ASTR 610
Theory of Galaxy Formation

Lecture 5: Newtonian Perturbation Theory
II. Baryonic Perturbations

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Yale University, Spring 2024

Structure Formation: The Linear Regime

Thus far we have focussed on an unperturbed Universe. In this lecture we examine how small perturbations grow and evolve in a FRW metric (i.e., in a expanding space-time).

Topics that will be covered include:

- Newtonian Perturbation Theory
- Equation of State
- Jeans Criterion
- Horizons
- Linear Growth Rate
- Damping (Silk & Free Streaming)
- Meszaros Effect

The Fluid Equations

In Lecture 4 we derived the following fluid equations in comoving coordinates:

continuity equation
$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla \cdot [(1+\delta)\,\vec{v}] = 0$$

Euler equations
$$\frac{\partial \vec{v}}{\partial t} + \frac{\dot{a}}{a}\vec{v} + \frac{1}{a}(\vec{v}\cdot\nabla)\,\vec{v} = -\frac{\nabla\Phi}{a} - \frac{c_{\rm s}^2}{a}\frac{\nabla\delta}{(1+\delta)} - \frac{2T}{3a}\nabla S$$

Poisson equation
$$\nabla^2\Phi = 4\pi G\bar{\rho}a^2\delta$$

The Linearized Fluid Equations

After linearization:

continuity equation

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla \vec{v} = 0$$

Euler equations

$$\frac{\partial \vec{v}}{\partial t} + \frac{\dot{a}}{a}\vec{v} = -\frac{\nabla \Phi}{a} - \frac{c_s^2}{a}\nabla \delta - \frac{2\bar{T}}{3a}\nabla S$$

Poisson equation

$$\nabla^2 \Phi = 4\pi G \bar{\rho} a^2 \delta$$

The Master Equation

Differentiating continuity equation, and substituting Euler & Poisson Equations:

Master Equation: $$\frac{\partial^2 \delta}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial \delta}{\partial t} = 4\pi G \bar{\rho} \delta + \frac{c_s^2}{a^2}\nabla^2\delta + \frac{2}{3}\frac{\bar{T}}{a^2}\nabla^2 S$$

`Hubble drag' term, expresses how expansion suppresses perturbation growth

gravitational term, expresses how gravity promotes perturbation growth

pressure terms, expressing how pressure gradients due to spatial gradients in density and/or entropy influence perturbation growth

The Master Equation in Fourier Space

After Fourier Transformation:

Master Equation: $$\frac{\mathrm{d}^2\delta_{\vec{k}}}{\mathrm{d}t^2} + 2\frac{\dot{a}}{a}\frac{\mathrm{d}\delta_{\vec{k}}}{\mathrm{d}t} = \left[4\pi G\bar{\rho} - \frac{k^2 c_\mathrm{s}^2}{a^2}\right]\delta_{\vec{k}} - \frac{2}{3}\frac{\bar{T}}{a^2}k^2\,S_{\vec{k}}$$

$$\delta(\vec{x},t) = \sum_k \delta_{\vec{k}}(t)e^{+i\vec{k}\cdot\vec{x}} \qquad \delta_S(\vec{x},t) \equiv \frac{S(\vec{x},t) - \bar{S}(t)}{\bar{S}(t)} = \sum_k S_{\vec{k}}(t)e^{+i\vec{k}\cdot\vec{x}}$$

NOTE: in linear theory, all modes evolve independently

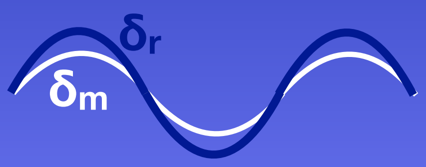
Types of Perturbations

Isentropic Perturbations $\delta_S = 0$ (pure density perturbations)

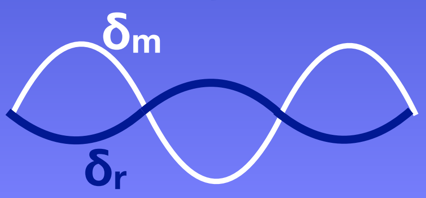
Isocurvature Perturbations $\delta = 0$ (pure entropy perturbations)

$$\boxed{\delta_S = \frac{\partial S}{S} = \frac{1}{S}\left[\frac{\partial S}{\partial \rho_{\rm r}}\,\partial\rho_{\rm r} + \frac{\partial S}{\partial \rho_{\rm m}}\,\partial\rho_{\rm m}\right] = \frac{3}{4}\delta_{\rm r} - \delta_{\rm m}}$$

Isentropic perturbations: $\delta_{\rm r}/\delta_{\rm m} = 4/3$



Isocurvature perturbations: $\delta_{\rm r}/\delta_{\rm m} = -(\bar{\rho}_{\rm m}/\bar{\rho}_{\rm r}) = -(a/a_{\rm eq})$



- Isentropic perturbations are often called adiabatic perturbations. However, it is better practice to reserve `adiabatic' to refer to an evolutionary process
- If evolution is adiabatic, isentropic perturbations remain isentropic. If not, the non-adiabatic processes create non-zero ∇S
- At early times, isocurvature perturbations obey approximately $\delta_{\rm r} = 0$. which is why they are sometimes called isothermal perturbations

Different Matter Components

The matter perturbations that we are describing consist of both baryons and dark matter (assumed to be collisionless).

In what follows we will first treat these separately.

- We start by considering a Universe without dark matter (only baryons + radiation).
- Next we considering a Universe without baryons (only dark matter + radiation).
- We end with discussing a more realistic Universe (radiation + baryons + dark matter)

Gravitational Instability: the Jeans criterion

Consider adiabatic evolution of isentropic perturbations ⇒ $\delta_S = 0$ at all times.

If we ignore for the moment the expansion of the Universe ($\dot{a} = 0$), then our linearized equation in Fourier space reduces to a wave equation:

$$\frac{\mathrm{d}^2\delta_{\vec{k}}}{\mathrm{d}t^2} = -\omega^2 \delta_{\vec{k}} \qquad \text{where} \qquad \omega^2 = \frac{k^2 c_\mathrm{s}^2}{a^2} - 4\pi G\bar{\rho}$$

The special case $\omega = 0$ defines a characteristic mode, k_J, which translates into a characteristic scale

the Jeans length $$\boxed{\lambda_\mathrm{J}^\mathrm{prop} = a(t)\lambda_\mathrm{J}^\mathrm{com} = a(t)\frac{2\pi}{k_\mathrm{J}} = c_\mathrm{s}\sqrt{\frac{\pi}{G\bar{\rho}}}}$$

Hence, we have the following Jeans criterion:

$\lambda < \lambda_\mathrm{J}$ ⇒ $\omega^2 > 0$ ⇒ $\delta_{\vec{k}}(t) \propto e^{\pm i\omega t}$ sound wave, propagating w. sound speed

