# ASTR 610
# Theory of Galaxy Formation

## Lecture 15: Heating & Cooling

Frank van den Bosch
Yale University, Spring 2024

# Heating & Cooling

In this lecture we address heating and cooling of gas inside dark matter haloes. After discussing shock heating & hydrostatic equilibrium, we introduce the concept of `virial temperature', discuss radiative cooling processes and introduce the cooling function. We discuss the link between cooling and galaxy formation, and end with a discussion of photo-ionization heating.

Topics that will be covered include:

- shock heating
- hydrostatic equilibrium
- virial temperature
- radiative cooling
- cooling function & cooling time
- ionization equilibrium
- photo-ionization heating

# Shock Heating

Consider a gas cloud of mass $M_{\rm gas}$ falling into a halo of mass $M_{\rm h}$ with velocity $v_{\rm in}$

At some point the gas is shocked; either close to center, where flow lines converge, or at the accretion shock, which is typically located close to the virial radius.

If we assume that the shock thermalizes all the kinetic energy of the gas cloud, so that $\langle v_{\rm gas}\rangle \simeq 0$ after it is shocked (a reasonable assumption), and that $v_{\rm in}^2 \gg \frac{k_{\rm B} T_{\rm in}}{\mu m_{\rm p}}$ (so that internal energy of infalling gas can be ignored) then the internal energy of the shocked gas is equal to the kinetic energy of the gas at infall:

$$E_{\rm int,sh} = \frac{3}{2} N\, k_{\rm B}\, T_{\rm sh} = \frac{1}{2} M_{\rm gas} v_{\rm in}^2$$

where $N = M_{\rm gas}/(\mu m_{\rm p})$ is the number of gas particles, and we have assumed a mono-atomic gas, for which $\gamma = 5/3$

$$\Rightarrow \quad T_{\rm sh} = \frac{\mu m_{\rm p}}{3 k_{\rm B}}\, v_{\rm in}^2$$

If the gas falls in from large distance (where $\Phi(r) \simeq 0$), and has negligible, initial velocity, then

$$v_{\rm in} \simeq v_{\rm esc}(r_{\rm sh}) = \sqrt{2|\Phi(r_{\rm sh})|}$$

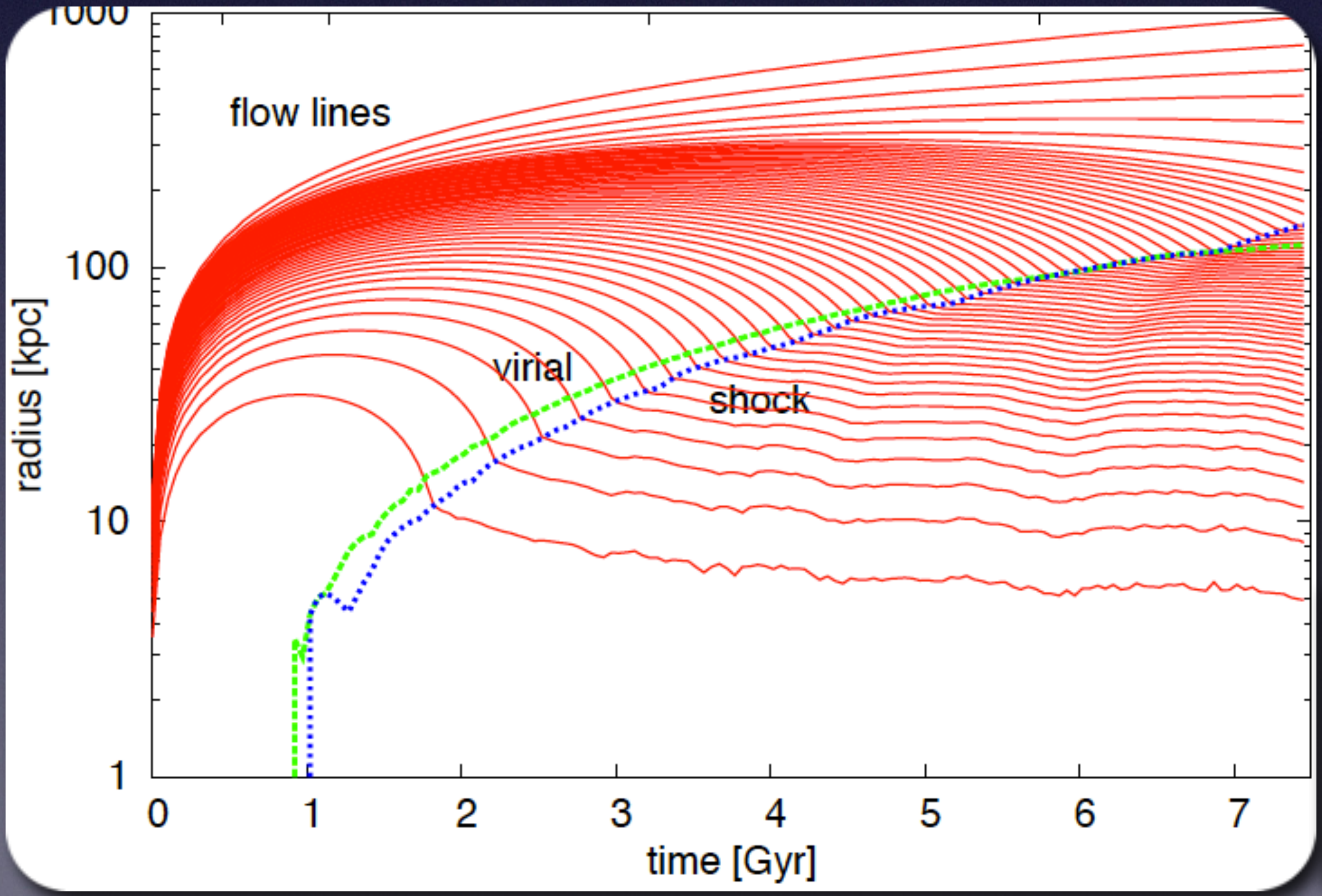
# Shock Heating

If we assume that $r_{\rm sh} = r_{\rm vir}$ (a common assumption), then $v_{\rm in}^2 = \zeta \frac{G M_{\rm vir}}{r_{\rm vir}} = \zeta V_{\rm vir}^2$

Here $\zeta = \mathcal{O}(1)$ is a parameter that depends on the detailed density profile of the halo.

The temperature of the shocked gas in a halo with virial velocity $V_{\rm vir}$ is

$$T_{\rm sh} \simeq \frac{\zeta}{3} \frac{\mu m_{\rm p}}{k_{\rm B}} V_{\rm vir}^2$$




Source: Birnboim & Dekel, 2003, MNRAS, 345, 349

The build-up of a virial shock (discontinuity in velocity) at around the virial radius in a collapsing structure. Based on 1D calculations in an expanding Universe...

# Hydrostatic Equilibrium

If we assume that gas is non-radiative (cannot cool except via adiabatic expansion, and cannot be heated by radiation), then the shocked gas will settle in

**Hydrostatic Equilibrium** $$\nabla P(r) = -\rho_{\rm gas} \nabla \Phi(r)$$

**Spherical Symmetry**

$$\nabla \Phi = \frac{\mathrm{d}\Phi}{\mathrm{d}r} = \frac{G\,M(r)}{r^2}$$

**Ideal Gas**

$$\nabla P = \frac{\mathrm{d}P}{\mathrm{d}r} = \frac{k_{\rm B}}{\mu m_{\rm p}} \frac{\mathrm{d}}{\mathrm{d}r}(\rho T)$$

$$M(r) = M_{\rm gas}(r) + M_{\rm DM}(r) = -\frac{k_{\rm B}\,T(r)\,r}{\mu m_{\rm p}\,G} \left[\frac{\mathrm{d}\ln\rho_{\rm gas}}{\mathrm{d}\ln r} + \frac{\mathrm{d}\ln T}{\mathrm{d}\ln r}\right]$$

If one knows $T(r)$ and $\rho_{\rm gas}(r)$ one can infer the total mass profile $M_{\rm tot}(r)$

# Hydrostatic Equilibrium

- **NOTE:** we have made the assumption that $P = P_{\rm thermal}$. In general, one can also have non-thermal pressure support from magnetic fields, cosmic rays and/or turbulence. When these are present we have that

$$M_{\rm tot}(r) = -\frac{k_{\rm B}\,T(r)\,r}{\mu m_{\rm p}\,G}\left[\frac{{\rm d}\ln\rho_{\rm gas}}{{\rm d}\ln r} + \frac{{\rm d}\ln T}{{\rm d}\ln r} + \frac{P_{\rm nt}}{P_{\rm th}}\frac{{\rm d}\ln P_{\rm nt}}{{\rm d}\ln r}\right]$$

contribution due to non-thermal pressure

Unfortunately, accurate measurements of $P_{\rm nt}(r)$ are extremely difficult to obtain....

