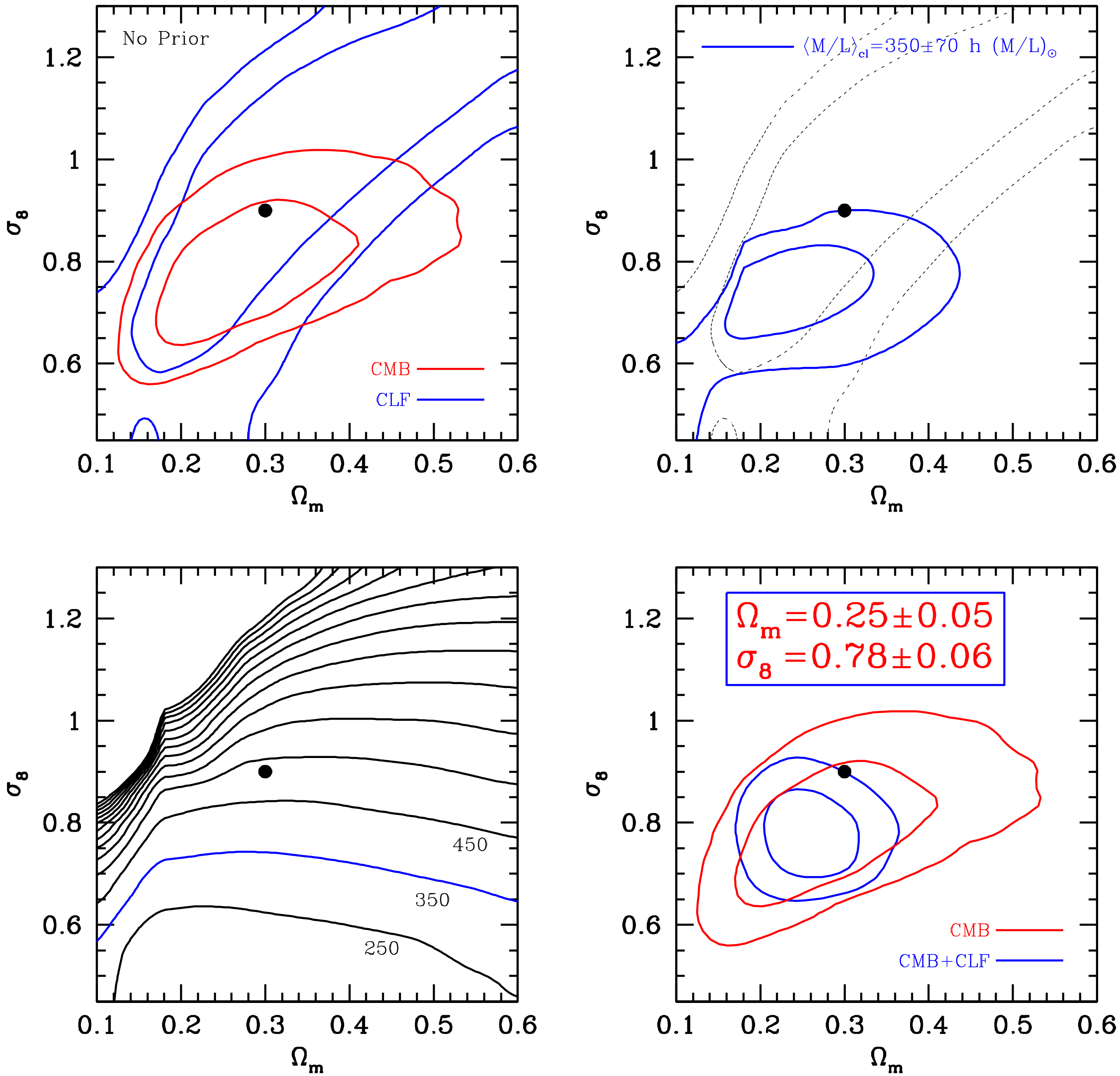
- Parameterize $\tilde{\Phi}^*$, $\tilde{L}^*$ and $\tilde{\alpha}$. In total our model has 8 free parameters
- Construct Monte-Carlo Markov Chain to sample posterior distribution of free parameters. ($N_{\mathrm{eq}} = 10^4$, $N_{\mathrm{step}} = 4 \times 10^7$, $N_{\mathrm{chain}} = 2000$)
- Use MCMC to put confidence levels on derived quantities such as $\langle M/L\rangle(M)$ and $\tilde{\alpha}(M)$.
- Use MCMC to explore degeneracies and correlations between various parameters.

The Relation between Light & Mass



vdB, Yang, Mo & Norberg, 2005, MNRAS, 356, 1233

Cosmological Constraints



vdB, Mo & Yang, 2003, MNRAS, 345, 923

See also Tinker et al. 2005; Vale & Ostriker 2005

HODs from Galaxy Groups

Halo Occupation Statistics can also be obtained directly from galaxy groups

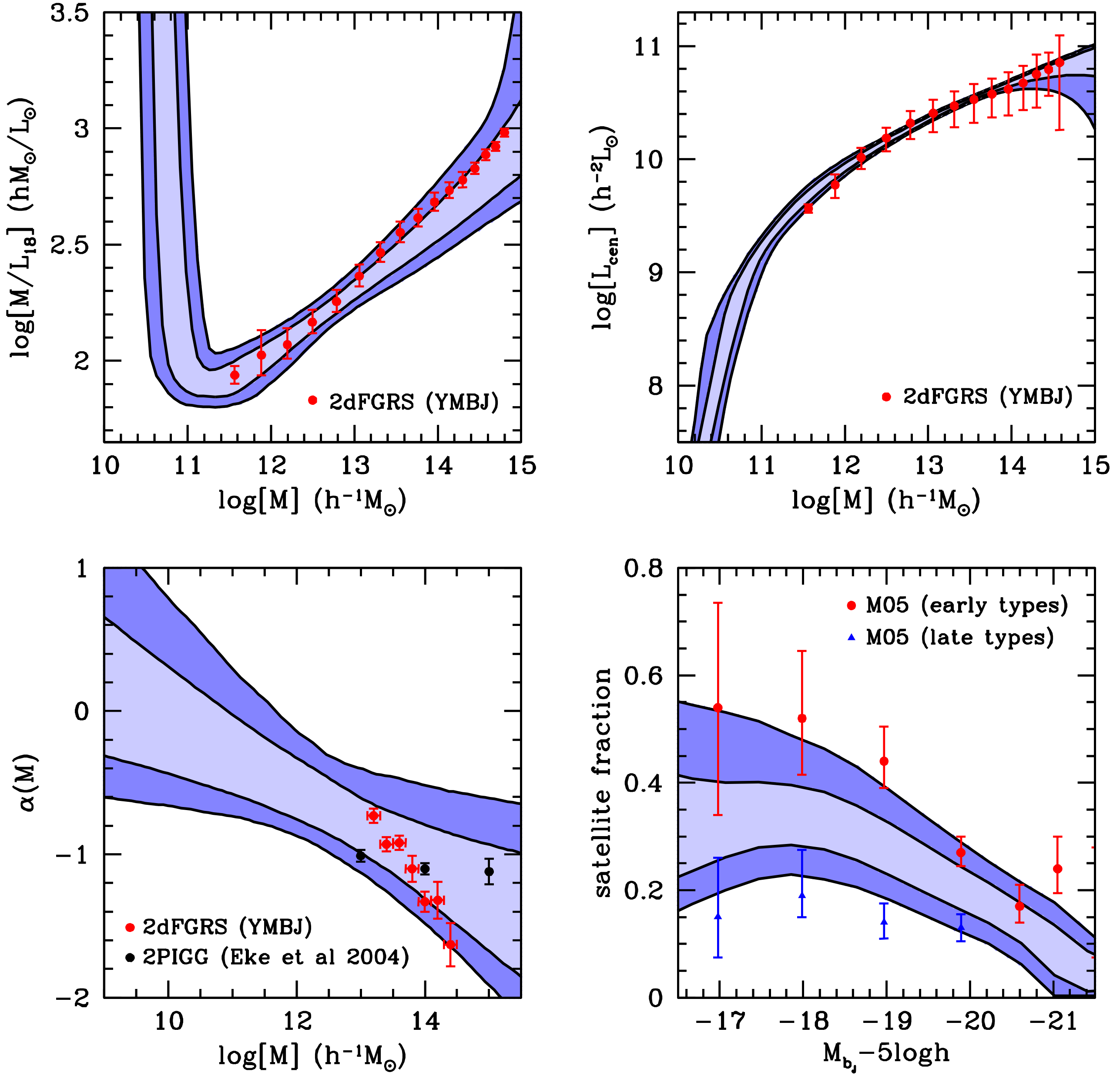
Potential Problems: interlopers, (in)completeness, mass estimates

We developed new, iterative group finder, using an adaptive filter modeled after halo virial properties Yang, Mo, vdB, Jing 2005, MNRAS, 356, 1293

- Calibrated & Optimized with Mock Galaxy Redshift Surveys
- Low interloper fraction ($\lesssim 20\%$).
- High completeness of members ($\gtrsim 90\%$).
- Masses estimated from group luminosities.
  More accurate than using velocity dispersion of members.
- Can also detect “groups” with single member
  ▷ Large dynamic range ($11.5 \lesssim \log[M] \lesssim 15$).

Group finder has been applied to both the 2dFGRS (completed survey) and to the SDSS (DR2, NYU-VAGC; Blanton et al. 2005)

The Relation between Light & Mass



YMBJ = Yang, Mo, vdB & Jing, 2005

vdB et al. 2006, in prep.

M05 = Mandelbaum et al. 2005

Galaxy Ecology

Many studies have investigated relation between various galaxy properties (morphology / SFR / colour) and environment

(e.g., Dressler 1980; Balogh et al. 2004; Goto et al. 2003; Hogg et al. 2004)

Environment estimated using galaxy overdensity (projected) to $n^{\rm th}$ nearest neighbour, Σ_n or using fixed, metric aperture, Σ_R.

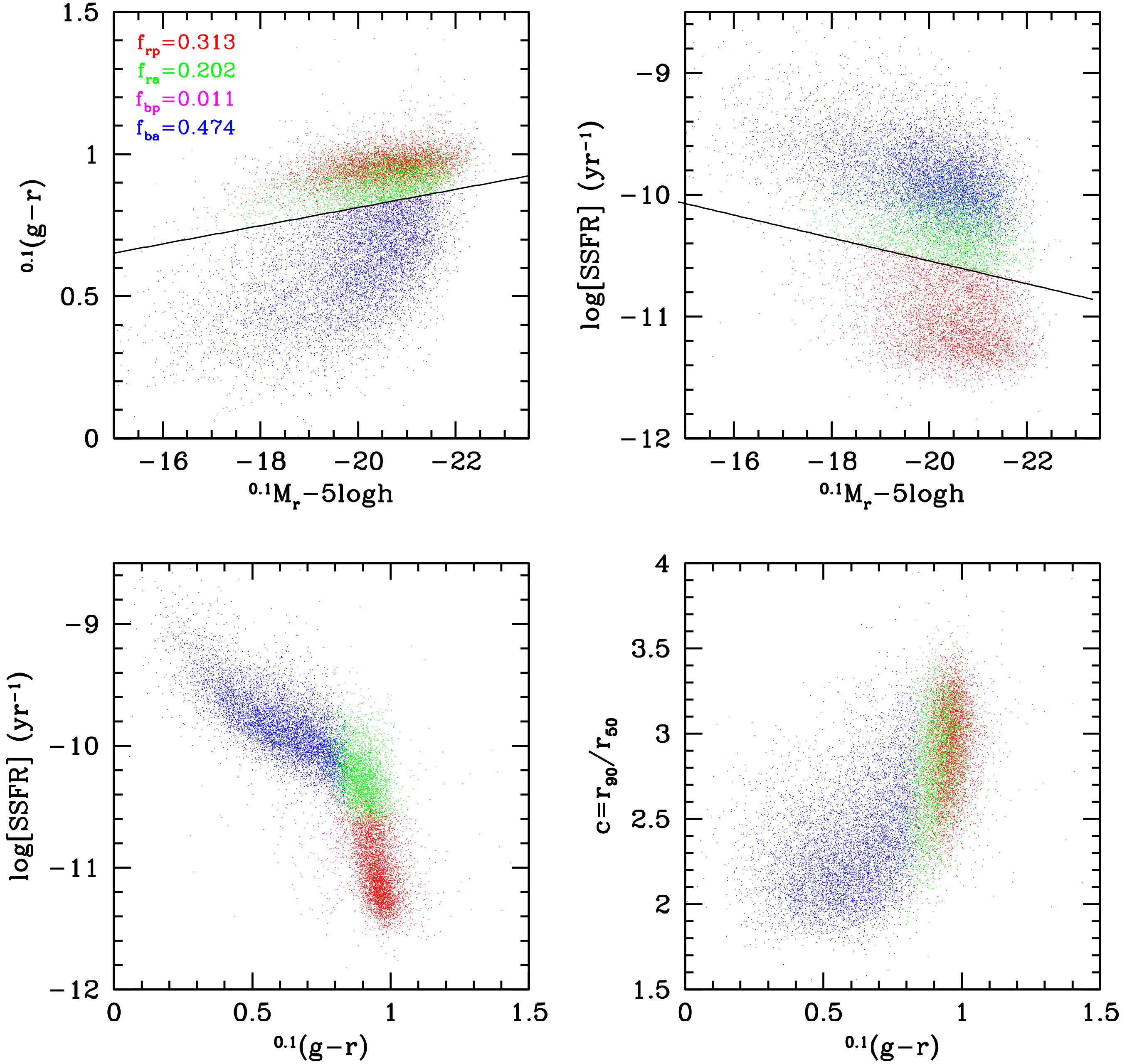
- Fraction of early types increases with density
- There is a characteristic density ($\sim$ group-scale) below which environment dependence vanishes
- Groups and Clusters reveal radial dependence:
  late type fraction increases with radius
- No radial dependence in groups with $M \lesssim 10^{13.5} h^{-1}\,{\rm M}_{\odot}$

Danger: Physical meaning of Σ_n and Σ_R depends on environment.

Physically more meaningful to investigate halo mass dependence of galaxy properties. This requires galaxy group catalogues.

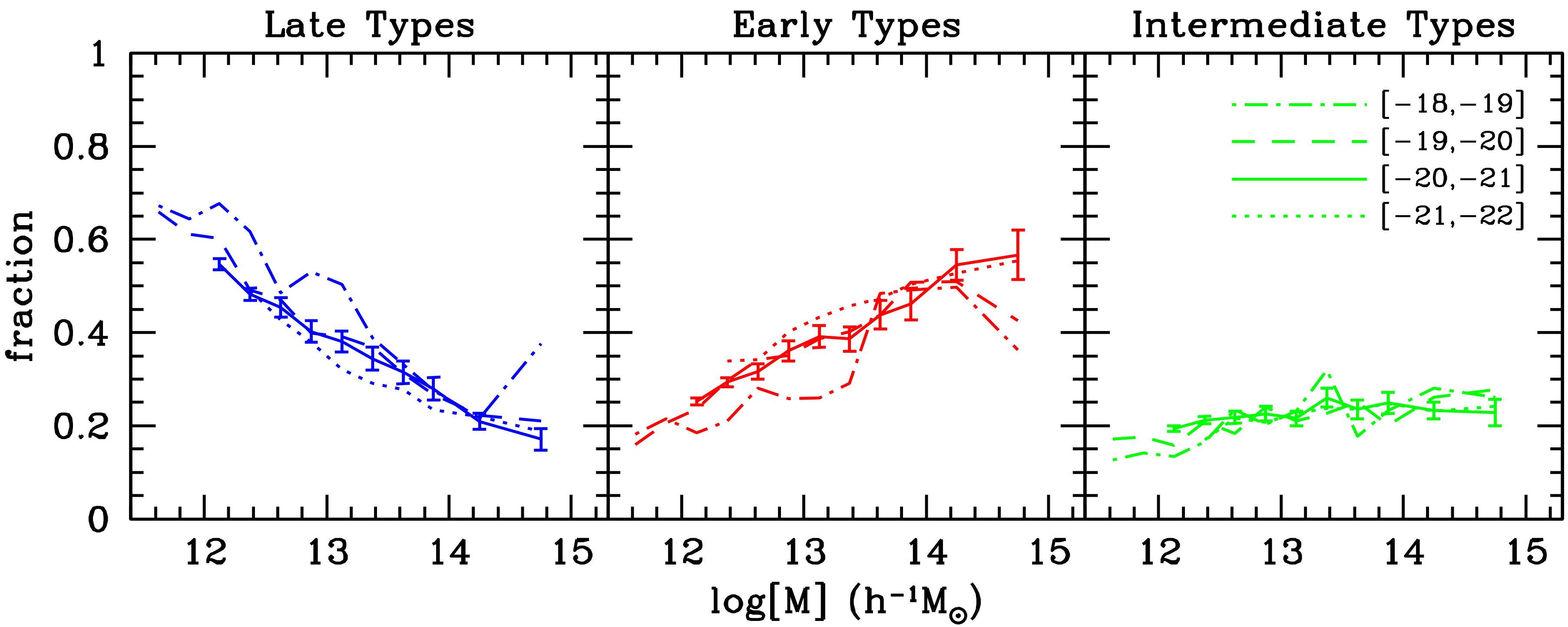
Important: Separate luminosity dependence from halo mass dependence.

Defining Galaxy Types



Data from NYU-VAGC (Blanton et al. 2005): SSFRs from Kauffmann et al. (2003) and Brinchmann et al. (2004)

Halo Mass Dependence



The fractions of early and late type galaxies depend strongly on halo mass.

At fixed halo mass, there is virtually no luminosity dependence.

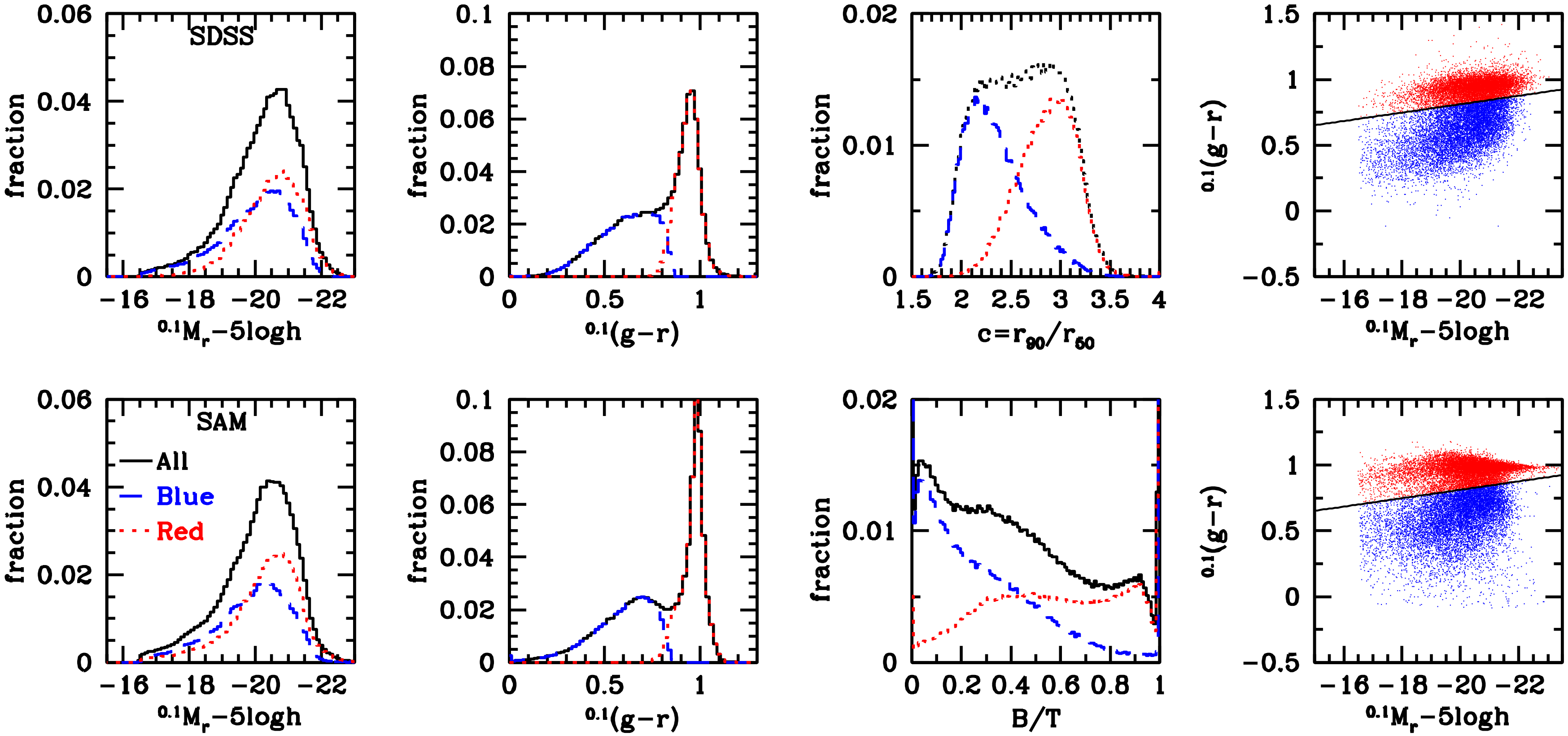
The mass dependence is smooth: there is no characteristic mass scale

The intermediate type fraction is independent of luminosity and mass.

(Weinmann, vdB, Yang & Mo, 2006)

Comparison with Semi-Analytical Model

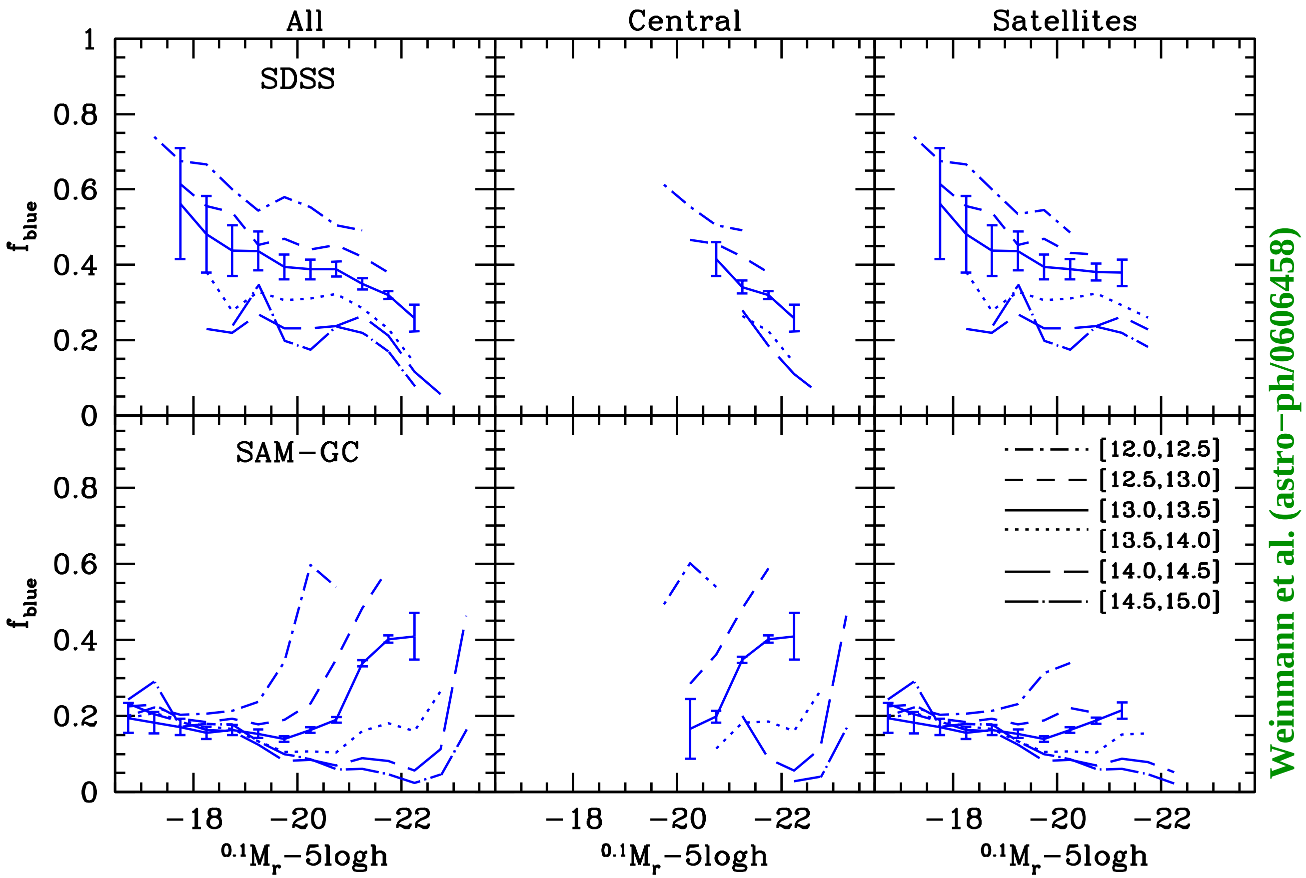
Comparison of Group Occupation Statistics with Semi-Analytical Model of Croton et al. 2006. Includes ‘radio-mode’ AGN feedback.