$\lambda > \lambda_\mathrm{J}$ ⇒ $\omega^2 < 0$ ⇒ $\delta_{\vec{k}}(t) \propto e^{\pm i\omega t}$ static mode, growing or decaying exponentially with time

The Jeans Mass

Prior to recombination: photon-baryon fluid $c_s = \frac{c}{\sqrt{3}}\left[\frac{3}{4}\frac{\rho_b(t)}{\rho_r(t)} + 1\right]^{-1/2}$ (see problem set 2)

$$\left.\begin{array}{l}\rho_b \propto a^{-3} \\ \rho_r \propto a^{-4}\end{array}\right\} \Leftrightarrow \left\{\begin{array}{l} t < t_{eq} \Rightarrow c_s = c/\sqrt{3} \propto a^0 \\ t > t_{eq} \Rightarrow c_s \propto a^{-1/2}\end{array}\right.$$

WARNING: the fluid equations that we used above are only valid for a non-relativistic fluid; our Newtonian treatment is not valid for the tightly coupled photon-baryon fluid. However, we may still use the Newtonian Jeans criterion for an order-of-magnitude analysis, which is what we will do here and in what follows. A proper treatment requires solving the Boltzmann equation in a perturbed space-time metric, which is beyond the scope of these lectures. (see MBW §4.2 for details if interested)

The Jeans Mass

Prior to recombination: photon-baryon fluid $c_{\rm s} = \frac{c}{\sqrt{3}} \left[\frac{3}{4} \frac{\rho_{\rm b}(t)}{\rho_{\rm r}(t)} + 1 \right]^{-1/2}$

$$\left.\begin{matrix} \rho_{\rm b} \propto a^{-3} \\ \rho_{\rm r} \propto a^{-4} \end{matrix}\right\} \Longleftrightarrow \left\{ \begin{matrix} t < t_{\rm eq} & \Rightarrow & c_{\rm s} = c/\sqrt{3} \propto a^0 \\ t > t_{\rm eq} & \Rightarrow & c_{\rm s} \propto a^{-1/2} \end{matrix} \right.$$

After recombination: baryon fluid is `ideal gas' $c_{\rm s} = (\partial P/\partial \rho)^{1/2} \propto T^{1/2}$

$$T \propto a^{-2} \Rightarrow c_{\rm s} \propto a^{-1}$$

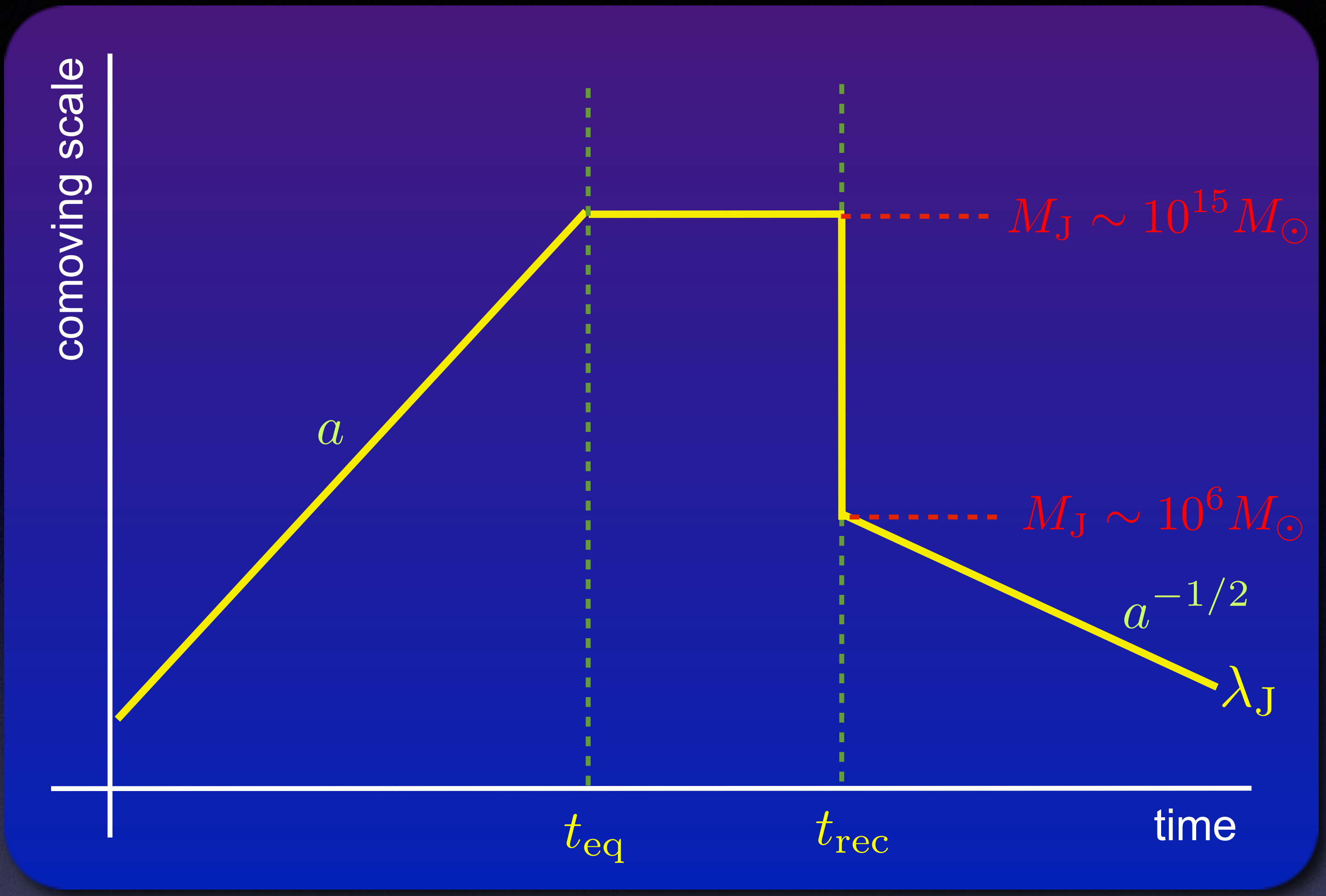
Using Jeans length, we can also define Jeans mass: $\boxed{M_{\rm J} = \frac{4\pi}{3} \bar{\rho} \left(\frac{\lambda_{\rm J}}{2} \right)^3 = \frac{\pi}{6} \bar{\rho}\, \lambda_{\rm J}^3}$

- Immediately after recombination, $M_{\rm J} = 1.5 \times 10^5 (\Omega_{\rm b,0} h^2)^{-1/2} M_\odot$

  while at matter-radiation equality, $M_{\rm J} = 1.5 \times 10^{16} (\Omega_{\rm b,0} h^2)^{-2} M_\odot$

- At recombination, photons decouple from baryons,which dramatically reduces the pressure, causing a huge drop in the Jeans mass...