- **NOTE:** simply stating that the gas is in HE is not sufficient to determine its density or temperature profiles. To make progress one often makes simplifying assumptions. Examples of such assumptions are

  - **isothermal gas** → $T(r) = T$ $\quad P_{\rm nt} = 0$
  - **polytropic gas** → $P(r) \propto \rho_{\rm gas}^{\Gamma}$

# Hydrostatic Equilibrium of Polytropic Gas

A polytropic gas has an equation of state: $P \propto \rho^{\Gamma}$ 

$\Gamma$ = polytropic index

In this case, hydrostatic equilibrium implies

$$k_{\rm B} T(r) = \frac{1-\Gamma}{\Gamma} \mu m_{\rm p} \, \Phi(r)$$

(temperature profile reflects gravitational potential)

Using the ideal gas law, according to which $P \propto \rho \, T$, we also have that

$$\rho(r) \propto [T(r)]^{\frac{1}{\Gamma-1}} \qquad P(r) \propto [\rho(r)]^{\Gamma}$$

An important example of a polytropic gas is an isentropic gas for which $\Gamma = \gamma$

For a mono-atomic, isentropic gas, we have that $\gamma = 5/3$, and thus $\rho(r) \propto T(r)^{3/2}$

# The Virial Temperature

In the absence of a full solution for T(r), one can get a rough estimate of the temperature of the gas using the virial theorem:

$$\frac{1}{2}\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} = 2K + W + \Sigma$$

I = moment of inertia
K = kinetic energy
W = potential energy
Σ = surface pressure

The system is said to be in virial equilibrium if $2K + W + \Sigma = 0$. If the system is not in virial equilibium, it either expands ($\mathrm{d}^2 I/\mathrm{d}t^2 > 0$) or contracts ($\mathrm{d}^2 I/\mathrm{d}t^2 < 0$)

The gas in a halo of mass $M_{\rm vir}$ and radius $r_{\rm vir}$ is in virial equilibrium if

$$3\frac{M_{\rm gas}}{\mu m_{\rm p}} k_{\rm B} T_{\rm vir} - \zeta \frac{G M_{\rm gas} M_{\rm vir}}{r_{\rm vir}} - 4\pi r_{\rm cl}^3 P_{\rm ext} = 0$$

where we have assumed that the gas is ideal and mono-atomic, and the halo is spherical.

If we ignore the external pressure ($P_{\rm ext} = 0$), this defines the virial temperature:

**virial temperature**

$$T_{\rm vir} = \frac{\zeta}{3}\frac{\mu m_{\rm p}}{k_{\rm B}} V_{\rm vir}^2$$

which is exactly the same as the temperature of the shocked gas defined before...

# The Virial Temperature

For a truncated, singular isothermal sphere of gas (no dark matter), the virial theorem implies a virial temperature

$$T_{\rm vir} = \frac{\mu m_{\rm p}}{2\,k_{\rm B}} V_{\rm vir}^2 \simeq 3.6 \times 10^5\,{\rm K} \left(\frac{V_{\rm vir}}{100\,{\rm km/s}}\right)^2$$

where we have assumed that $\mu = 0.59$, appropriate for a primordial gas (X,Y,Z) = (¾,¼,0)

➡ This is the definition of virial temperature most often adopted in the literature, and is identical to that defined above under the `assumption' that $\zeta = 3/2$

**CAUTION:** in general, gas inside a (virialized) dark matter halo will have a temperature profile, and cannot be described merely by a single temperature. Nevertheless, the concept of `virial temperature' is useful for order of magnitude estimates in galaxy formation theory, and is frequently used.

# Radiative Cooling

Thus far we have ignored radiative processes, which can cause both heating and cooling of the gas. In what follows we investigate how these radiative processes impact the gas in a virialized dark matter halo, focussing first on cooling.

Let $\mathcal{H}$ and $\mathcal{C}$ be the volumetric heating and cooling rates, respectively.

$$[\mathcal{H}] = [\mathcal{C}] = \mathrm{erg\,s^{-1}\,cm^{-3}}$$

In what follows we ignore heating ( $\mathcal{H} = 0$ ), and we assume that the gas is optically thin, so that any photon that it emits escapes the system.

It is useful to define the **cooling function**:

$$\Lambda(T, Z) \equiv \frac{\mathcal{C}}{n_{\rm H}^2}$$

which depends on the temperature, T, and composition (metallicity Z) of the gas, but not on its density.

$$[\Lambda] = \mathrm{erg\,s^{-1}\,cm^{+3}}$$

# Cooling Time

The cooling time, the time it takes the gas to radiate away its internal energy, is given by

$$t_{\rm cool} \equiv \frac{\rho\,\varepsilon}{\mathcal{C}} = \frac{\rho\,\varepsilon}{n_{\rm H}^2\,\Lambda(T)}$$

where for the sake of brevity we don't explicitely write down the metallicity dependence of the cooling function.

Assuming an ideal, monoatomic gas, for which $\varepsilon = \frac{1}{\gamma-1}\frac{k_{\rm B}T}{\mu m_{\rm p}}$ with $\gamma = 5/3$ this yields

$$t_{\rm cool} = \frac{3n\,k_{\rm B}T}{2\,n_{\rm H}^2\,\Lambda(T)} \simeq 3.3\times10^9\,{\rm yr}\left(\frac{T}{10^6\,{\rm K}}\right)\left(\frac{n}{10^{-3}\,{\rm cm}^{-3}}\right)^{-1}\left(\frac{\Lambda(T)}{10^{-23}\,{\rm erg\,s^{-1}\,cm^3}}\right)^{-1}$$

where we have assumed a completely ionized gas of primordial composition, for which $n_{\rm H} = 4/9\,n$, with $n = \rho/(\mu m_{\rm p})$ the number density of gas particles, which can be written as

$$n \simeq 9\times10^{-5}\,{\rm cm}^{-3}\left(\frac{f_{\rm gas}}{0.15}\right)\left(\frac{1+\delta}{200}\right)\left(\frac{\Omega_{\rm m,0}h^2}{0.15}\right)(1+z)^3$$

# Cooling Time

$$t_{\rm cool} = \frac{3n\,k_{\rm B}T}{2\,n_{\rm H}^2\,\Lambda(T)}$$ → $$t_{\rm cool} \propto n^{-1} \propto \rho^{-1}$$ → denser gas cools faster...

In order to assess the impact of cooling on a system, we compare the cooling time to two other timescales:

- the age of the Universe, which is roughly the Hubble time

$$t_{\rm H} = \frac{1}{H(z)} \propto \frac{1}{(G\bar{\rho})^{1/2}} \qquad \bar{\rho} = \Omega_{\rm m}\rho_{\rm crit}$$

- the dynamical time (or `free-fall time') of the system

$$t_{\rm ff} = \left(\frac{3\pi}{32 G\bar{\rho}_{\rm sys}}\right)^{1/2} \propto \frac{1}{(G\,\bar{\rho}_{\rm sys})^{1/2}} \qquad \bar{\rho}_{\rm sys} = \bar{\rho}_{\rm gas} + \bar{\rho}_{\rm DM}$$

**NOTE:** the free-fall time is timescale on which gas cloud collapses in absence of pressure, and timescale on which system restores hydrodynamic equilibrium if disturbed.

**NOTE:** $\bar{\rho}_{\rm sys} \sim 200\bar{\rho}$ → $t_{\rm ff} \sim t_{\rm H}/10$

# Cooling Time

We distinguish three regimes:

$t_{\rm cool} > t_{\rm H}$ Cooling is not important. Gas is in hydrostatic equilibrium, unless it was recently disturbed

$t_{\rm ff} < t_{\rm cool} < t_{\rm H}$ System is in quasi-hydrostatic equilibrium. It evolves on cooling time scale. Gas contracts slowly as it cools, but system has sufficient time to continue to re-establish hydrostatic equilibrium.

$t_{\rm cool} < t_{\rm ff}$ Cooling is catastropic. Gas cannot respond fast enough to loss of pressure. Since cooling time decreases with increasing density, cooling proceeds faster and faster (=catastrophic). Gas falls to center of dynamic system on free-fall time...

