

ASTR 610

Theory of Galaxy Formation

Lecture 13: The Halo Model & Halo Occupation Statistics

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Halo Model & Halo Occupation Statistics

In this lecture we discuss the “halo model”, an analytical model to describe the (dark matter) mass distribution on small, non-linear scales. We also introduce the concept of halo occupation statistics and discuss how data can constrain these. We end with some discussion on the galaxy-dark matter connection

Topics that will be covered include:

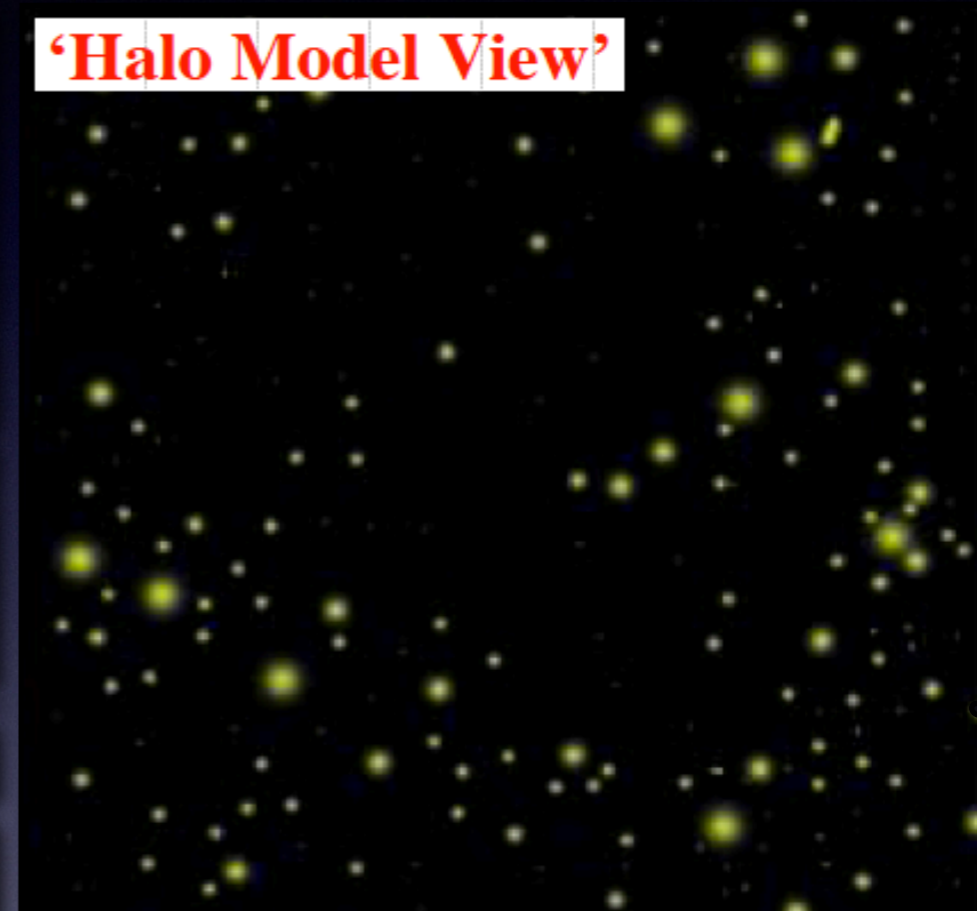
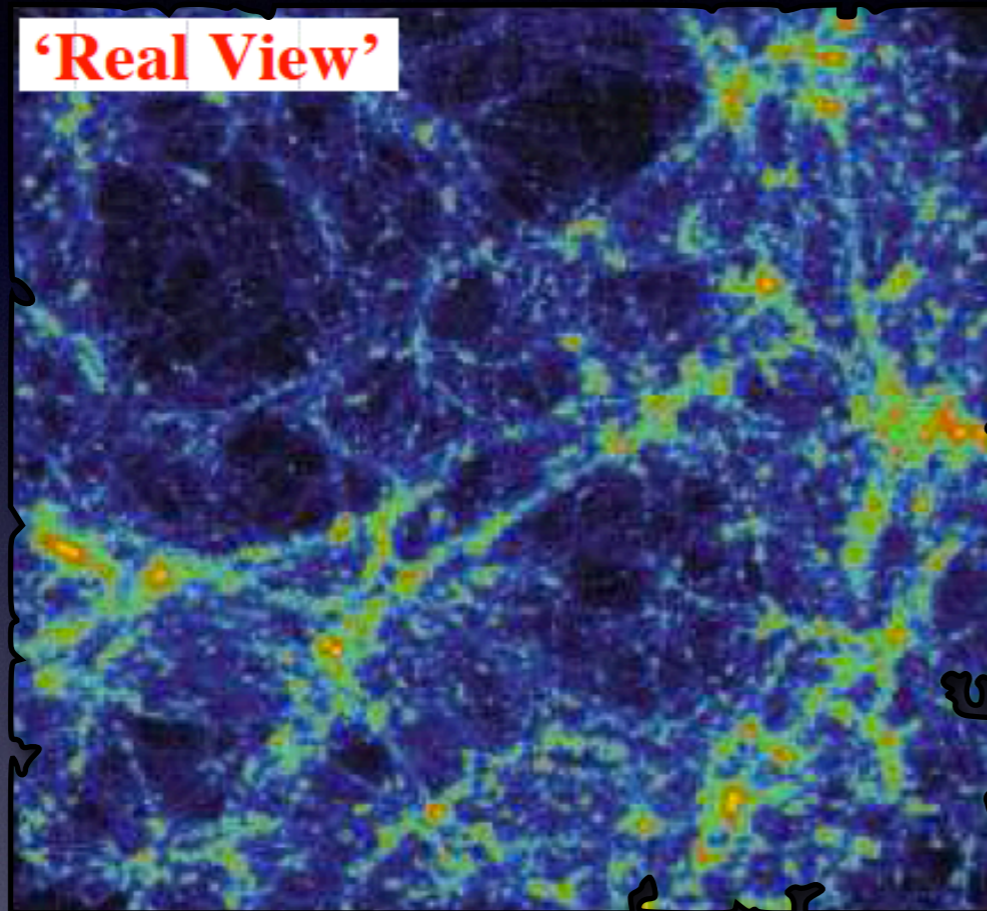
- The Halo Model
- 1-halo vs 2-halo terms
- Halo Occupation Models
- Halo Exclusion
- Conditional Luminosity Function
- Poisson statistics
- Galaxy-Dark Matter Connection

