ASTR 610
Theory of Galaxy Formation

Lecture 6: Newtonian Perturbation Theory
III. Dark Matter

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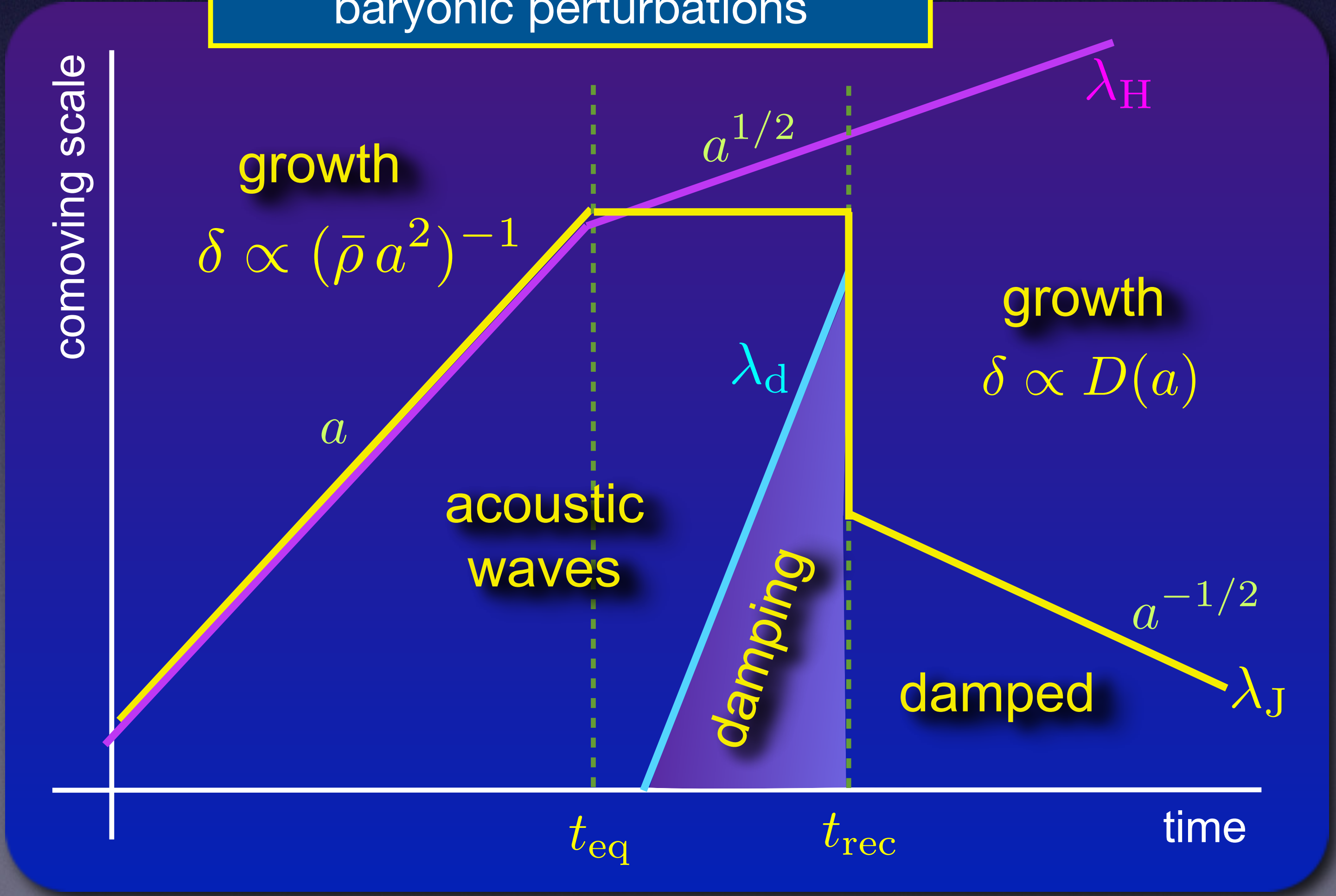
The Evolution of Baryonic Perturbations

Master Equation: $$\frac{d^2\delta_{\vec{k}}}{dt^2} + 2\frac{\dot{a}}{a}\frac{d\delta_{\vec{k}}}{dt} = \left[4\pi G\bar{\rho} - \frac{k^2 c_s^2}{a^2}\right]\delta_{\vec{k}} - \frac{2}{3}\frac{\bar{T}}{a^2}k^2\,S_{\vec{k}}$$

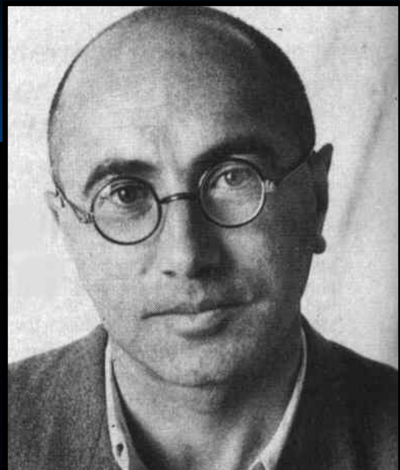
Describes evolution of individual modes in Fourier space, sourced by gravity & pressure (i.e., by isentropic and isocurvature perturbations).

Adiabatic evolution of isentropic, baryonic perturbations



- Perturbations smaller than Jeans scale undergo acoustic oscillations
- Silk damping arises from photon-diffusion close to recombination.
- After recombination, perturbations larger than Jeans length grow according to linear growth rate, D(a)

The Adiabatic, Baryonic Model



Yakov Zel'dovich

If matter is purely baryonic, and perturbations are isentropic (`adiabatic'), structure formation proceeds top-down by fragmentation of perturbations larger than Silk damping scale at recombination $M_{\rm d} \sim 10^{13} M_{\odot}$

This was the picture for structure formation developed by Zel'dovich and his colleagues during the 1960's in Moscow. However, it soon became clear that this picture was doomed....

To allow sufficient time for fragmentation, the large-scale perturbations need large amplitudes in order to collapse sufficiently early. At recombination one requires $|\delta_{\rm m}| > 10^{-3}$

Using that $\delta_T = 1/4\delta_{\rm r} = 1/3\delta_{\rm m}$, which follows from fact that perturbations are isentropic and from $\rho_{\rm r} \propto T^4$, this model implies CMB fluctuations

$$\Delta T/T > 10^{-3}$$

Such large fluctuations were already ruled out in early 1980s (e.g., Uson & Wilkinson 1984)