As we will see, in the latter case the assumption of a virial shock at the halo's virial radius which heats the gas to the halo's virial temperature is too simplistic...

**NOTE:**

$$t_{\rm cool} \propto \rho_{\rm gas}^{-1} \propto (1+z)^{-3}$$

$$t_{\rm ff} \propto \rho^{-1/2} \propto (1+z)^{-3/2}$$

⇒ cooling is generally more efficient at higher redshifts

# Cooling Processes

The primary cooling processes relevant for galaxy formation are two-body radiative processes in which gas loses energy through the emission of photons as a consequence of two-body interactions.

**Four processes are important:**

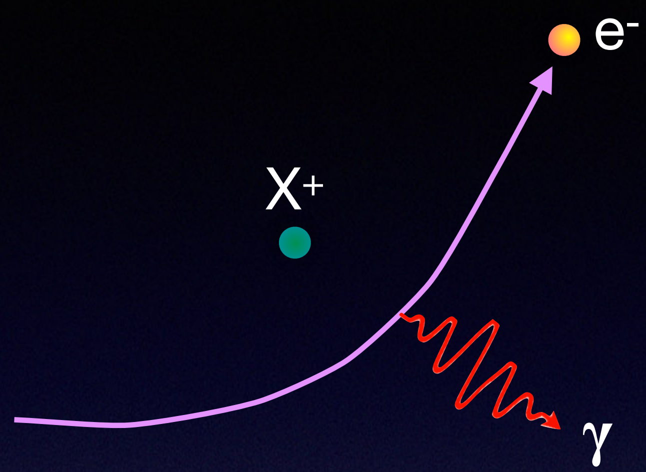
| | type | reaction | name |
|---|---|---|---|
| 1 | free-free | $e^- + X^+ \rightarrow e^- + X^+ + \gamma$ | bremsstrahlung |
| 2 | free-bound | $e^- + X^+ \rightarrow X + \gamma$ | recombination |
| 3 | bound-free | $e^- + X \rightarrow X^+ + 2e^-$ | collisional ionization |
| 4 | bound-bound | $e^- + X \rightarrow e^- + X' \rightarrow e- + X + \gamma$ | collisional excitation |

**NOTE:** all these processes require the presence of free electrons...

Throughout we assume that the gas is optically thin, so that every photon generated escapes the gas, thus contributing to its cooling...

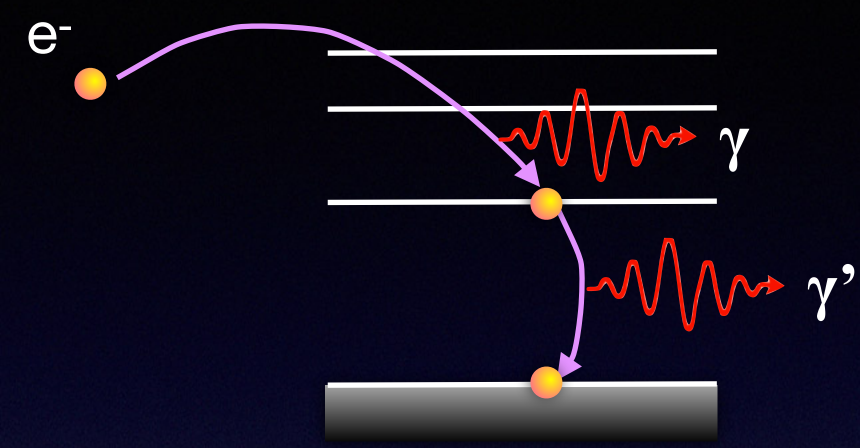
# Cooling Processes

## 1) free-free (bremsstrahlung)



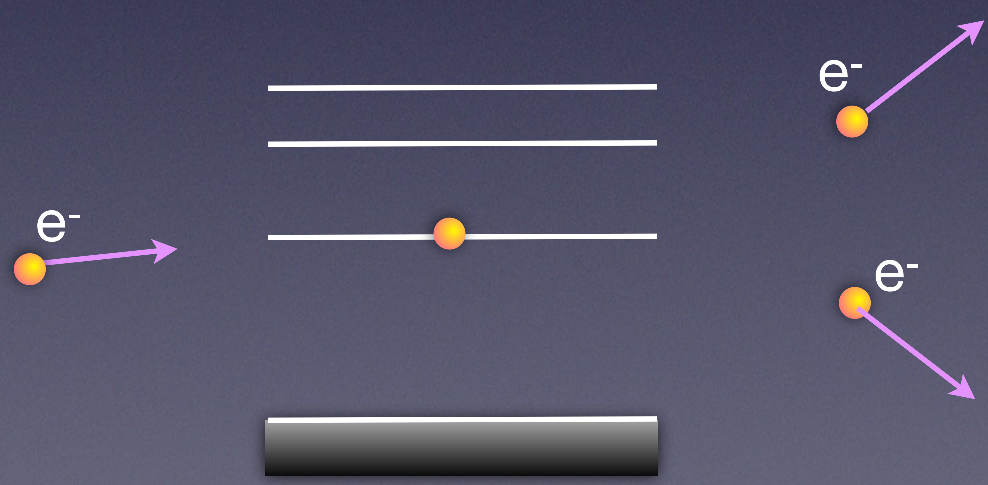

Free electron is accelerated by ion. Accelerated charges emit photons, resulting in cooling. For bremsstrahlung, $\Lambda \propto T^{1/2}$

## 2) free-bound (recombination)



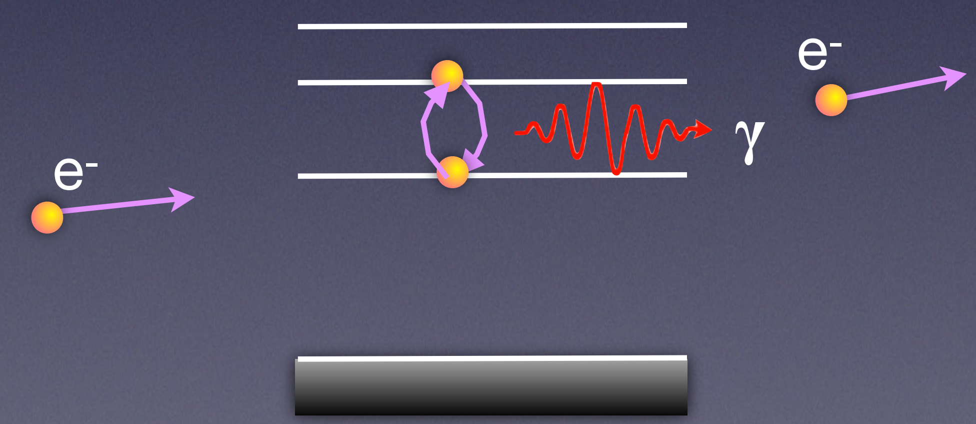

Free electron recombines with ion. Binding energy plus free electron's kinetic energy are radiated away. If capture into an excited state, subsequent (line) emission may result as electron cascades down to ground level.

## 3) bound-free (collisional ionization)




Impact of free electron ionizes a formerly bound electron, taking (kinetic) energy from the free electron

## 4) bound-bound (collisional excitation)




Impact of free electron knocks bound electron to excited state. As it decays, it emits a photon. Note, in case of collisional de-excitation, no photon is emitted (no net cooling)

# The Cooling Function

In order to compute the cooling function $\Lambda(T) \equiv \mathcal{C}/n_{\rm H}^2$ for a certain gas, one first needs to determine the densities of the various ionic species. In the case of a pure H/He mixture (the simplest, relevant case), these are $n_{\rm e}, n_{\rm H_0}, n_{\rm H^+}, n_{\rm He_0}, n_{\rm He^+}, n_{\rm He^{++}}$

At fixed total gas densities, these densities are governed by differential equations such as

$$\frac{{\rm d}n_{\rm H_0}}{{\rm d}t} = \alpha_{\rm H^+}(T)\, n_{\rm H^+}\, n_{\rm e} - \Gamma_{\rm eH_0}(T)\, n_{\rm e}\, n_{\rm H_0} - \Gamma_{\gamma {\rm H_0}}\, n_{\rm H_0}$$

where

$\alpha_{\rm H^+}(T)$ = Hydrogen recombination coefficient [cm$^3$ s$^{-1}$]

$\Gamma_{\rm eH_0}(T)$ = collisional ionization rate [cm$^3$ s$^{-1}$]