- **SAM matches global statistics of SDSS**
- **Luminosity function, bimodal color distribution, and overall blue fraction**
- **But what about statistics as function of halo mass?**

Constraining Star Formation Truncation

To allow for fair comparison, we run our Group Finder over SAM.



Satellites: red fraction too large: ▷ **strangulation too efficient as modelled**

Centrals: $f_{\rm blue}(L|M)$ **wrong:** ▷ **AGN feedback/dust modelling wrong**

$f_{\rm blue}(L, M)$ **useful to constrain SF truncation mechanism**

Conclusions

Galaxy Bias = Halo Bias + Halo Occupation Statistics

Halo Occupation Statistics can be modeled & constrained using:

- Halo Occupation Distribution (HOD) $P(N|M)$
- Conditional Luminosity Function (CLF) $\Phi(L|M)$

or it can be ‘measured’ directly using galaxy groups

Halo Model and/or Halo Occupation Statistics can:

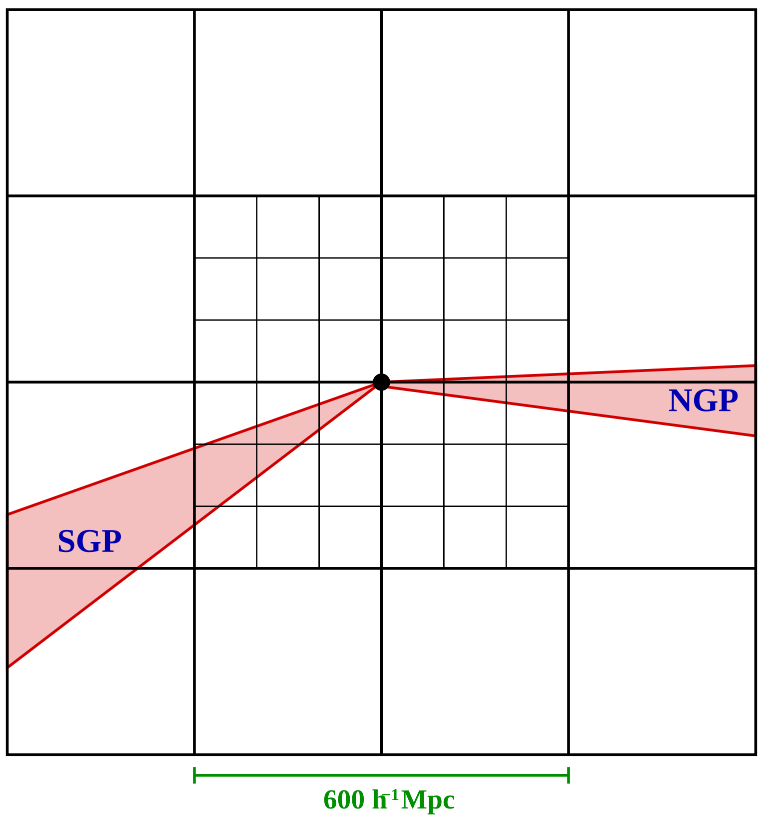
- Constrain Cosmological Parameters
- Constrain Galaxy Formation

In the near future we will be able to

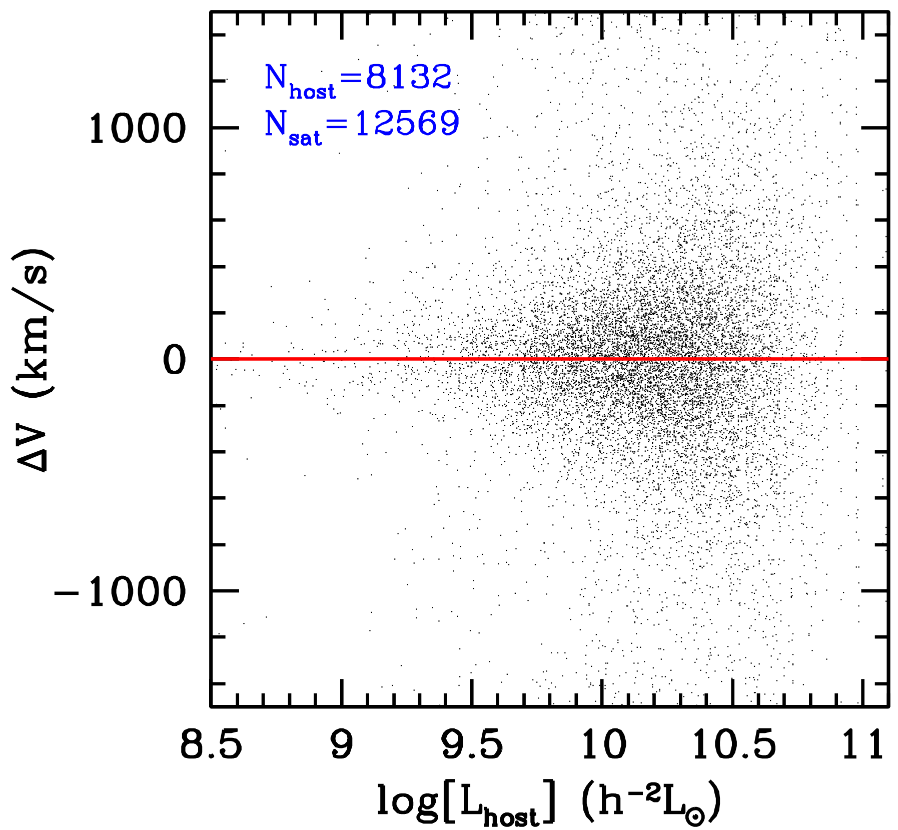
- Constrain galaxy bias as function of redshift
- Obtain independent constraints from galaxy-galaxy lensing

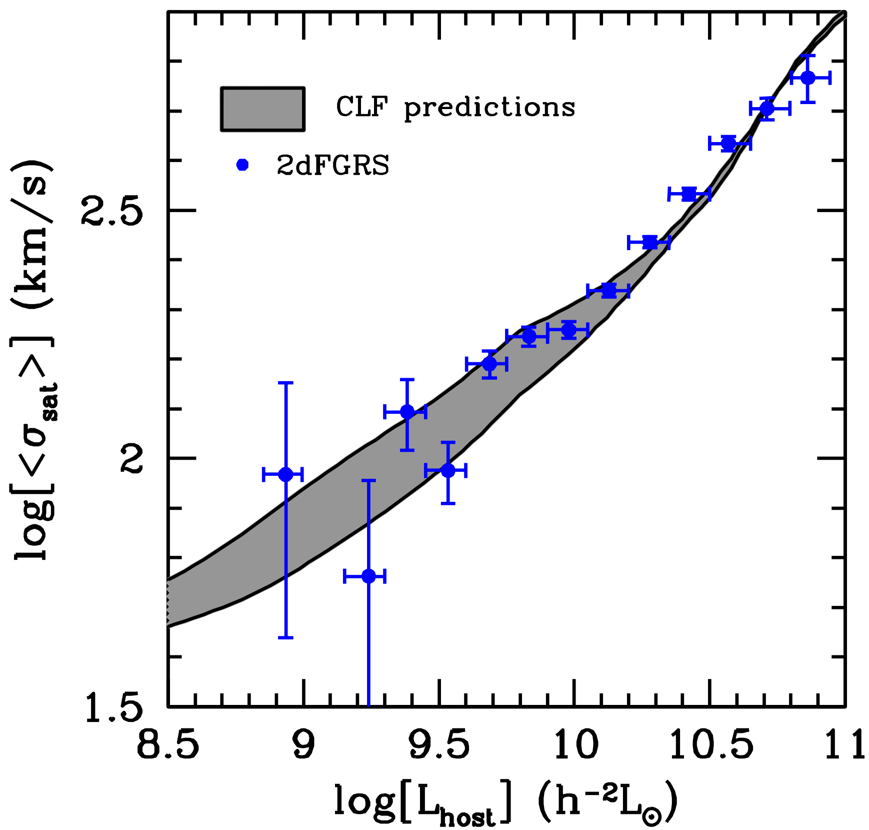
Constructing Mock Surveys

- Run **numerical simulations**: ΛCDM concordance cosmology (WMAP1) $L_{\rm box} = 100h^{-1}$ Mpc and $300h^{-1}$ Mpc with 512^3 CDM particles each.
- Identify **dark matter haloes** with (FOF algorithm.
- **Populate haloes** with galaxies using CLF.
- Stack boxes to create **virtual universe** and mimick observations (magnitude limit, completeness, geometry, fiber collisions)



Satellite Kinematics in the 2dFGRS



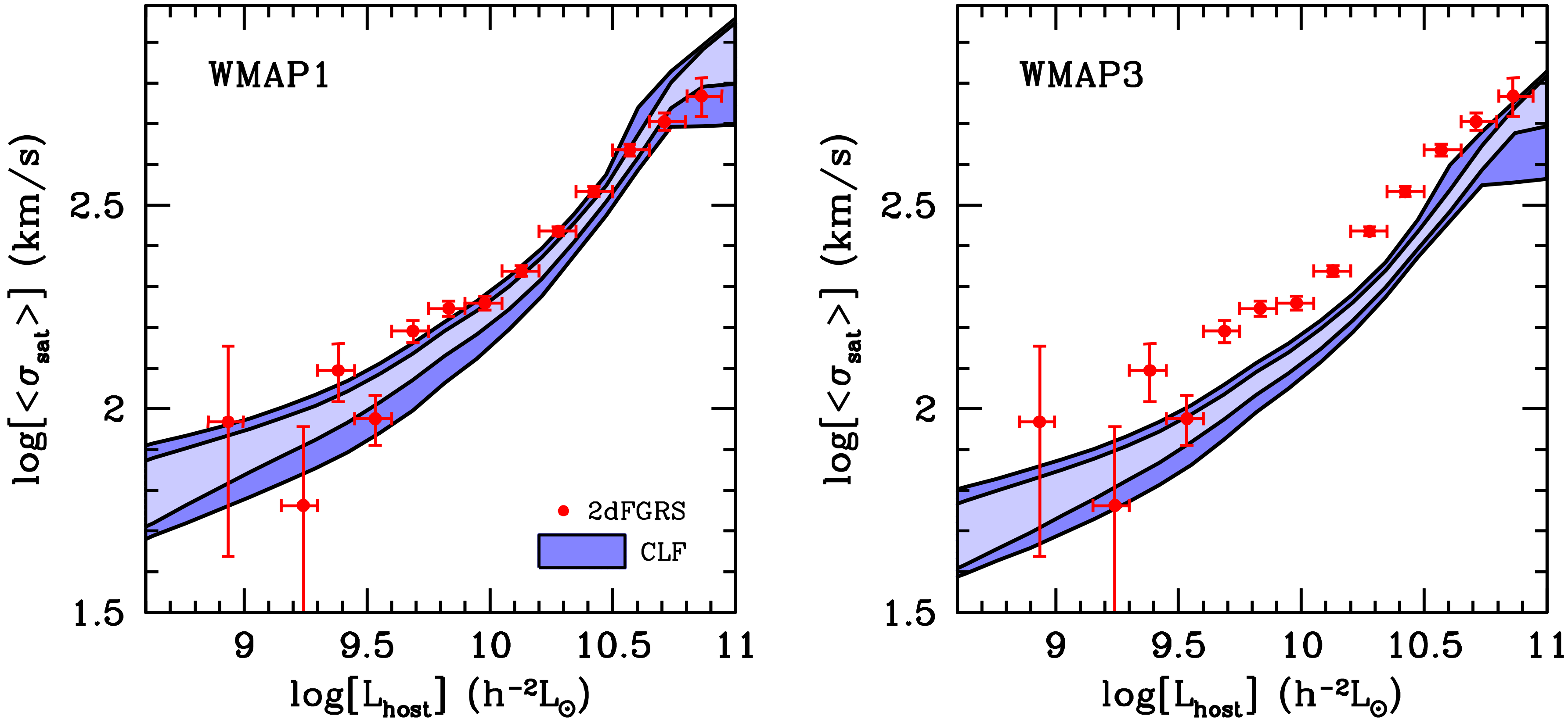


- **Mocks are used to optimize host-satellite selection criteria**
- **Using an iterative, adaptive selection criterion minimizes interlopers**
- **Application to 2dFGRS yields 12569 satellites & 8132 hosts**
- **Independent dynamical evidence to support WMAP1-CLF results**

vdB, Norberg, Mo & Yang, 2004, MNRAS, 352, 1302

vdB, Yang, Mo & Norberg, 2005, MNRAS, 356, 1233

Problems for the WMAP3 Cosmology?



- In **WMAP3** cosmology, haloes have lower mass-to-light ratios and are less concentrated.
- **WMAP3-CLF** underpredicts satellite velocity dispersions by $\sim 30\%$
- But, $L_{\rm cen}(M)$ in good agreement with group-data....
- Central galaxies do **not** reside at rest at center of halo.

vdB et al. 2006, in preparation

Brightest Halo Galaxies

Paradigm: Brightest Galaxy in halo resides at rest at center

In order to test this **Central Galaxy Paradigm**, we compare the velocity of central galaxy to the average velocity of the satellites. Define

$$\mathcal{R} = \frac{N_s(v_c - \bar{v}_s)}{\hat{\sigma}_s}$$

with $\bar{v}_s = \frac{1}{N_s}\sum_{i=1}^{N_s} v_i$ and $\hat{\sigma}_s = \sqrt{\frac{1}{N_s-1}\sum_{i=1}^{N_s}(v_i - \bar{v}_s)^2}$.

If **Central Galaxy Paradigm** is correct, $P(\mathcal{R})$ follows a Student t-distribution with $N_s - 1$ degrees of freedom

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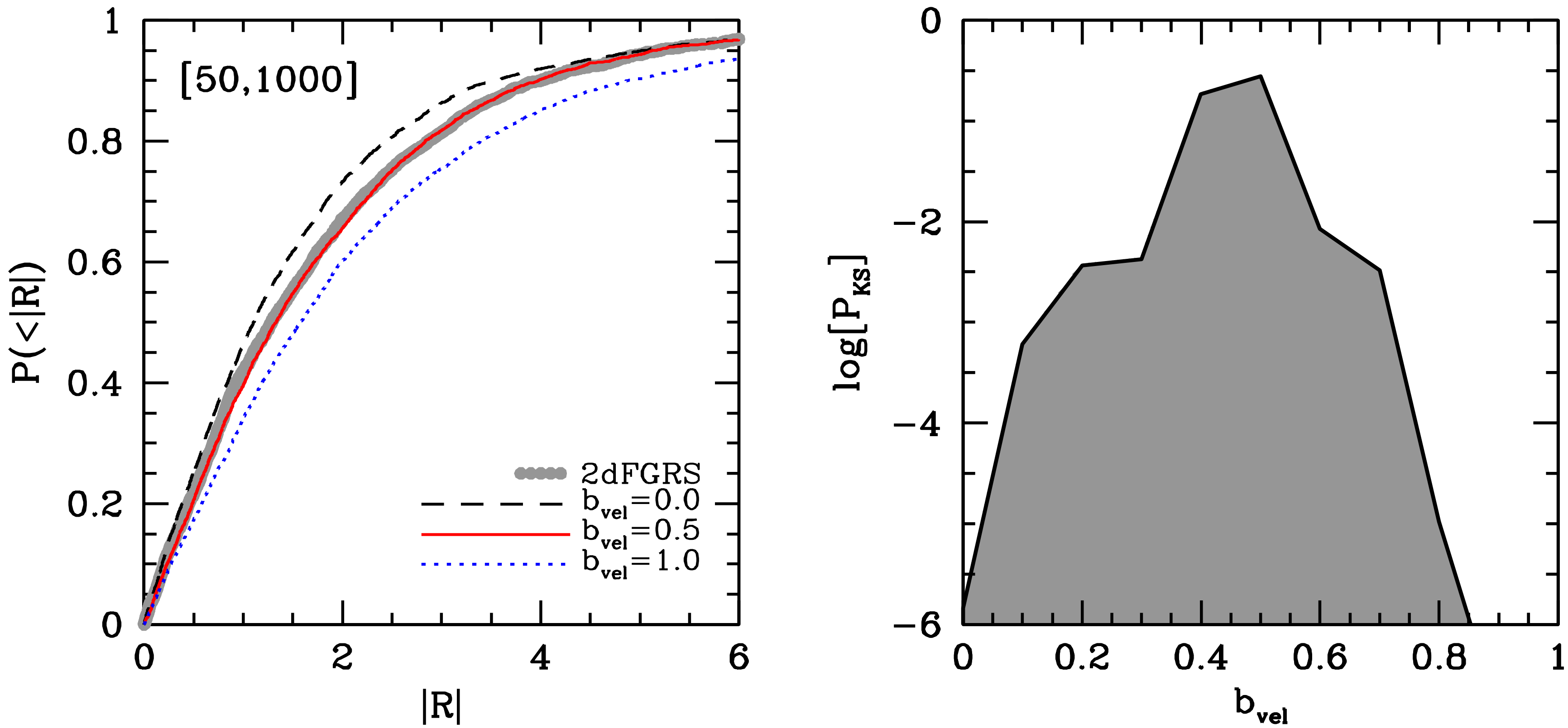
IMPORTANT: Applicability of this $\mathcal{R}$-test depends strongly on ability to find those galaxies that belong to same halo.

PROBLEM: Interlopers and incompleteness effects

SOLUTION: Use halo-based group finder and mock galaxy redshift surveys

DATA: Both 2dFGRS (Final Data Release) and SDSS (DR2, NYU-VAGC)

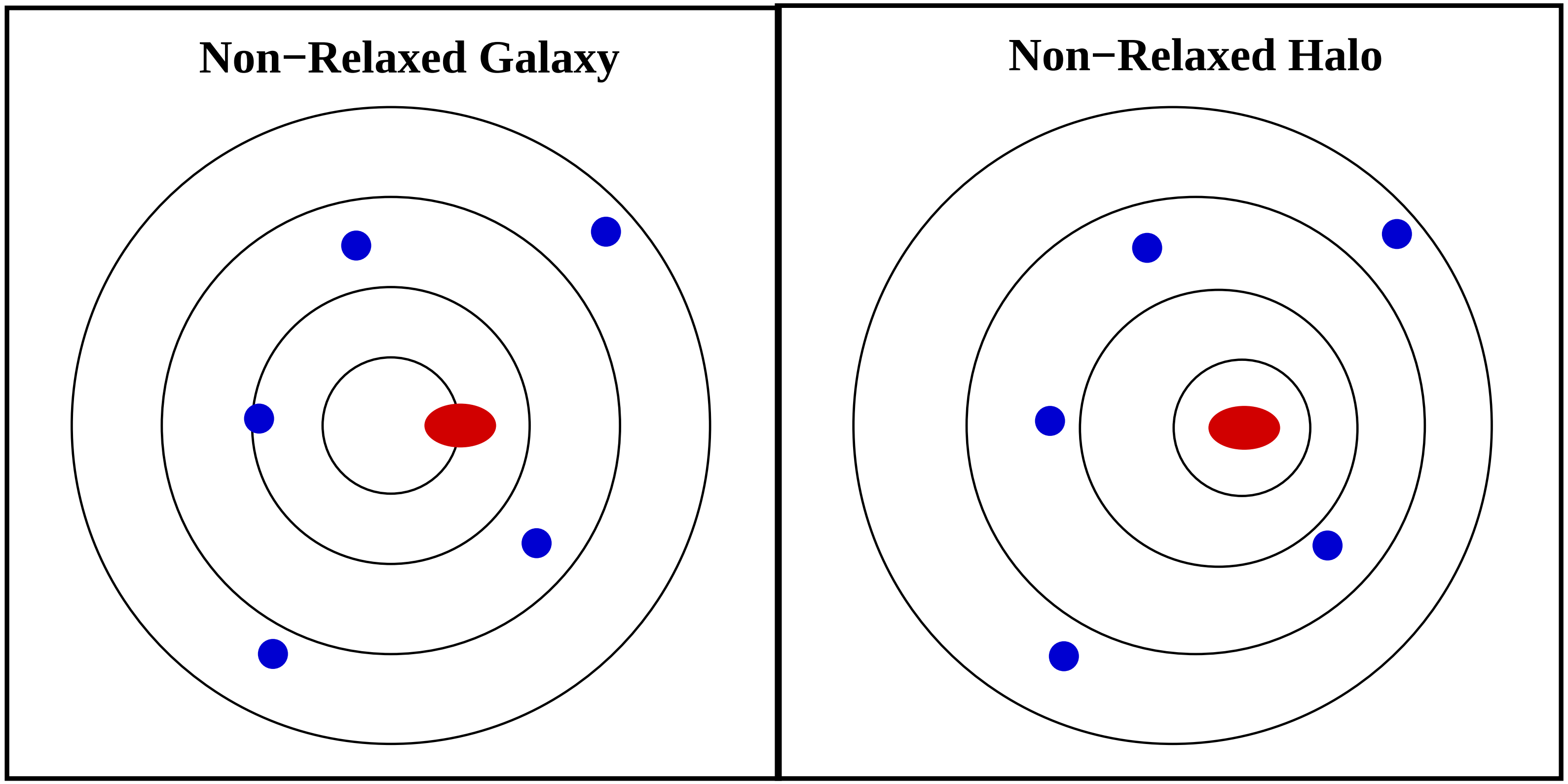
Evidence against Central Galaxy Paradigm



- We construct ten MGRSs, that only differ in the velocity bias ($b_{\rm vel}$) of the brightest halo galaxy
- The $P(\mathcal{R})$ of 2dFGRS is best reproduced by MGRS with $b_{\rm vel} = 0.5$
- The null-hypothesis of the Central Galaxy Paradigm is ruled out at strong confidence: $P_{\rm KS} = 1.5 \times 10^{-6}$
- Best-fit value of $b_{\rm vel} = 0.5$ suggests that specific kinetic energy of central galaxies is $\sim$ 25% of that of satellites

vdB et al. 2005, MNRAS, 361, 1203

Implications



- Brightest halo galaxy either oscillates in relaxed halo, or resides at potential minimum of non-relaxed halo.
- Strong gravitational lensing (external shear?)
- Distortions in disk galaxies (lopsidedness & bars)
- Satellite kinematics $\sigma_{\rm sat} = \sqrt{1 + b_{\rm vel}}\,\sigma_{\rm dm}$
  with $b_{\rm vel} = \langle |v_{\rm cen}|^2 \rangle / \sigma_{\rm sat}$

Large Scale Structure: Theory

Galaxy redshift surveys yield $\xi(r_p, \pi)$ with r_p and π the pair separations perpendicular and parallel to the line-of-sight.