Nature 1984

Improved limits on small-scale anisotropy in cosmic microwave background

Juan M. Uson & David T. Wilkinson

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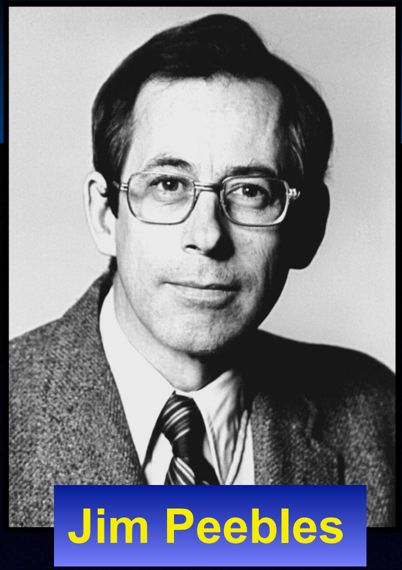
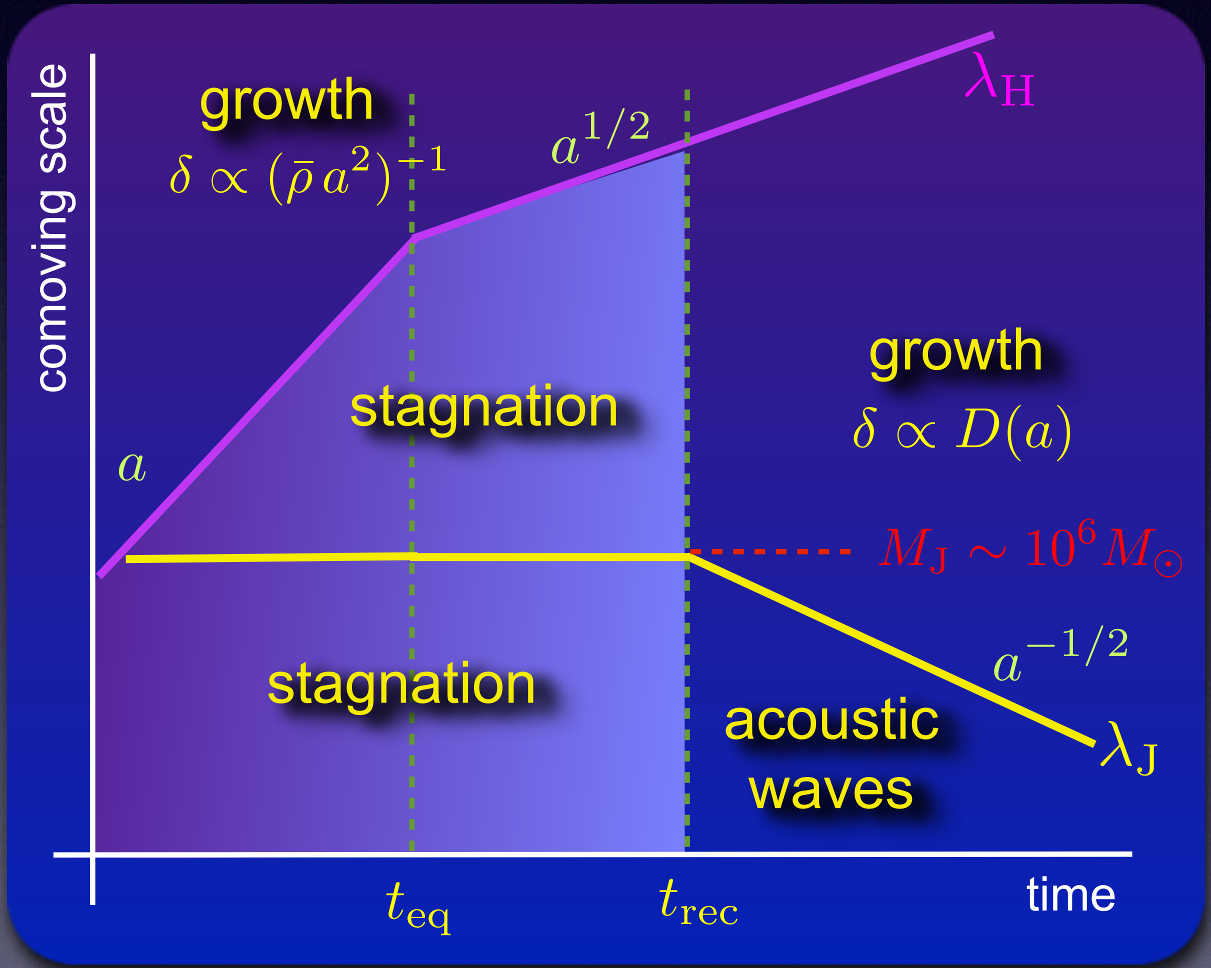
As a remnant of the early Universe, the cosmic microwave background provides unique information on the initial conditions from which matter has evolved to form the structures we see today. All efforts to detect small-scale structure in this radiation have so far been unsuccessful (ref. 1 and refs therein)[1–4]. Nevertheless, upper limits set on possible underlying fluctuations restrict the range of physical models for perturbations of the density in the early Universe. Our search for small-scale anisotropy in the background radiation has now resulted in a lowering of the upper limit on root-mean-square fluctuations ($\Delta T_{\rm r.m.s.}$) observed at an angular scale of ~4 arc min to $\Delta T_{\rm r.m.s.}/T < 2.1 \times 10^{-5}$ at the 95% confidence level (where $T = 2.7$ K, the temperature of the background radiation). The actual limits deduced from our experiment depend on the model assumed for the unseen fluctuations. Several possibilities are discussed as well as the implications this new measurement has for various cosmological models.

The Isothermal, Baryonic Model



While Zel'dovich was working on his adiabatic model, Peebles and his colleagues at Princeton developed an alternative model for structure formation, in which the perturbations were assumed to be isothermal (i.e., $\delta_{\rm r} = 0$), which is a good approximation for isocurvature perturbations prior to the matter era.

In this model, sound speed prior to recombination is much lower, resulting in much lower Jeans mass. Also, since there are no radiation perturbations, there is NO Silk damping. All perturbations with $M > M_{\rm J} \sim 10^6 M_{\odot}$ survive, and structure formation proceeds hierarchical (bottom-up).



Prior to recombination, radiation drag prevents perturbations from growing (they are `frozen'), but at least they are not damped...

Similar to adiabatic model, this isothermal baryonic model requires large temperature fluctuations in CMB to explain observed structure

In addition, isothermal perturbations are fairly "unnatural".
All these problems dissapear when considering a separate matter component: dark matter

Collisionless Dark Matter

We now turn our attention to Collisionless Dark Matter. Here `Dark' indicates that this matter has no EM interaction (no interaction with photons).

In what follows we use X to indicate some generic particle species produced in the early Universe. We call such particles `cosmic relics'.

Cosmic Relics

Non-thermal Relics

not produced in thermal equilibrium (TE) with rest of Universe.

e.g., axions (FDM), monopoles, cosmic strings

Thermal Relics

are held in TE with other components of Universe until they `decouple', which happens when interaction rate $\Gamma = n\langle\sigma v\rangle$ drops below expansion rate $H(a)$

Hot Dark Matter

particles are still relativistic at decoupling, i.e., $3k_{\rm B}T_{\rm d} > m_{\rm X}c^2$

e.g., massive neutrinos

Cold Dark Matter

particles are non-relativistic at decoupling i.e., $3k_{\rm B}T_{\rm d} < m_{\rm X}c^2$

e.g., WIMPS

Collisionless Fluids

In order to describe evolution of collisionless dark matter, in general one cannot use the fluid description. Instead, one has to resort to the Collisionless Boltzmann Equation (CBE):

Let $f(\vec{x}, \vec{v}, t) = \mathrm{d}N/\mathrm{d}^3\vec{x}\,\mathrm{d}^3\vec{v}$ be the distribution function, which expresses the number of particles per unit volume in phase-space.

For a collisionless system we have that $\frac{\mathrm{d}f}{\mathrm{d}t} = 0$

This is the CBE, which expresses that in a collisionless system the phase-space density around each particle is conserved (i.e., there is no diffusion or scattering).

Using that $\mathrm{d}f = \frac{\partial f}{\partial t}\mathrm{d}t + \frac{\partial f}{\partial x_i}\mathrm{d}x_i + \frac{\partial f}{\partial v_i}\mathrm{d}v_i$ (Einstein summation convention)

and that $\mathrm{d}v_i/\mathrm{d}t = -\partial\Phi/\partial x_i$ we can write the CBE as:

$$\frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} - \frac{\partial \Phi}{\partial x_i}\frac{\partial f}{\partial v_i} = 0$$

Collisionless Fluids

Rather than solving CBE itself, one normally focusses on the moment equations. These are obtained by multiplying all terms by v^k and integrating over velocity space. When using comoving coordinates in an expanding space with scale factor $a(t)$, one obtains, for $k=0$

$$\frac{\partial \delta}{\partial t} + \frac{1}{a} \sum_j \frac{\partial}{\partial x_j} \left[(1+\delta)\, \langle v_j \rangle\right] = 0$$

continuity equation

Here $\rho_{\rm com}(\vec{x}) = \int f \, {\rm d}^3\vec{v} = \bar{\rho}\, a^3 \left[1 + \delta(\vec{x})\right]$, and $\langle v_i \rangle$ is the mean streaming motion in the i-direction.