$\Gamma_{\gamma {\rm H_0}} \equiv \int_{\nu_{\rm T}}^{\infty} \frac{4\pi J(\nu)}{h_{\rm P}\nu}\,\sigma(\nu)\,{\rm d}\nu$ = photo-ionization rate [s$^{-1}$]

$\nu_{\rm T}$ = ionization threshold (e.g., 13.6 eV/$h_{\rm P}$ for H)

$\sigma(\nu)$ = ionization cross section [cm$^{-2}$]

$J(\nu)$ = radiation background intensity [erg s$^{-1}$ cm$^{-2}$ sr$^{-1}$ Hz$^{-1}$]

# Ionization Equilibrium

The typical timescale for photo-ionization, for a typical ionizing background, is

$$t_{\rm photo} \sim \frac{1}{\Gamma_{\gamma {\rm H}_0}} \sim 3 \times 10^4 {\rm yr}$$

which is very short (much shorter than the typical dynamical times involved). Hence, the timescale on which $n_{{\rm H}_0}$ evolves is dominated by photoionization. However, even in the absence of a photo-ionizing background, $n_{{\rm H}_0}$ evolves on a timescale

$$\frac{1}{n_{\rm e}(\alpha_{{\rm H}+} - \Gamma_{{\rm eH}_0})} \sim 10^6 \, {\rm yr} \left( \frac{n_{\rm e}}{10^{-5} \, {\rm cm}^{-3}} \right)^{-1}$$

Both these timescales are typically short compared to the (hydro)-dynamical times. Hence, in most (but not all) cases, it is safe to assume that the system has equilibrated the destruction and creation rates. Such an equilibrium is called ionization equilibrium.

If photo-ionization is ignored (i.e., there is no ionizing radiation). and one still has equilibrium, this is called collisional ionization equilibrium (CIE).

# Ionization Equilibriun

In ionization equilibrium, the ionic abundances are determined by simple algebraic equations (much easier than differential equations):

Examples are:
$$\Gamma_{\rm eH_0}\, n_{\rm e}\, n_{\rm H_0} + \Gamma_{\gamma {\rm H_0}}\, n_{\rm H_0} = \alpha_{\rm H^+}\, n_{\rm e}\, n_{\rm H^+}$$
$$n_{\rm H^+} + n_{\rm H_0} = n_{\rm H}$$
$$n_{\rm H^+} + n_{\rm He^+} + 2n_{\rm He^{++}} = n_{\rm e}$$
etc.

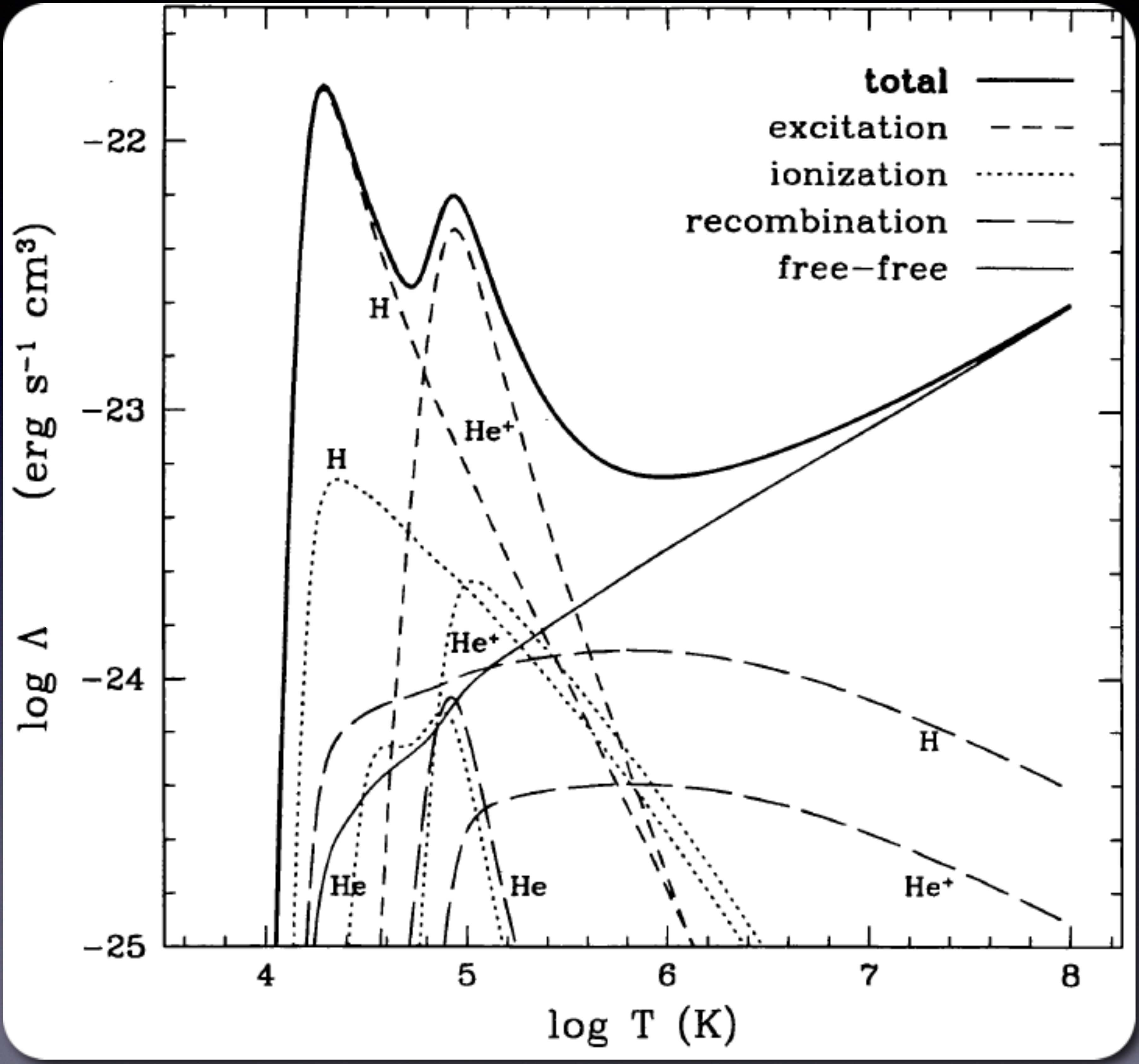
In numerical simulations and/or analytical calculations, one must decide whether ionization equilibrium is valid or not. If not, one needs to solve the differential equations in order to infer the various ionic abundances...

If there is no photo-ionization (i.e., $J(\nu) = 0$), then, under CIE, the relative abundances of ionic species depend only on temperature

$$\Lambda = \frac{\mathcal{C}}{n_{\rm H}^2} = \Lambda(T)$$

This is the situation most often assumed in semi-analytical models for galaxy formation