Evolution of the Jeans Length



$$\lambda_{\rm J}^{\rm prop} = c_{\rm s} \sqrt{\frac{\pi}{G\bar{\rho}}}$$

Using how the sound speed and density scale with the scale factor we obtain the following evolution of the Jeans length.

Baryon perturbations with $\lambda > \lambda_{\rm J}$ grow, while those with $\lambda < \lambda_{\rm J}$ become acoustic waves.

$t < t_{\rm eq} \Rightarrow \bar{\rho} \propto a^{-4},\ c_{\rm s} \propto a^{0} \Rightarrow \lambda_{\rm J}^{\rm prop} \propto a^2 \Rightarrow \lambda_{\rm J}^{\rm com} \propto a$

$t_{\rm eq} < t < t_{\rm rec} \Rightarrow \bar{\rho} \propto a^{-3},\ c_{\rm s} \propto a^{-1/2} \Rightarrow \lambda_{\rm J}^{\rm prop} \propto a \Rightarrow \lambda_{\rm J}^{\rm com} \propto a^{0}$

$t > t_{\rm rec} \Rightarrow \bar{\rho} \propto a^{-3},\ c_{\rm s} \propto a^{-1} \Rightarrow \lambda_{\rm J}^{\rm prop} \propto a^{1/2} \Rightarrow \lambda_{\rm J}^{\rm com} \propto a^{-1/2}$

Perturbation Growth in expanding Space-Time

According to the Jeans criterion discussed above, which ignored the expansion of the Universe, baryonic perturbations with $\lambda > \lambda_{\rm J}$ will grow exponentially.

In the case of expansion, there are two modifications:

- Hubble drag
- Horizons

As is evident from linearized fluid equation, Hubble drag resists perturbation growth:
If $\lambda > \lambda_{\rm J}$ perturbations do not grow exponentially, but as a power-law $\delta_{\vec{k}}(t) \propto t^{a}$
The index a depends on cosmology, EoS and epoch, as discussed at a later stage.

In addition, in an expanding space-time, one needs to account for the presence of horizons, and distinguish sub-horizon perturbations (those with $\lambda < \lambda_{\rm H}$) from super-horizon perturbation ($\lambda > \lambda_{\rm H}$)....

Super-Horizon Perturbations

In an expanding space-time, one needs to account for the presence of horizons.

Inflation creates perturbations on all scales, including on super-horizon scales.

A proper treatment of super-horizon perturbations requires general relativistic perturbation theory, which is beyond the scope of this course (see MBW §4.2)

Crudely speaking, a super-horizon perturbation ($\lambda > \lambda_{\rm H}$) doesn't "know" it is a perturbation as it has no causal knowledge of the metric on scales larger than itself $\Rightarrow$ metric perturbations with $\lambda > \lambda_{\rm H}$ don't evolve (`frozen').

$$\left.\begin{array}{l} \Phi(\vec{x},t) = \sum_k \Phi_{\vec{k}}(t) e^{+i\vec{k}\cdot\vec{x}} \\ -k^2 \Phi_{\vec{k}} = 4\pi G a^2 \bar{\rho} \delta_{\vec{k}} \end{array}\right\} \Rightarrow$$

$\Phi_{\vec{k}}$ is constant implies that $\delta_{\vec{k}} \propto (\bar{\rho} a^2)^{-1}$

Growth of super-horizon density-perturbations is governed by conservation of the associated potential perturbations.....Jeans criterion does NOT apply for them.

Horizons

Recall: a photon moves along a geodesic (ds=0). Since we can always choose our coordinate system such that the photon moves along $\mathrm{d}\theta = \mathrm{d}\phi = 0$, the FRW metric implies that a photon path is characterized by $\mathrm{d}\chi = \mathrm{d}\tau$.

Using that the conformal time interval $\mathrm{d}\tau = c\,\mathrm{d}t/a(t)$, and that $\mathrm{d}t = \mathrm{d}a/\dot{a}$

$$\chi(r_\mathrm{e}) = \int_{t_\mathrm{e}}^{t_0} \frac{c\,\mathrm{d}t}{a} = \int_{a_\mathrm{e}}^{a_0} \frac{c\,\mathrm{d}a}{a\,\dot{a}}$$

where the subscripts e and o refer to `emitted' and `observed'.

If $\chi(r_\mathrm{e})$ converges to a finite value χ_H when $a_\mathrm{e} \rightarrow 0$, then there are events for which $\chi > \chi_\mathrm{H}$ and from which no communication can have reached the observer.

$$\chi_\mathrm{H} = \int_0^{t_0} \frac{c\,\mathrm{d}t}{a}$$ is called the comoving particle horizon

If $\lim_{t_0 \rightarrow \infty} \chi_\mathrm{H}$ converges to a finite value, it means that there are fundamental observers with whom the observer can never communicate. Such fundamental observers are said to lie outside the comoving event horizon.

In what follows we are mainly concerned with particle horizons only....

Horizons

Using the Friedmann equation $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{K c^2}{a^2}$ (see Lecture 3)

we can write the dependence of the comoving (particle) horizon on scale factor as

$$\boxed{\chi_{\rm H}(a) = \int_0^a \frac{c\,{\rm d}a}{a\,\dot{a}} = \int_0^a \frac{{\rm d}a}{a} \left[\frac{8\pi G\rho\, a^2}{3\, c^2} - K\right]^{-1/2}}$$