NOTE: summation signs have been added for clarity, but can be removed according to the Einstein summation convention

Similarly, for $k=1$ one obtains

$$\frac{\partial \langle v_i \rangle}{\partial t} + \frac{\dot{a}}{a} \langle v_i \rangle + \frac{1}{a} \sum_j \langle v_j \rangle \frac{\partial \langle v_i \rangle}{\partial x_j} = -\frac{1}{a} \frac{\partial \Phi}{\partial x_i} - \frac{1}{\bar{\rho} a (1+\delta)} \sum_j \frac{\partial \rho \sigma_{ij}^2}{\partial x_j}$$

These are called the Jeans equations, and are the collisionless analog of the Euler equations for a fluid. The only difference is that the stress tensor $\rho\sigma_{ij}^2$ now plays the role of the pressure...

Collisionless Fluids

The velocity dispersion tensor is defined as $\sigma_{ij}^2 = \langle v_i \, v_j \rangle - \langle v_i \rangle \, \langle v_j \rangle$

Since this tensor is manifest symmetric, i.e., $\sigma_{ij} = \sigma_{ji}$ it is characterized by 6 independent numbers. Thus, whereas fluid pressure is characterized by a single scalar quantity, the stress tensor has 6 unknowns.

This also means that a simple `Equation-of-State' will NOT suffice to close the set of equations → in general the Jeans equations cannot be solved.

One typically proceeds by making a number of simplifying assumptions (see MBW §5.4). In what follows, we will do the same and assume that the velocity dispersion is isotropic → $\sigma_{ij} = \sigma \, \delta_{ij}$

If we further assume that σ is independent of location, then $\frac{\nabla \rho \sigma^2}{\bar{\rho}} = \sigma^2 \nabla \delta$

Substituting this in the Jeans equations, and comparing the result to the Euler equations, it is clear that the velocity disperion $\sigma = \langle v_i^2 \rangle^{1/2}$ now plays the same role as the sound speed, $c_{\rm s}$, in the fluid equations.

Collisional vs. Collisionless Fluids

Moments of Collisionless Boltzmann Equation

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\sum_j \frac{\partial}{\partial x_j}\left[(1+\delta)\langle v_j\rangle\right] = 0$$

$$\frac{\partial\langle v_i\rangle}{\partial t} + \frac{\dot{a}}{a}\langle v_i\rangle + \frac{1}{a}\sum_j \langle v_j\rangle \frac{\partial\langle v_i\rangle}{\partial x_j} = -\frac{1}{a}\frac{\partial\Phi}{\partial x_i} - \frac{1}{\bar{\rho}a(1+\delta)}\sum_j \frac{\partial \rho\sigma_{ij}^2}{\partial x_j}$$

vector notation

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla\cdot\left[(1+\delta)\,\vec{v}\right] = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \frac{\dot{a}}{a}\vec{v} + \frac{1}{a}(\vec{v}\cdot\nabla)\,\vec{v} = -\frac{\nabla\Phi}{a} - \frac{1}{a\,(1+\delta)}\frac{\nabla\rho\sigma^2}{\bar{\rho}}$$

σ independent of location

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla\cdot\left[(1+\delta)\,\vec{v}\right] = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \frac{\dot{a}}{a}\vec{v} + \frac{1}{a}(\vec{v}\cdot\nabla)\,\vec{v} = -\frac{\nabla\Phi}{a} - \frac{\sigma^2}{a}\frac{\nabla\delta}{(1+\delta)}$$

continuity equation

Jeans equations

Euler equations

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla\cdot\left[(1+\delta)\,\vec{v}\right] = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \frac{\dot{a}}{a}\vec{v} + \frac{1}{a}(\vec{v}\cdot\nabla)\,\vec{v} = -\frac{\nabla\Phi}{a} - \frac{c_{\rm s}^2}{a}\frac{\nabla\delta}{(1+\delta)} - \frac{2T}{3a}\nabla S$$

Fluid Equations (see Lecture 4)

The Jeans Length of a Collisionless Fluid

Thus, a collisionless fluid with isotropic and homogeneous velocity dispersion can be described by exactly the same continuity and momentum equations as a collisional fluid, but with the sound speed $c_{\rm s}$ replaced by $\sigma = \langle v_i^2 \rangle^{1/2}$

Hence we can define the collisionless analog of the Jeans length:

$$\lambda_{\rm J}^{\rm prop} = a(t)\lambda_{\rm J}^{\rm com} = a(t)\frac{2\pi}{k_{\rm J}} = \sigma\sqrt{\frac{\pi}{G\bar{\rho}}}$$

However, there is one very important difference wrt a collisional fluid:

Collisional Fluid	
$\lambda > \lambda_{\rm J}$	gravitational collapse
$\lambda < \lambda_{\rm J}$	acoustic oscillations

Collisionless Fluid	
$\lambda > \lambda_{\rm J}$	gravitational collapse
$\lambda < \lambda_{\rm J}$	free streaming damping

Free streaming damping arises because particles disperse in random directions due to their non-zero velocity dispersion. This \`erases' perturbations with $\lambda < \lambda_{\rm J}$.

Free Streaming Damping

The fact that perturbations with $\lambda < \lambda_{\rm J}$ damp due to `free streaming' is easy to understand:

The time for collisionless particles to disperse over a distance λ is $\tau_{\rm disp} = \frac{\lambda}{\sigma}$.

Hence, the criterion $\lambda < \lambda_{\rm J}$ implies that $\tau_{\rm disp} < \sqrt{\frac{\pi}{G\bar{\rho}}}$, where we have used the definition of the Jeans length.

The Friedmann equation for a flat Universe (without cosmological constant) is:

$$H(a) = \sqrt{\frac{8\pi G\bar{\rho}}{3}}$$

from which we see that (roughly) $\tau_{\rm disp} < \tau_{\rm H} = 1/H(a)$

Hence for perturbations with $\lambda < \lambda_{\rm J}$ the dispersion time is shorter than the Hubble time, and those perturbations will thus have dispersed (=damped).

Free Streaming Damping

See MBW §3.3.3 for more details

As an example, consider massive neutrinos with a mass $m_\nu c^2 \sim 30\mathrm{eV}$

This was actually a popular dark matter candidate during early 1980s for two reasons:

- There was a (false) experimental claim that neutrinos indeed were this massive.
- One can show that $\Omega_\nu \sim 0.32 h^{-2} (m_\nu c^2 / 30\,\mathrm{eV})$ (of order unity, as required).

These neutrinos become non-relativistic when $3 k_\mathrm{B} T_\gamma \sim m_\nu c^2$

Using that $T_\gamma = 2.7\,\mathrm{K}(1+z)$ this yields $(1+z_\mathrm{NR}) \simeq 4.3 \times 10^4$

A more accurate calculation that accounts for fact that after decoupling $T_\nu \neq T_\gamma$ yields $(1+z_\mathrm{NR}) \simeq 6.0 \times 10^4$

For comparison, neutrinos decouple (`freeze-out') at $(1+z_\mathrm{dec}) \sim 4.3 \times 10^9$ so that we clearly have $z_\mathrm{NR} \ll z_\mathrm{dec}$ ➡ massive neutrinos are HDM

We now proceed to estimate how far these neutrinos could have free-streamed up to the time of equality. We define the free-streaming length as the comoving distance travelled, i.e.,

$$\lambda_\mathrm{fs}^\mathrm{com} = \int_0^{t_\mathrm{eq}} \frac{v(t')}{a(t')} \mathrm{d}t'$$

Free Streaming Damping

See MBW §3.3.3 for more details

Using that the peculiar velocity of neutrinos is $v \sim c$ prior to $t_{\rm NR}$ and $v \propto a^{-1}$ thereafter, one obtains that at the time of equality

$$\lambda_{\rm fs}^{\rm com} = \frac{2\,c\,t_{\rm NR}}{a_{\rm NR}} \left[1 + \ln(a_{\rm eq}/a_{\rm NR})\right]$$

Where we have used that $a(t) \propto t^{1/2}$ during the radiation dominated era.

Using that $1/a_{\rm NR} = (1 + z_{\rm NR}) \simeq 6 \times 10^4$ one obtains a (comoving) free-streaming length of $\lambda_{\rm fs}^{\rm com} \sim 30\,{\rm Mpc}$

The corresponding free-streaming mass is

$$M_{\rm fs} \equiv \frac{\pi}{6}\bar{\rho}_{\rm m,0}\,\left(\lambda_{\rm fs}^{\rm com}\right)^3 \simeq 1.3 \times 10^{15} M_\odot$$

Thus, all perturbations on scales below that of a massive cluster will have been erased by the time of equality. ‡

Structure formation in HDM cosmologies proceeds top-down.