# The CIE Cooling Function




Source: Katz, Weinberg & Hernquist, 1996, ApJS, 105, 19

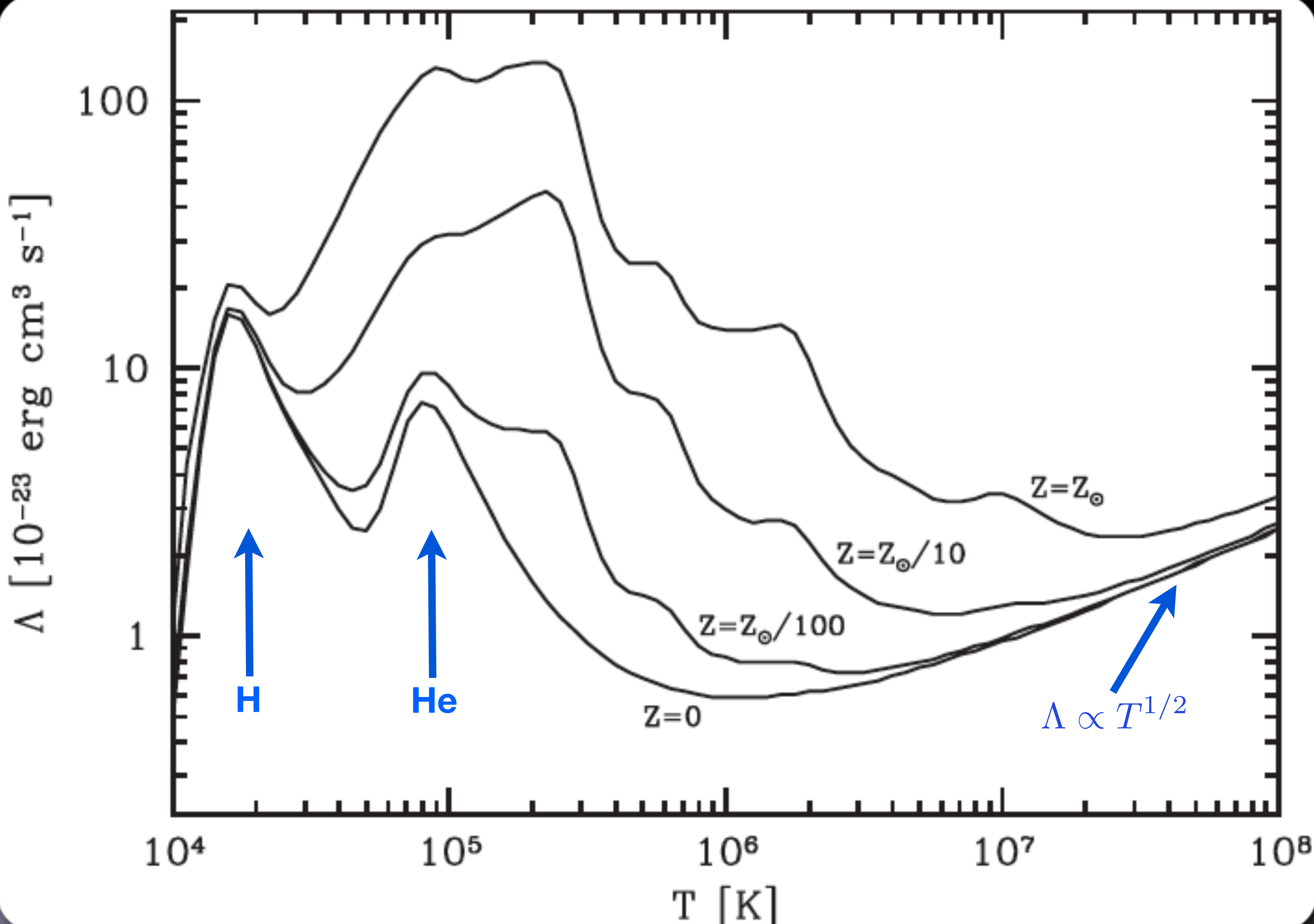
# The CIE Cooling Function

Consider Collisional Ionization Equilibrium: what will the cooling function look like?

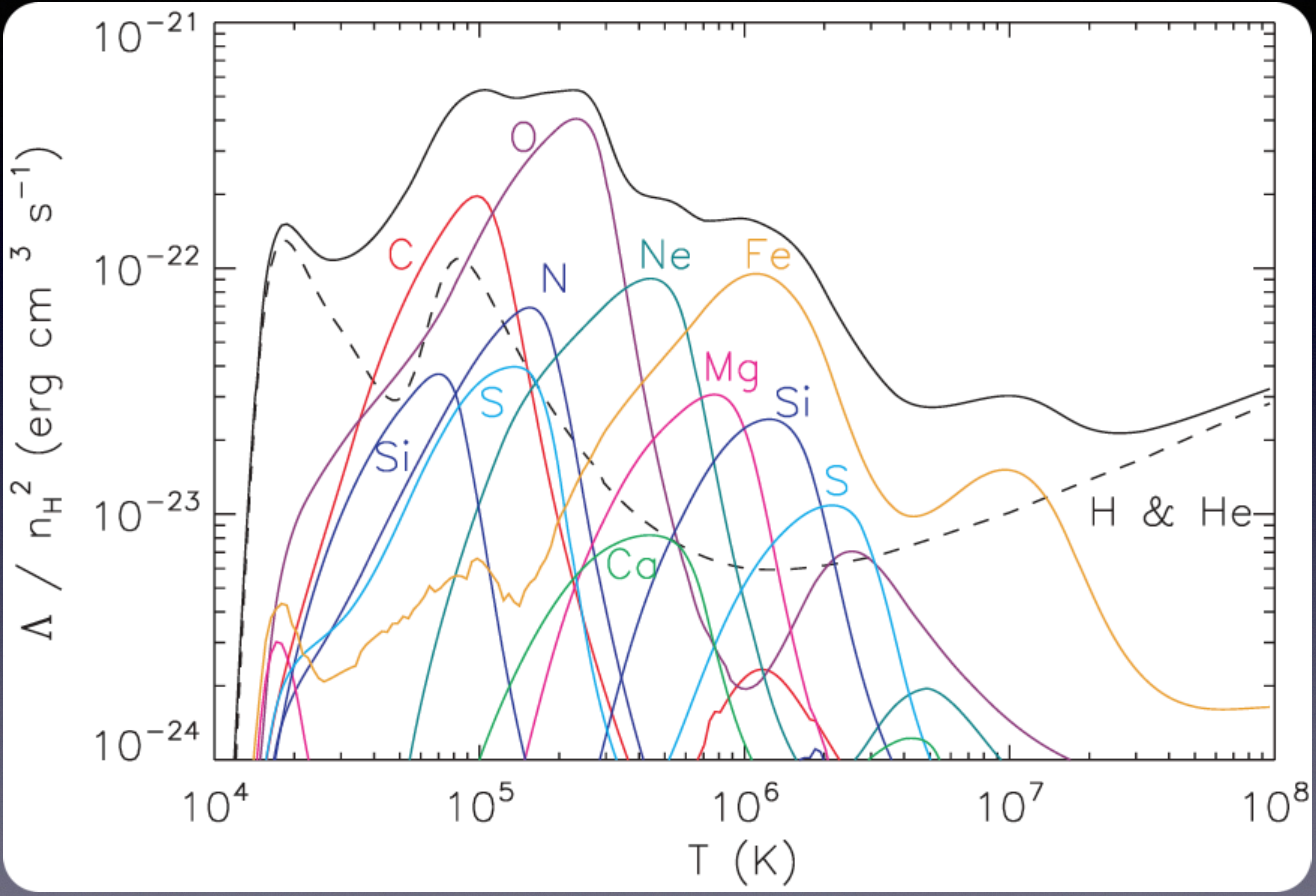
- at high T, gas is fully ionized ⇨ only bremsstrahlung contributes ⇨ $\Lambda \propto T^{1/2}$
- at $T < 10^4\,\mathrm{K}$ , all the gas is neutral ⇨ no ions ⇨ no bremsstrahlung

  at sufficiently low T, the residual free electrons do not have enough energy to excite H to its first excited state (which requires 10.2 eV)
- if T > few x $10^4$K all H is ionized ⇨ H no longer contributes to cooling causing a local drop in Λ(T)
- He is responsible for a second peak in Λ(T) at around T ~ $10^5$K
- when metals are present, many new cooling channels are available, mainly between ~$10^4$K and ~$10^7$K, greatly increasing Λ(T) . For $Z = Z_\odot$ the cooling rate at $10^6$K is boosted by a factor ~100 with respect to a primordial gas!

