ASTR 610: Problem Set 4

This problem set consists of two problems,
both of which require you to write some code.
Due date: thursday Dec 7, 2022

Problem 1: Mass Variance `[40 points]`
Write a computer program that computes the mass variance, $\sigma(M)$, for any flat ΛCDM cosmology with an initial power spectrum $P(k) \propto k^n$. Use a top-hat filter. As input, the program should read the following cosmological parameters; the present-day matter density $\Omega_{\rm m,0} = 1 - \Omega_{\Lambda,0}$, the present-day density of baryons, $\Omega_{\rm b,0}$, the Hubble parameter, $h = H_0/(100\,{\rm km\,s^{-1}\,Mpc^{-1}})$, the spectral index, n, and the power-spectrum normalization parameter σ_8. As output the program should write out $\sigma(M)$ for $\log[M/(h^{-1}\,{\rm M}_\odot)]$ ranging from 5.0 to 15.0 in steps of $\Delta \log M = 0.2$. As transfer function, use the fitting function of BBKS (Bardeen et al. 1986), which is given by

$$T(k) = \frac{\ln(1+2.34q)}{2.34q}\,\Big[1.0 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4\Big]^{-1/4}$$

where

$$q = \frac{1}{\Gamma}\,\left(\frac{k}{h\,{\rm Mpc^{-1}}}\right)$$

and

$$\Gamma = \Omega_{\rm m,0} h \exp\Big[-\Omega_{\rm b,0}(1+\sqrt{2h}/\Omega_{\rm m,0})\Big]$$

(this exponential correction factor for baryons is due to Sugiyama 1995).

- Plot the mass variance (in a log-log plot) for a ΛCDM cosmology with $(\Omega_{\rm m,0}, \Omega_{\rm b,0}, h, n, \sigma_8) = (0.3, 0.04, 0.7, 1.0, 0.8)$ [hereafter the 'fiducial model']. Overplot the results for cosmologies in which $\Omega_{\rm m,0}$ is changed to 0.2 and 0.4, while all other parameters are kept fixed to their fiducial value. Halo masses are to be expressed in $h^{-1}M_\odot$.

- Plot the mass variance (in a log-log plot) for the fiducial model, and overplot the results for cosmologies in which $\Omega_{\rm b,0}$ is changed to 0.01 and 0.08, while all other parameters are kept fixed to their fiducial value.

- Plot the mass variance (in a log-log plot) for the fiducial model, and overplot the results for cosmologies in which n is changed to 0.9 and 0.95, while all other parameters are kept fixed to their fiducial value.

- Plot the mass variance (in a log-log plot) for the fiducial model, and overplot the results for cosmologies in which σ_8 is changed to 0.7 and 0.9, while all other parameters are kept fixed to their fiducial value.

- Print out the code that you wrote for this exersize, and include it with your work.

Problem 2: Halo Mass Function `[20 points]`
Using the code of problem 1, write additional code that computes, for the same flat ΛCDM cosmology, the $z = 0$ halo mass function $\mathrm{d}N/\mathrm{d}\ln M$ as predicted by Press-Schecter theory. You can assume that the critical density for collapse is $\delta_{\rm c} = 1.686$ (ignore the weak dependence on cosmology) and you can approximate the linear growth rate as $D(z) = g(z)/(1+z)$ with

$$g(z) = \frac{5}{2}\Omega_{\rm m}(z)\left\{\Omega_{\rm m}^{4/7}(z) - \Omega_\Lambda(z) + [1+\Omega_{\rm m}(z)/2]\,[1+\Omega_\Lambda(z)/70]\right\}^{-1}$$

which is due to Carroll et al. (1992). You can use Eq. (3.77) in MBW to compute $\Omega_{\rm m}(z)$ and $\Omega_\Lambda(z)$.

- Plot the halo mass function (in a log-log plot) for the fiducial cosmology defined under Problem 1, and show how it changes under the same modifications of the cosmological parameters as under problem 1. Halo masses are to be expressed in $h^{-1}M_\odot$ and the abundance of haloes has to be in units of $h^3{\rm Mpc}^{-3}$.

- Print out the code that you wrote for this exersize, and include it with your work.