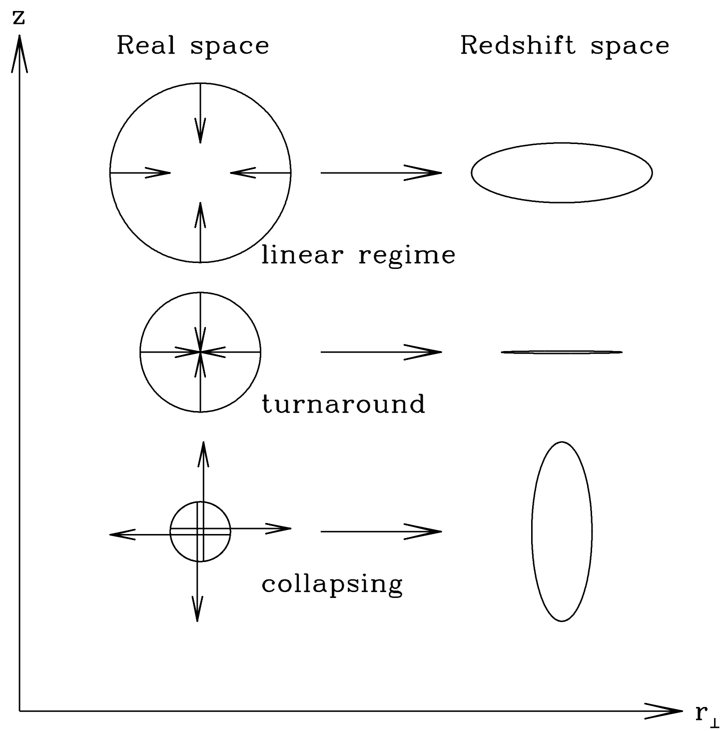
redshift space CF: $\xi(s)$ with $s = \sqrt{r_p^2 + \pi^2}$

projected CF: $$w_p(r_p) = \int_{-\infty}^{\infty} \xi(r_p, \pi) d\pi = 2 \int_{r_p}^{\infty} \xi(r) \frac{r\, dr}{\sqrt{r^2 - r_p^2}}$$

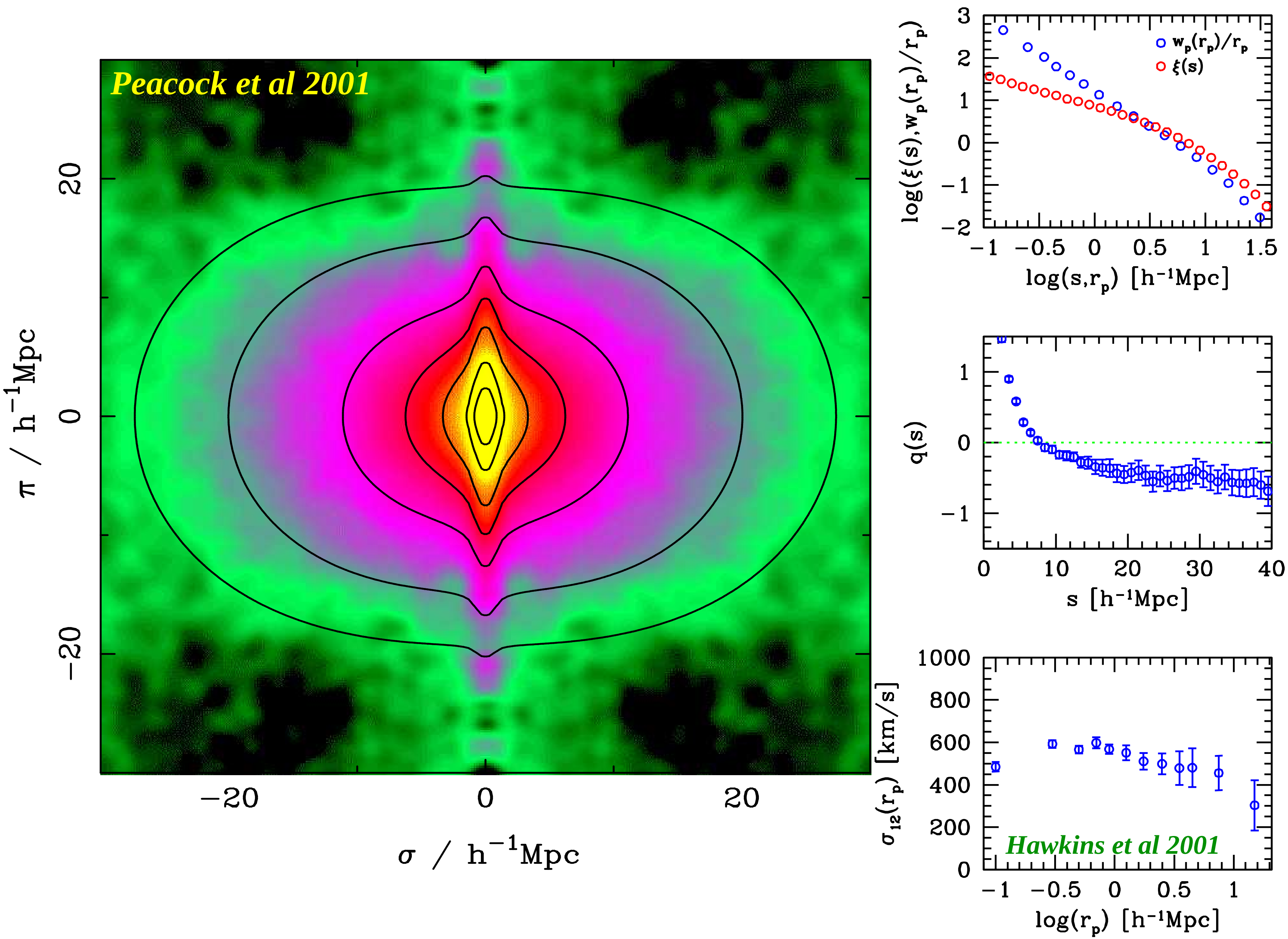
Peculiar velocities cause $\xi(r_p, \pi)$ to be anisotropic.
Consequently, $\xi(s) \neq \xi(r)$.

In particular, there are two effects:

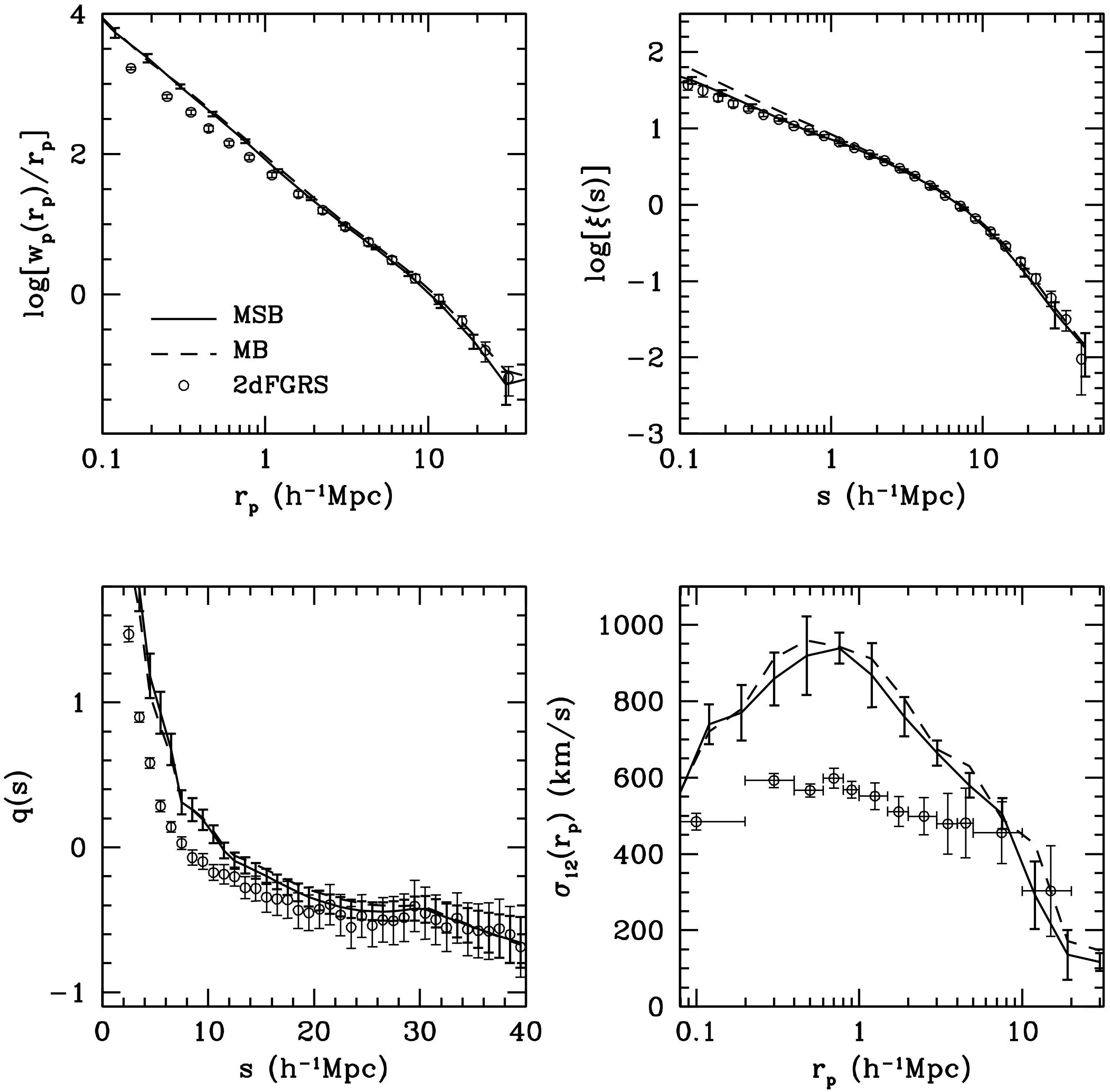
- **Large Scales: Infall ("Kaiser Effect")**
- **Small Scales: "Finger-of-God-effect"**



Large Scale Structure: The 2dFGRS

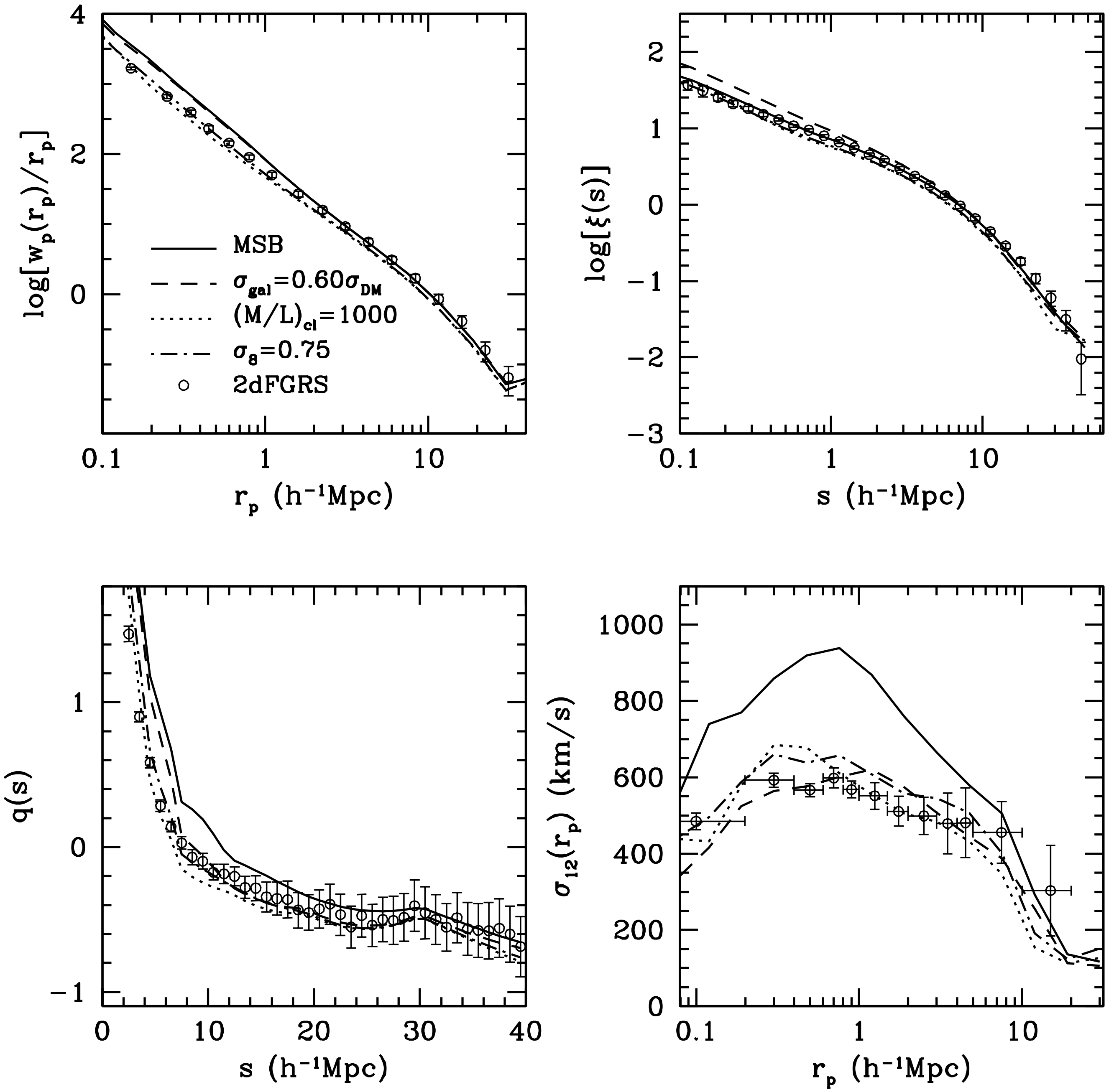


Mock versus 2dFGRS: round 1



Yang, Mo, Jing, vdB & Chu, 2004, MNRAS, 350, 1153

Mock versus 2dFGRS: round 2

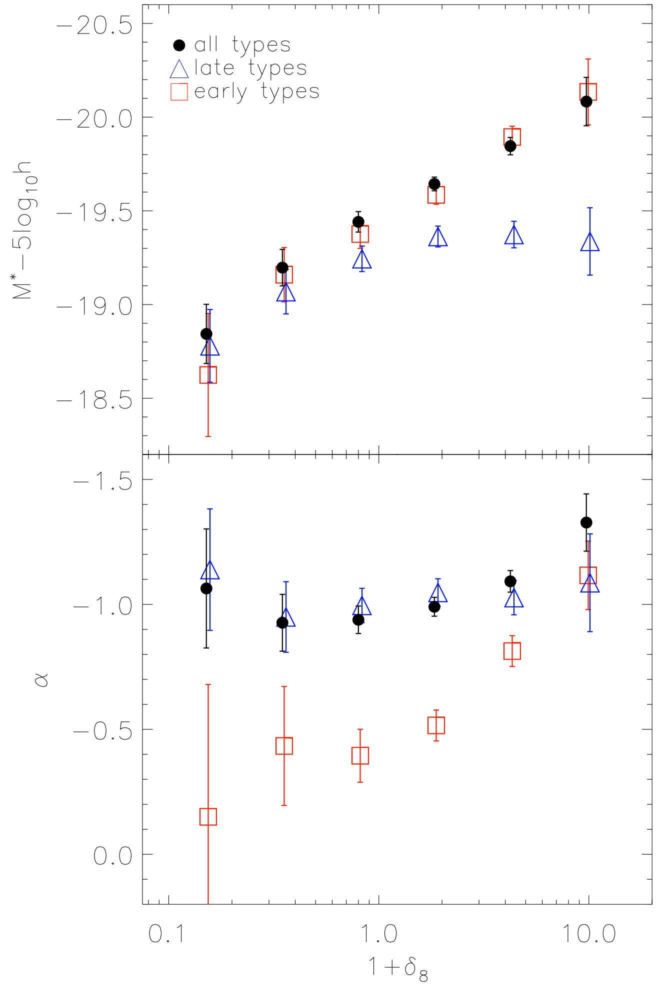


Yang, Mo, Jing, vdB & Chu, 2004, MNRAS, 350, 1153

Large-Scale Environment Dependence

Inherent to CLF formalism is assumption that L depends only on M.

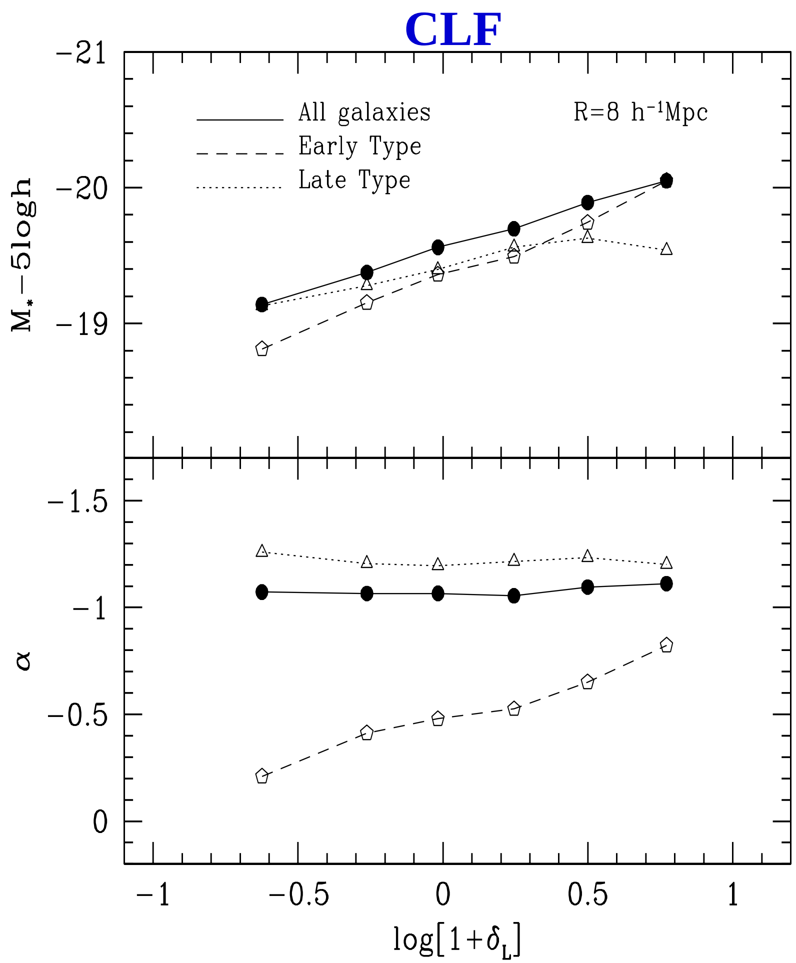
But $\Phi(L)$ has been shown to depend on large scale environment

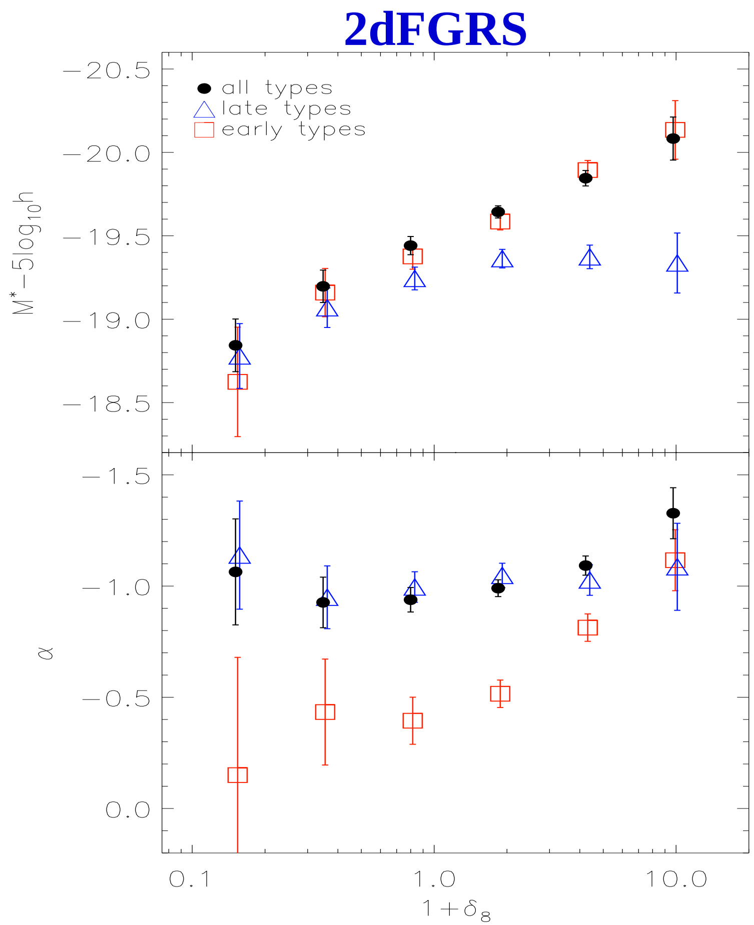


Croton et al. 2005

Does this violate the implicit assumptions of the CLF formalism?

Large-Scale Environment Dependence





Populate haloes in N-body simulations with galaxies using $\Phi(L|M)$

Compute $\Phi(L)$ as function of environment and type as in Croton et al. (2005)

Because $n(M)$ depends on environment, we reproduce observed trend

There is no environment dependence, only halo-mass dependence

Theoretical Expectations

From the fact that

$$\delta_h(m) \equiv \frac{n(m|\delta)}{n(m)} - 1 = b(m)\delta$$

we obtain that

$$n(m|\delta) = [1 + b(m)\delta]\; n(m)$$

Since the halo bias $b(m)$ is an increasing function of halo mass, the abundance of more massive haloes is more strongly boosted in overdense regions than that of less massive haloes

In other words; massive haloes live in overdense regions

If more massive haloes host more luminous galaxies, we thus expect that the luminosity function of galaxies also depends on environment

Correlation Functions

Define the dimensionless density perturbation field: $\delta(\vec{x}) = \frac{\rho(\vec{x})-\bar{\rho}}{\bar{\rho}}$

For a Gaussian random field, the one-point probability function is:

$$P(\delta)\mathrm{d}\delta = \frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{\delta^2}{2\sigma^2}\right]\mathrm{d}\delta$$

$$\begin{aligned} \langle\delta\rangle &= \int \delta P(\delta)\mathrm{d}\delta &= 0 \\ \langle\delta^2\rangle &= \int \delta^2 P(\delta)\mathrm{d}\delta &= \sigma^2 \end{aligned}$$

Define n-point probability function: $P_n\left(\delta_1, \delta_2, \cdots, \delta_n\right)\, \mathrm{d}\delta_1\, \mathrm{d}\delta_2 \cdots\, \mathrm{d}\delta_n$

Gravity induces correlations between δ_i so that

$$P_n\left(\delta_1, \delta_2, \cdots, \delta_n\right) \neq \prod_{i=1}^{n} P(\delta_i)$$

Correlations are specified via n-point correlation function:

$$\langle\delta_1\delta_2\cdots\delta_n\rangle = \int \delta_1\delta_2\cdots\delta_n P_n\left(\delta_1, \delta_2, \cdots, \delta_n\right)\, \mathrm{d}\delta_1\, \mathrm{d}\delta_2 \cdots\, \mathrm{d}\delta_n$$

In particular, we will often use the two-point correlation function

$$\xi(x) = \langle\delta_1\delta_2\rangle \qquad \text{with } x = |\vec{x}_1 - \vec{x}_2|$$

Power Spectrum

It is useful to write $\delta(\vec{x})$ as a Fourier series:

$$\delta(\vec{x}) = \sum_{\vec{k}} \delta_{\vec{k}} \mathrm{e}^{i\vec{k}\cdot\vec{x}} \qquad \delta_{\vec{k}} = \frac{1}{V}\int \delta(\vec{x})\mathrm{e}^{-i\vec{k}\cdot\vec{x}}\mathrm{d}^3\vec{x}$$

Note that $\delta_{\vec{k}}$ are complex quantities: $\delta_{\vec{k}} = |\delta_{\vec{k}}|\mathrm{e}^{i\theta_{\vec{k}}}$

Decomposition in Fourier modes is preserved during linear evolution, so that

$$P_n\left(\delta_{\vec{k}_1}, \delta_{\vec{k}_2}, \cdots, \delta_{\vec{k}_n}\right) = \prod_{i=1}^{n} P(\delta_{\vec{k}_i})$$

Thus, statistical properties of $\delta(\vec{x})$ completely specified by $P(\delta_{\vec{k}})$

A Gaussian random field is completely specified by first two moments:

$$\begin{aligned} \langle \delta_{\vec{k}} \rangle &= 0 \\ \langle |\delta_{\vec{k}}|^2 \rangle &= P(k) \qquad \text{Power Spectrum} \\ \langle \delta_{\vec{k}} \delta_{\vec{p}} \rangle &= 0 \qquad (\text{for } k \neq p) \end{aligned}$$

The power spectrum is Fourier Transform of two-point correlation function:

$$\xi(r) = \frac{1}{(2\pi)^3}\int P(k) e^{i\vec{k}\cdot\vec{r}} \mathrm{d}^3\vec{k} = \frac{1}{2\pi^2}\int_0^\infty P(k)\frac{\sin kr}{kr} k^2 \mathrm{d}k$$

Mass Variance

Let $\delta_M(\vec{x})$ be the density field $\delta(\vec{x})$ smoothed (convolved) with a filter of size $R_f \propto [M/\bar{\rho}]^{1/3}$.

Since convolution is multiplication in Fourier space, we have that

$$\delta_M(\vec{x}) = \sum_{\vec{k}} \delta_{\vec{k}} \, \widehat{W}_M(\vec{k}) \, \mathrm{e}^{i\vec{k}\cdot\vec{x}}$$

with $\widehat{W}_M(\vec{k})$ the FT of the filter function $W_M(\vec{x})$.

The mass variance is simply

$$\boxed{\sigma^2(M) = \langle \delta_M^2 \rangle = \frac{1}{2\pi^2} \int P(k) \widehat{W}_M^2(k) \, k^2 \mathrm{d}k}$$

Note that $\sigma^2(M) \rightarrow 0$ if $M \rightarrow \infty$.

Press-Schechter Formalism

In **CDM** universes, density perturbations grow, turn around from Hubble expansion, collapse, and virialize to form **dark matter halo**.

According to **spherical collapse model** the collapse occurs when

$$\delta_{\rm lin} = \delta_{\rm sc} \simeq \frac{3}{20}(12\pi)^{2/3} \simeq 1.686$$

$\delta_{\rm lin}$ is linearly extrapolated density perturbation field

$\delta_{\rm sc}$ is critical overdensity for spherical collapse.

Press-Schechter ansatz: if $\delta_{{\rm lin},M}(\vec{x}) > \delta_{\rm sc}$ then $\vec{x}$ is located in a halo with mass $> M$.