The background of the slide is a deep blue image of the cosmic web, showing a complex network of glowing blue filaments and dark voids, with several bright yellow-white points representing galaxies or star clusters.

The Halo Model

The Halo Model

The **Halo model** is an analytical model that describes dark matter density distribution in terms of its **halo building blocks**, under ansatz that all dark matter is partitioned over haloes.



Throughout we assume that all dark matter haloes are spherical, and have a density distribution that only depends on halo mass:

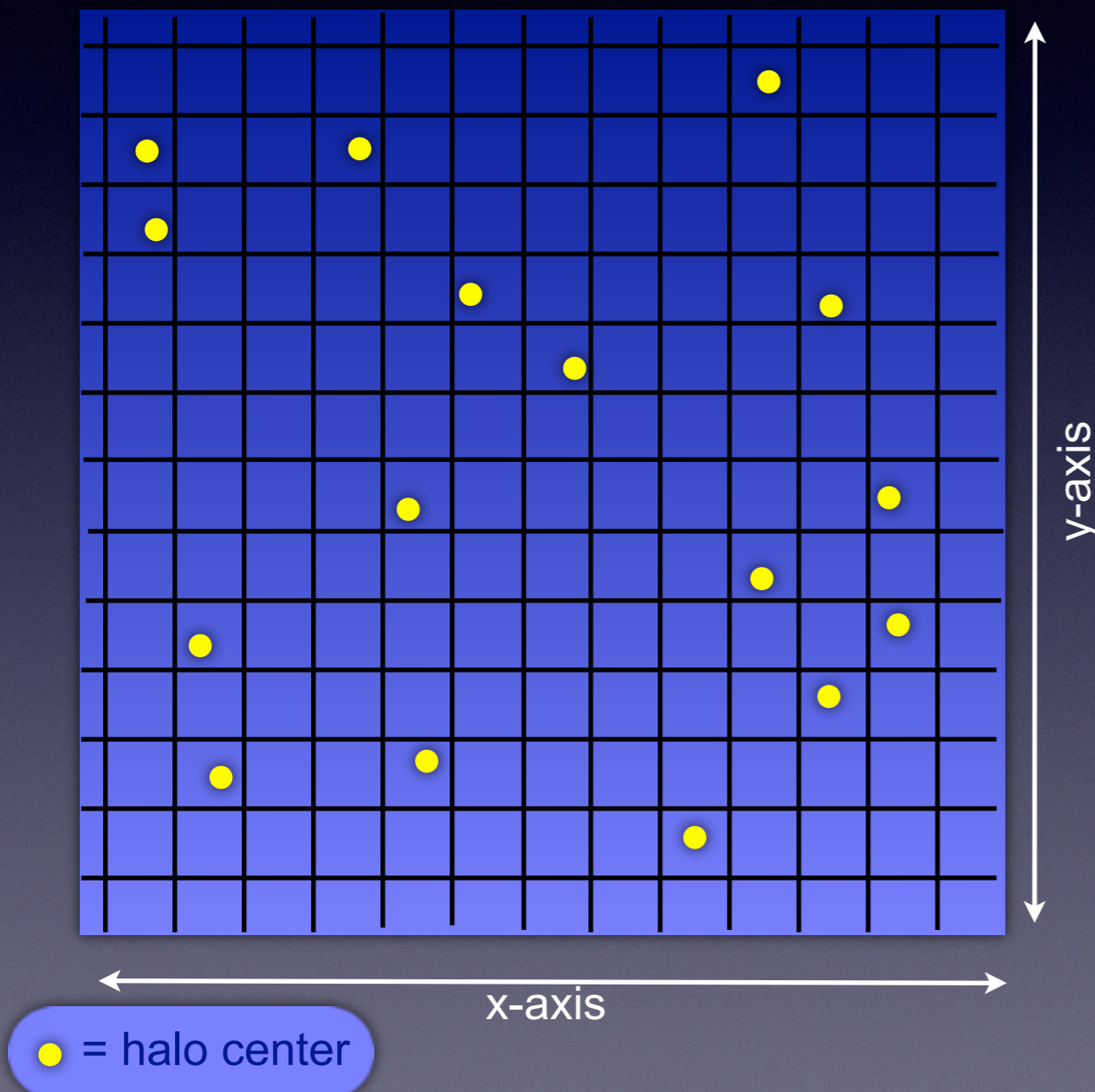
$$\rho(r|M) = M u(r|M)$$

Here $u(r|M)$ is the normalized density profile:

$$\int d^3\vec{x} u(\vec{x}|M) = 1$$

The Halo Model

Imagine space divided into many small volumes, ΔV_i , which are so small that none of them contain more than one halo center.