# The CIE Cooling Function



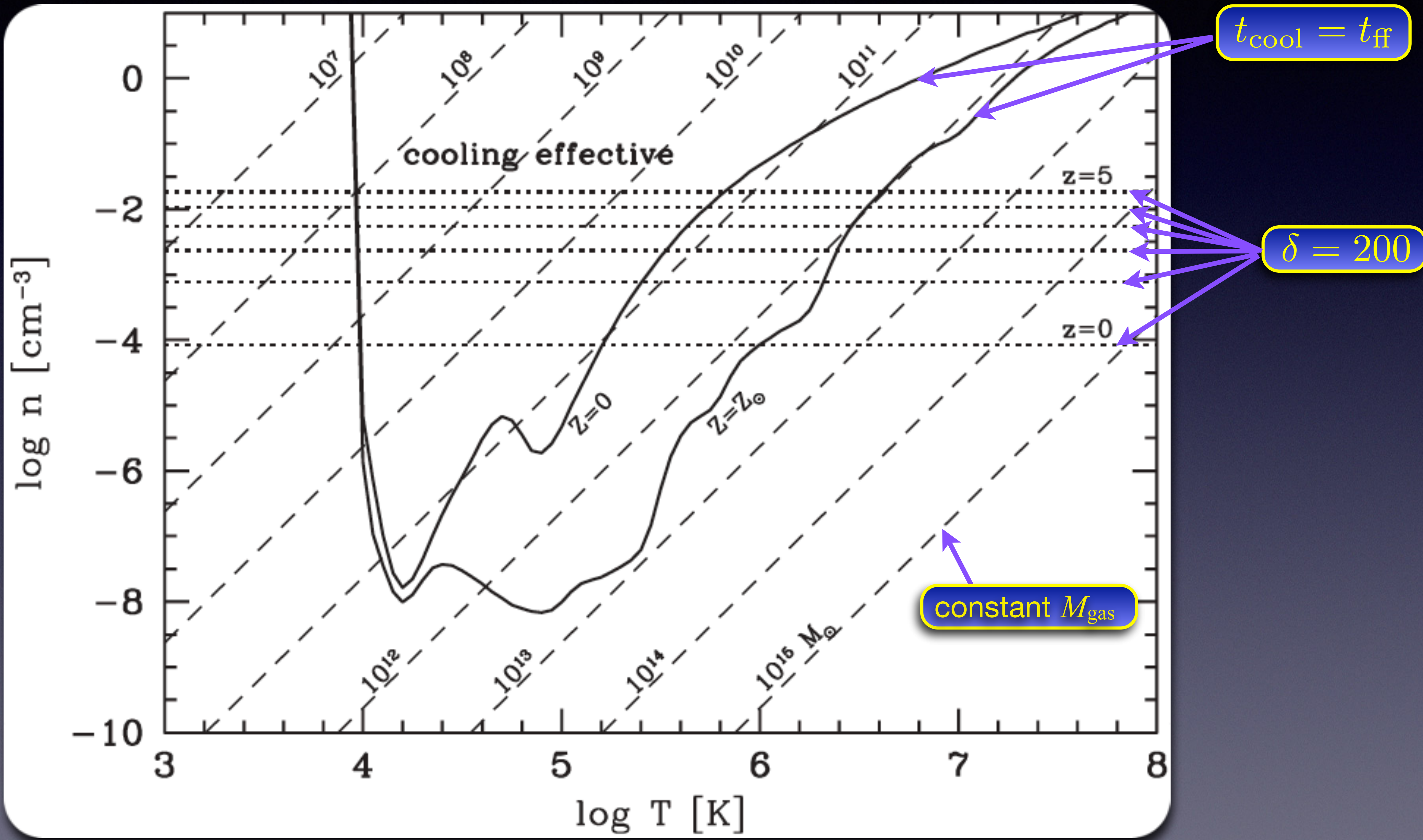
# The CIE Cooling Function



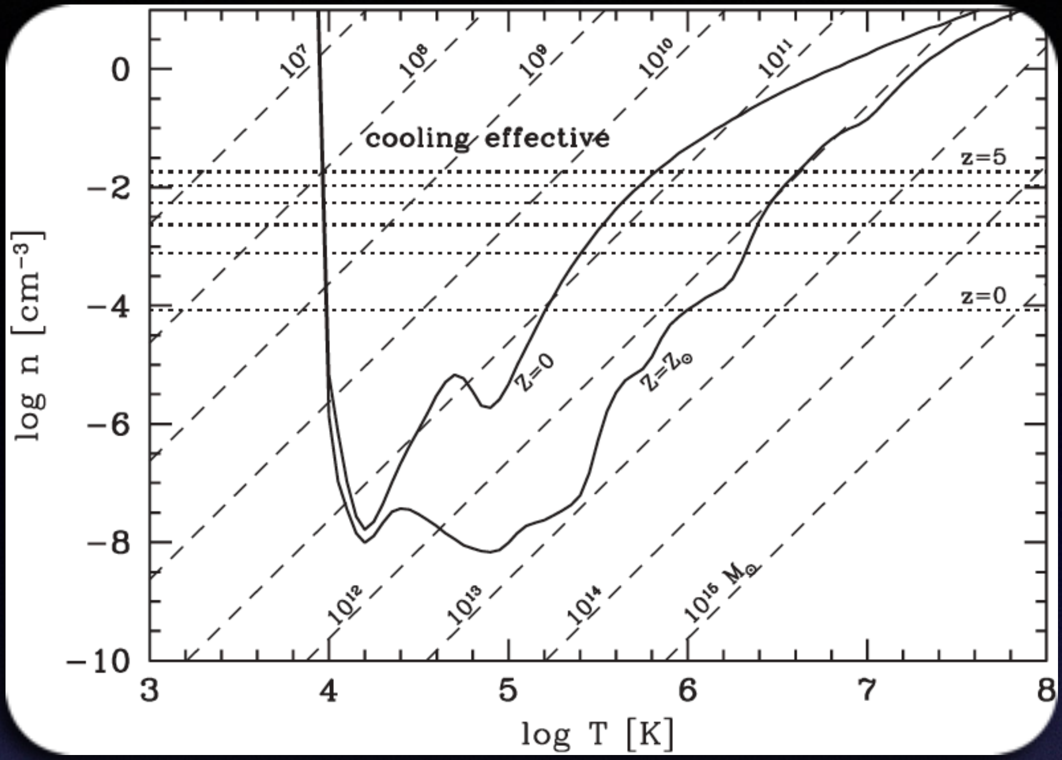

Source: Wiersma, Schaye & Smith, 2009, MNRAS, 393, 99

# Cooling & Galaxy Formation

Under the assumption of CIE, we can compute $\frac{t_{\rm cool}}{t_{\rm ff}} = \frac{t_{\rm cool}}{t_{\rm ff}}(n, T, Z)$



# Cooling & Galaxy Formation




$$M_{\rm gas} = 0.15 M_{\rm vir} \;\Rightarrow\; M_{\rm vir} \simeq 6.6 M_{\rm gas}$$

- Haloes with $M_{\rm vir} < 10^9 M_\odot$ $(V_{\rm vir} < 20\,{\rm km/s})$ can't cool their gas (except by molecular cooling...)
- Haloes with $M_{\rm vir} > 10^{12} M_\odot (Z=0)$
  $M_{\rm vir} > 10^{13} M_\odot (Z=Z_\odot)$
  can't cool their gas efficiently either...

In early papers (and textbooks) on galaxy formation, this mass scale of $10^{12} - 10^{13} M_\odot$ was invoked to exlain the exponential cut-off in the luminosity/stellar mass function of galaxies; more massive galaxies can't form because they can't efficiently cool their gas...

(e.g., Binney, 1977, ApJ, 215, 483; Silk, 1977, ApJ, 211, 683; Ostriker & Rees, 1977, MNRAS, 179, 541)

However, this argument is seriously flawed for two reasons:

- Haloes and galaxies form hierarchically ⇒ the progenitors of massive haloes can cool, especially at higher redshifts...
- The curve $t_{\rm cool} = t_{\rm ff}$ is calculated for an overdensity $\delta = 200$. The gas in a halo typically has a density profile, and can have $\delta \gg 200$ near the center.
  ⇒ at least some fraction of the gas should have cooled...

# Cooling & Galaxy Formation

White & Rees (1978, MNRAS, 183, 341), in a seminal paper, showed that taking into account that the gas accumulates & condensates in dark matter haloes which form hierarchically, results in a prediction that most of the gas should have cooled and formed stars (it vastly overpredicts the number density of faint galaxies). This is the

**overcooling problem**

which calls for some extra processes in galaxy formation that can heat the gas!!!

In order to better account of the fact that realistic haloes have both density and temperature profiles, (semi-)analytical models (SAMs) of galaxy formation normally adopt the concept of a cooling radius, defined as the radius at which the cooling time

$$t_{\rm cool}(r) = \frac{3\, n(r)\, k_{\rm B}\, T(r)}{2\, n_{\rm H}^2(r)\, \Lambda(T)}$$