The probability that $\vec{x}$ is in a halo of mass $> M$ therefore is

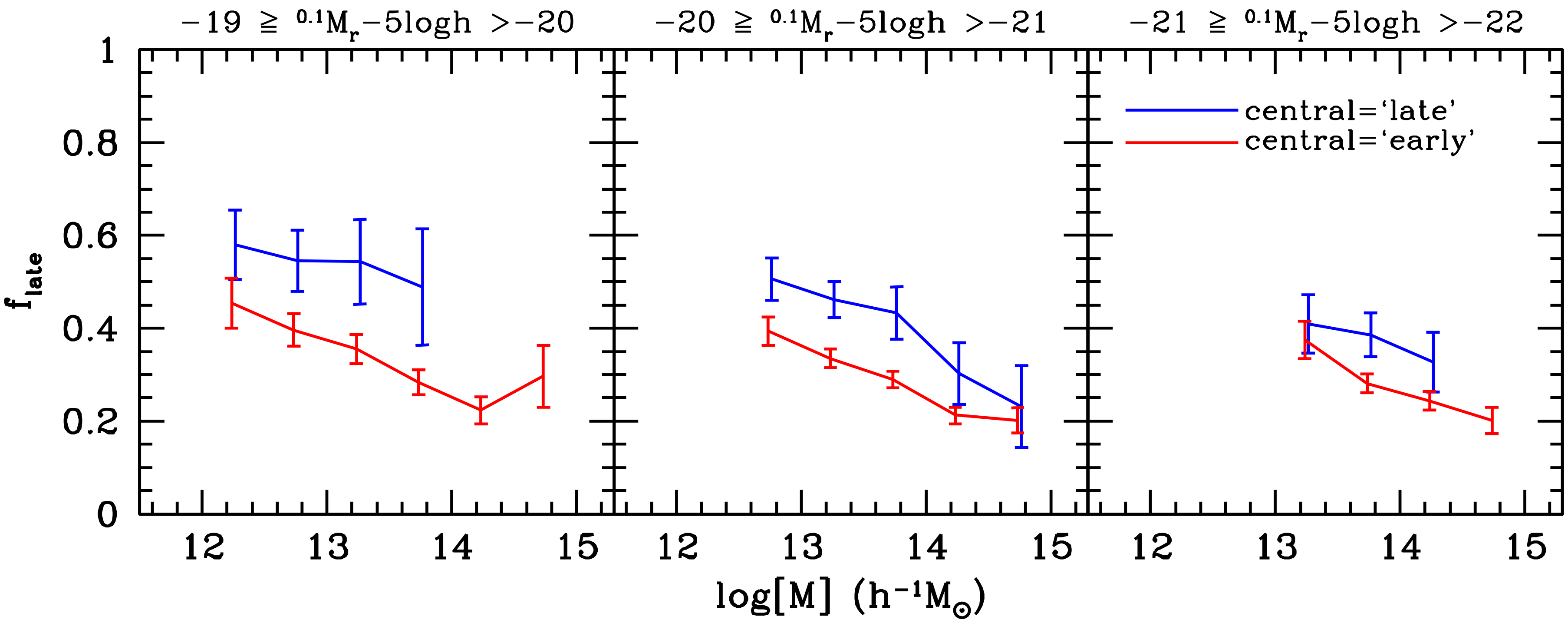
$$P(\delta_{{\rm lin},M} > \delta_{\rm sc}) = \frac{1}{\sqrt{2\pi}\sigma(M)} \int_{\delta_{\rm sc}}^{\infty} \exp\left(-\frac{\delta^2}{2\sigma^2(M)}\right) {\rm d}\delta$$

The **Halo Mass Function**, then becomes

$$n(M){\rm d}M = \frac{\bar{\rho}}{M}\frac{{\rm d}P}{{\rm d}M}{\rm d}M = \sqrt{\frac{2}{\pi}}\frac{\bar{\rho}}{M^2}\left|\frac{{\rm d}\ln\sigma}{{\rm d}\ln M}\right|\sqrt{\nu}{\rm e}^{-\nu/2}$$

where $\nu = \delta_{\rm sc}^2/\sigma^2(M)$, and a ‘fudge-factor’ 2 has been added.

Galactic Conformity



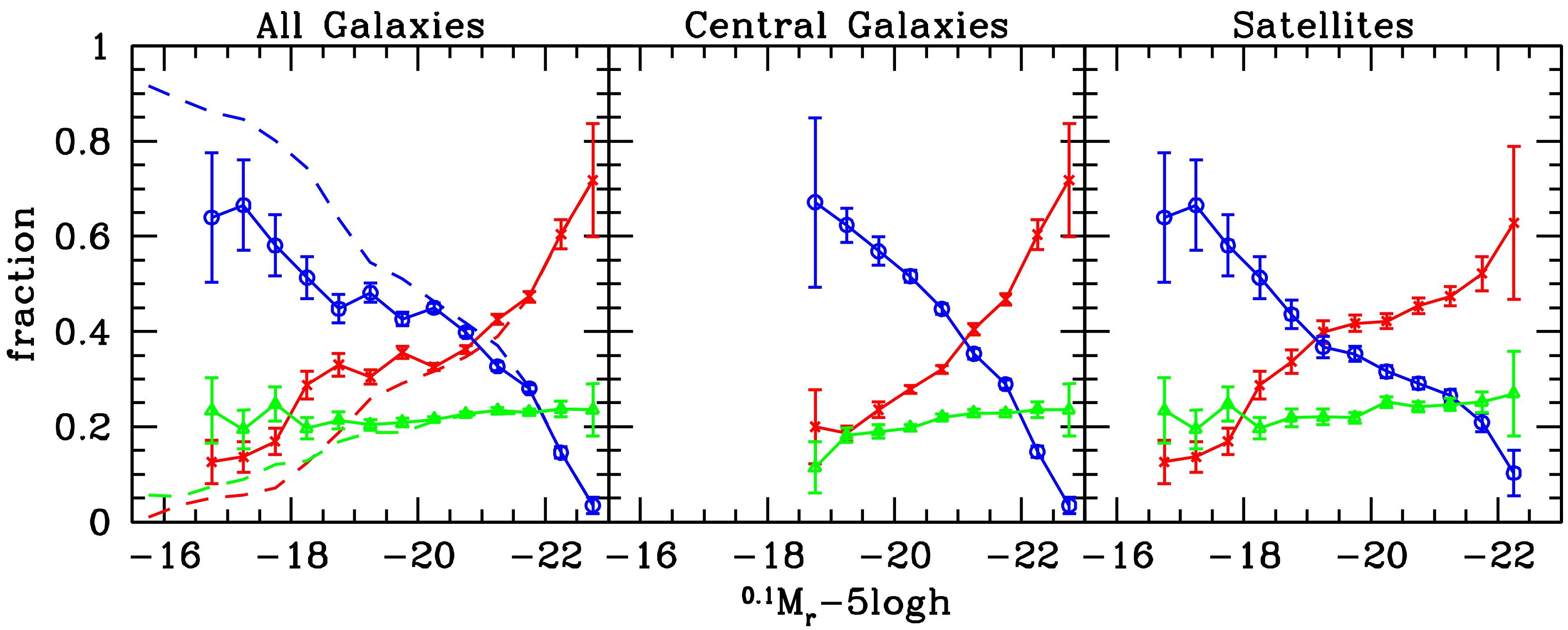
Late type 'centrals' have preferentially late type satellites, and vice versa.

Satellite galaxies 'adjust' themselves to properties of their central galaxy

Galactic Conformity present over large ranges in luminosity and halo mass.

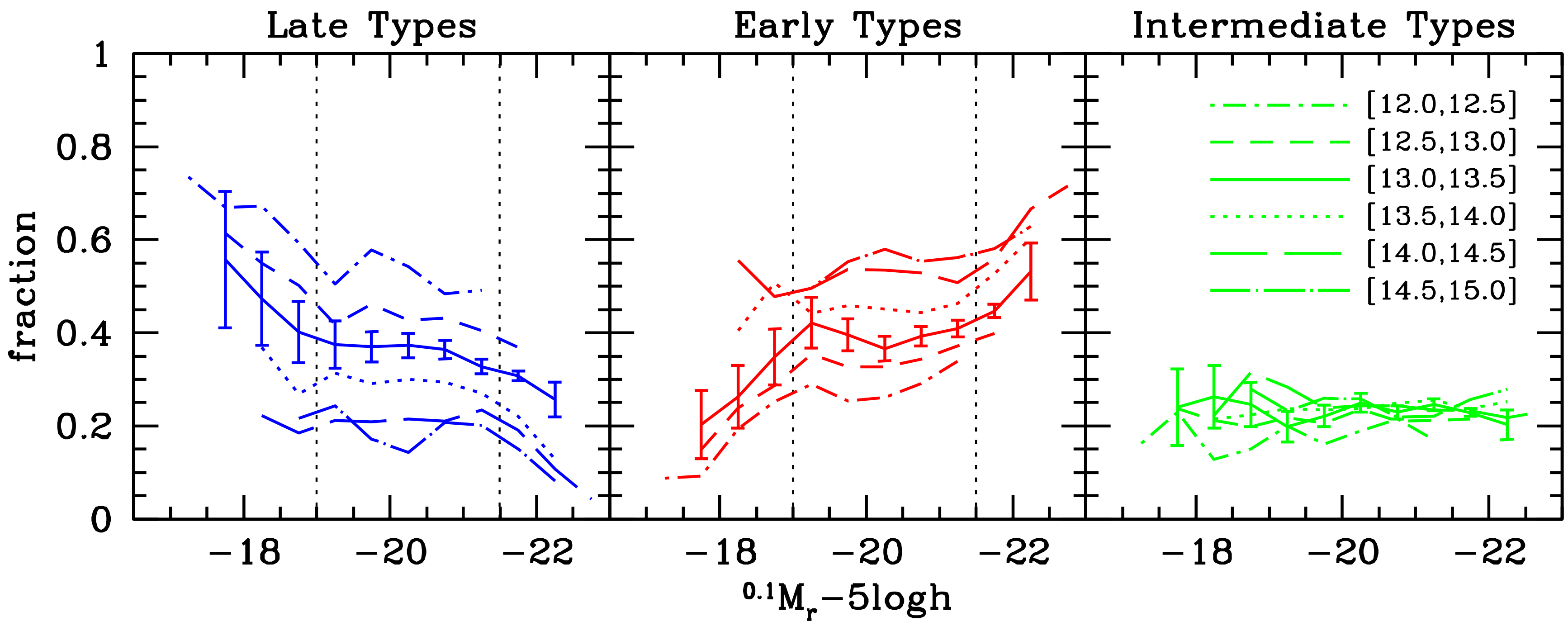
(Weinmann, vdB, Yang & Mo, 2006)

Luminosity Dependence



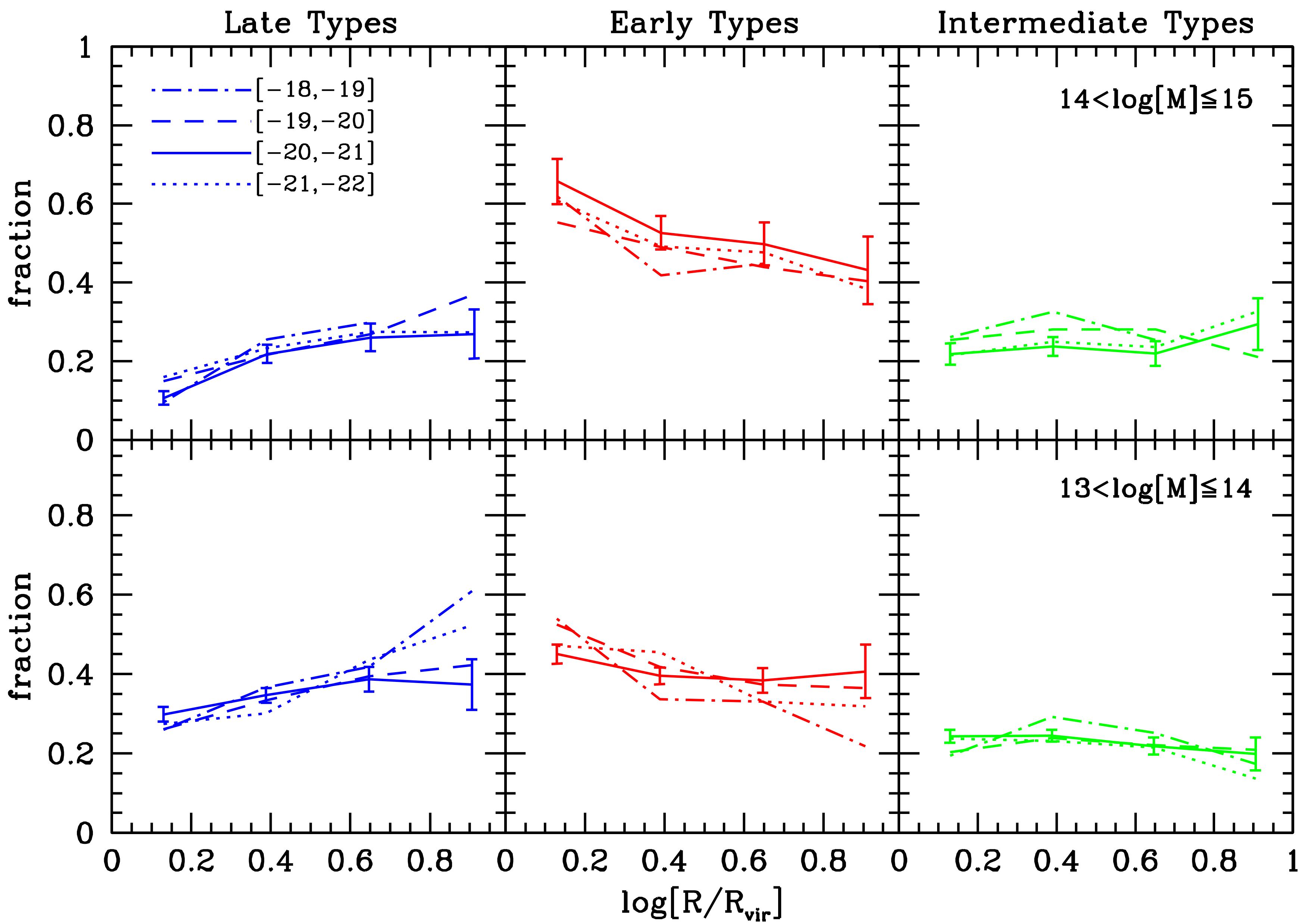
(Weinmann, vdB, Yang & Mo, 2005)

Mass-Luminosity Dependence



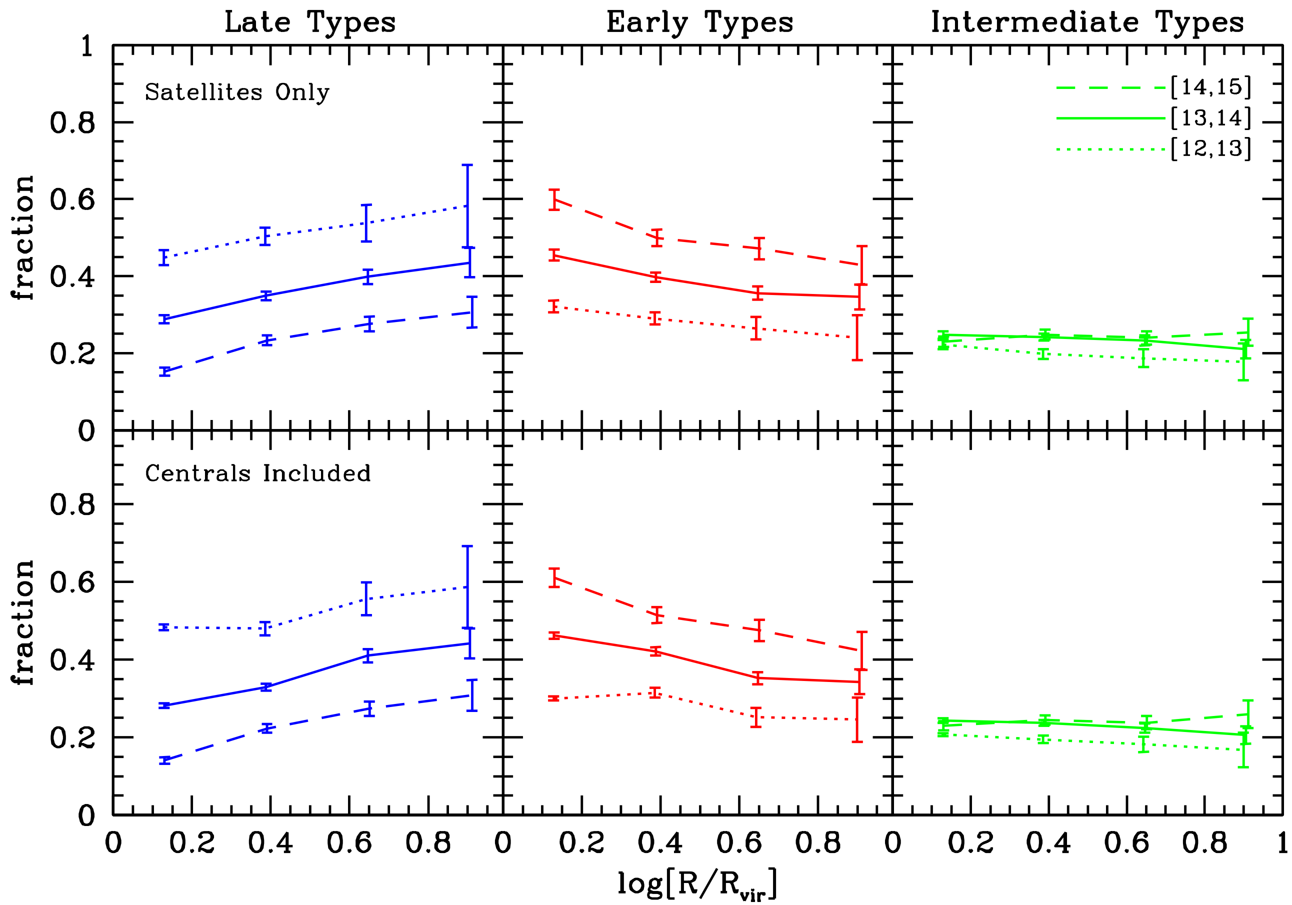
(Weinmann, vdB, Yang & Mo, 2005)

Radial Dependence



(Weinmann, vdB, Yang & Mo, 2005)

Dependence on Group-centric Radius



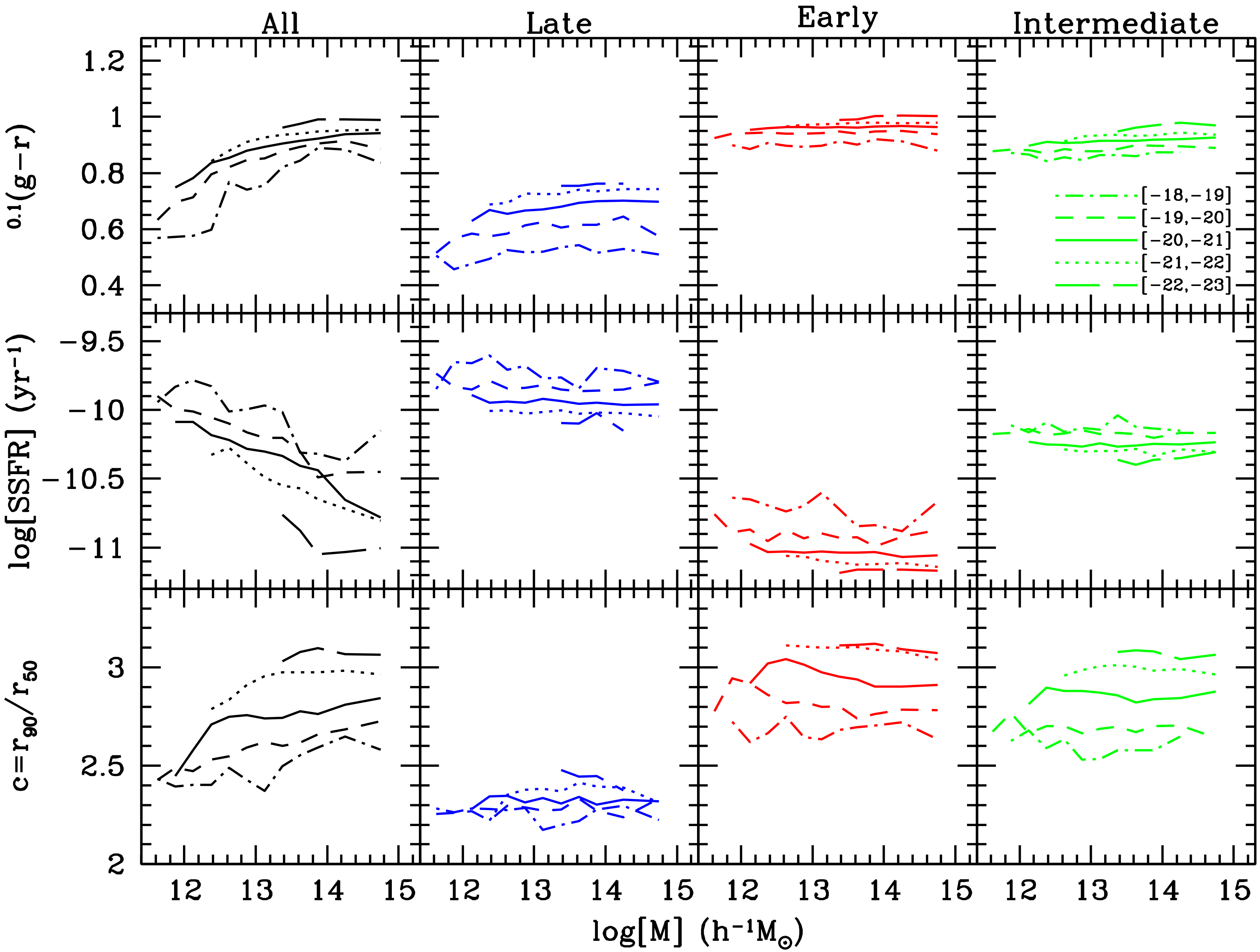
As noticed before, the late type fraction of satellites increases with radius. This trend is independent of halo mass!

Inconsistent with previous studies, but these included central galaxies.

Our results rule out group- and cluster-specific processes such as ram-pressure stripping and harassment: nature rather than nurture!

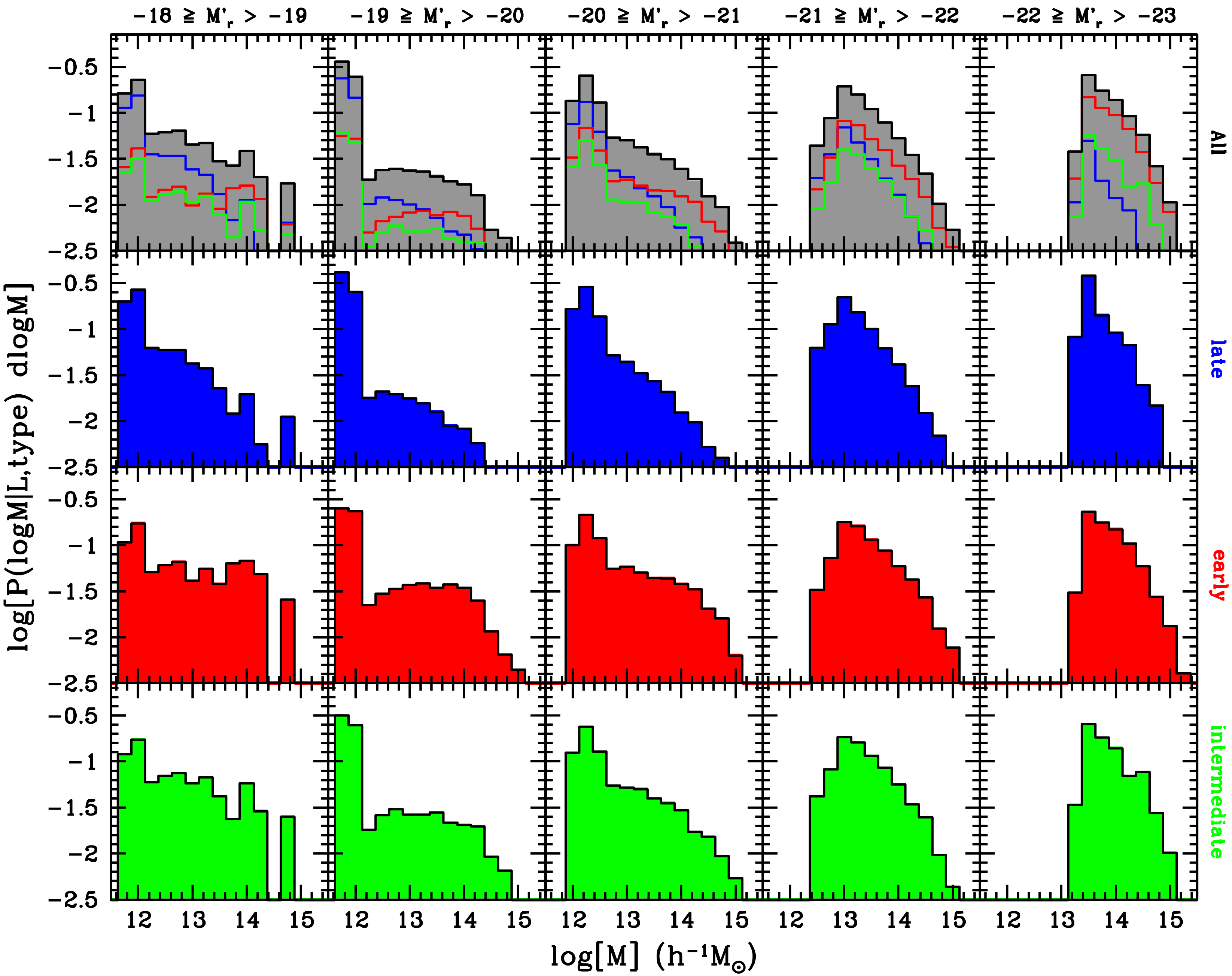
(Weinmann, vdB, Yang & Mo, 2005)