Let \mathcal{N}_i be the occupation number of dark matter haloes in cell i

Then we have that $\mathcal{N}_i = 0, 1$
and therefore $\mathcal{N}_i = \mathcal{N}_i^2 = \mathcal{N}_i^3 =$

This allows us to write the matter density field as a summation:

$$\rho(\vec{x}) = \sum_i \mathcal{N}_i M_i u(\vec{x} - \vec{x}_i | M_i)$$

The Halo Model

$$\rho(\vec{x}) = \sum_i \mathcal{N}_i M_i u(\vec{x} - \vec{x}_i | M_i)$$

$$\bar{\rho} = \int \rho(\vec{x}) d^3\vec{x} \stackrel{\text{ergodicity}}{=} \langle \rho(\vec{x}) \rangle = \left\langle \sum_i \mathcal{N}_i M_i u(\vec{x} - \vec{x}_i | M_i) \right\rangle$$

ergodicity

$$= \sum_i \langle \mathcal{N}_i M_i u(\vec{x} - \vec{x}_i | M_i) \rangle$$

halo mass function

$$= \sum_i \int dM M n(M) \Delta V_i u(\vec{x} - \vec{x}_i | M)$$

$$= \int dM M n(M) \int d^3\vec{y} u(\vec{x} - \vec{y} | M)$$

$$= \int dM M n(M)$$

$$= \bar{\rho}$$

Q.E.D.

The Halo Model

$$\rho(\vec{x}) = \sum_i \mathcal{N}_i M_i u(\vec{x} - \vec{x}_i | M_i)$$

Now that we can write the density field in terms of the halo building blocks, let's focus on two-point statistics: $\xi_{\text{mm}}(r) \equiv \langle \delta(\vec{x}) \delta(\vec{x} + \vec{r}) \rangle = \frac{1}{\bar{\rho}^2} \langle \rho(\vec{x}) \rho(\vec{x} + \vec{r}) \rangle - 1$

$$\langle \rho(\vec{x}) \rho(\vec{x} + \vec{r}) \rangle = \left\langle \sum_i \mathcal{N}_i M_i u(\vec{x}_1 - \vec{x}_i | M_i) \cdot \sum_j \mathcal{N}_j M_j u(\vec{x}_2 - \vec{x}_j | M_j) \right\rangle$$

$$= \sum_i \sum_j \langle \mathcal{N}_i \mathcal{N}_j M_i M_j u(\vec{x}_1 - \vec{x}_i | M_i) u(\vec{x}_2 - \vec{x}_j | M_j) \rangle$$

$$\vec{x}_2 = \vec{x}_1 + \vec{r}$$

We split this in two parts: the 1-halo term ($i = j$), and the 2-halo term ($i \neq j$)

For the 1-halo term we obtain:

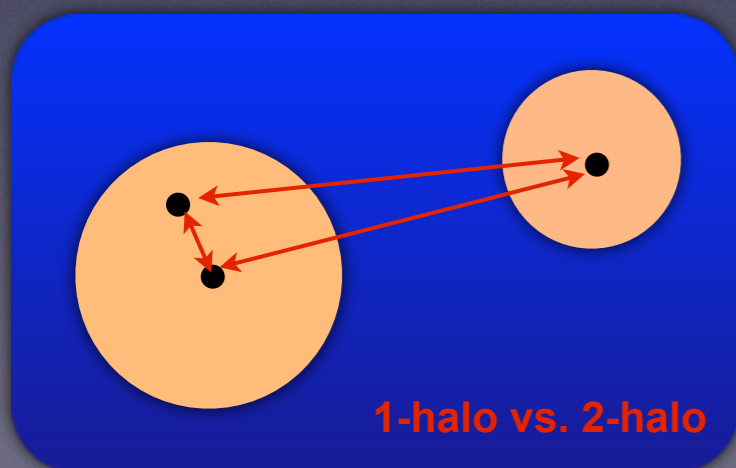
$$\mathcal{N}_i^2 = \mathcal{N}_i$$

$$\langle \rho(\vec{x}) \rho(\vec{x} + \vec{r}) \rangle_{1\text{h}} = \sum_i \langle \mathcal{N}_i M_i^2 u(\vec{x}_1 - \vec{x}_i | M_i) u(\vec{x}_2 - \vec{x}_i | M_i) \rangle$$

$$= \sum_i \int dM M^2 n(M) \Delta V_i u(\vec{x}_1 - \vec{x}_i | M) u(\vec{x}_2 - \vec{x}_i | M)$$

$$= \int dM M^2 n(M) \int d^3 \vec{y} u(\vec{x}_1 - \vec{y} | M) u(\vec{x}_2 - \vec{y} | M)$$

convolution integral



The Halo Model

$$\rho(\vec{x}) = \sum_i \mathcal{N}_i M_i u(\vec{x} - \vec{x}_i | M_i)$$

For the 2-halo term we obtain:

$$\begin{aligned} \langle \rho(\vec{x}) \rho(\vec{x} + \vec{r}) \rangle_{2h} &= \sum_i \sum_{j \neq i} \langle \mathcal{N}_i \mathcal{N}_j M_i M_j u(\vec{x}_1 - \vec{x}_i | M_i) u(\vec{x}_2 - \vec{x}_j | M_j) \rangle \\ &\stackrel{?}{\neq} \sum_i \sum_{j \neq i} \int dM_1 M_1 n(M_1) \int dM_2 M_2 n(M_2) \Delta V_i \Delta V_j \times \\ &\quad u(\vec{x}_1 - \vec{x}_i | M_1) u(\vec{x}_2 - \vec{x}_j | M_2) = \bar{\rho}^2 \end{aligned}$$

NO: dark matter haloes themselves are clustered, i.e., have a non-zero two point correlation function. This needs to be taken into account.