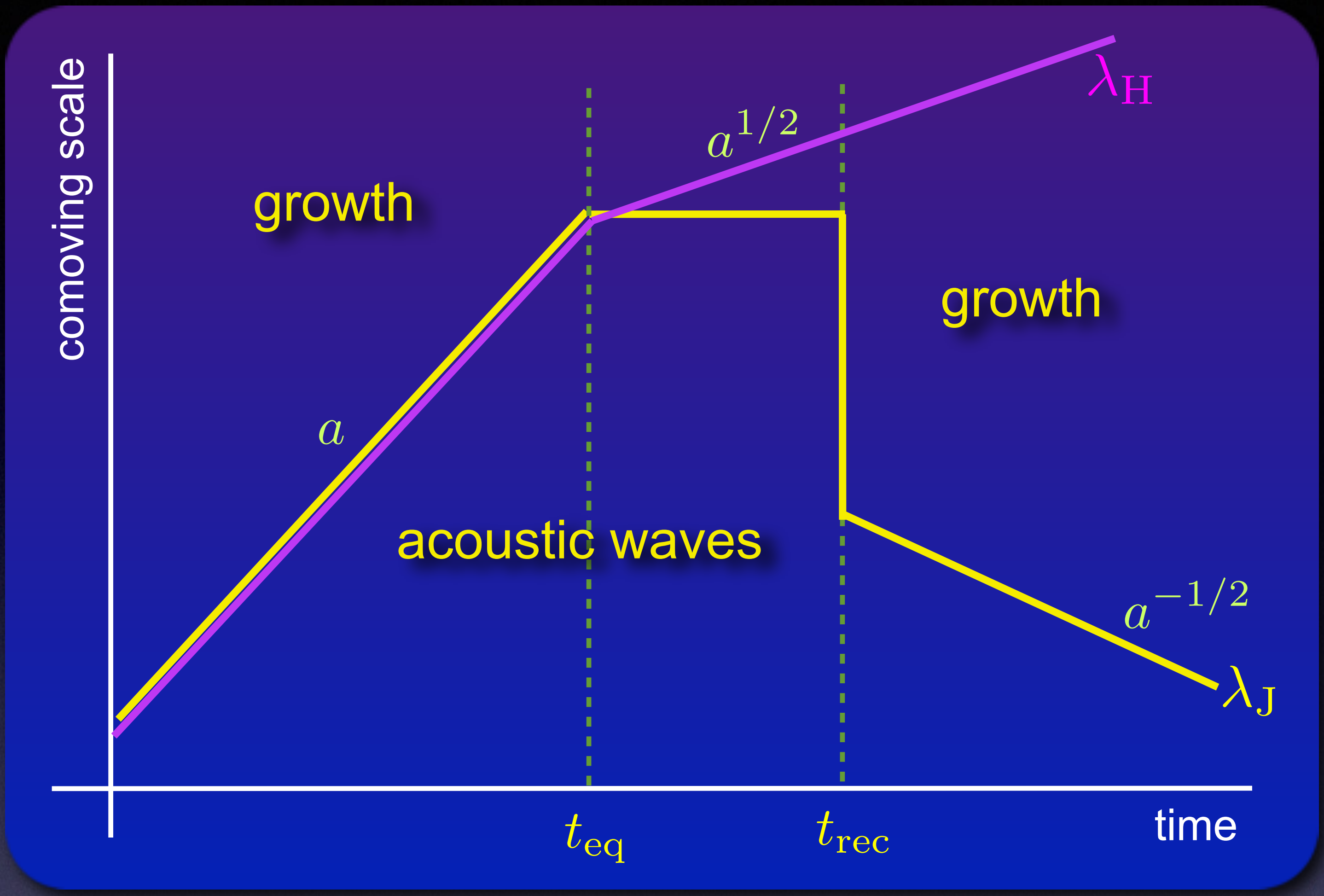
For a flat Universe (K=0), this reduces to $\chi_{\rm H}(a) = \left(\frac{3c^2}{8\pi G}\right)^{1/2} \int_0^a \frac{{\rm d}a}{a^2\sqrt{\rho}}$

$t \ll t_{\rm eq}$ ⇒ $\rho \propto a^{-4}$ ⇒ $\chi_{\rm H}(a) \propto \int_0^a {\rm d}a \propto a$

$t \gg t_{\rm eq}$ ⇒ $\rho \propto a^{-3}$ ⇒ $\chi_{\rm H}(a) \propto \int_0^a a^{-1/2}{\rm d}a \propto a^{1/2}$

Λ dominates ⇒ $\rho \propto a^0$ ⇒ $\chi_{\rm H}(a) \propto \int_0^a a^{-2}{\rm d}a \propto a^{-1}$

The Evolution of Baryonic Perturbation



It can be shown that the proper (particle) horizon during the radiation dominated era follows

$$\boxed{\lambda_H^{prop} = 2\,c\,t}$$

which is the same as the Jeans length during this era, except for a factor of order unity....

(see problem set 1)

For comparison, in a matter-dominated Universe we have that $\boxed{\lambda_H^{prop} = 3\,c\,t}$

Using that super-horizon perturbations experience growth, we can distinguish three different regions in our scale-size diagram for the evolution of baryonic perturbations in an expanding space-time...

However, this picture is not yet complete....

Silk Damping

Before decoupling photons & baryons are tightly coupled via Compton scattering. However, this coupling is imperfect as the photons have a mean-free path $\lambda = (\sigma_{\rm T}\, n_{\rm e})^{-1}$ that is not zero.

→ photon diffusion

→ damping of perturbations in photon distribution

→ damping of acoustic oscillations (also in the baryons)

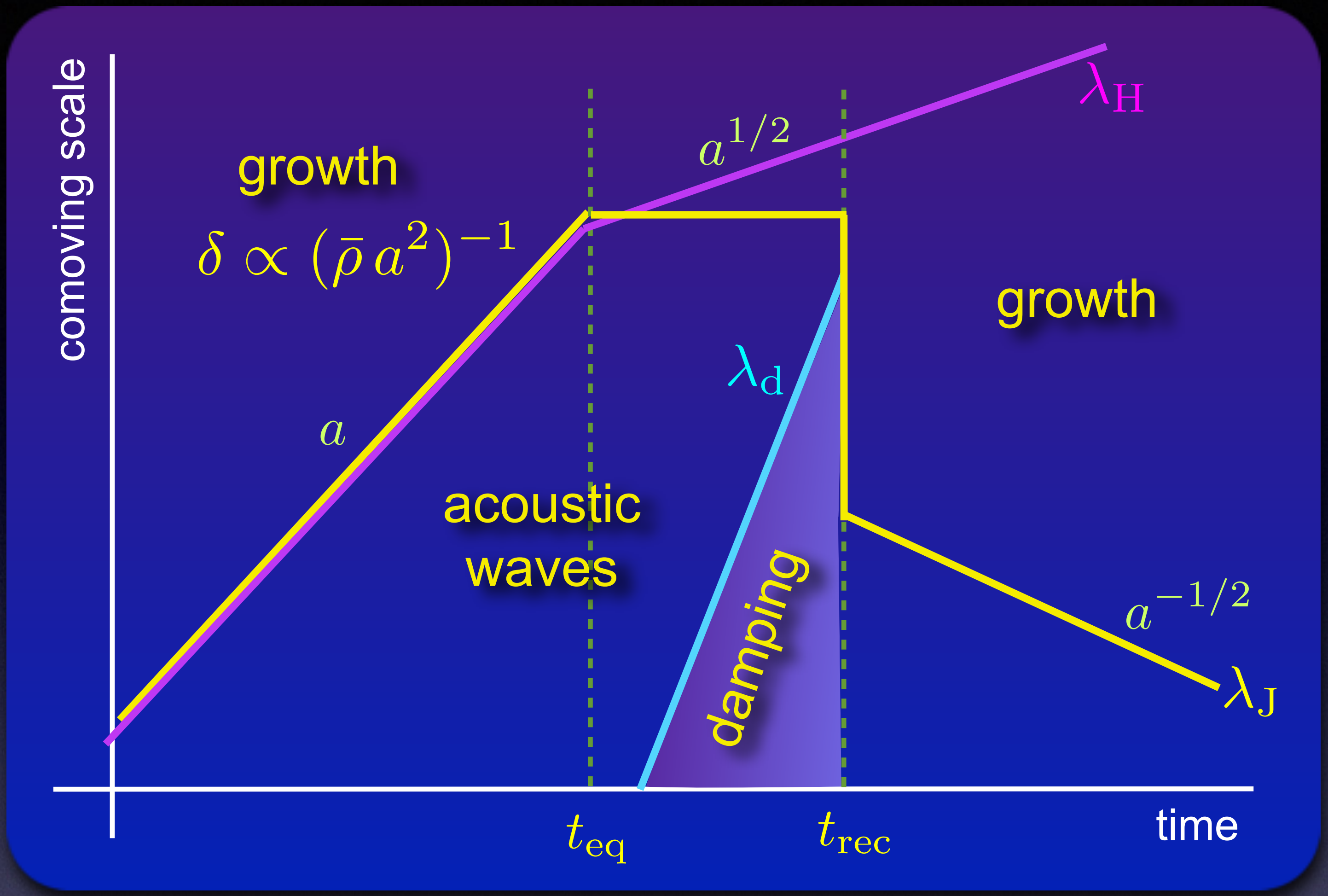
This damping mechanism is known as Silk damping.