Median Properties

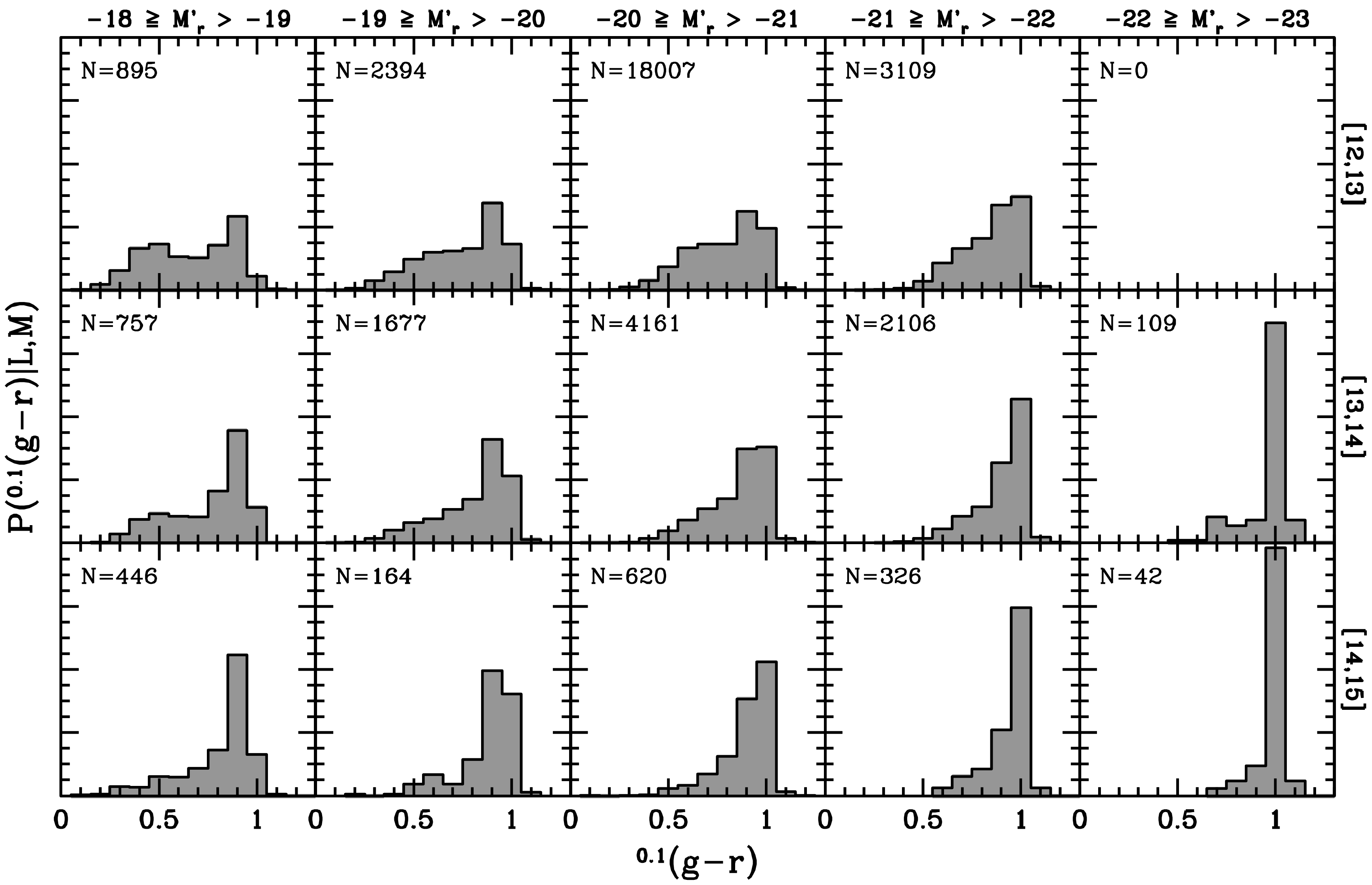


(Weinmann, vdB, Yang & Mo, 2005)

Conditional Mass Funcion

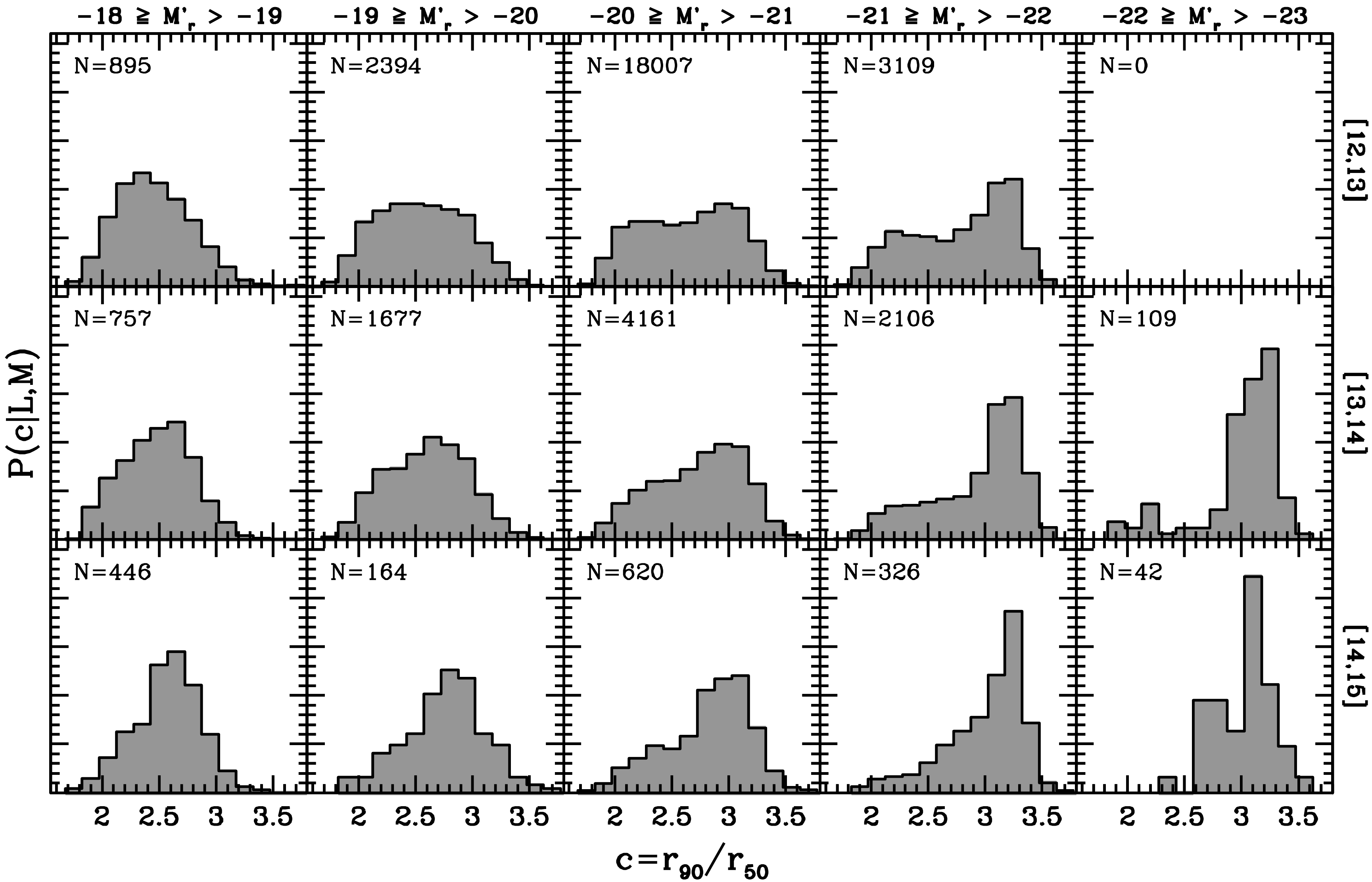


Conditional Colour Funcion



(Weinmann, vdB, Yang & Mo, 2005)

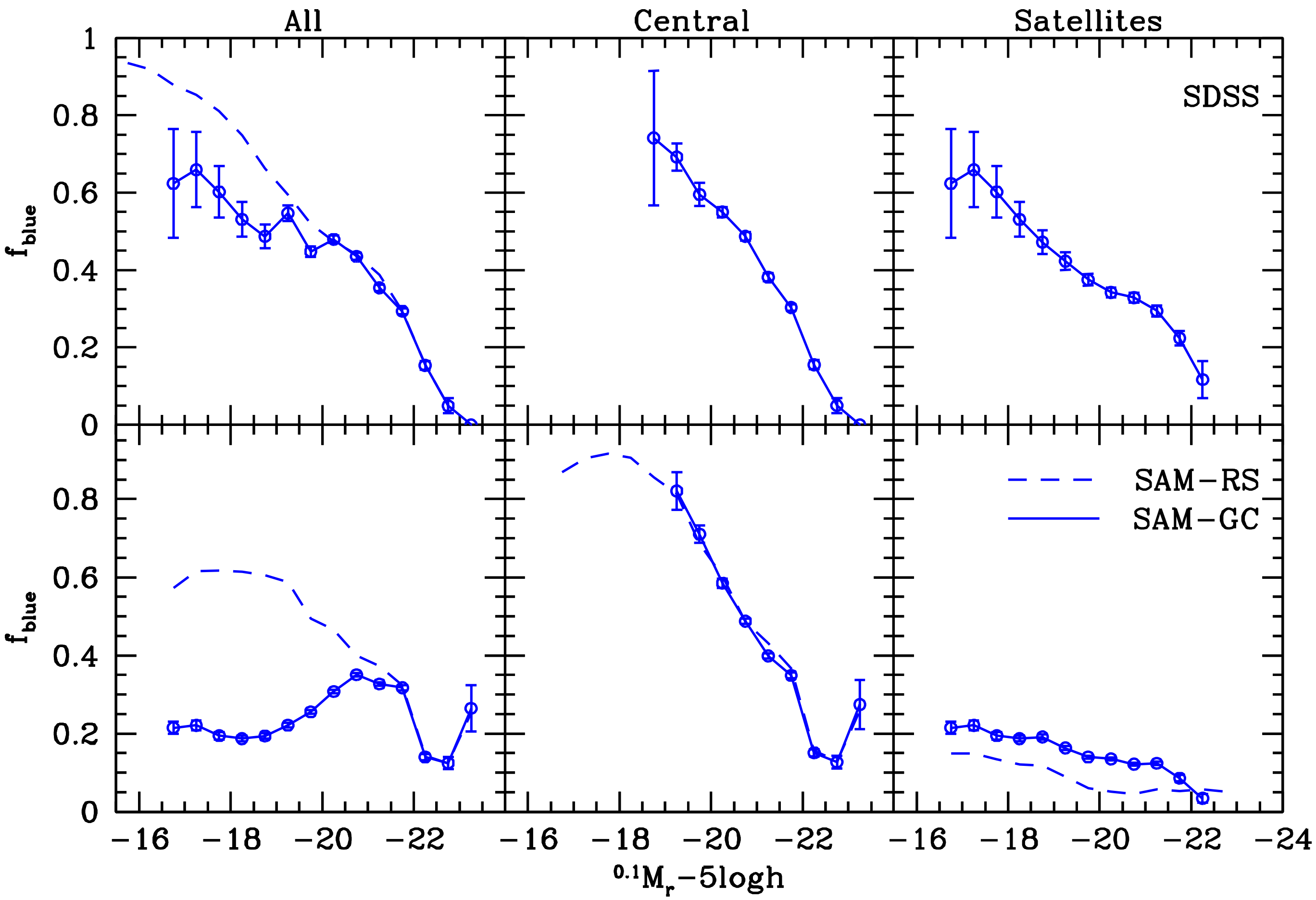
Conditional Concentration Funcion



(Weinmann, vdB, Yang & Mo, 2005)

Blue Fraction as Function of Luminosity

In SAM virtually all satellites are red, contrary to SDSS, where the fraction of red satellites decreases with luminosity.

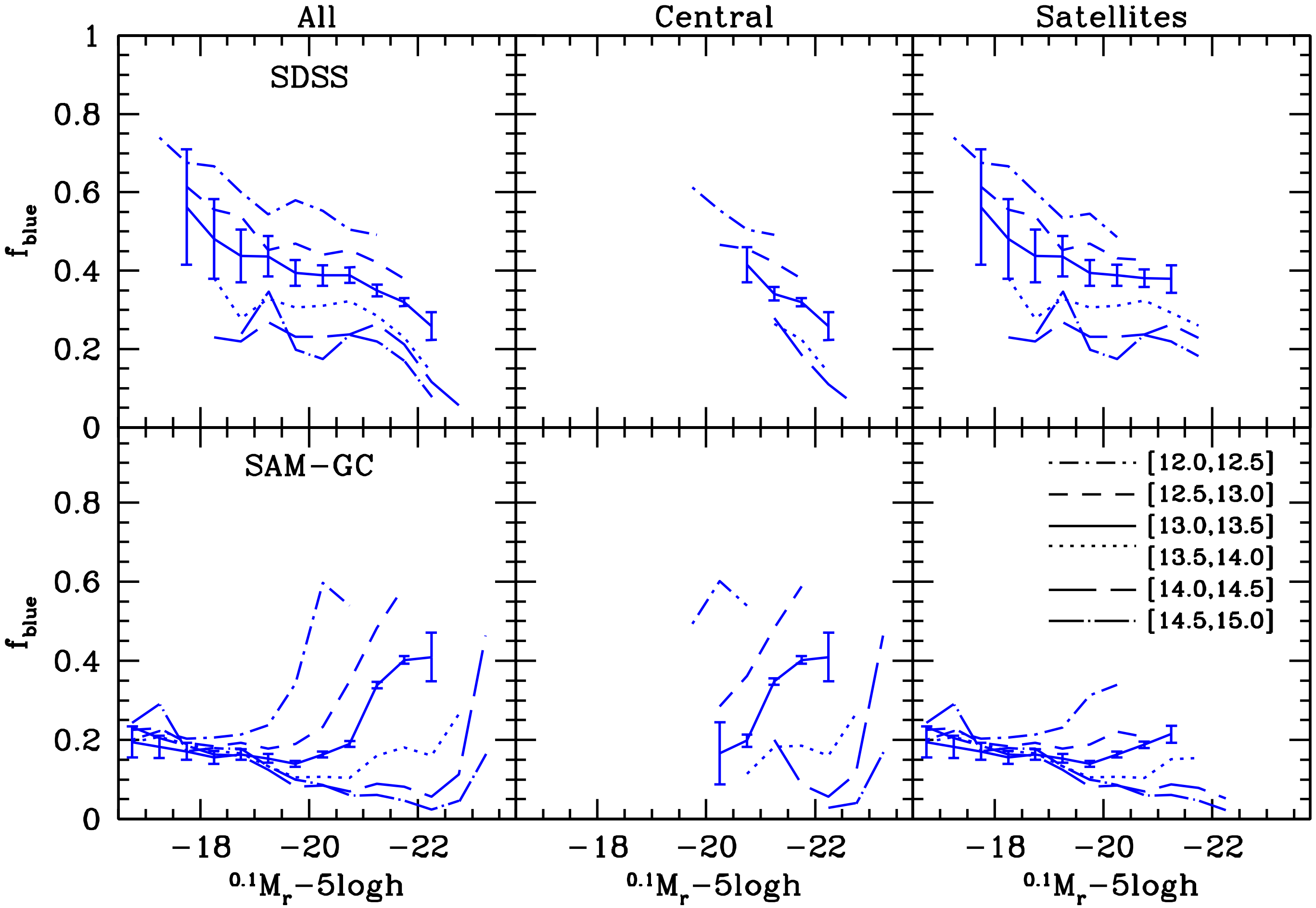


In SAM, satellites stripped of hot gas halo immediately after accretion

▷ **This strangulation is too efficient.**

Blue Fraction as Function of Halo Mass

To allow for fair comparison, we run our Group Finder over SAM.



Problem with central galaxies: At fixed halo mass, blue fraction increases with L in SAM, but decreases with L in SDSS.

▷ **Modelling of AGN feedback is not yet entirely correct**

The Importance of Satellite Galaxies

Kinematics

- **Satellites sample large radii $\Rightarrow$ virial mass estimator**
- **Only few satellites per halo $\Rightarrow$ Need to stack many host/satellite pairs**
- **Beware of Interlopers & Observational Biases**

Abundances

- **Halo Occupation Numbers $\Leftrightarrow$ Conditional Luminosity Function**

  (Yang, Mo & van den Bosch 2003; van den Bosch, Yang & Mo 2003)

- **Comparison with dark matter subhaloes**

  (Moore et al. 1999; Klypin et al. 1999; Vale & Ostriker 2004; Kravtsov et al. 2004)

Radial Distribution

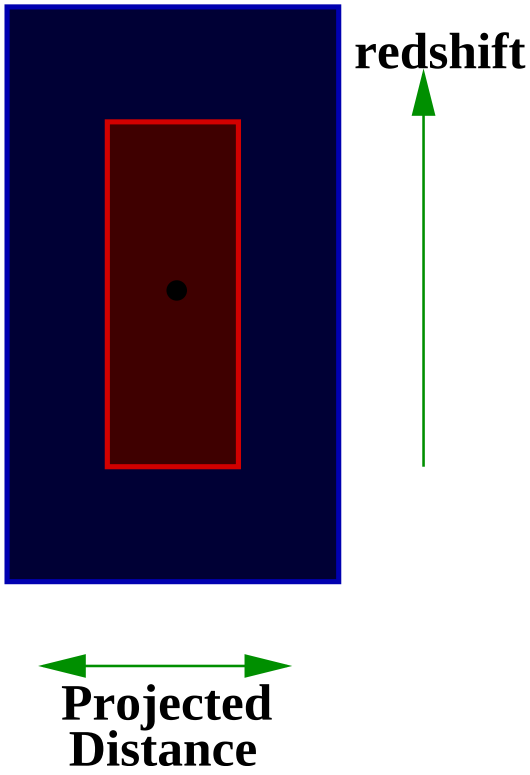
- **Dark Matter Subhaloes show spatial anti-bias**

  (Klypin et al. 1999; Ghigna et al. 2000; De Lucia et al. 2003; Diemand et al. 2004)

- **Impact of tidal stripping, dynamical friction, harassment etc.**

  (Moore et al. 1998; Mayer et al. 2001; Taffoni et al. 2003)

Selecting hosts & satellites



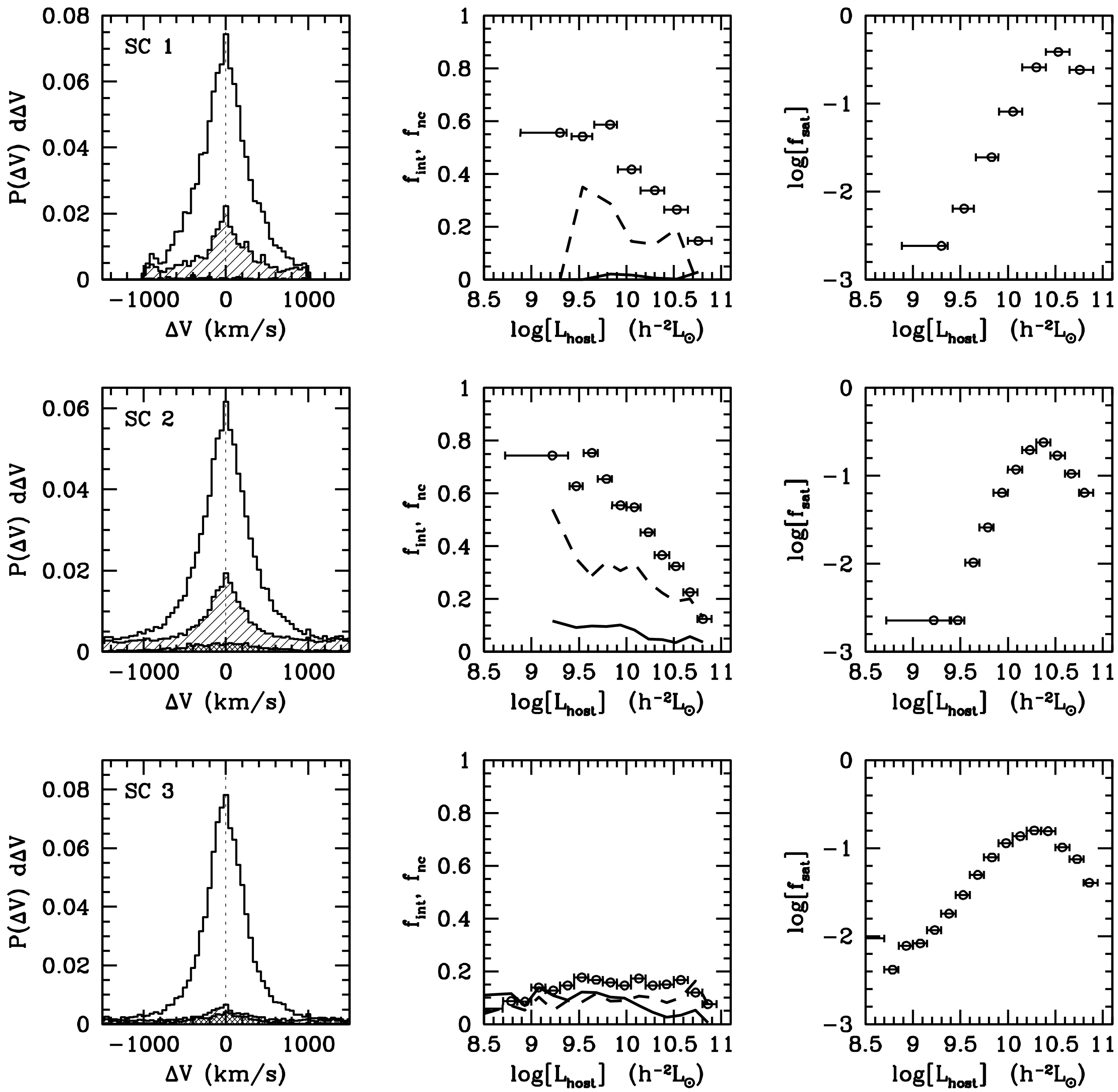
HOSTS ***At least f_h times brighter than any other galaxy in blue volume.***

SATS ***In red volume and at least f_s times fainter than host.***

All previous studies used very conservative selection criteria: $f_h = 2$, $f_s = 4$, $\Delta V_h = \Delta V_s = 1000\,\mathrm{km\,s^{-1}}$, $R_h = 2h^{-1}\,\mathrm{Mpc}$, **and** $R_s = 0.5h^{-1}\,\mathrm{Mpc}$. (McKay et al. 2002; Brainerd & Specian 2003; Prada et al. 2003)

Use Mock Galaxy Redshift Surveys, constructed from CLF to test selection criteria.