Clustering of dark matter haloes is characterized by halo-halo correlation function:

$$\xi_{hh}(r | M_1, M_2) = b(M_1) b(M_2) \xi_{mm}^{\text{lin}}(r)$$

Here $b(M)$ is the halo bias function. **Note:** the above description of the halo-halo correlation function is only valid on large (linear) scales! On small scales non-linearities and halo exclusion become important (see below).



The Halo Model

$$\rho(\vec{x}) = \sum_i \mathcal{N}_i M_i u(\vec{x} - \vec{x}_i | M_i)$$

For the 2-halo term we obtain:

$$\begin{aligned} \langle \rho(\vec{x}) \rho(\vec{x} + \vec{r}) \rangle_{2h} &= \sum_i \sum_{j \neq i} \langle \mathcal{N}_i \mathcal{N}_j M_i M_j u(\vec{x}_1 - \vec{x}_i | M_i) u(\vec{x}_2 - \vec{x}_j | M_j) \rangle \\ &= \sum_i \sum_{j \neq i} \int dM_1 M_1 n(M_1) \int dM_2 M_2 n(M_2) \Delta V_i \Delta V_j \times \\ &\quad [1 + \xi_{hh}(\vec{x}_i - \vec{x}_j | M_1, M_2)] u(\vec{x}_1 - \vec{x}_i | M_1) u(\vec{x}_2 - \vec{x}_j | M_2) \\ &= \bar{\rho}^2 + \int dM_1 M_1 n(M_1) \int dM_2 M_2 n(M_2) \times \\ &\quad \int d^3 \vec{y}_1 \int d^3 \vec{y}_2 u(\vec{x}_1 - \vec{y}_1 | M_1) u(\vec{x}_2 - \vec{y}_2 | M_2) \xi_{hh}(\vec{y}_1 - \vec{y}_2 | M_1, M_2) \\ &= \bar{\rho}^2 + \int dM_1 M_1 b(M_1) n(M_1) \int dM_2 M_2 b(M_2) n(M_2) \times \\ &\quad \int d^3 \vec{y}_1 \int d^3 \vec{y}_2 u(\vec{x}_1 - \vec{y}_1 | M_1) u(\vec{x}_2 - \vec{y}_2 | M_2) \xi_{mm}^{\text{lin}}(\vec{y}_1 - \vec{y}_2) \end{aligned}$$

convolution integral

The Halo Model: Summary

$$\xi(r) = \xi^{1h}(r) + \xi^{2h}(r)$$

$$\xi^{1h}(r) = \frac{1}{\bar{\rho}^2} \int dM M^2 n(M) \int d^3\vec{y} u(\vec{x} - \vec{y}|M) u(\vec{x} + \vec{r} - \vec{y}|M)$$

$$\xi^{2h}(r) = \frac{1}{\bar{\rho}^2} \int dM_1 M_1 b(M_1) n(M_1) \int dM_2 M_2 b(M_2) n(M_2) \times \\ \int d^3\vec{y}_1 \int d^3\vec{y}_2 u(\vec{x} - \vec{y}_1|M_1) u(\vec{x} + \vec{r} - \vec{y}_2|M_2) \xi_{mm}^{lin}(\vec{y}_1 - \vec{y}_2)$$

Halo Model Ingredients:

- the halo density profiles $\rho(r|M) = M u(r|M)$
- the halo mass function $n(M)$
- the halo bias function $b(M)$
- the linear correlation function of matter $\xi_{mm}^{lin}(r)$

All of these are (reasonably) well calibrated against numerical simulations.