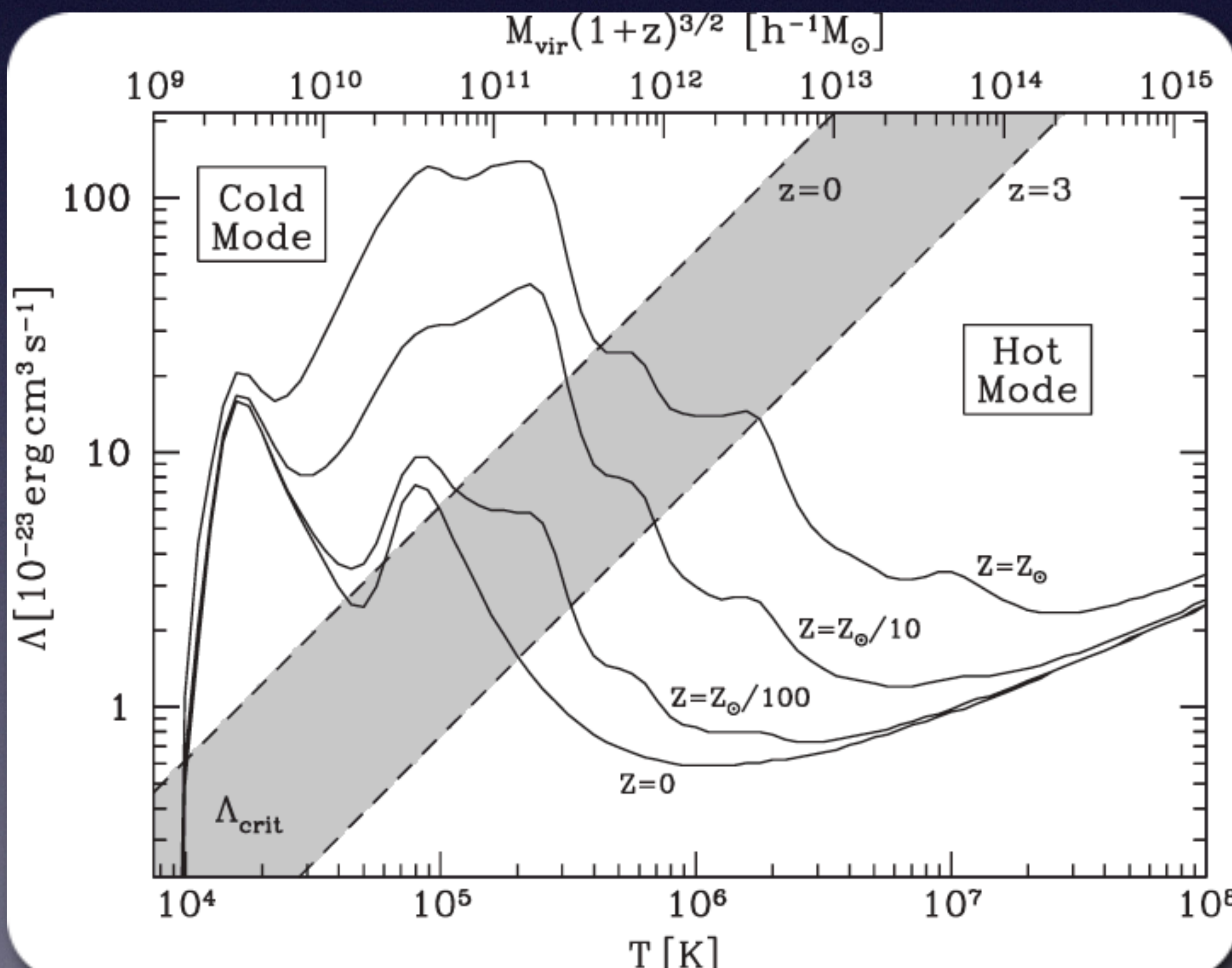
is equal to the free-fall time, $t_{\rm ff}$ , or the Hubble time, $t_{\rm H}$ (depending on the SAM)
With the cooling radius thus defined, the cooling rate is simply

$$\dot{M}_{\rm cool}(t) = 4\pi \rho_{\rm gas}(r_{\rm cool})\, r_{\rm cool}^2 \, \frac{{\rm d}r_{\rm cool}}{{\rm d}t}$$

# Hot Mode vs. Cold Mode

- When $r_{\rm cool} \gg r_{\rm vir}$ we are in the regime of `catastropic cooling' as all the gas within the halo is expected to have cooled. If there is no hot gas in the halo, there can also be no accretion shock (close to the virial radius). Hence, any new gas that is accreted will not be shock heated.....it will fall to the center cold. This is called **cold mode accretion**.

- When $r_{\rm cool} \ll r_{\rm vir}$ only the inner gas can cool. In this case, halo will have hot atmosphere, and an accretion shock close to the virial radius. Newly accreted gas is shock heated to close to the virial temperature, and then slowly cools, in quasi-hydrostatic equilibrium. This is called **hot mode accretion**.




The band $\Lambda_{\rm crit}$ indicates the boundary between hot mode and cold mode accretion. Halos with masses $M_{\rm vir} < 10^{10} h^{-1} M_\odot$ always accrete their gas in the cold mode, while haloes with $M_{\rm vir} > 10^{12} h^{-1} M_\odot$ have a stable accretion shock close to the virial radius causing hot mode accretion. Whether haloes in the intermediate mass range experience hot or cold mode accretion depends on redshift and the metallicity of the gas (see MBW §8.4.4).

# Photo-Ionization

In the discussion of cooling & galaxy formation above, we ignored the presence of a radiation field. At $z < 6$, and perhaps higher, the Universe is pervaded by a UV radiation background, produced by QSOs and (star-forming) galaxies. The intensity of this UV background radiation is ~$10^{-22}$ erg $s^{-1}$ $cm^{-2}$ $sr^{-1}$ $Hz^{-1}$ at the Lyman limit at z~2.

Close to a QSO or to young stars, the local radiation field may be orders of magnitude higher than this background average.

**UV radiation causes photo-ionization, which impacts cooling in two ways:**

**1)** it severely suppresses the cooling rate of low density gas

**2)** it heats the gas

Because UV radiation above Lyman limit ionizes low density gas, it eliminates collisional ionization & line excitation as cooling processes. Although recombination cooling increases, the net effect is a severe reduction of the overall cooling rate of low density gas.

- At high densities, where recombination rate becomes comparable to or larger than photoionization rate, cooling rate are similar to under CIE.
- For gas near the cosmic mean density, the photoionization rates corresponding to typical UV background, are much higher than corresponding recombination rates, causing both hydrogen and helium to be fully ionized...

# Photo-Ionization

Photoionization also heats the gas because the photoelectrons carry off residual energy:

The heating rate is:

$$\mathcal{H} = n_{\rm H_0}\,\varepsilon_{\rm H_0} + n_{\rm He_0}\,\varepsilon_{\rm He_0} + n_{\rm He^+}\,\varepsilon_{\rm He^+}$$

where $$\varepsilon_i = \int_{\nu_i}^{\infty} \frac{4\pi\,J(\nu)}{h_{\rm P}\,\nu}\,\sigma_i(\nu)\,[h_{\rm P}\nu - h_{\rm P}\nu_i]\,{\rm d}\nu$$

photo-ionization cross-section

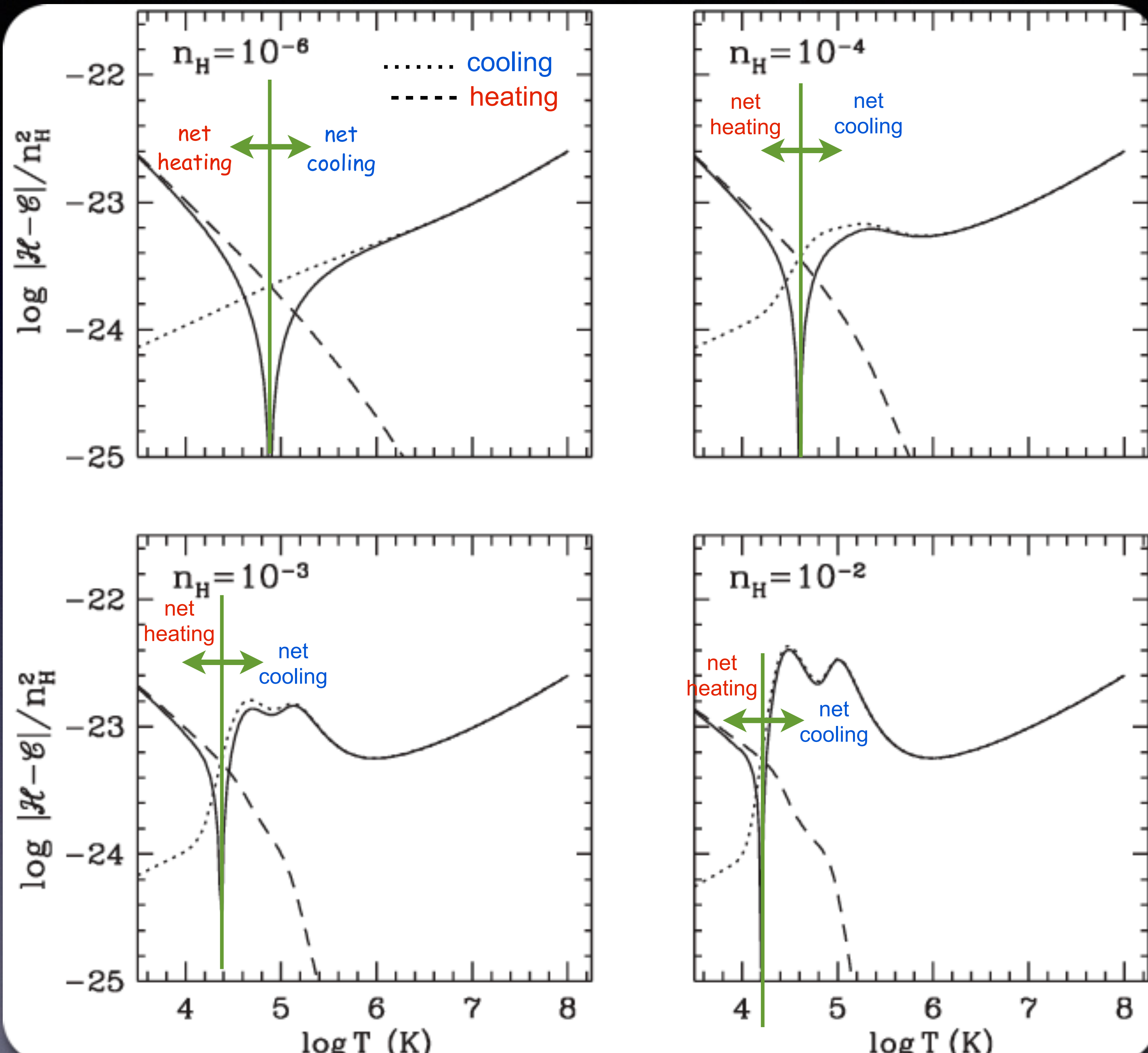
The heating rate decreases with increasing temperature, because the recombination rates, and hence the neutral "targets" for the photons, decline.