Selecting hosts & satellites



N_h=1851
N_s=3876

f_h=2.0
f_s=4.0
R_h=2.0 chimp
R_s=0.5 chimp
dV_h=1000 km/s
dV_s=1000 km/s

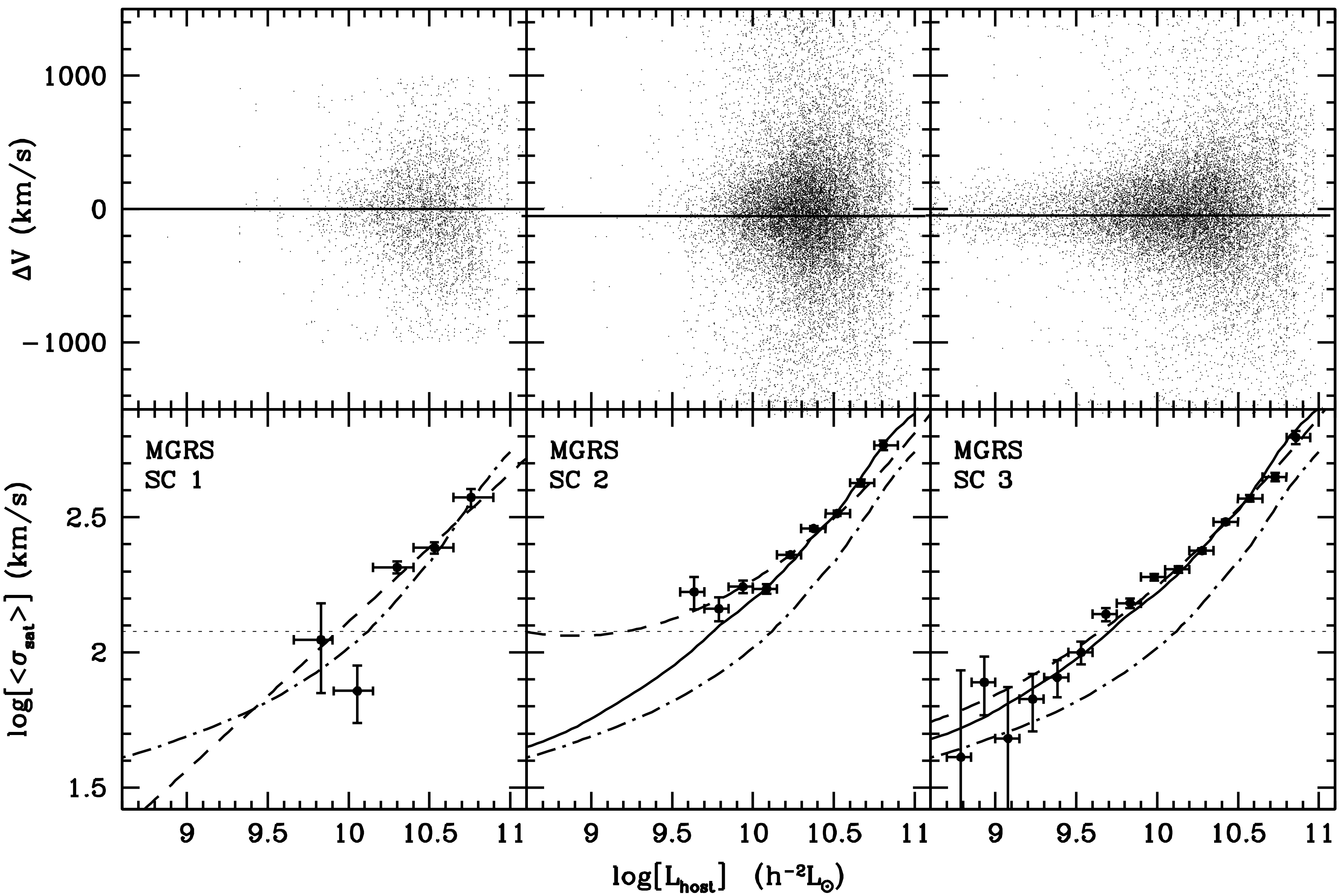
N_h=7863
N_s=19099

f_h=1.0
f_s=1.0
R_h=2.0 chimp
R_s=0.5 chimp
dV_h=2000 km/s
dV_s=2000 km/s

N_h=10483
N_s=16750

f_h=1.0
f_s=1.0
assume sigma(L)
iterate

Measuring Satellite Kinematics



Interpreting Satellite Kinematics

Velocity dispersion of satellite galaxies follows from Jeans Equation:

$$\sigma^2_{\rm sat}(r) = \frac{1}{n_{\rm sat}(r)} \int_r^\infty n_{\rm sat}(r') \, \frac{\partial \Phi}{\partial r}(r') \, {\rm d}r'$$

The halo averaged expectation value:

$$\langle \sigma_{\rm sat} \rangle_M = \frac{4\pi}{\langle N_{\rm sat} \rangle_M} \int_0^{r_{\rm vir}} n_{\rm sat}(r) \, \sigma_{\rm sat}(r) \, r^2 \, {\rm d}r$$

Expectation value for the number of satellites follows from CLF:

$$\langle N_{\rm sat} \rangle_M = \int_{L_1}^\infty \Phi(L|M) \, {\rm d}L - 1$$

Scatter in relation between M and host luminosity L_h:

$$\langle \sigma_{\rm sat} \rangle (L_h) = \int_0^\infty P(M|L_h) \langle \sigma_{\rm sat} \rangle_M {\rm d}M$$

Stacking host-satellite pairs yields satellite-weighted mean:

$$\langle \sigma_{\rm sat} \rangle (L_h) = \frac{\int_0^\infty P(M|L_h) \, \langle \sigma_{\rm sat} \rangle_M \, \langle N_{\rm sat} \rangle_M \, {\rm d}M}{\int_0^\infty P(M|L_h) \, \langle N_{\rm sat} \rangle_M \, {\rm d}M}$$

Accouting for flux-limited surveys:

$$\langle \sigma_{\rm sat} \rangle (L_h) = \frac{1}{V} \int_0^\Omega {\rm d}\Omega \int_0^{z_{\rm max}} {\rm d}z \, \frac{{\rm d}V}{{\rm d}\Omega \, {\rm d}z} \, \langle \sigma_{\rm sat}(L_h, z) \rangle$$

You need CLF to properly interpret satellite kinematics

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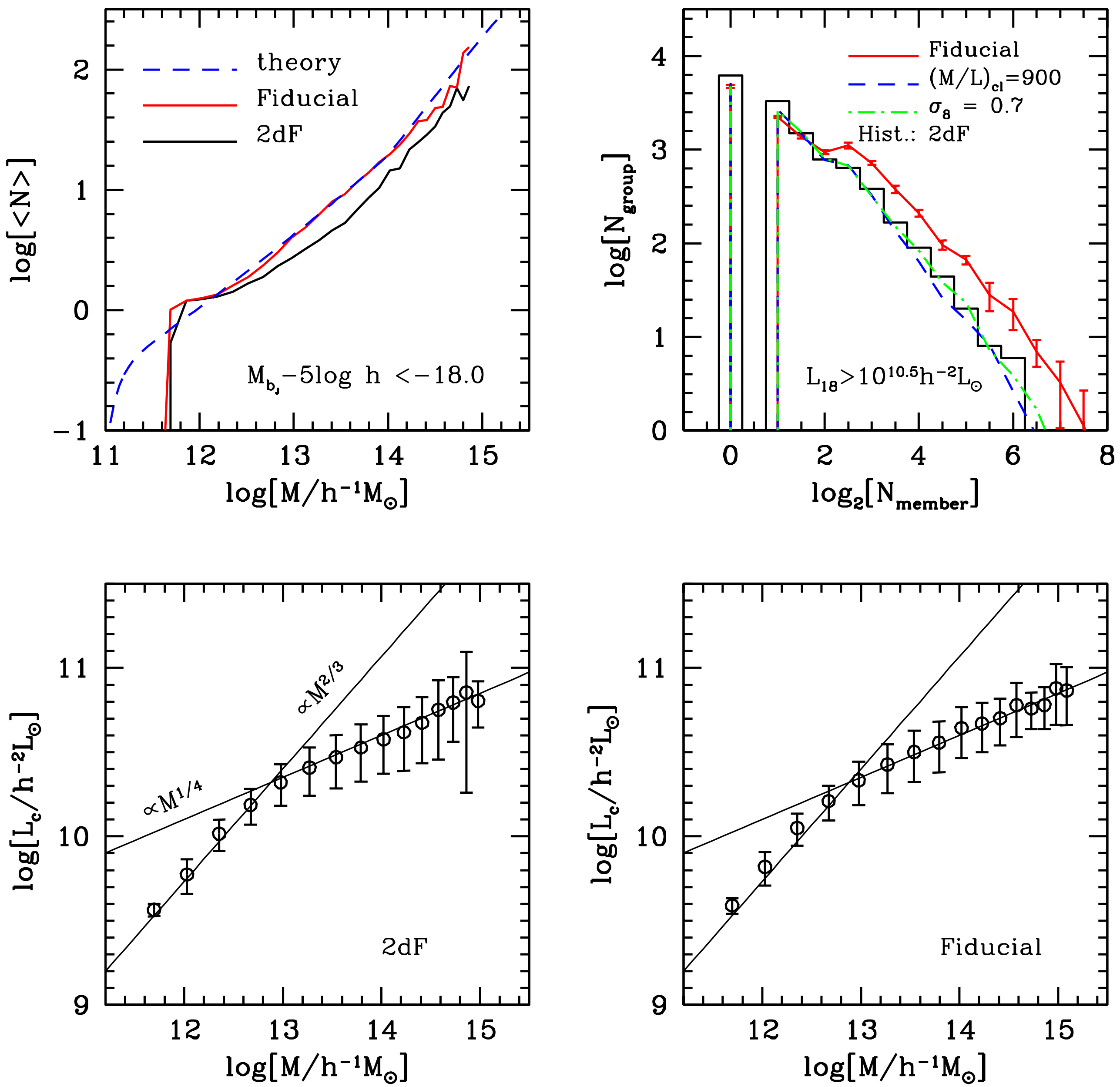
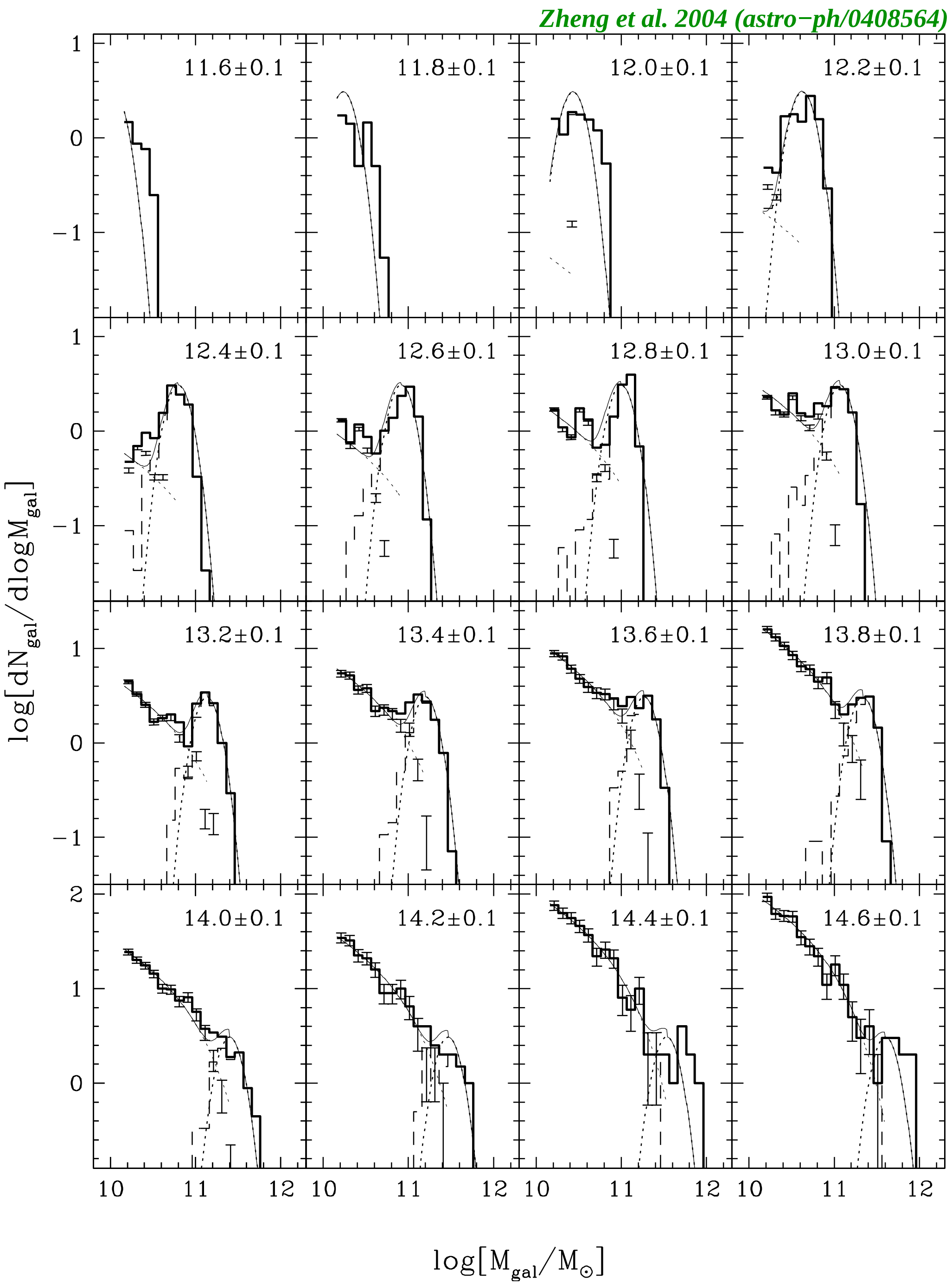
Various Statistics of Galaxy Groups



Yang, Mo, vdB & Jing 2005, MNRAS, 356, 1293

The shape of the CLF



Using semi−analytical models Zheng et al (2004) computed the conditional baryonic mass function (CMF).

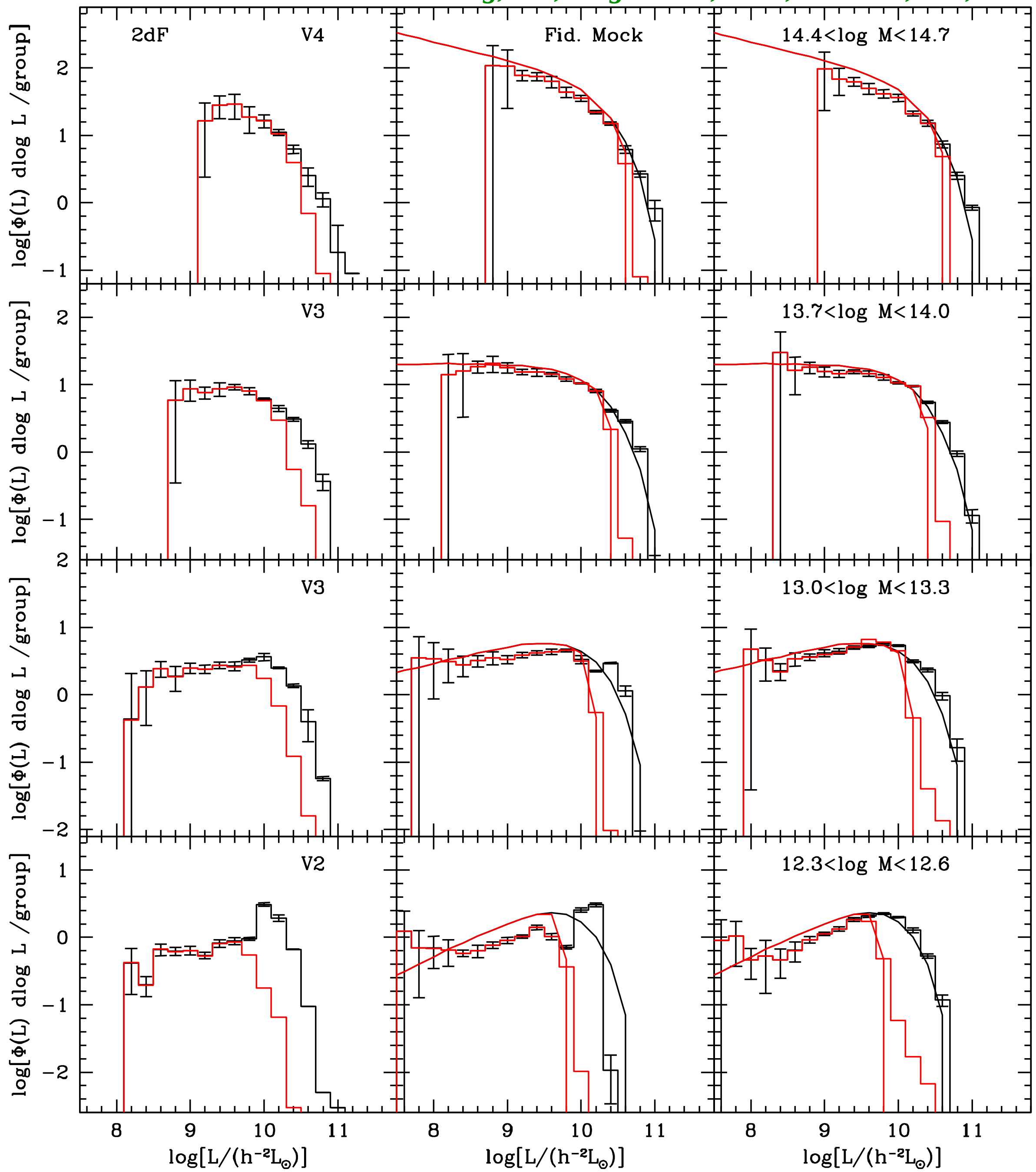
For group−sized haloes the CMF does not follow a Schechter function.

Instead, the CMF is better fit with a Gaussian (for the central galaxies) plus a power−law describing the satellite galaxies.

Does the true CLF have a similar shape??

Direct Determination of CLF from Groups

Yang, Mo, Jing & vdB, 2005, MNRAS, 358, 217



We determined CLF directly from groups in 2dFGRS & mocks.

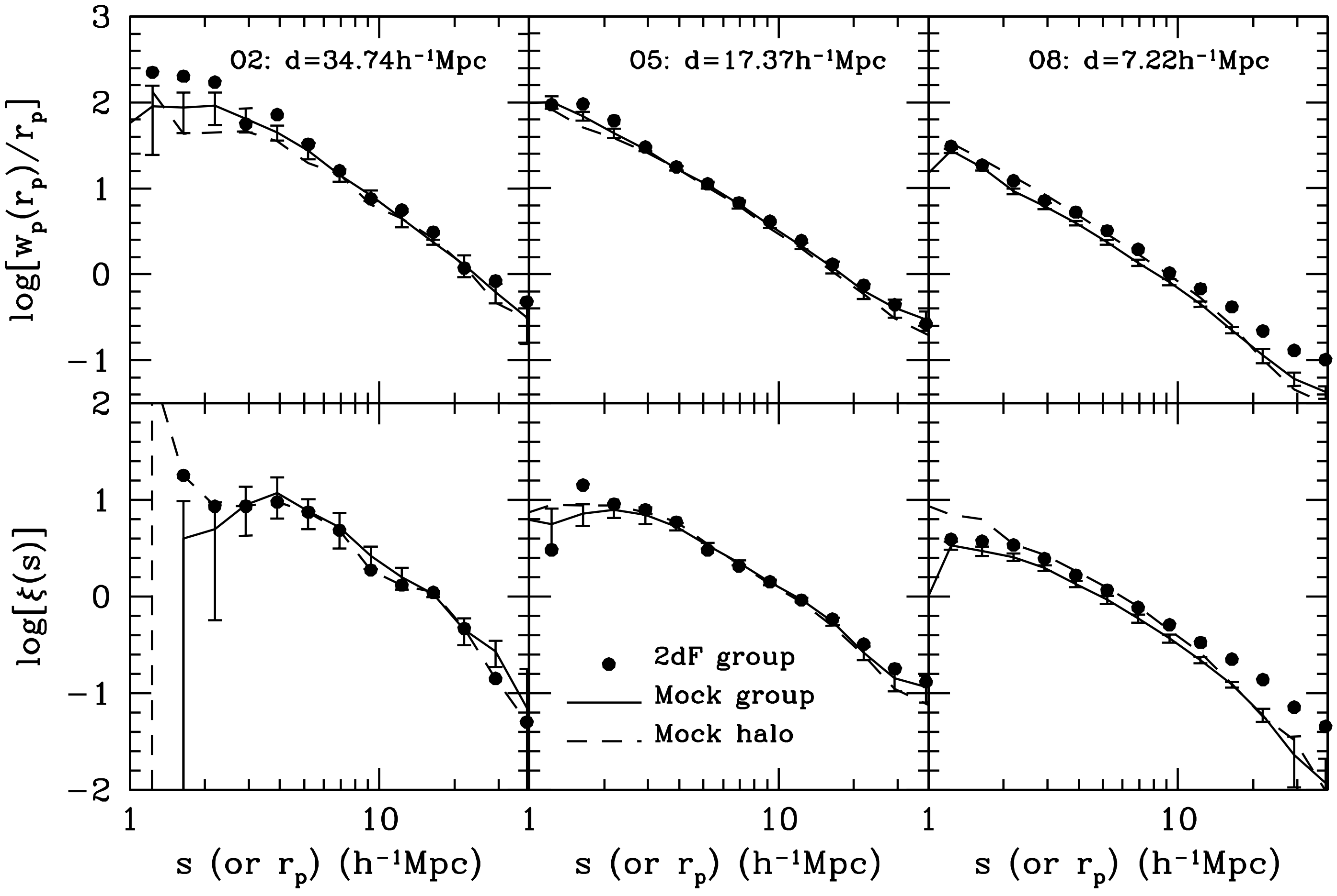
Group−sized haloes have Schechter CLF.

Galaxy−sized haloes reveal Gaussian peak of central galaxies.

Mocks show similar behavior, pointing to an artefact due to mass estimator.

Data is consistent with Schechter CLF!

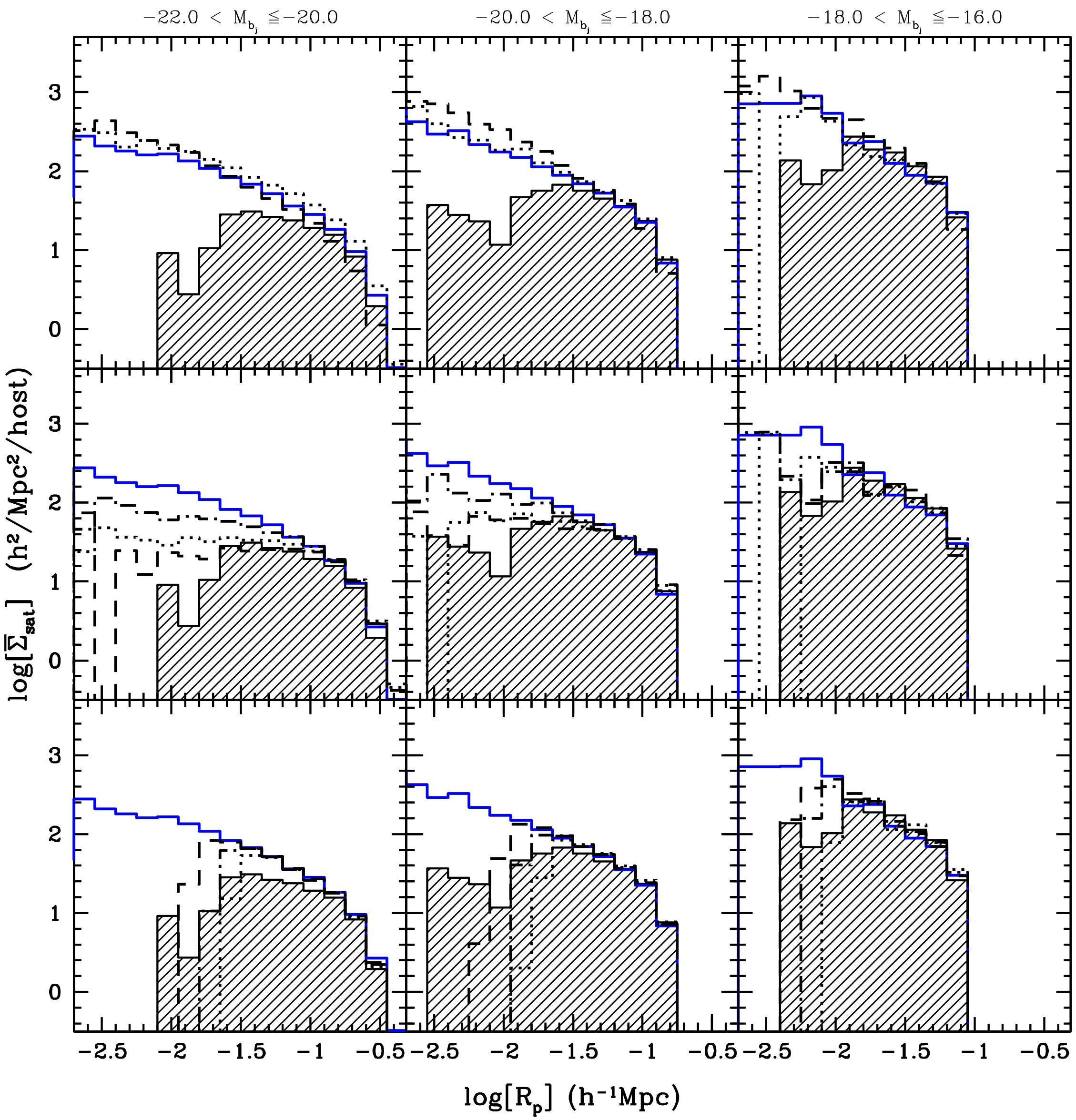
Probing Clustering of Dark Matter Haloes



Yang, Mo, vdB & Jing 2005, MNRAS, 357, 608

- **The group-group correlation function directly reflects the halo-halo correlation function (no issues with galaxy bias)**
- **Promising tool to constrain cosmological parameters**

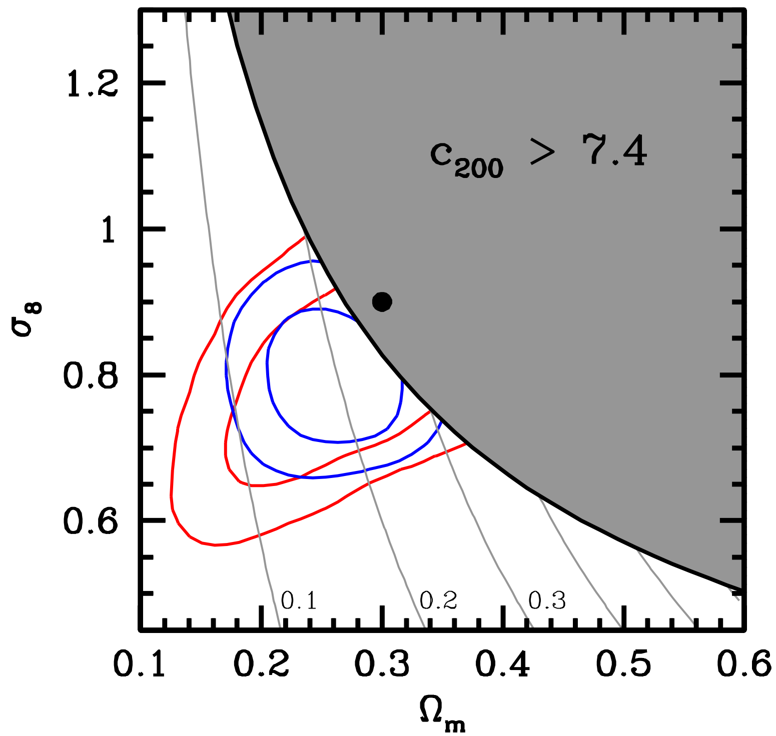
Radial Distribution of Satellite Galaxies