The Halo Model in Fourier Space

$$P(k) = P^{1h}(k) + P^{2h}(k)$$

$$P^{1h}(k) = \frac{1}{\bar{\rho}^2} \int dM M^2 n(M) |\tilde{u}(k|M)|^2$$

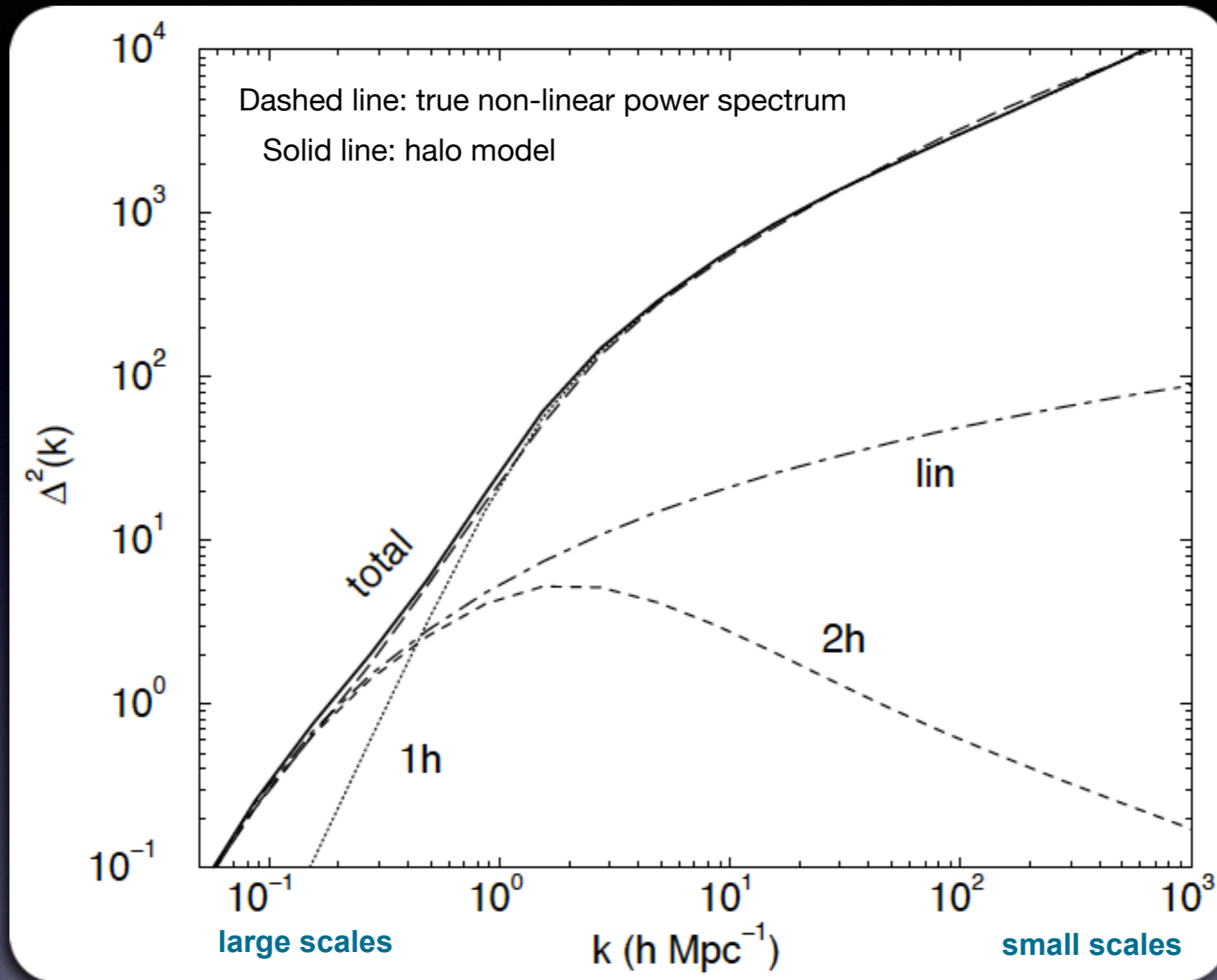
$$P^{2h}(k) = P^{\text{lin}}(k) \left[\frac{1}{\bar{\rho}} \int dM M b(M) n(M) \tilde{u}(k|M) \right]^2$$

$$P^{\text{lin}}(k) = P_{\text{i}}(k) T^2(k) = k^{n_s} T^2(k)$$

$$\tilde{u}(\vec{k}|M) = \int u(\vec{x}|M) e^{-i\vec{k}\cdot\vec{x}} d^3\vec{x} = 4\pi \int_0^\infty u(r|M) \frac{\sin kr}{kr} r^2 dr$$

Since convolutions in **real-space** become multiplications in **Fourier space**, the halo model expression for the power spectrum is much easier. Therefore, in practice, one computes $P(k)$ and then uses Fourier transformation to obtain two-point correlation function $\xi(r)$

The Halo Model in Fourier Space



Source: Cooray & Sheth (2002)

$$\Delta^2(k) = \frac{1}{2\pi^2} k^3 P(k)$$

Dimensionless power spectrum

The Halo Model: complications

$$P^{1h}(k) = \frac{1}{\bar{\rho}^2} \int dM M^2 n(M) |\tilde{u}(k|M)|^2$$

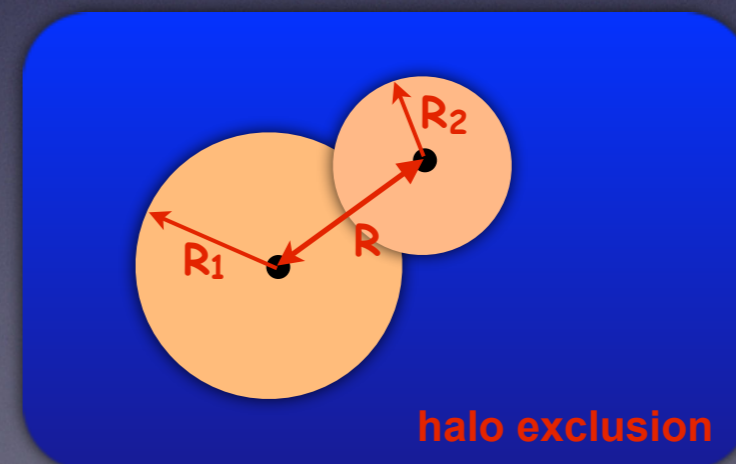
$$P^{2h}(k) = P^{\text{lin}}(k) \left[\frac{1}{\bar{\rho}} \int dM M b(M) n(M) \tilde{u}(k|M) \right]^2$$

However, this is ONLY true under the simplifying assumption that

$$\xi_{\text{hh}}(r|M_1, M_2) = b(M_1) b(M_2) \xi_{\text{mm}}^{\text{lin}}(r)$$


In reality, on small scales, in the (quasi)-linear regime, this description of the halo-halo correlation function becomes inadequate for two reasons:

- $\xi_{\text{mm}}^{\text{lin}}(r)$ is no longer adequate
- halo exclusion



Properly accounting for this is complicated

(if interested, see [vdBosch+13](#))

The background of the slide is a deep blue image showing the cosmic web, with glowing filaments and clusters of galaxies. A semi-transparent blue rectangle with rounded corners and a thin black border is centered on the slide, containing the title text.