‡ **Note:** In problem set 2 you will compute $M_{\rm fs}$ for a CDM model, and find it to be negligible....

Velocity Dispersion of Dark Matter

In order to compute the Jeans length, and thus also the Jeans mass, for dark matter, all we need is to determine the velocity dispersion as a function of time.

- When particles are still relativistic: $v_X \simeq c$
- Particles that are non-relativistic, but still coupled to photons: $v_X \propto a^{-1/2}$ ‡
- Particles that are non-relativistic, and decoupled: $v_X \propto a^{-1}$

‡ This follows from the fact that $3k_{\rm B}T_\gamma = m_X v_X^2$ and that $T_\gamma \propto a^{-1}$.

Hot Relics	Cold Relics
$t < t_{\rm NR} \Rightarrow \sigma_X = \frac{c}{\sqrt{3}}$	$t < t_{\rm NR} \Rightarrow \sigma_X = \frac{c}{\sqrt{3}}$
$t > t_{\rm NR} \Rightarrow \sigma_X = \frac{c}{\sqrt{3}}\frac{a_{\rm NR}}{a}$	$t_{\rm NR} < t < t_{\rm dec} \Rightarrow \sigma_X = \frac{c}{\sqrt{3}}\left(\frac{a_{\rm NR}}{a}\right)^{1/2}$
	$t > t_{\rm dec} \Rightarrow \sigma_X = \frac{c}{\sqrt{3}}\left(\frac{a_{\rm NR}}{a_{\rm dec}}\right)^{1/2}\frac{a_{\rm dec}}{a}$

The Jeans Length of Dark Matter

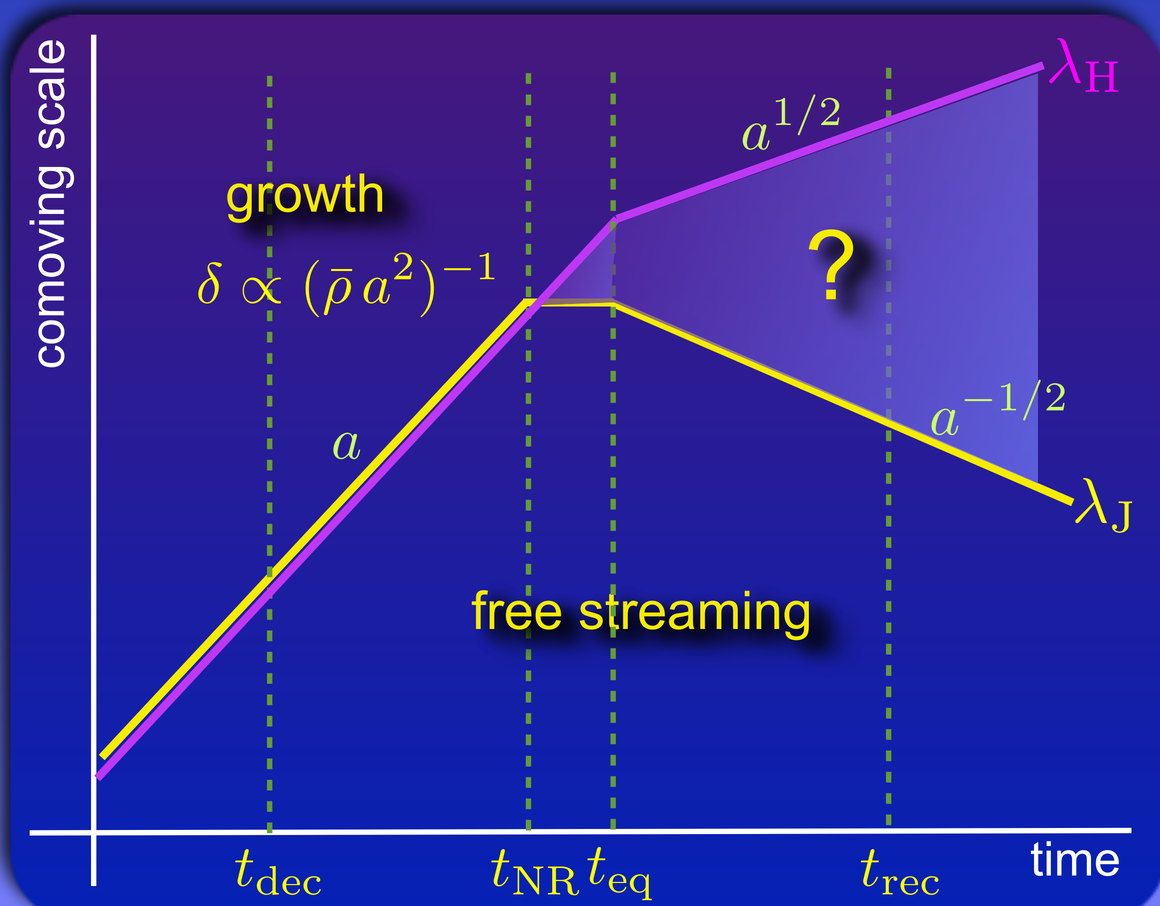
$$\lambda_J^{com} = \frac{\sigma}{a(t)} \sqrt{\frac{\pi}{G\bar{\rho}}}$$

Hot Relics (HDM)

$t < t_{NR}$ ⇨ $\lambda_J^{com} \propto a$

$t_{NR} < t < t_{eq}$ ⇨ $\lambda_J^{com} \propto a^0$

$t > t_{eq}$ ⇨ $\lambda_J^{com} \propto a^{-1/2}$



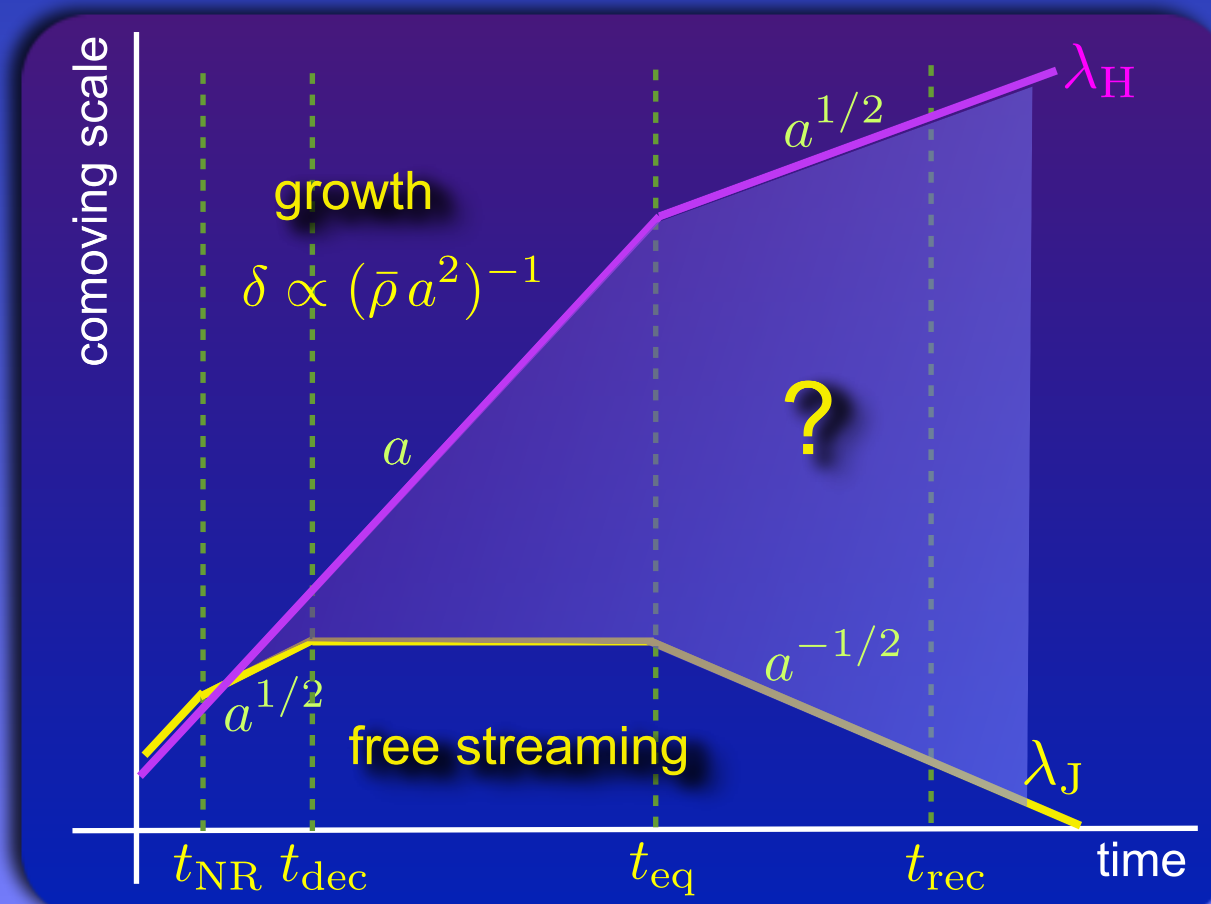
Cold Relics (CDM)