The Silk damping scale, $\lambda_{\rm d}$, is typical distance photon can diffuse in Hubble time.

- Let photon path be a random walk with mean step length λ
- During a time t the photon takes on average $N = c\,t/\lambda$ steps
- Kinetic Theory → $\lambda_{\rm d} \simeq (N/3)^{1/2}\lambda = \left(\frac{c\,t}{3\sigma_{\rm T}\, n_{\rm e}}\right)^{1/2}$

$\sigma_{\rm T}$ is Thomson cross-section

The Evolution of Baryonic Perturbations



We can now complete our picture of evolution of baryonic perturbations:

Silk damping will `erase' all baryonic fluctuations on scales $\lambda < \lambda_{\rm d}$

Consequently, if matter is purely baryonic, after recombination there are no perturbations left on small scales ⇨ structure formation has to proceed in top-down fashion....

Detailed calculations, accounting for the evolution in ionization fraction, $X_{\rm e}$, yield

at recombination: $\lambda_{\rm d}^{\rm com} \sim 5.7 (\Omega_{\rm m,0} h^2)^{-3/4} \left(\frac{\Omega_{\rm b,0}}{\Omega_{\rm m,0}}\right)^{-1/2} \left(\frac{X_{\rm e}}{0.1}\right)^{-1/2} \left(\frac{1+z_{\rm dec}}{1100}\right)^{-5/4}$ Mpc

$$M_{\rm d} = \frac{4\pi}{3}\bar{\rho}\left(\frac{\lambda_{\rm d}^{\rm phys}}{2}\right)^3 \sim 2.7 \times 10^{13} (\Omega_{\rm m,0} h^2)^{-5/4} \left(\frac{\Omega_{\rm b,0}}{\Omega_{\rm m,0}}\right)^{-3/2} \left(\frac{X_{\rm e}}{0.1}\right)^{-3/2} \left(\frac{1+z_{\rm dec}}{1100}\right)^{-15/4} M_\odot$$

Perturbation Growth during Matter Era

Having explored baryonic perturbations prior to recombination, we now focus on their evolution past recombination, when the baryonic matter can be treated as a pressureless fluid ($c_{\rm s} = 0$). We will focus on the adiabatic evolution of isentropic perturbations ($\delta_S = 0$), and since we are now in the matter-dominated era, we can ignore radiation.

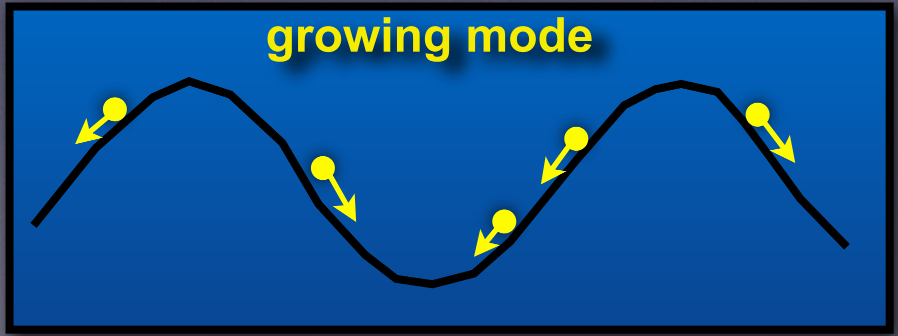
$$\begin{array}{l} \delta_S = 0 \\ c_{\rm s} = 0 \end{array} \Rightarrow \frac{{\rm d}^2\delta_{\vec{k}}}{{\rm d}t^2} + 2\frac{\dot{a}}{a}\frac{{\rm d}\delta_{\vec{k}}}{{\rm d}t} = 4\pi G\bar{\rho}_{\rm m}\,\delta_{\vec{k}}$$

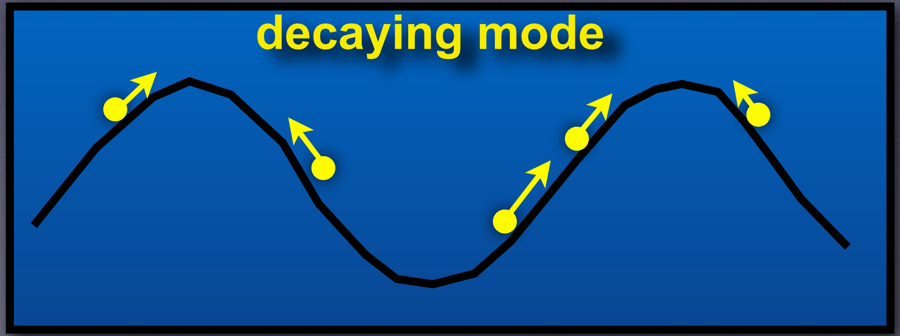
Without derivation (see MBW §4.1.6a), there are two solutions to this equation:

decaying mode: $\delta_- \propto H(t)$

growing mode: $\delta_+ \propto H(t) \int_0^t \frac{{\rm d}t'}{a^2(t')\,H^2(t')}$

Question: what, physically, do these different modes represent?





The Linear Growth Rate

In general, the solution is a linear combination of the growing and decaying modes

$$\boxed{\delta(t) = \mathcal{C}_1\,\delta_+ + \mathcal{C}_2\,\delta_-}$$

Since, by definition, the decaying mode dissapears with time, we shall only be concerned with the growing mode, and simply write $\delta(t) \propto \delta_+$ in what follows.

In what follows, we write that $\delta_+ \propto D(z) \propto \frac{g(z)}{1+z}$ with $D(z)$ the linear growth rate.