Halo Occupation Modelling

The Galaxy Power Spectrum

$$P^{1h}(k) = \frac{1}{\bar{\rho}^2} \int dM M^2 n(M) |\tilde{u}(k|M)|^2$$


$$P^{2h}(k) = P^{lin}(k) \left[\frac{1}{\bar{\rho}} \int dM M b(M) n(M) \tilde{u}(k|M) \right]^2$$

The above equations describe the halo model predictions for the **matter power spectrum**

The same formalism can also be used to compute the galaxy power spectrum:

simply replace:

$\frac{M}{\bar{\rho}}$	\rightarrow	$\frac{\langle N \rangle_M}{\bar{n}_g}$
$\frac{M^2}{\bar{\rho}^2}$	\rightarrow	$\frac{\langle N(N-1) \rangle_M}{\bar{n}_g^2}$
$\tilde{u}(k M)$	\rightarrow	$\tilde{u}_g(k M)$



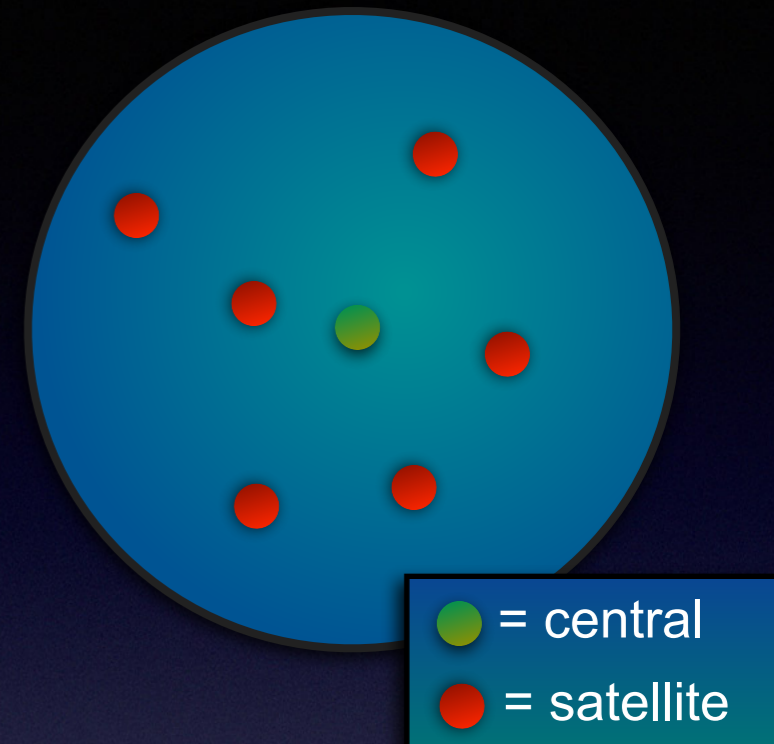
Here $\langle N \rangle_M$ describes average number of galaxies (with certain properties) that reside in a halo of mass M , \bar{n}_g is the average number density of those galaxies, and $u_g(r|M)$ is the normalized, radial number density distribution of galaxies in haloes of mass M .

Halo Occupation Statistics

When describing halo occupation statistics, it is important to treat central and satellite galaxies separately.

Central Galaxies: those galaxies that reside at the center of their dark matter (host) halo

Satellite Galaxies: those galaxies that reside at the center of a dark matter sub-halo, and are orbiting inside a larger host halo.



Central Galaxies

$$\langle N_c \rangle_M = \sum_{N_c=0}^1 N_c P(N_c|M) = P(N_c = 1|M)$$

$$\langle N_c^2 \rangle_M = \sum_{N_c=0}^1 N_c^2 P(N_c|M) = P(N_c = 1|M) = \langle N_c \rangle_M$$

$$u_c(r|M) = \delta^D(r)$$

Satellite Galaxies

$$\langle N_s \rangle_M = \sum_{N_s=0}^{\infty} N_s P(N_s|M)$$

$$\langle N_s^2 \rangle_M = \sum_{N_s=0}^{\infty} N_s^2 P(N_s|M)$$

$$u_s(r|M) = \text{TBD}$$

Halo Occupation Statistics

Central Galaxies

$$\langle N_c \rangle_M = \sum_{N_c=0}^1 N_c P(N_c|M) = P(N_c = 1|M)$$

$$\langle N_c^2 \rangle_M = \sum_{N_c=0}^1 N_c^2 P(N_c|M) = P(N_c = 1|M) = \langle N_c \rangle_M$$

$$u_c(r|M) = \delta^D(r)$$

Satellite Galaxies

$$\langle N_s \rangle_M = \sum_{N_s=0}^{\infty} N_s P(N_s|M)$$

$$\langle N_s^2 \rangle_M = \sum_{N_s=0}^{\infty} N_s^2 P(N_s|M)$$

$$u_s(r|M) = \text{TBD}$$

Calculating galaxy-galaxy correlation functions requires following halo occupation statistic ingredients:

Halo occupation distribution for centrals	$P(N_c M)$
Halo occupation distribution for satellites	$P(N_s M)$
Radial number density profile of satellites	$u_s(r M)$

In principle, as we will see, one also requires the probability function $P(N_c, N_s|M)$, but it is common practice to assume that the occupation statistics of **centrals** and **satellites** are independent, i.e., that $P(N_c, N_s|M) = P(N_c|M) \times P(N_s|M)$

Halo Occupation Statistics

Consider a **luminosity threshold sample**; all galaxies brighter than some threshold luminosity. The halo occupation statistics for such a sample are typically parameterized as follows:

$$\langle N_c \rangle_M = \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{\log M - \log M_{\min}}{\sigma_{\log M}} \right) \right]$$

$$\langle N_s \rangle_M = \begin{cases} \left(\frac{M}{M_1} \right)^\alpha & \text{if } M > M_{\text{cut}} \\ 0 & \text{if } M < M_{\text{cut}} \end{cases}$$

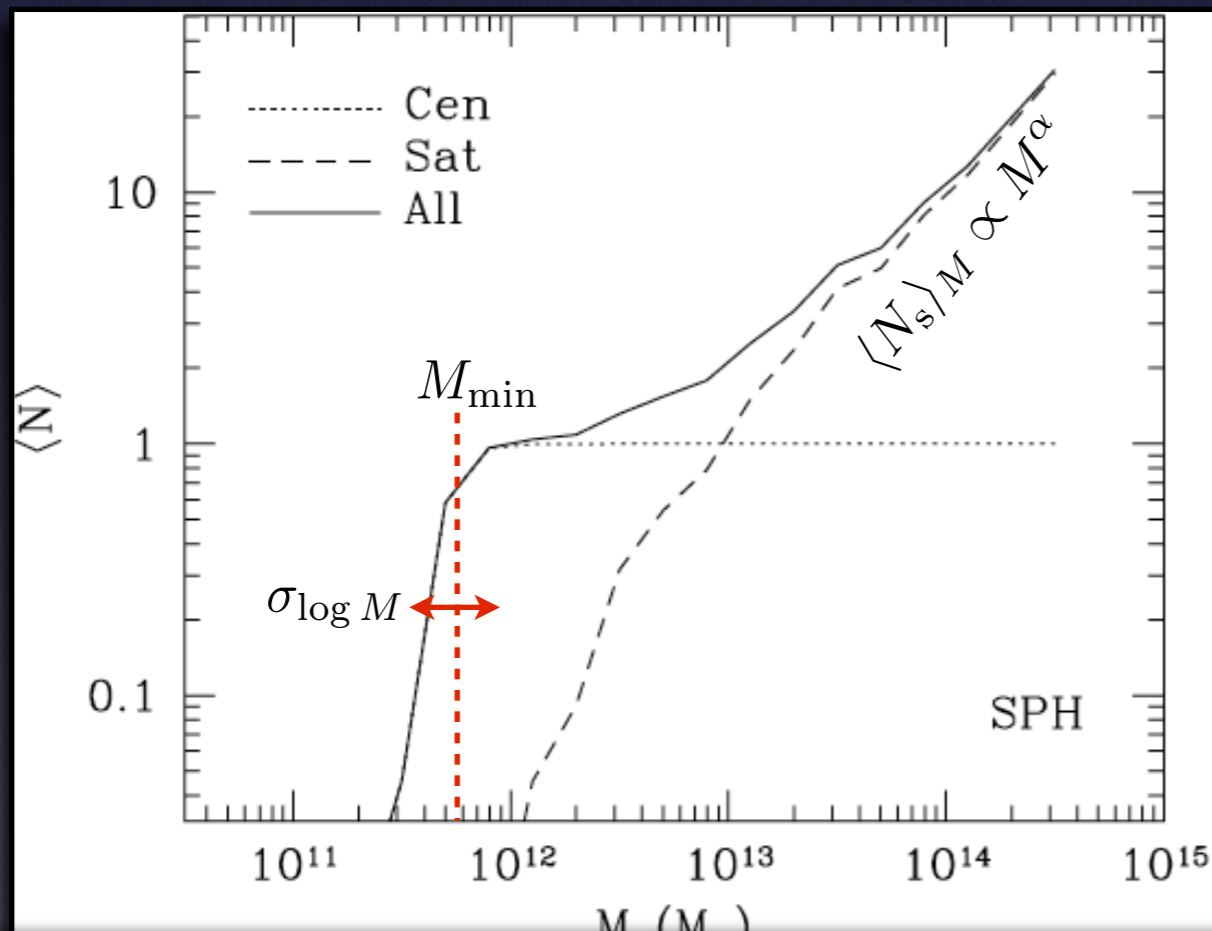
M_{\min} = characteristic minimum mass of haloes that host centrals above luminosity threshold

$\sigma_{\log M}$ = characteristic transition width due to scatter in L-M relation of centrals

M_{cut} = cut-off mass below which you have zero satellites above luminosity threshold