Because of this heating, in the presence of photo-ionization what is important is the net heating/cooling rate, $(\mathcal{C} - \mathcal{H})/n_{\rm H}^2$

Unlike in the case of CIE, in the presence of photo-ionization, this net rate is NOT only a function of temperature. Instead, it also depends on density. This arises because of the competition between photo-ionization & recombination.

In the presence of photo-ionization, the "cooling function" $$\Lambda \equiv \frac{\mathcal{C} - \mathcal{H}}{n_{\rm H}^2} = \Lambda(T, n_{\rm H})$$

# Photo-Ionization




The net heating/cooling rates as a function of temperature for gas of primordial composition in ionization equilibrium with a UV radiation background of intensity

$$J(\nu) = 10^{-22}\,(\nu_H/\nu)\,\mathrm{erg/s/cm^2/sr/Hz}$$

Results are shown for 4 different $n_H$, as indicated (in $cm^{-3}$). Dotted and dashed lines show the cooling and heating rates, respectively, while the solid curves show the absolute value of the net cooling rate. Note how heating becomes dominant at low temperatures, and how photo-ionization suppresses the H and He cooling peaks in low density gas....

# Lecture 15

## SUMMARY

# Summary: key words & important facts

| Key words | |
|---|---|
| Hydro-static Equilibrium | Overcooling Problem |
| Accretion Shock | Cold mode vs. Hot mode |
| Virial Temperature | Ionization equilibrium |
| Cooling Function | Photo-ionization heating |

- Mass estimates based on the assumption of hydrostatic equilibrium need to correct for non-thermal pressure sources (turbulence, magnetic fields, cosmic rays)
- Gas infalling in a halo through an accretion shock is heated to the virial temperature at which the gas is in hydrostatic, virial equilibrium with the halo potential.
- Low mass halos ($M_h < 10^{12}\,M_\odot$) are predicted to experience cold mode accretion (via streams), as they can't support an accretion shock.
- When ignoring photo-ionizations, it is typically assumed that the gas is in collisional ionization equilibrium (CIE) → one uses CIE cooling functions
- In the presence of photo-ionization, the net heating/cooling rate, $(\mathcal{C} - \mathcal{H})/n_{\rm H}^2$, is a function of both temperature and density. This arises because of the competition between photo-ionization & recombination.

# Summary: key equations & expressions

## Hydrostatic Equilibrium

$$\nabla P(r) = -\rho_{\rm gas} \nabla \Phi(r)$$

**Spherical Symmetry**

$$\nabla \Phi = \frac{{\rm d}\Phi}{{\rm d}r} = \frac{G\,M(r)}{r^2}$$

**Ideal Gas**

$$\nabla P = \frac{{\rm d}P}{{\rm d}r} = \frac{k_{\rm B}}{\mu m_{\rm p}} \frac{\rm d}{{\rm d}r}(\rho T)$$

$$M(r) = M_{\rm gas}(r) + M_{\rm DM}(r) = -\frac{k_{\rm B}\,T(r)\,r}{\mu m_{\rm p}\,G}\left[\frac{{\rm d}\ln\rho_{\rm gas}}{{\rm d}\ln r} + \frac{{\rm d}\ln T}{{\rm d}\ln r}\right]$$

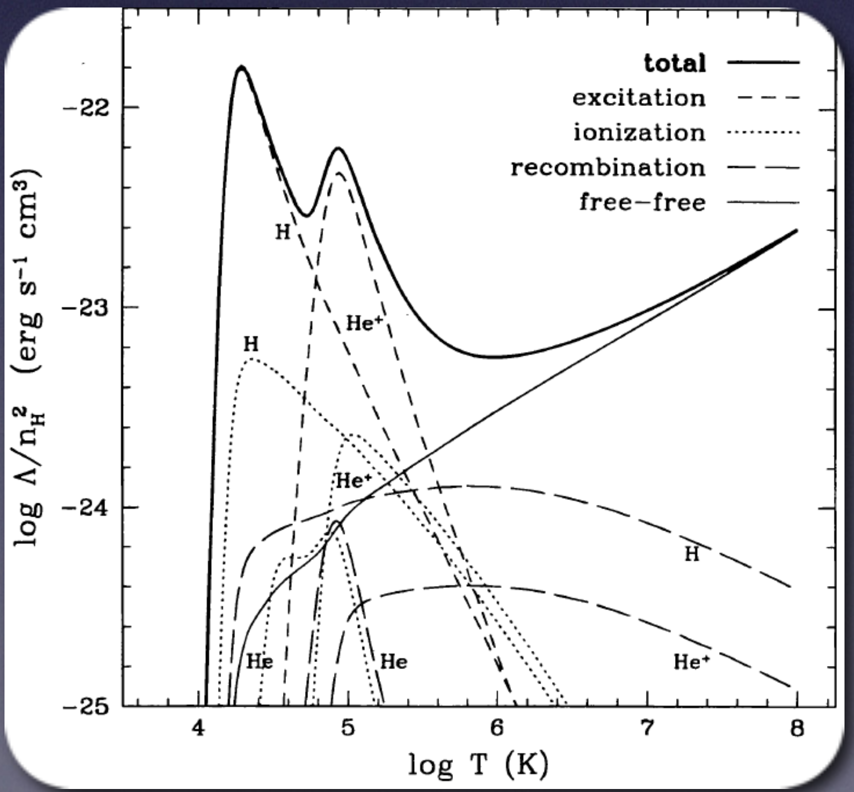
## Cooling Time

$$t_{\rm cool} = \frac{3n\,k_{\rm B}T}{2\,n_{\rm H}^2\,\Lambda(T)}$$

## Cooling Function

$$\Lambda \equiv \frac{\mathcal{C} - \mathcal{H}}{n_{\rm H}^2} = \Lambda(T, n_{\rm H})$$

in presence of heating




## virial temperature

$$T_{\rm vir} = \frac{\mu m_{\rm p}}{2\,k_{\rm B}} V_{\rm vir}^2 \simeq 3.6 \times 10^5\,{\rm K}\left(\frac{V_{\rm vir}}{100\,{\rm km/s}}\right)^2$$

## Photo-ionization heating rate:

$$\mathcal{H} = n_{\rm H_0}\,\varepsilon_{\rm H_0} + n_{\rm He_0}\,\varepsilon_{\rm He_0} + n_{\rm He^+}\,\varepsilon_{\rm He^+}$$

where $$\varepsilon_i = \int_{\nu_i}^{\infty} \frac{4\pi\,J(\nu)}{h_{\rm P}\,\nu}\,\sigma_i(\nu)\,[h_{\rm P}\nu - h_{\rm P}\nu_i]\,{\rm d}\nu$$