$t < t_{NR}$ ⇨ $\lambda_J^{com} \propto a$

$t_{NR} < t < t_{dec}$ ⇨ $\lambda_J^{com} \propto a^{1/2}$

$t_{dec} < t < t_{eq}$ ⇨ $\lambda_J^{com} \propto a^0$

$t > t_{eq}$ ⇨ $\lambda_J^{com} \propto a^{-1/2}$



Perturbations Growth during the Matter Era

In order to complete our description of the growth of dark matter perturbations we now focus on perturbations with $\lambda > \lambda_{\rm J}$ during the matter-dominated era:

Since $\sigma_{\rm X} \propto a^{-1}$ we have that the `equivalent of pressure' $P_{\rm X} = \rho_{\rm X}\,\sigma_{\rm X}^2 \propto a^{-5}$
Hence, the `pressure' drops like a rock, and we may thus treat our collisionless fluid as a pressureless fluid (but only for $\lambda > \lambda_{\rm J}$!!!)

As we have already seen, the (linearized) fluid equation describing perturbation growth of pressureless fluid is

$$\frac{{\rm d}^2\delta_{\vec{k}}}{{\rm d}t^2} + 2\frac{\dot{a}}{a}\frac{{\rm d}\delta_{\vec{k}}}{{\rm d}t} = 4\pi G\bar{\rho}_{\rm m}\,\delta_{\vec{k}}$$

When discussing baryonic perturbation growth, we have already seen that the growing mode solution for such perturbations is given by

$$\delta \propto D(a)$$

➡ This linear growth rate applies to both baryonic matter and collisionless dark matter!

EdS cosmology	➡	$D(a) \propto a$
ΛCDM cosmology	➡	$D(a) \propto a^{\gamma} \quad (\gamma < 1)$

Once the cosmological constant starts to dominate, the accelerated expansion suppresses structure formation. This is simply a consequence of the increased Hubble drag.

Perturbations Growth during the Matter Era

During radiation-dominated era, but after decoupling, we have that $P_{\rm X} = \rho_{\rm X}\,\sigma_{\rm X}^2 \propto a^{-6}$, so once again we may treat our collisionless fluid as a pressureless fluid (but only for $\lambda > \lambda_{\rm J}$!!!)

However, since now $\bar{\rho}_{\rm r} \gg \bar{\rho}_{\rm m}$ we need to modify our linearized fluid equation to:

$$\frac{{\rm d}^2\delta_{\vec{k}}}{{\rm d}t^2} + 2\frac{\dot{a}}{a}\frac{{\rm d}\delta_{\vec{k}}}{{\rm d}t} = 4\pi G(\bar{\rho}_{\rm m} + \bar{\rho}_{\rm r})\,\delta_{\vec{k}} \qquad \text{(for } t < t_{\rm eq}\text{)}$$

new term

Since $\bar{\rho}_{\rm m} \propto a^{-3}$ whereas $\bar{\rho}_{\rm r} \propto a^{-4}$, one has different growth rates during the matter- and radiation-dominated periods. Introducing the new time variable $\zeta \equiv \bar{\rho}_{\rm m}/\bar{\rho}_{\rm r} = a/a_{\rm eq}$ this can be written as:

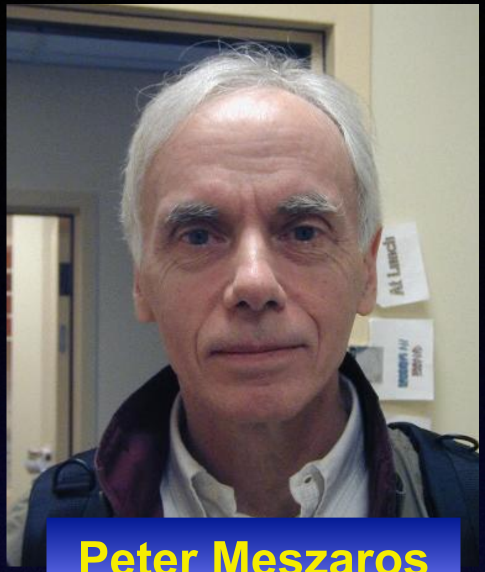
$$\frac{{\rm d}^2\delta_{\vec{k}}}{{\rm d}\zeta^2} + \frac{(2+3\zeta)}{2\zeta(1+\zeta)}\frac{{\rm d}\delta_{\vec{k}}}{{\rm d}\zeta} = \frac{3}{2}\,\frac{\delta_{\vec{k}}}{\zeta(1+\zeta)}$$

for which the growing mode solution is $\delta_+ \propto 1 + \frac{3}{2}\zeta$

$t \ll t_{\rm eq}$ → $\zeta \ll 1$ → δ_+ constant → stagnation

$t \gg t_{\rm eq}$ → $\zeta \gg 1$ → $\delta_+ \propto a$ → growth

Stagnation: the "Meszaros Effect"



Peter Meszaros

The stagnation of growth in pressureless matter perturbations during radiation dominated era is known as the Meszaros effect

This differs from stagnation due to radiation drag that plays role in isothermal, baryonic perturbations. After all, CDM experiences no radiation drag (i.e., has no EM interaction).

The Meszaros effect is simply a manifestation of the fact that the Hubble drag term during the radiation dominated era is larger than during the matter dominated era. Consider the following qualitative argument:

Characteristic time for growth (`collapse') of perturbation of pressureless material (e.g., dark matter) is the free-fall time $\tau_{\rm ff} \propto (G\rho_{\rm m})^{-1/2}$

For comparison, the characteristic time for the expansion of the Universe is the Hubble time $\tau_{\rm H} = 1/H$. Using the Friedmann equation, one immediately sees that $\tau_{\rm H} \propto (G\rho_{\rm r})^{-1/2}$ during radiation era, and $\tau_{\rm H} \propto (G\rho_{\rm m})^{-1/2}$ during matter era.