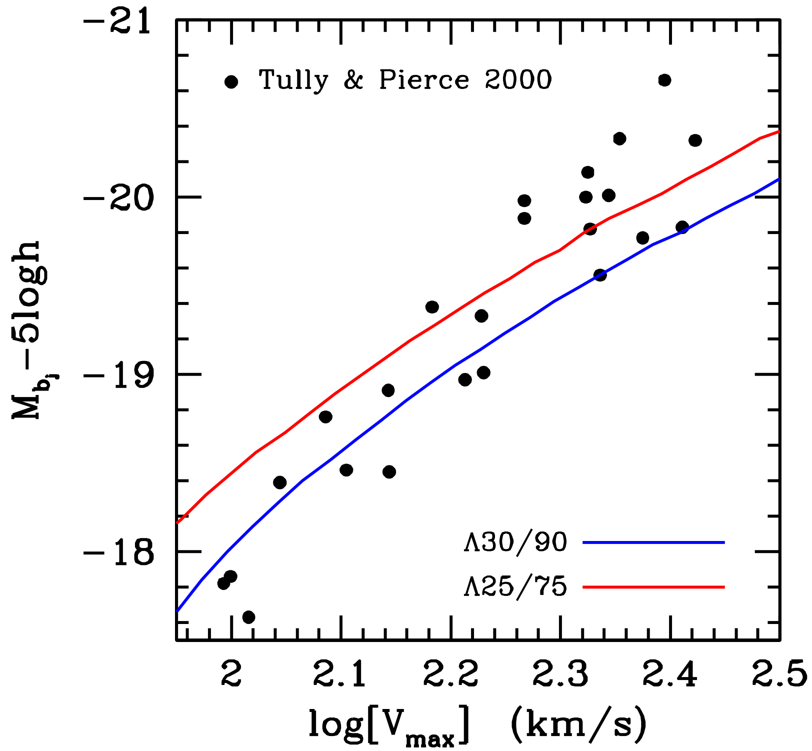


Consistent with no spatial bias, but only if $R_{\rm gal} \simeq 15h^{-1}\,{\rm kpc}L_{10}^{1/3}$

Concordance on Galactic Scales?

$$\Omega_m = 0.25 \pm 0.04 \qquad \sigma_8 = 0.78 \pm 0.06$$





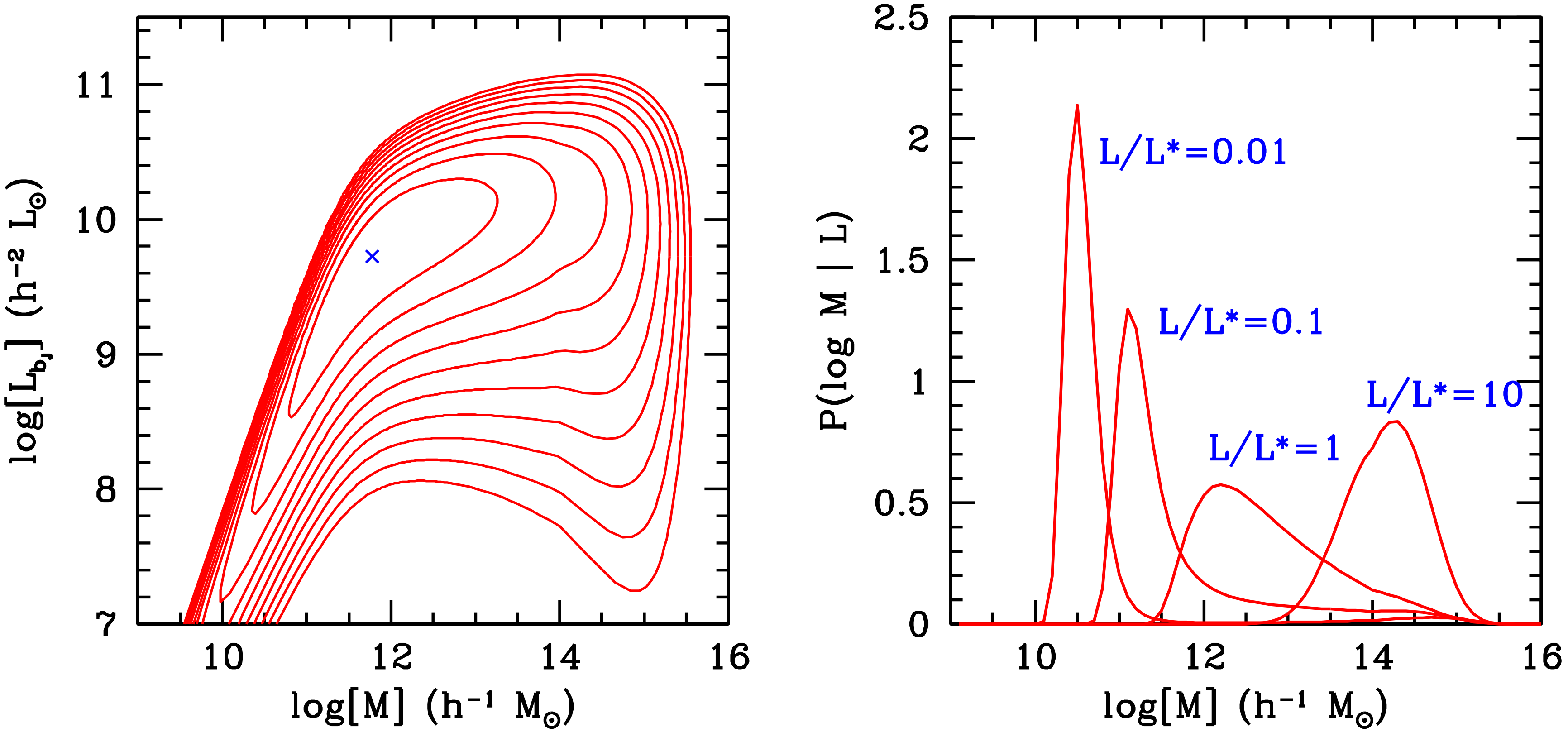
Cosmologies with lower Ω_m and lower σ_8 yield dark matter haloes that are significantly less concentrated. This

- **Alleviates problem with rotation curves of dwarf and LSB galaxies.**
- **Results in a TF zero-point that is $\sim 0.3 - 0.5$ magnitudes brighter.**

The Galaxy Phone Book

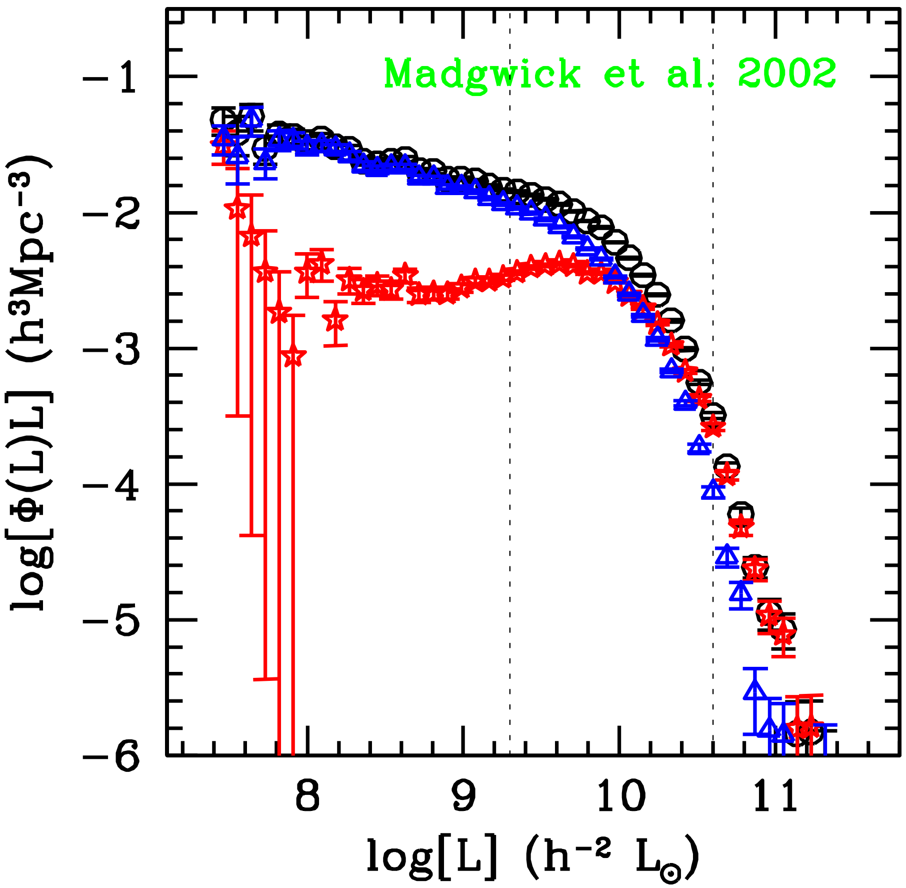
$$P(L,M)\,\mathrm{d}L\,\mathrm{d}M = \frac{1}{\bar{\rho}_L}\,n(M)\,\Phi(L|M)\,L\,\mathrm{d}L\,\mathrm{d}M$$

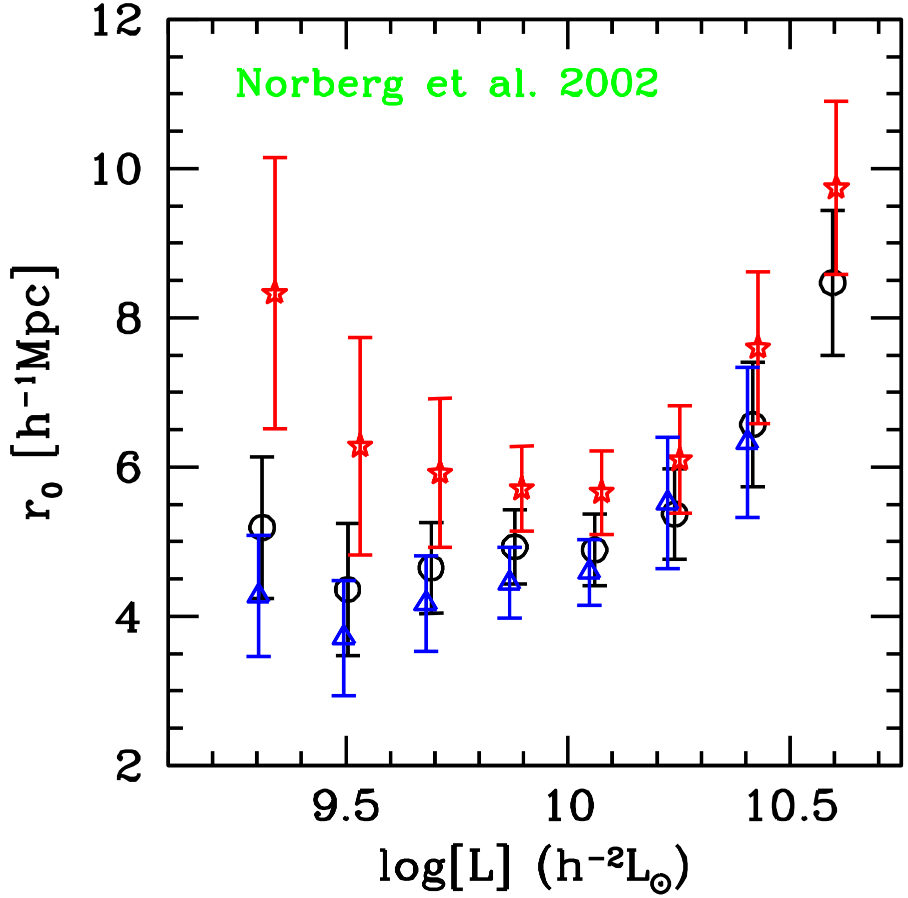
$$P(M|L)\mathrm{d}M = \frac{\Phi(L|M)\,n(M)\,\mathrm{d}M}{\Phi(L)}$$



50% of light is produced in haloes $M \lesssim 2 \times 10^{12} h^{-1}\,\mathrm{M}_\odot$.

Luminosity & Correlation Functions



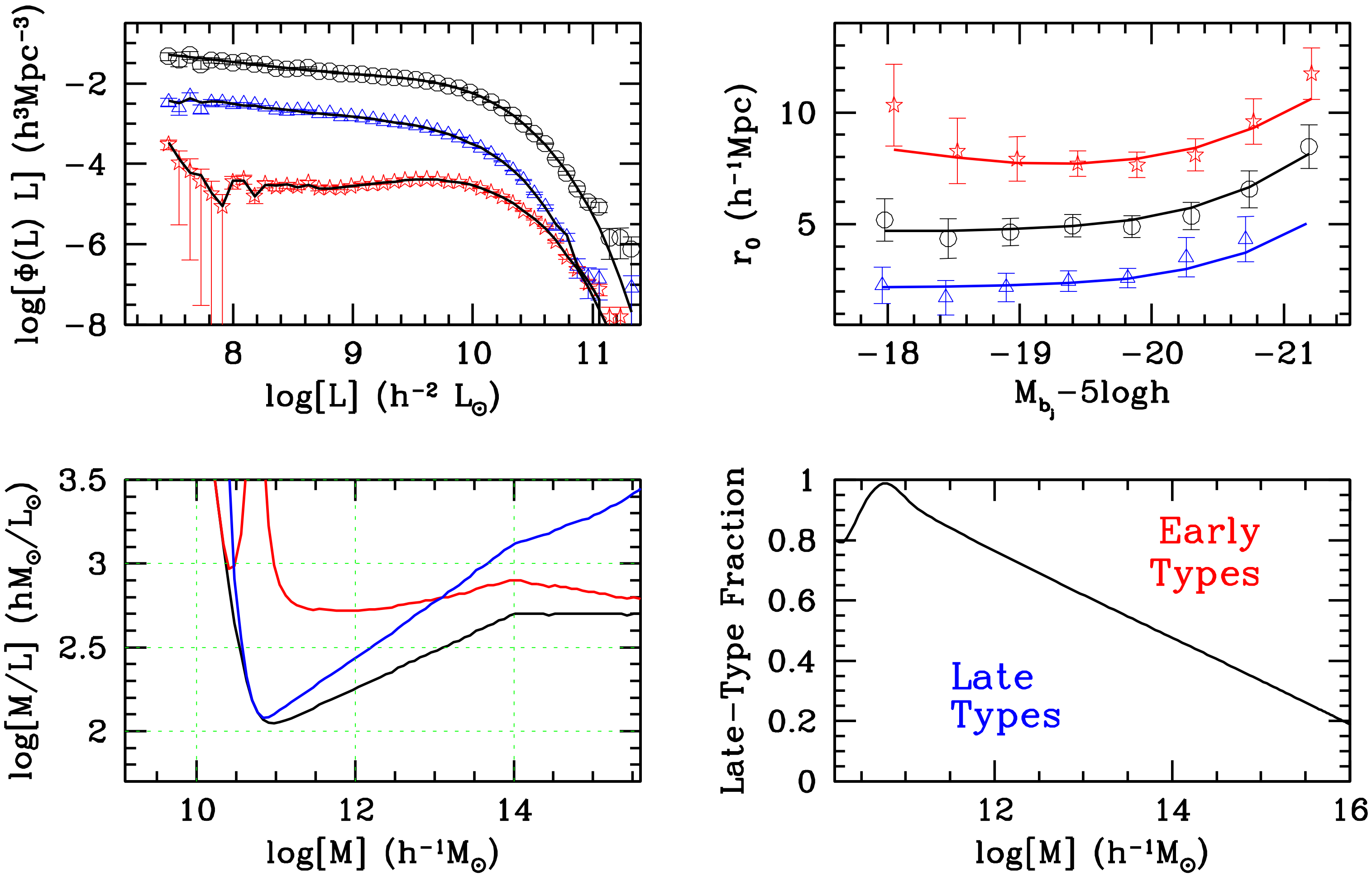


- **On average, early-type galaxies are more luminous and more strongly clustered than late-type galaxies.**
- **In general, more luminous galaxies are more strongly clustered.**

REMINDER: Correlation length r_0 defined by $\xi(r_0) = 1$

The Concordance Cosmology

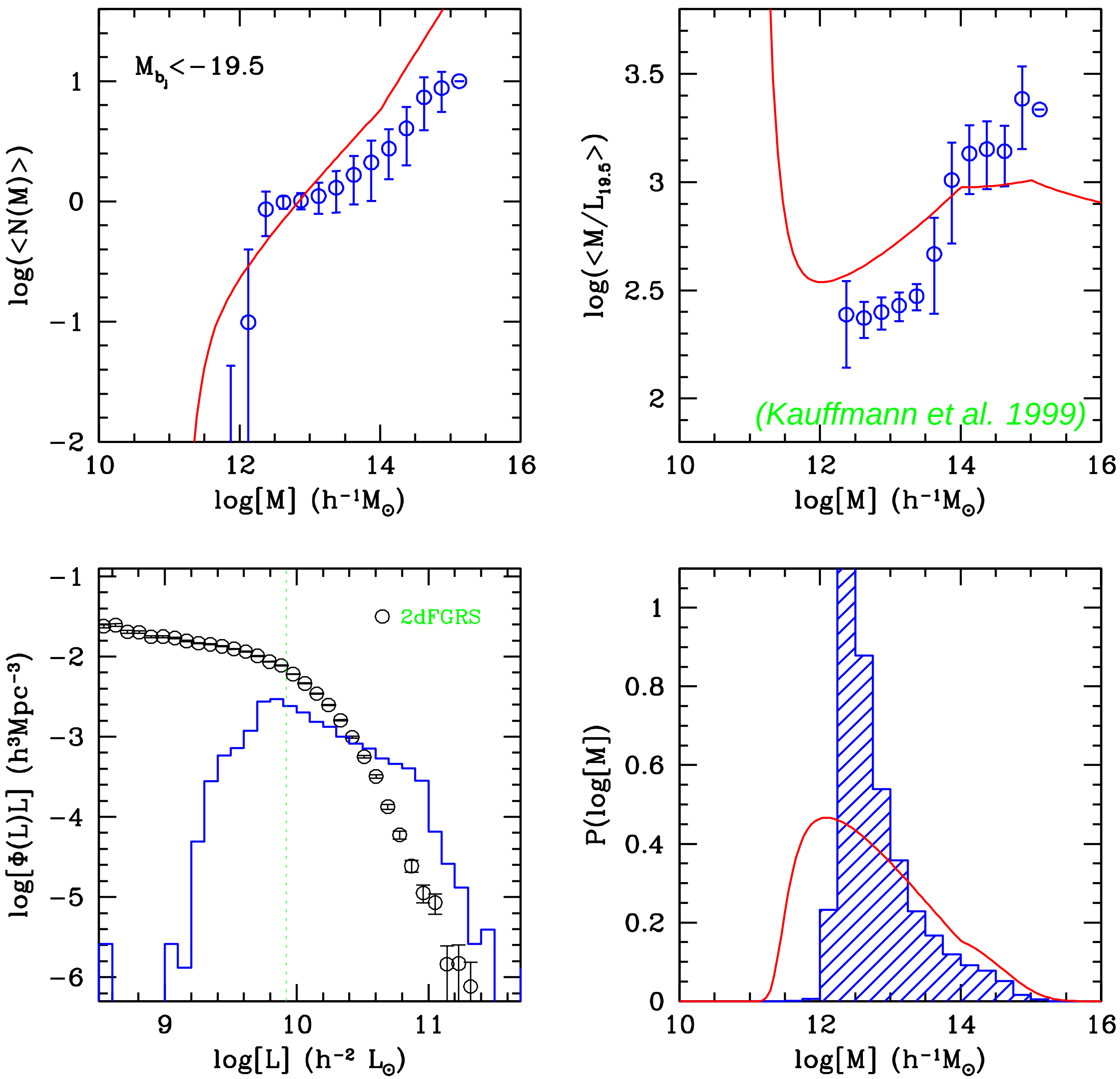
$$\Omega_m = 0.3;\ \Omega_\Lambda = 0.7,\ h = 0.7,\ \sigma_8 = 0.9,\ n = 1.0$$



vdB, Yang & Mo, 2003, MNRAS, 340, 771

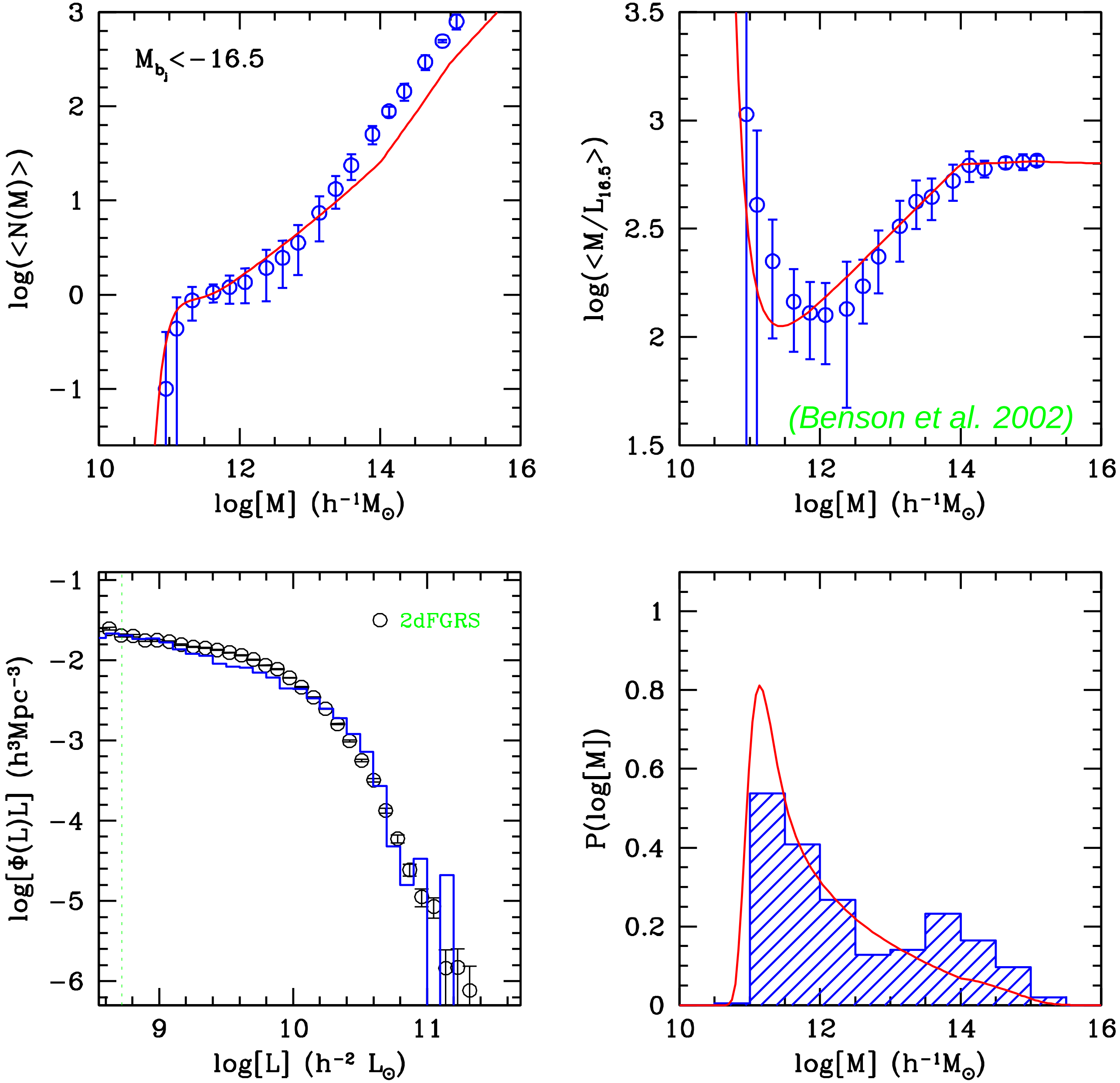
Concordance model fits $\Phi(L)$ and $r_0(L)$ of both early- and late-type galaxies.