An accurate approximation for the linear growth rate (Carroll et al. 1992) has

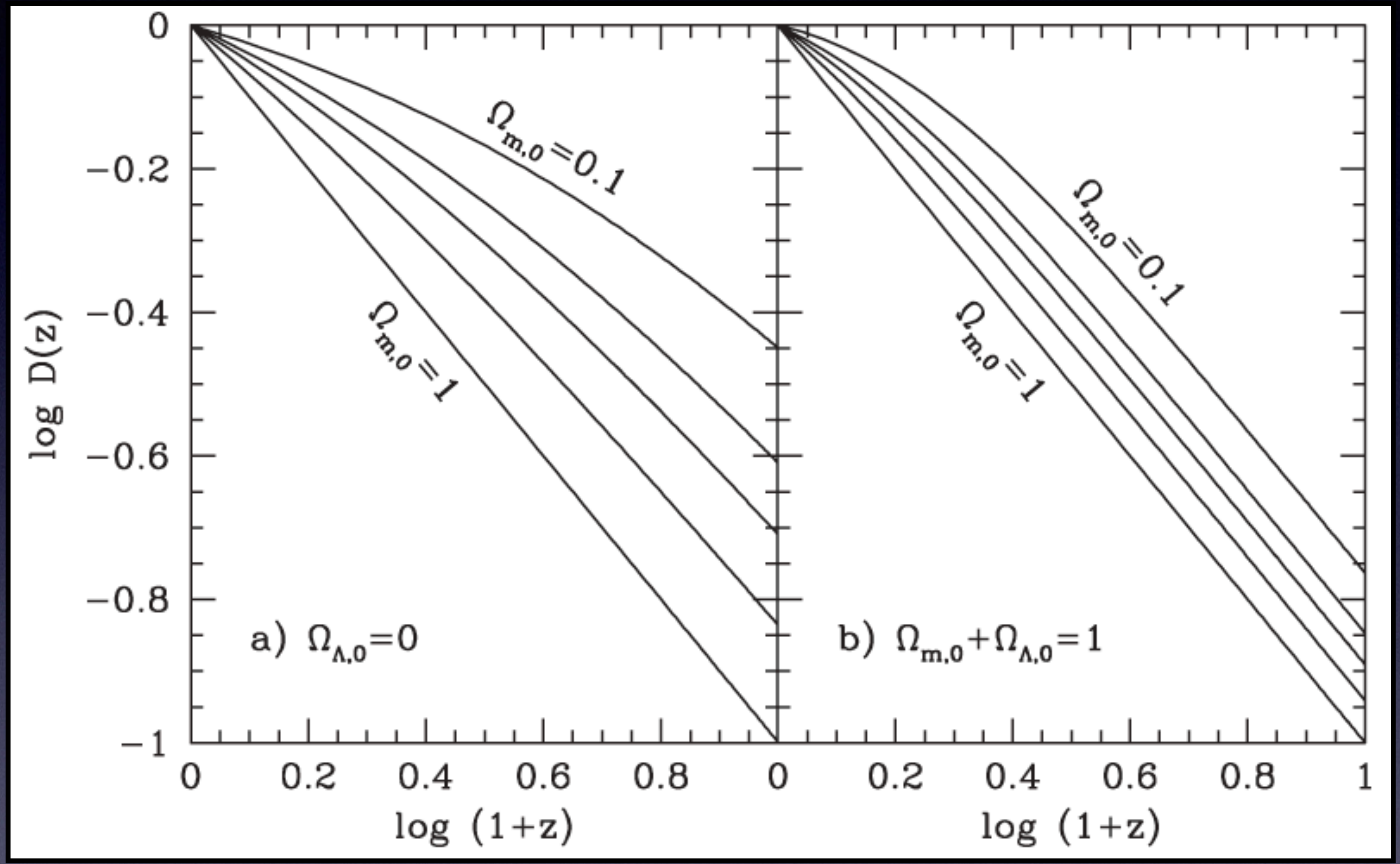
$$\boxed{g(z) \simeq \frac{5}{2}\Omega_{\rm m}(z)\left\{\Omega_{\rm m}^{4/7}(z) - \Omega_\Lambda(z) + \left[1 + \frac{\Omega_{\rm m}(z)}{2}\right]\left[1 + \frac{\Omega_\Lambda(z)}{70}\right]\right\}^{-1}}$$

For an Einstein-de Sitter (EdS) cosmology $(\Omega_{\rm m,0}, \Omega_{\Lambda,0}) = (1,0)$ the solutions are particularly simple: $\delta_+ \propto a \propto t^{2/3}$ and $\delta_- \propto t^{-1}$

Note that, as already mentioned before, because of the expansion of the Universe, the growth-rate is not exponential (as in static case), but a power-law.

The Linear Growth Rate

$$g(z) \simeq \frac{5}{2}\Omega_{\rm m}(z)\left\{\Omega_{\rm m}^{4/7}(z) - \Omega_\Lambda(z) + \left[1 + \frac{\Omega_{\rm m}(z)}{2}\right]\left[1 + \frac{\Omega_\Lambda(z)}{70}\right]\right\}^{-1}$$