Hence, $\tau_{\rm H}/\tau_{\rm ff} \propto (\rho_{\rm m}/\rho_{\rm r})^{1/2}$ during radiation era, strongly suppressing structure growth. During matter era, on the other hand, $\tau_{\rm H}/\tau_{\rm ff} \simeq 1$

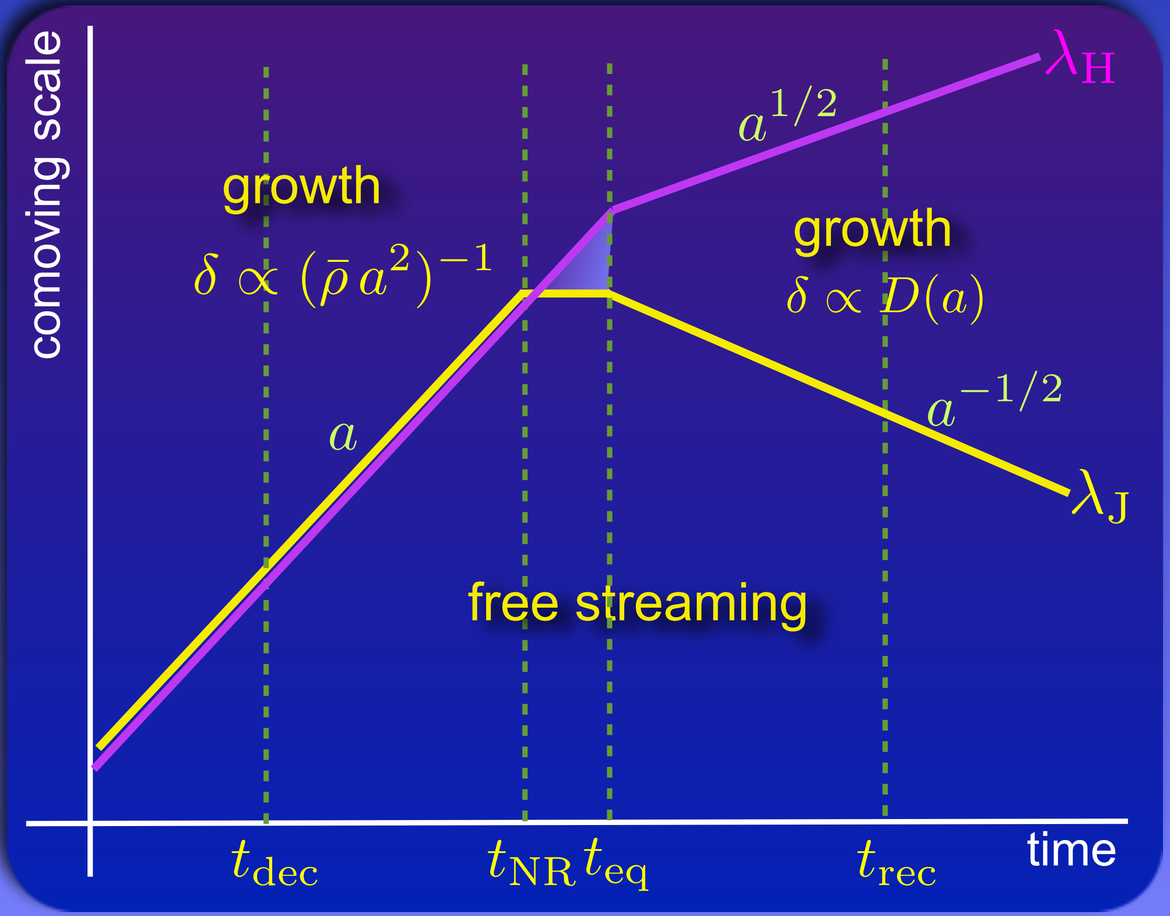
Growth of Dark Matter Perturbations

Hot Relics (HDM)

$t < t_{\rm NR}$ ➡ $\lambda_J^{\rm com} \propto a$

$t_{\rm NR} < t < t_{\rm eq}$ ➡ $\lambda_J^{\rm com} \propto a^0$

$t > t_{\rm eq}$ ➡ $\lambda_J^{\rm com} \propto a^{-1/2}$



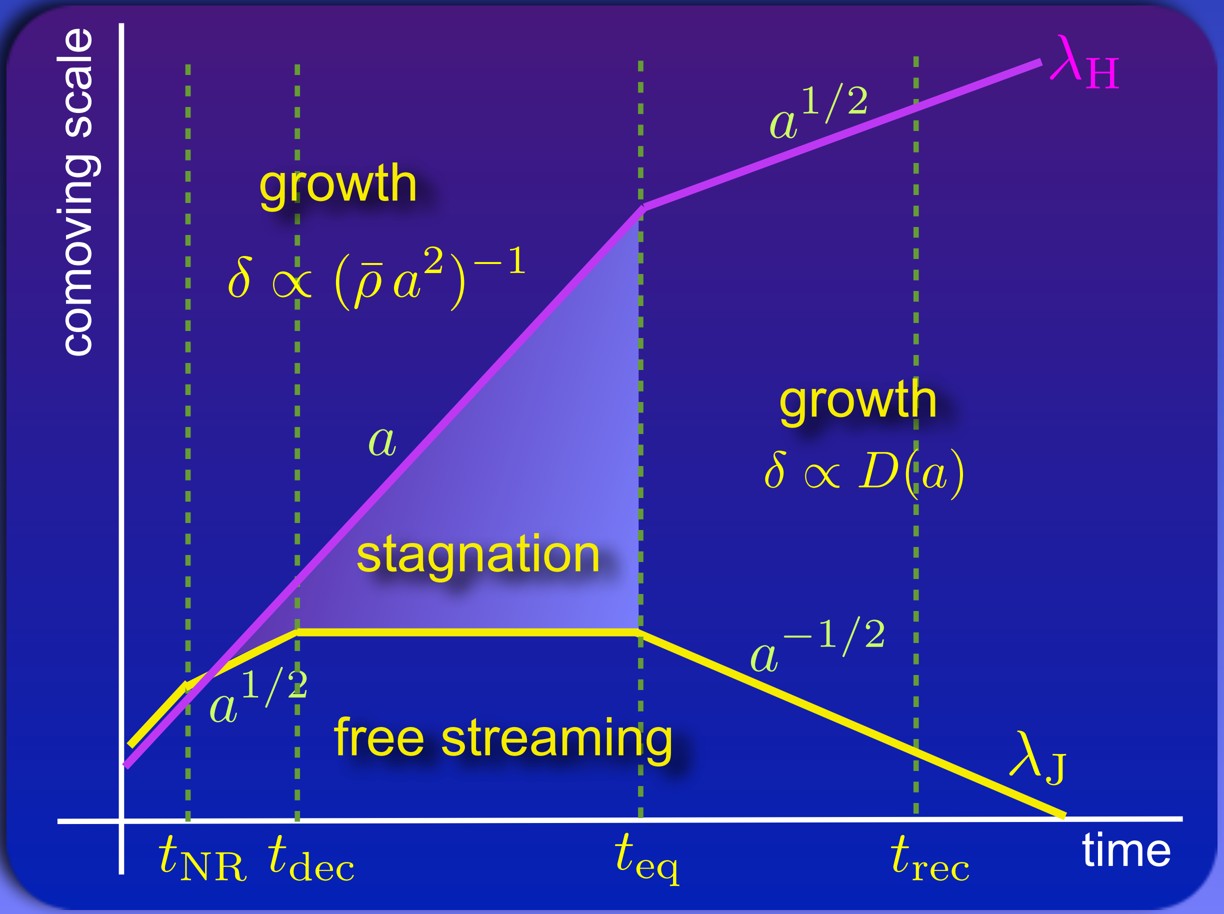
Cold Relics (CDM)