Semi-Analytical Models I



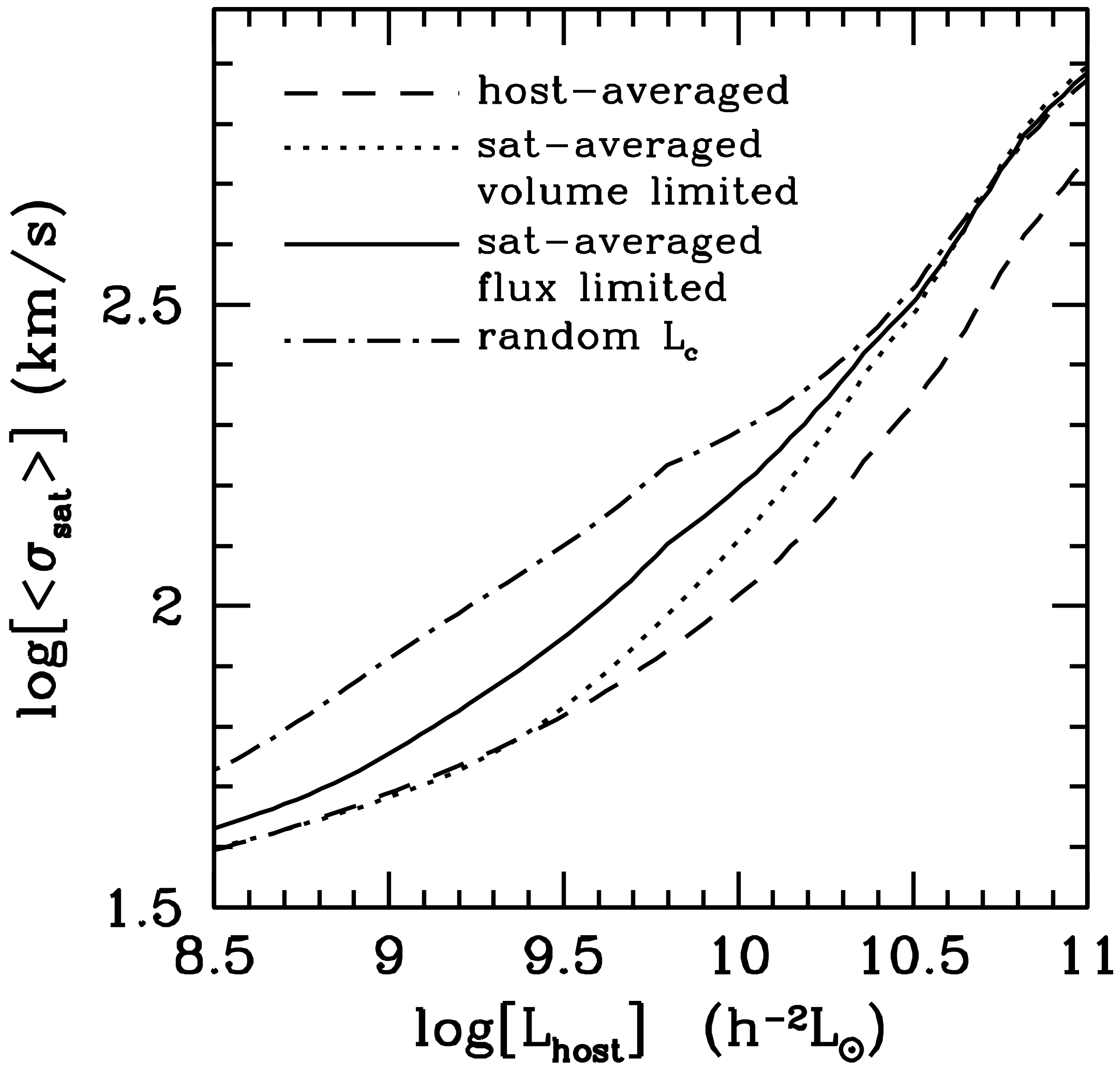
Poor agreement with CLF; but SAM doesn't fit LF

Semi-Analytical Models II



Good agreement with SAMs that fit LF

Bias in Satellite Kinematics



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The Galaxy Correlation Function

The two-point galaxy-galaxy correlation function can be split in a 1-halo and a 2-halo term

$$\xi_{\rm gg}(r) = \xi_{\rm gg}^{\rm 1h}(r) + \xi_{\rm gg}^{2h}(r) = \xi_{\rm gg}^{\rm 1h}(r) + \bar{b}^2 \xi_{\rm hh}^{\rm 2h}(r)$$

Here $\bar{b}$ and $\xi_{\rm hh}^{2h}(r)$ are computed as follows:

- $\bar{b} = \frac{1}{\bar{n}_g} \int_0^\infty n(M) \langle N(M) \rangle \, b(M) \, {\rm d}M$ (Berlind & Weinberg 2002)
- $\langle N(M) \rangle = \int_{L_1}^{L_2} \Phi(L|M) \, {\rm d}L$
- $\xi_{\rm hh}^{2h}(r; M_1, M_2) = b(M_1) \, b(M_2) \, \xi_{\rm dm}^{\rm 2h}(r)$ (Mo & White 1996)
- $\xi_{\rm dm}^{\rm 2h}(r) = \xi_{\rm dm}(r) - \xi_{\rm dm}^{1h}(r)$ (Ma & Fry 2000)
- $\xi_{\rm dm}(r) = \int_0^\infty \Delta_{\rm nl}^2(k) \, \frac{\sin(kr)}{kr} \, \frac{{\rm d}k}{k}$

NOTE: $\xi_{\rm gg}^{\rm 1h}(r)$ can be ignored at large r

Derivation of Halo Bias II

According to PS formalism haloes are associates with regions with an overdensity $\delta > \delta_{\rm sc}$

Therefore, we can compute $n(m,z|M,V) = n(m,z|\delta)$ by simply replacing $\delta_{\rm sc}$ with $\delta_{\rm sc} - \delta$ (Peak-Background split).

Using a Taylor Series expansion to first order, we write that

$$n(m,z|\delta) = n(m,z) + (\delta_{\rm sc} - \delta - \delta_{\rm sc})\left(\frac{\partial n}{\partial \delta_{\rm sc}}\right)_{\delta_{\rm sc}}$$

which is sufficiently accurate as long as $\delta \ll \delta_{\rm sc}$.

Substitution in eq. for δ_h and only keeping terms to lowest order in δ yields

$$\delta_h = \delta\left[1 - \frac{1}{n}\left(\frac{\partial n}{\partial \delta_{\rm sc}}\right)_{\delta_{\rm sc}}\right]$$

Derivation of Halo Bias III

According to the PS formalism the halo mass function is

$$n(m,z) = \sqrt{\frac{2}{\pi}}\frac{\bar{\rho}}{m^2}\left|\frac{\mathrm{d}\ln\sigma}{\mathrm{d}\ln m}\right|\sqrt{\nu}\,\mathrm{e}^{-\nu/2}$$

with $\nu = \nu(m,z) = \delta_{\rm sc}^2(z)/\sigma^2(m)$

If we define the halo bias as $b(m,z) = \delta_h/\delta$**, one obtains that**

$$\boxed{b(m,z) = 1 + \frac{\nu - 1}{\delta_{\rm sc}}}$$

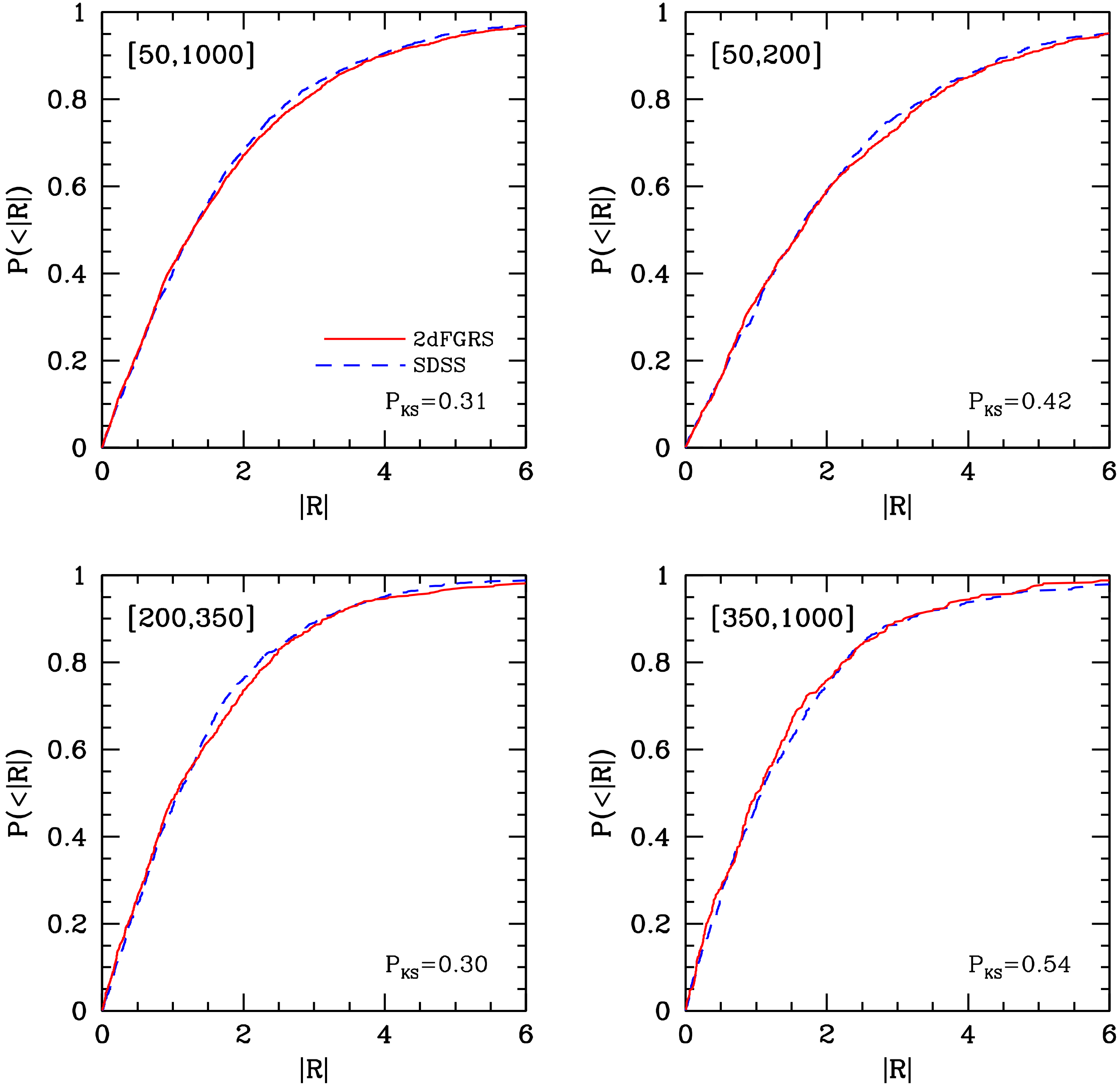
The characteristic mass m^* **is defined by** $\sigma(m^*) = \delta_{\rm sc}$**.**

This implies that $\nu(m^*) = 1$**, and thus that** $b(m^*) = 1$

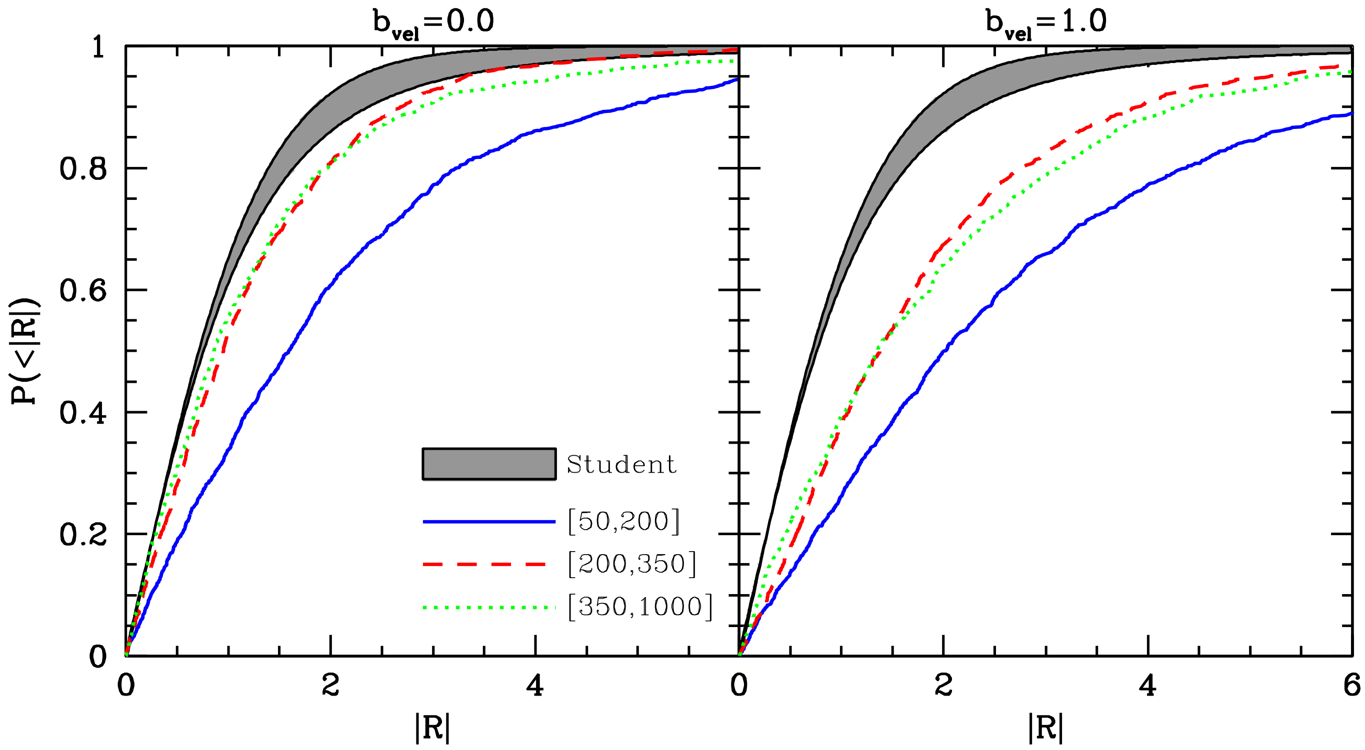
$b(m,z) > 1$	**if** $m > m^*$	**(biased)**
$b(m,z) = 1$	**if** $m = m^*$	**(unbiased)**
$1 - \frac{1}{\delta_{\rm sc}} < b(m,z) < 1$	**if** $m < m^*$	**(anti-biased)**

Note that there is an absolute minimim to the halo bias.

Comparing 2dFGRS with SDSS

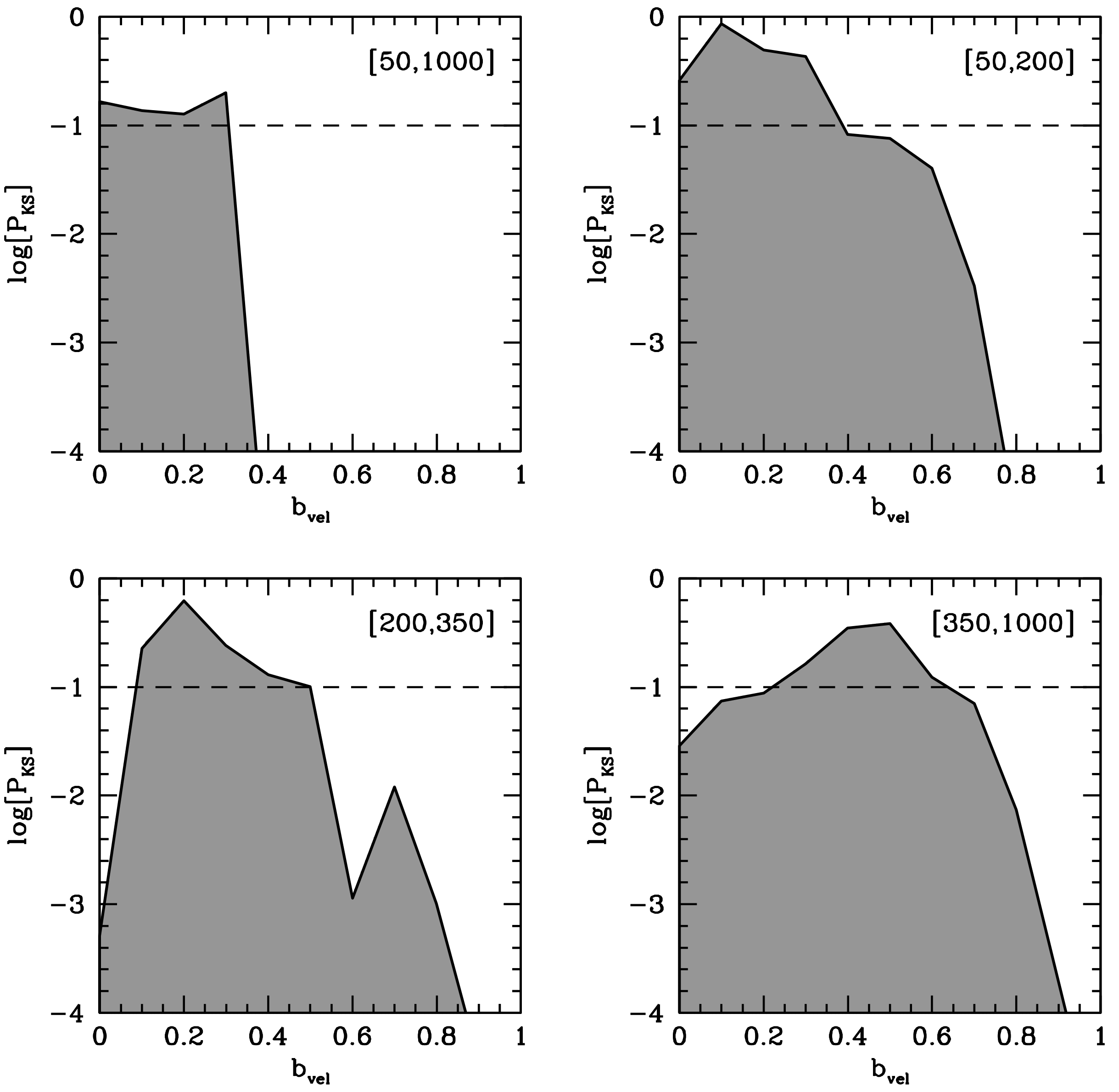


Testing $\mathcal{R}$-statistic with MGRSs



- We construct ten MGRSs, that only differ in the velocity bias ($b_{\rm vel}$) of the brightest halo galaxy
- Due to interlopers and incompleteness effects $P(\mathcal{R})$ is significantly broader than Student t-distribution, even for $b_{\rm vel} = 0$
- Different $b_{\rm vel}$ result in significantly different $P(\mathcal{R})$, allowing for a determination of $b_{\rm vel}$ despite interlopers and incompleteness.

Mass Dependence of b_{vel}



The velocity bias b_{vel} is larger for more massive haloes, in agreement with a hierarchical formation scenario.

Assumptions

- **All dark matter is collapsed into dark matter haloes**
- **All galaxies live in dark matter haloes**
- **Central galaxy resides at rest at center of halo**
- **Satellite galaxies follow dark matter distribition (NFW profile)**
- $P(N_{\rm sat}|M)$ **is Poissonian**
- **Halo bias:** $b = b(M)$ **; HOD:** $P(N) = P(N|M)$
- **Accuracy of** $b(M)$, $n(M)$, $\rho(r)$ **and** $\xi_{\rm dm}(r)$
- **Halo is a cow, and therefore spherical**
- **What can we learn from HOD as function of redshift?**

Assembly Bias: does it matter?

Assume that halo bias $b = b(M, c)$ with c the halo concentration, but that HOD does not depend on halo concentration, i.e., $\langle N \rangle = \langle N \rangle(M)$

$$b_g = \frac{1}{n_g} \int \mathrm{d}M \, \langle N \rangle(M) \, \int \mathrm{d}c \, b(M, c) \, n(M, c)$$

However, when ignoring this assembly bias we compute

$$b'_g = \frac{1}{n_g} \int \mathrm{d}M \, \langle N \rangle(M) \, b(M) \, n(M)$$

Since $b(M) = \frac{1}{n(M)} \int b(M, c) n(M, c) \mathrm{d}c$ one has that

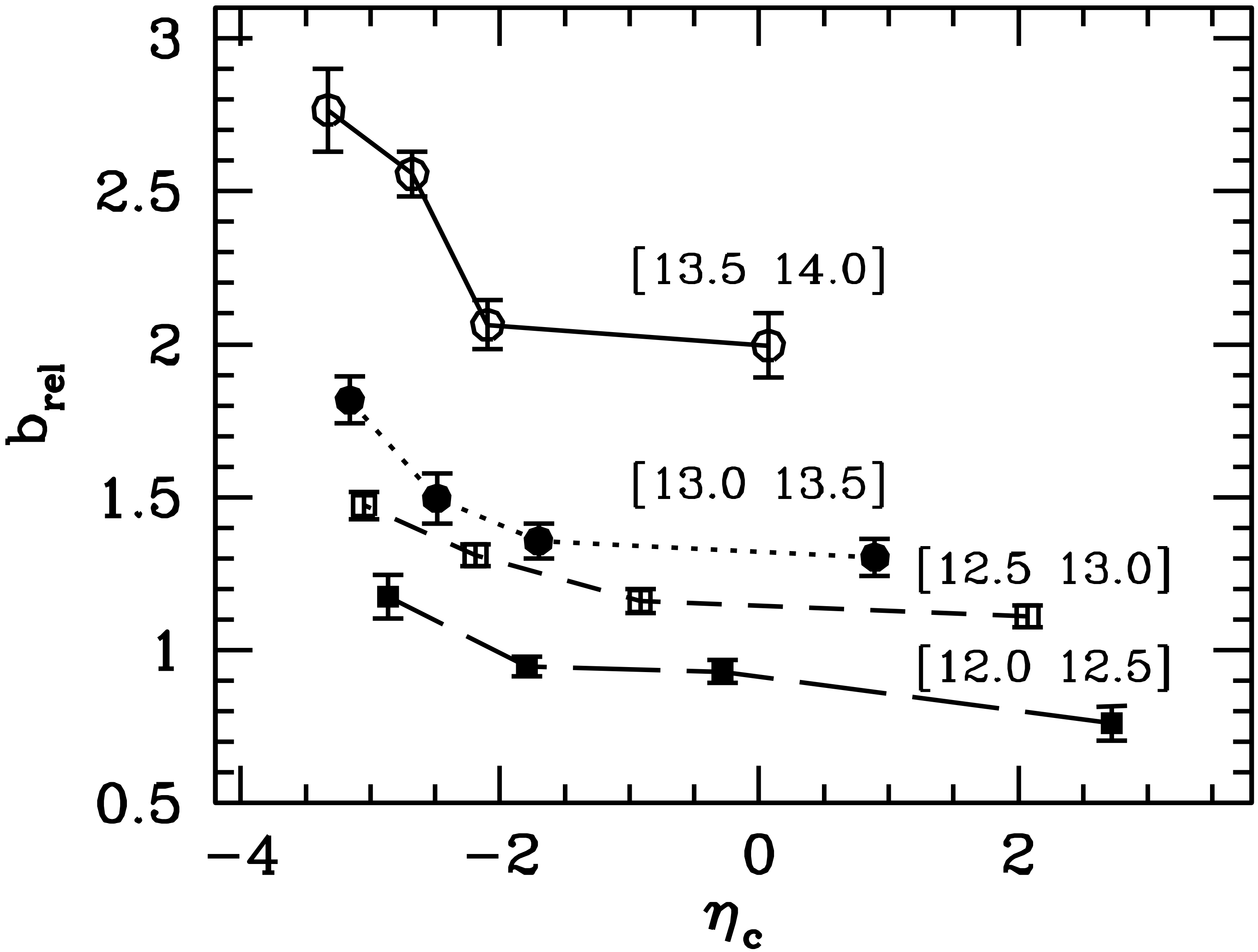
$$b'_g = \frac{1}{n_g} \int \mathrm{d}M \langle N \rangle(M) \, \int \mathrm{d}c \, b(M, c) \, n(M, c) = b_g$$

However, when $\langle N \rangle = \langle N \rangle(M, c)$ this is no longer true, and $b'_g \neq b_g$

Important questions:

- **What is $b(M, c)$?**
- **Do equal-mass haloes with different c have different galaxy properties?**

Assembly Bias??



Derivation of Halo Bias I

Define **halo bias** as $b(m) = \delta_h(m)/\delta$

Let $N(m|M,V)$ be the number of haloes of mass m in volume V.

The volume V has an overdensity δ so that $M = V\bar{\rho}(1+\delta)$ and initially was associated with a volume $V_0 = V(1+\delta)$.

The overdensity in the **number** of haloes of mass m is

$$\delta_h(m) = \frac{N(m|M,V)}{n(m)V} - 1$$

Here $n(m)$ is the (average) halo mass function.

To take account of the **dynamical** bias we write

$$N(m|M,V) = n(m|M,V)V_0 = n(m|M,V)V(1+\delta)$$

so that

$$\delta_h(m) = \frac{n(m|M,V)}{n(m)}(1+\delta) - 1$$

Derivation of Halo Bias II

PS ansatz: haloes are associates with regions with $\delta > \delta_{\rm sc}$

Therefore, we can compute $n(m|M,V) = n(m|\delta)$ by simply replacing $\delta_{\rm sc}$ with $\delta_{\rm sc} - \delta$ **(Peak-Background split)**.

Using that the **halo bias** is defined as $b(m) = \delta_h(m)/\delta$, one obtains that

$$b(m) = 1 + \frac{\nu - 1}{\delta_{\rm sc}}$$

where $\nu = \nu(m) = \delta_{\rm sc}^2/\sigma^2(m)$

Using that $\sigma(m^*) \equiv \delta_{\rm sc}$ we see that

$b(m) > 1$	if $m > m^*$	**(biased)**
$b(m) = 1$	if $m = m^*$	**(unbiased)**
$b(m) < 1$	if $m = m^*$	**(unbiased)**
$1 - \frac{1}{\delta_{\rm sc}} < b(m) < 1$	if $m < m^*$	**(anti-biased)**

Note that there is an absolute minimim to the halo bias: $b > 1 - \frac{1}{\delta_{\rm sc}}$.

Halo-Halo correlation function:

$$\xi_{\rm hh}(r) \equiv \langle \delta_{h_1} \delta_{h_2} \rangle = b(m_1) b(m_2) \langle \delta_1 \delta_2 \rangle = b(m_1) b(m_2) \xi(r)$$