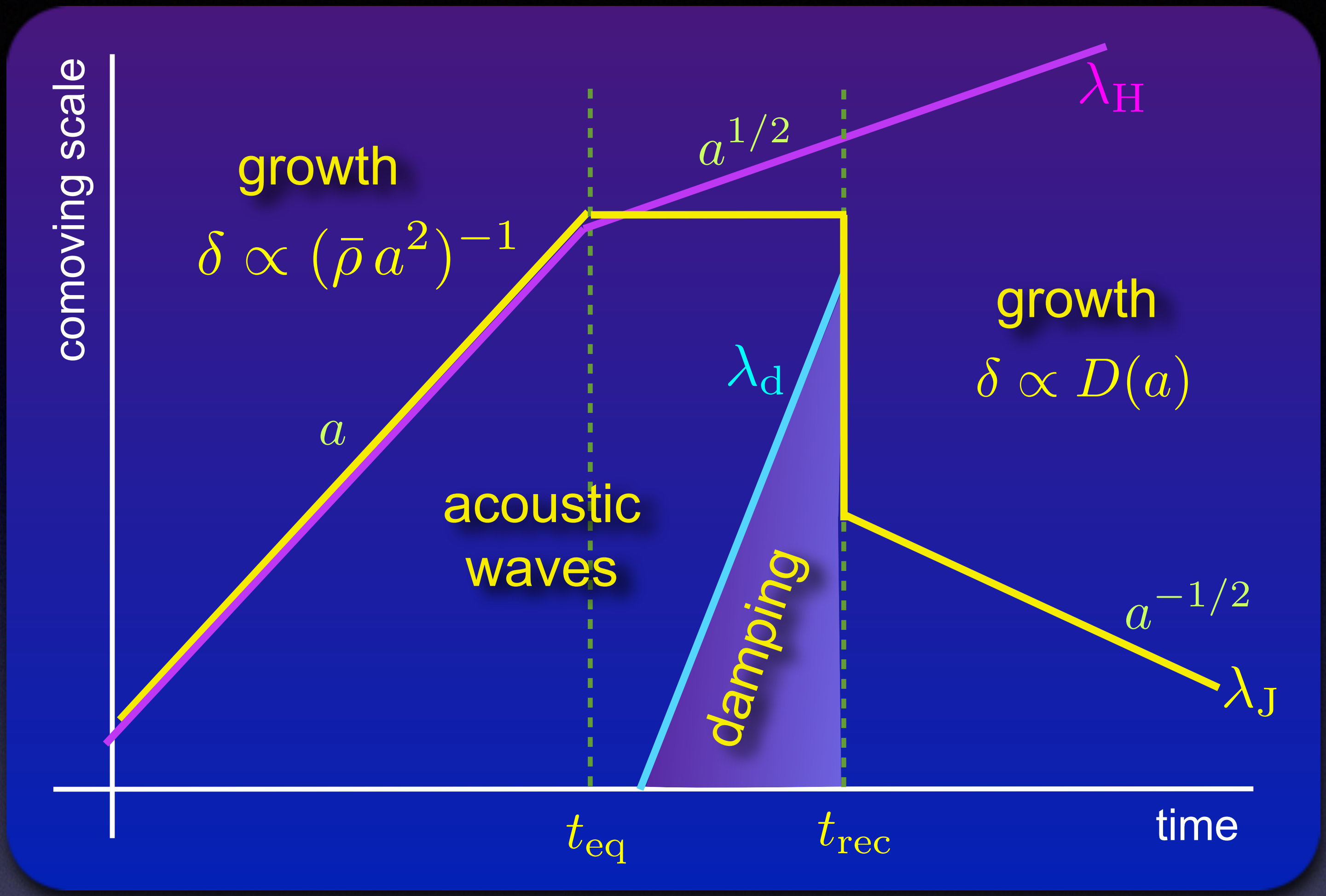


The linear growth rate, D(z), is normalized such that D(z=0) = 1.0

NOTE: perturbations grow faster in an EdS universe than in one with $\Omega_{\rm m,0} < 1$.

Why? Physically, this is due to fact that in open universe, or in one with non-zero cosmological constant, the expansion rate is larger than in EdS universe, causing a reduction of perturbation growth due to enhanced Hubble drag.

The Evolution of Baryonic Perturbations



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Consequently, if matter is purely baryonic, after recombination there are no perturbations left on small scales ➡ structure formation has to proceed in top-down fashion....

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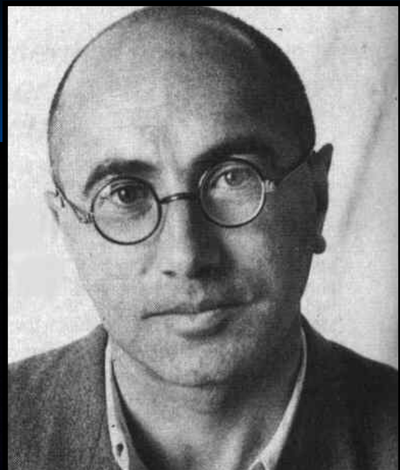
$$M_{\rm d} = \frac{4\pi}{3}\bar{\rho}\left(\frac{\lambda_{\rm d}^{\rm phys}}{2}\right)^3 \sim 2.7\times 10^{13}(\Omega_{\rm m,0}h^2)^{-5/4}\left(\frac{\Omega_{\rm b,0}}{\Omega_{\rm m,0}}\right)^{-3/2}\left(\frac{X_{\rm e}}{0.1}\right)^{-3/2}\left(\frac{1+z_{\rm dec}}{1100}\right)^{-15/4} M_\odot$$

The Adiabatic, Baryonic Model



Yakov Zel'dovich

If matter is purely baryonic, and perturbations are isentropic (`adiabatic'), structure formation proceeds top-down by fragmentation of perturbations larger than Silk damping scale at recombination $M_{\rm d} \sim 10^{13} M_{\odot}$

This was the picture for structure formation developed by Zel'dovich and his colleagues during the 1960's in Moscow. However, it soon became clear that this picture was doomed....

To allow sufficient time for fragmentation, the large-scale perturbations need large amplitudes in order to collapse sufficiently early. At recombination one requires $|\delta_{\rm m}| > 10^{-3}$

Using that $\delta_T = 1/4\delta_{\rm r} = 1/3\delta_{\rm m}$, which follows from fact that perturbations are isentropic and from $\rho_{\rm r} \propto T^4$, this model implies CMB fluctuations

$$\boxed{\Delta T/T > 10^{-3}}$$

Such large fluctuations were already ruled out in early 1980s (e.g., Uson & Wilkinson 1984)

Nature 1984

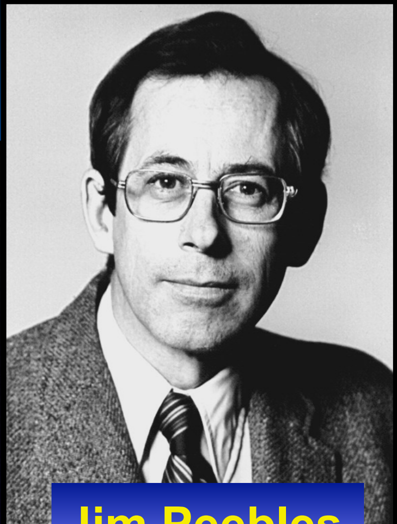
Improved limits on small-scale anisotropy in cosmic microwave background