$t < t_{\rm NR}$ ➡ $\lambda_J^{\rm com} \propto a$

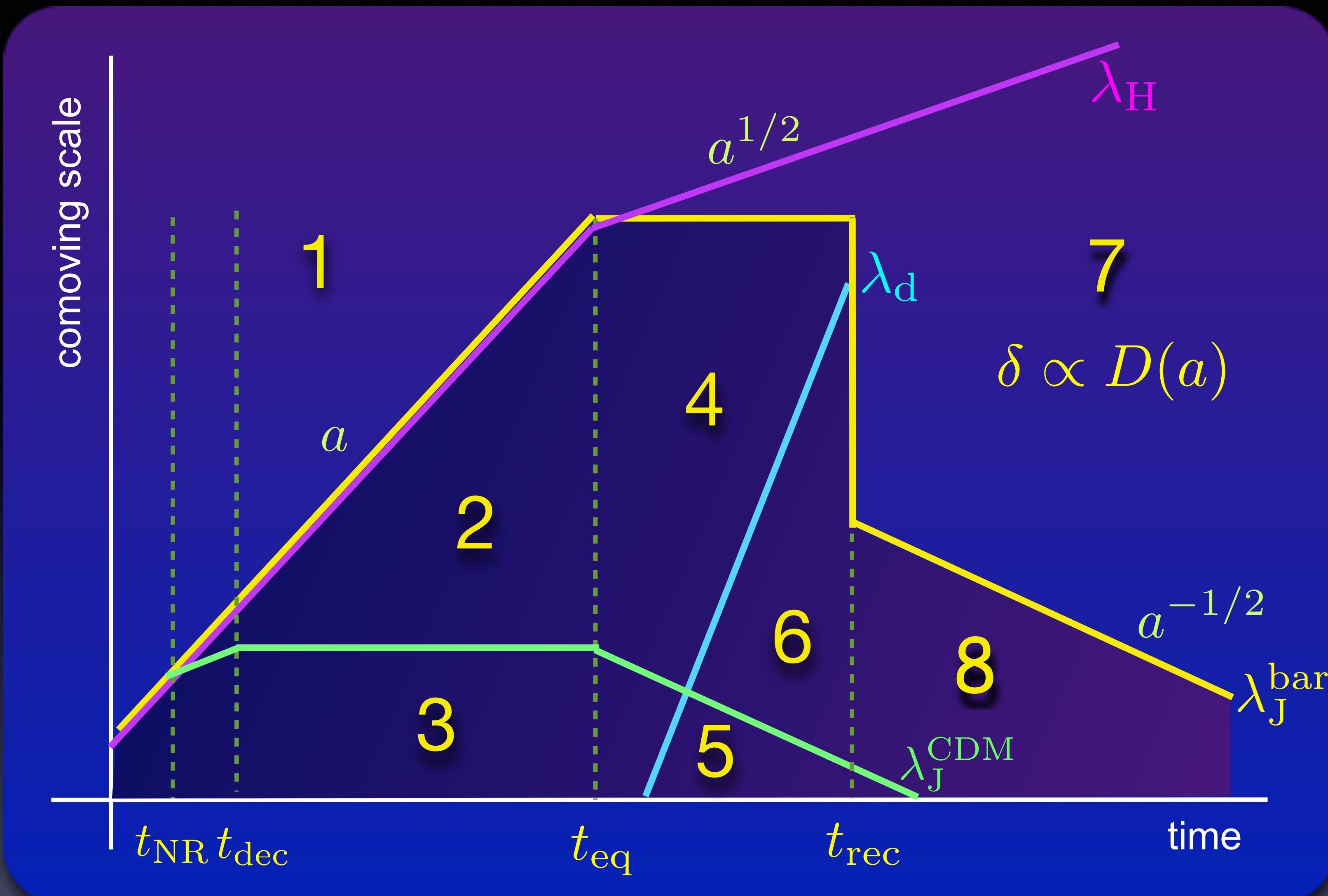
$t_{\rm NR} < t < t_{\rm dec}$ ➡ $\lambda_J^{\rm com} \propto a^{1/2}$

$t_{\rm dec} < t < t_{\rm eq}$ ➡ $\lambda_J^{\rm com} \propto a^0$

$t > t_{\rm eq}$ ➡ $\lambda_J^{\rm com} \propto a^{-1/2}$



Baryons & CDM combined

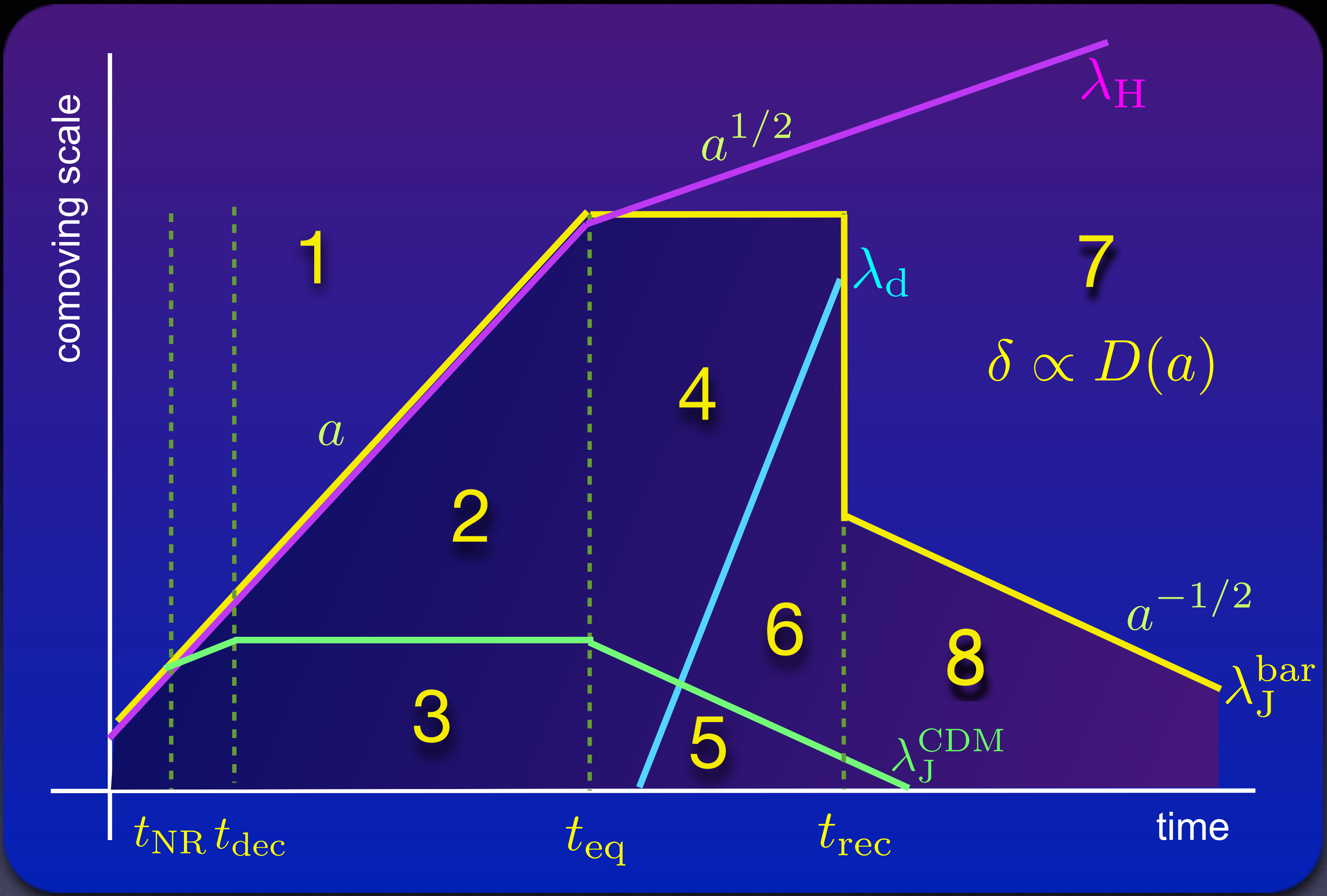


	Baryons	CDM
1		
2		
3		
4		
5		
6		
7		
8		

Growth of isentropic perturbations in a mixture of radiation, baryons and CDM.

- The acoustic oscillations of the baryons during 2 - 6 are now driven by the potential perturbations in the dark matter component. (see MBW §4.1.6c)
- After recombination, baryons fall in DM potential wells; gravity \`re-creates' the baryonic perturbations, un-doing the effect of Silk damping.

Baryons & CDM combined



	Baryons	CDM
1	growth	growth
2	oscillations	stagnation
3	oscillations	free-streaming
4	oscillations	growth
5	Silk-damping	free-streaming
6	Silk-damping	growth
7	growth	growth
8	oscillations	growth

Growth of isentropic perturbations in a mixture of radiation, baryons and CDM.

- The acoustic oscillations of the baryons during 2 - 6 are now driven by the potential perturbations in the dark matter component. (see MBW §4.1.6c)
- After recombination, baryons fall in DM potential wells; gravity `re-creates' the baryonic perturbations, un-doing the effect of Silk damping.

The Integrated Sachs-Wolfe Effect

The Poisson equation in Fourier space reads: $-k^2\Phi_{\vec{k}} = 4\pi G\bar{\rho}a^2\delta_{\vec{k}}$

In a matter dominated Universe $\bar{\rho} \propto a^{-3}$, so that $\Phi_{\vec{k}} \propto D(a)/a$

EdS cosmology → $D(a) \propto a$ → $\Phi_{\vec{k}} \propto \text{const}$

ΛCDM cosmology → $D(a) \propto a^{\gamma}$ → $\Phi_{\vec{k}} \propto a^{\gamma-1}$

Since $\gamma < 1$ for a typical ΛCDM cosmology, potential perturbations (in the linear regime), will decay with time.

A photon moving through (linear) perturbation will fall into deeper potential well than what it climbs out of: → it gains energy. This implies that large-scale (=linear) structure between last-scattering surface and us produce temperature fluctuations in CMB. This is called the Integrated Sachs Wolfe (ISW) effect.

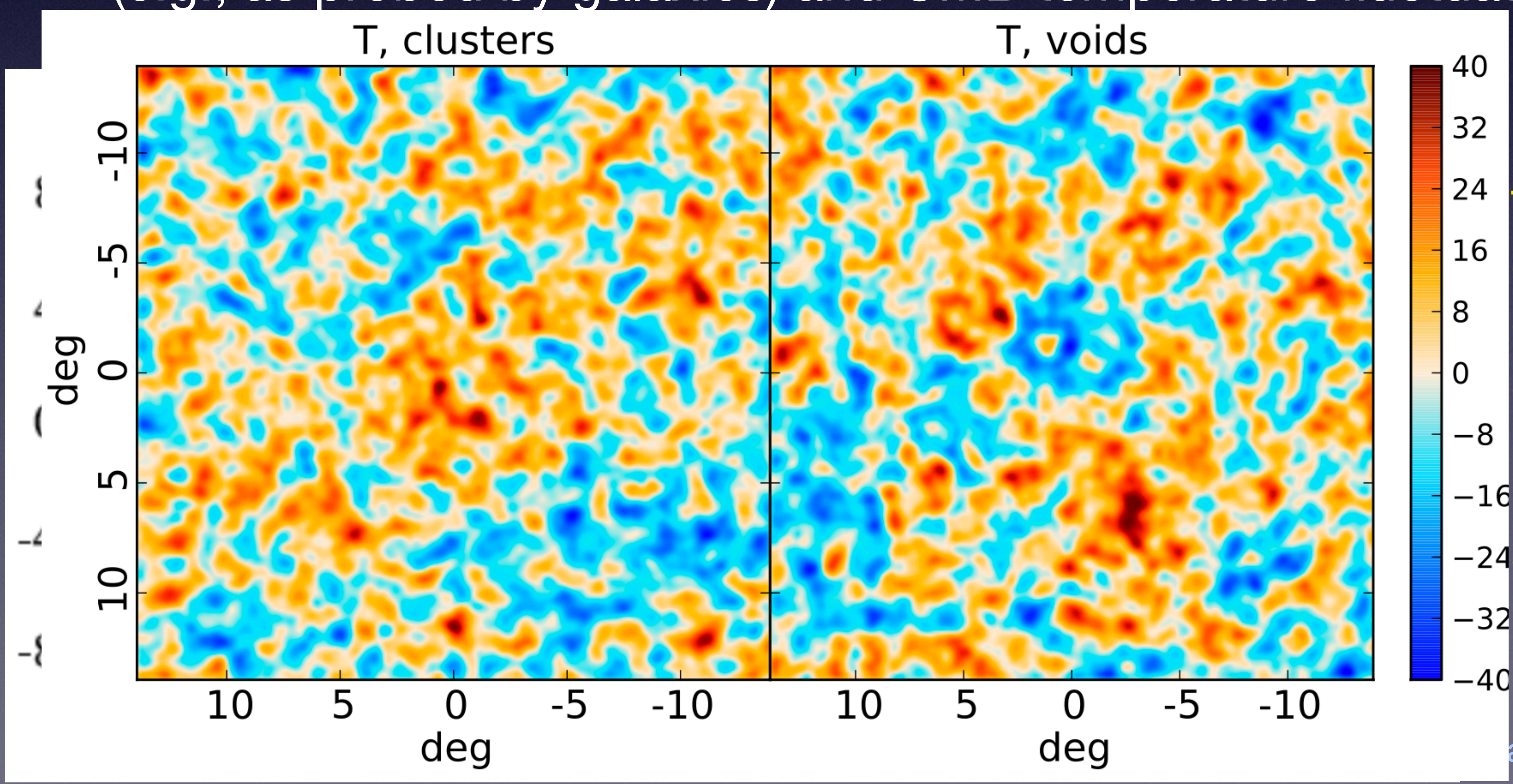
NOTE: In an EdS cosmology the ISW effect is absent!

The Integrated Sachs-Wolfe Effect

It is customary to distinguish

- "early-time" ISW effect: arises immediately after recombination, when radiation-contribution is still significant
- "late-time" ISW effect: arises at late times (close to z=0) due to impact of cosmological constant

ISW effect reveals itself in the form of cross-correlation between matter distribution (e.g., as probed by galaxies) and CMB temperature fluctuations.



Tens of studies have tried to detect (late-time) ISW effect. Most detections are mildly significant at best (< 5σ). One of the most intruiging detections is shown here, based on SDSS-LRG vs WMAP5

Planck 2015 results

anett, Neyrinck & Szapudi (2008)

Lecture 6

SUMMARY

Summary: key words & important facts

Key words

Thermal vs. Non-thernal relics	Freeze-out
Cold vs. Hot relics (CDM vs. HDM)	Meszaros effect
Collisionless Boltzmann equation	Free-streaming damping
Jeans equations	ISW effect

- A collisionless fluid with isotropic and homogeneous velocity dispersion is described by the same continuity and momentum equations as a collisional fluid, but with the sound speed $c_{\rm s}$ replaced by $\sigma = \langle v_i^2 \rangle^{1/2}$
- A collisionless fluid does not have an EoS → moment equations are not a closed set
- Collisionless dark matter and baryonic matter have the same linear growth rate.
- Collisional fluid: perturbations below Jeans mass undergo acoustic oscillations
  Collisionless fluid: perturbations below Jeans mass undergo free streaming
- After recombination, baryons fall in DM potential wells, thereby un-doing Silk damping.
- The integrated Sachs-Wolfe effect probes (linear) growth rate of structure.
  In an EdS cosmology $D(a) \propto a$ and the ISW effect is absent.

Summary: key equations & expressions

Collisionless Boltzmann Equation (CBE) $\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} - \frac{\partial \Phi}{\partial x_i}\frac{\partial f}{\partial v_i} = 0$

Moment equations: multiply all terms by v_i^k and integrate over all of velocity space

$k = 0$ ➡ Continuity equation $$\frac{\partial \delta}{\partial t} + \frac{1}{a}\sum_j \frac{\partial}{\partial x_j}\left[(1+\delta)\langle v_j \rangle\right] = 0$$

$k = 1$ ➡ Jeans equations $$\frac{\partial \langle v_i \rangle}{\partial t} + \frac{\dot{a}}{a}\langle v_i \rangle + \frac{1}{a}\sum_j \langle v_j \rangle \frac{\partial \langle v_i \rangle}{\partial x_j} = -\frac{1}{a}\frac{\partial \Phi}{\partial x_i} - \frac{1}{\bar{\rho} a (1+\delta)}\sum_j \frac{\partial \rho \sigma_{ij}^2}{\partial x_j}$$

Free-streaming scale $$\lambda_{\mathrm{fs}}^{\mathrm{com}} = \int_0^{t_{\mathrm{eq}}} \frac{v(t')}{a(t')}\mathrm{d}t'$$

Linear growth rate

EdS cosmology ➡ $D(a) \propto a$

ΛCDM cosmology ➡ $D(a) \propto a^{\gamma} \quad (\gamma < 1)$

Poisson equation in Fourier space: $-k^2 \Phi_{\vec{k}} = 4\pi G \bar{\rho} a^2 \delta_{\vec{k}}$

In matter dominated Universe: $\bar{\rho} \propto a^{-3}$

➡ $\Phi_{\vec{k}} \propto D(a)/a$