Juan M. Uson & David T. Wilkinson

Physics Department, Princeton University, Princeton, New Jersey 08544, USA

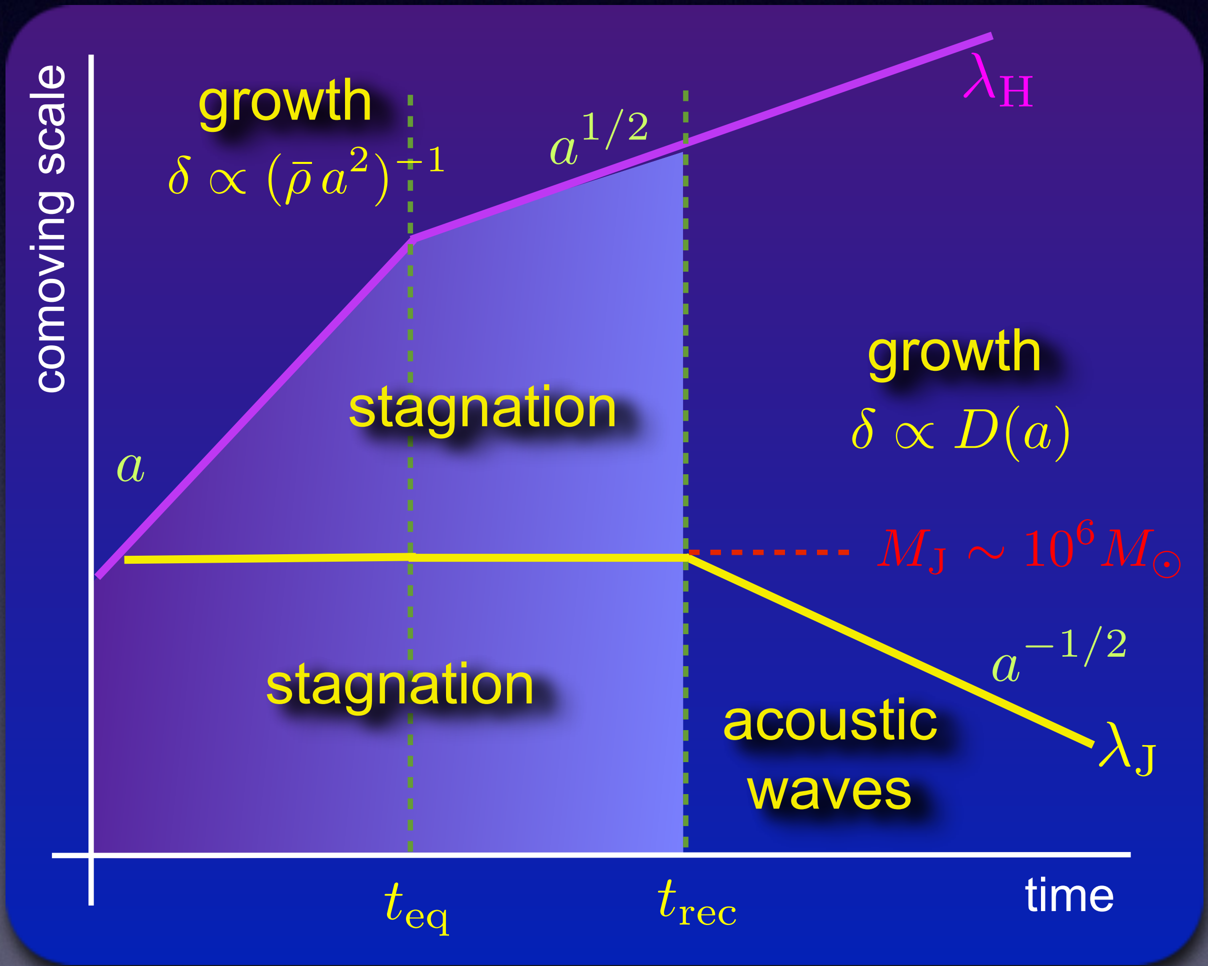
As a remnant of the early Universe, the cosmic microwave background provides unique information on the initial conditions from which matter has evolved to form the structures we see today. All efforts to detect small-scale structure in this radiation have so far been unsuccessful (ref. 1 and refs therein)[1–4]. Nevertheless, upper limits set on possible underlying fluctuations restrict the range of physical models for perturbations of the density in the early Universe. Our search for small-scale anisotropy in the background radiation has now resulted in a lowering of the upper limit on root-mean-square fluctuations ($\Delta T_{\rm r.m.s.}$) observed at an angular scale of ~4 arc min to $\Delta T_{\rm r.m.s.}/T < 2.1\times10^{-5}$ at the 95% confidence level (where $T = 2.7$ K, the temperature of the background radiation). The actual limits deduced from our experiment depend on the model assumed for the unseen fluctuations. Several possibilities are discussed as well as the implications this new measurement has for various cosmological models.

The Isothermal, Baryonic Model


Jim Peebles

While Zel'dovich was working on his adiabatic model, Peebles and his colleagues at Princeton developed an alternative model for structure formation, in which the perturbations were assumed to be isothermal (i.e., $\delta_{\rm r} = 0$), which is a good approximation for isocurvature perturbations prior to the matter era.

In this model, sound speed prior to recombination is much lower, resulting in much lower Jeans mass. Also, since there are no radiation perturbations, there is NO Silk damping. All perturbations with $M > M_{\rm J} \sim 10^6 M_{\odot}$ survive, and structure formation proceeds hierarchical (bottom-up).



Prior to recombination, radiation drag prevents perturbations from growing (they are `frozen'), but at least they are not damped...

Similar to adiabatic model, this isothermal baryonic model requires large temperature fluctuations in CMB to explain observed structure

In addition, isothermal perturbations are fairly "unnatural".
All these problems dissapear when considering a separate matter component: dark matter

Lecture 5

SUMMARY

Summary: key words & important facts

Key words

Jeans criterion	Linear growth rate
Jeans length	Silk damping
Horizons (particle vs. event)	Radiation drag

- Perturbations below the Jeans mass do not grow, but cause acoustic oscillations.

- At recombination photons decouple from baryons → huge drop in the Jeans mass.

- Hubble drag resists perturbation growth → perturbations above the Jeans mass do not grow exponentially, but as a power-law: $\delta_{\vec{k}}(t) \propto t^a$
  The index a depends on cosmology and EoS, as characterized by linear growth rate.

- Growth of super-horizon density-perturbations is governed by conservation of the associated perturbations in the metric.

- If matter is purely baryonic, at recombination Silk damping has erased all perturbations on relevant scales ($M_d \sim 10^{15}\, M_\odot$) → structure formation proceeds in top-down fashion.

Summary: key equations & expressions

prior to recombination: relativistic photon-baryon fluid

$$c_{\rm s} = \frac{c}{\sqrt{3}}\left[\frac{3}{4}\frac{\rho_{\rm b}(t)}{\rho_{\rm r}(t)} + 1\right]^{-1/2}$$

after recombination: baryon fluid is \`ideal gas'

$$c_{\rm s} = (\partial P/\partial \rho)^{1/2} \propto T^{1/2}$$

Jeans length & mass:

$$\lambda_{\rm J}^{\rm prop} = c_{\rm s}\sqrt{\frac{\pi}{G\bar{\rho}}} \qquad M_{\rm J} = \frac{4\pi}{3}\bar{\rho}\left(\frac{\lambda_{\rm J}}{2}\right)^3 = \frac{\pi}{6}\,\bar{\rho}\,\lambda_{\rm J}^3$$

comoving particle horizon: $\chi_{\rm H}(a) = \int_0^t \frac{c\,{\rm d}t}{a} = \int_0^a \frac{c\,{\rm d}a}{a\,\dot{a}}$ $\Rightarrow$ $\lambda_{\rm H}^{\rm prop} = \begin{cases} 2\,c\,t & \text{radiation era} \\ 3\,c\,t & \text{matter era}\end{cases}$

Poisson equation (Fourier space)

$$-k^2\Phi_{\vec{k}} = 4\pi G a^2\bar{\rho}\delta_{\vec{k}}$$

$\Rightarrow$ $\Phi_{\vec{k}}$ is constant implies that $\delta_{\vec{k}} \propto (\bar{\rho}a^2)^{-1}$