M_1 = normalization of satellite occupation numbers

α = slope of satellite occupation numbers



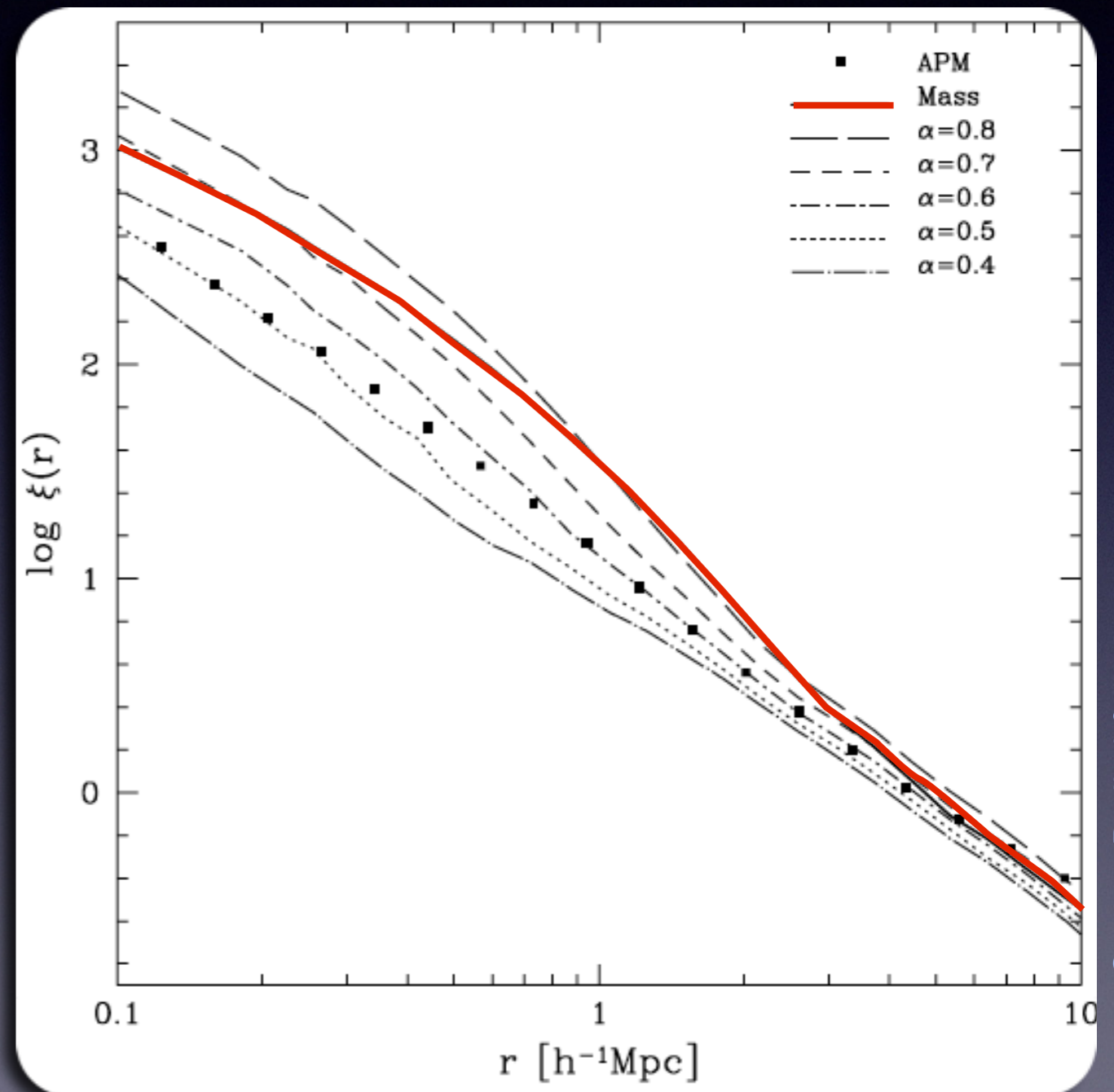
This particular **HOD** model, which is fairly popular in the literature, requires 5 parameters (M_{\min} , M_1 , M_{cut} , $\sigma_{\log M}$, α) to characterize the occupation statistics of a given **luminosity threshold sample**, and is (partially) motivated by the occupation statistics in **hydro simulations** of galaxy formation...

Halo Occupation Statistics

Increasing the slope $\alpha = d \log \langle N_s \rangle / d \log M$ boosts the 1-halo term of the correlation function. It also boosts the 2-halo term, but to a lesser extent.

The latter arises because a larger value of α implies that **satellites**, on average, reside in more massive haloes, which are more strongly biased.

The 1-halo term scales with satellite occupation numbers as $\langle N_s \rangle_M^2$ while the 2-halo term scales as $\langle N_s \rangle_M$. This means that the relative clustering strengths in the 1-halo and 2-halo regimes constrains the **satellite fractions**.



Source: Berlind & Weinberg, 2002, ApJ, 575, 587

Halo Occupation Statistics

An alternative parameterization, which has the advantage that it describes the occupation statistics for **any** luminosity sample (not only threshold samples), is the **conditional luminosity function**.

$$\Phi(L|M) = \Phi_c(L|M) + \Phi_s(L|M)$$

The **CLF** describes the average number of galaxies of luminosity **L** that reside in a dark matter halo of mass **M**.

$$\Phi(L) = \int_0^\infty \Phi(L|M) n(M) dM$$

CLF is the direct link between the halo mass function and the galaxy luminosity function.

$$\langle L \rangle_M = \int_0^\infty \Phi(L|M) L dL$$

CLF describes link between luminosity and mass

$$\langle N_x \rangle_M = \int_{L_1}^{L_2} \Phi_x(L|M) dL$$

CLF describes first moments of halo occupation statistics of **any** luminosity sample



The Conditional Luminosity Function

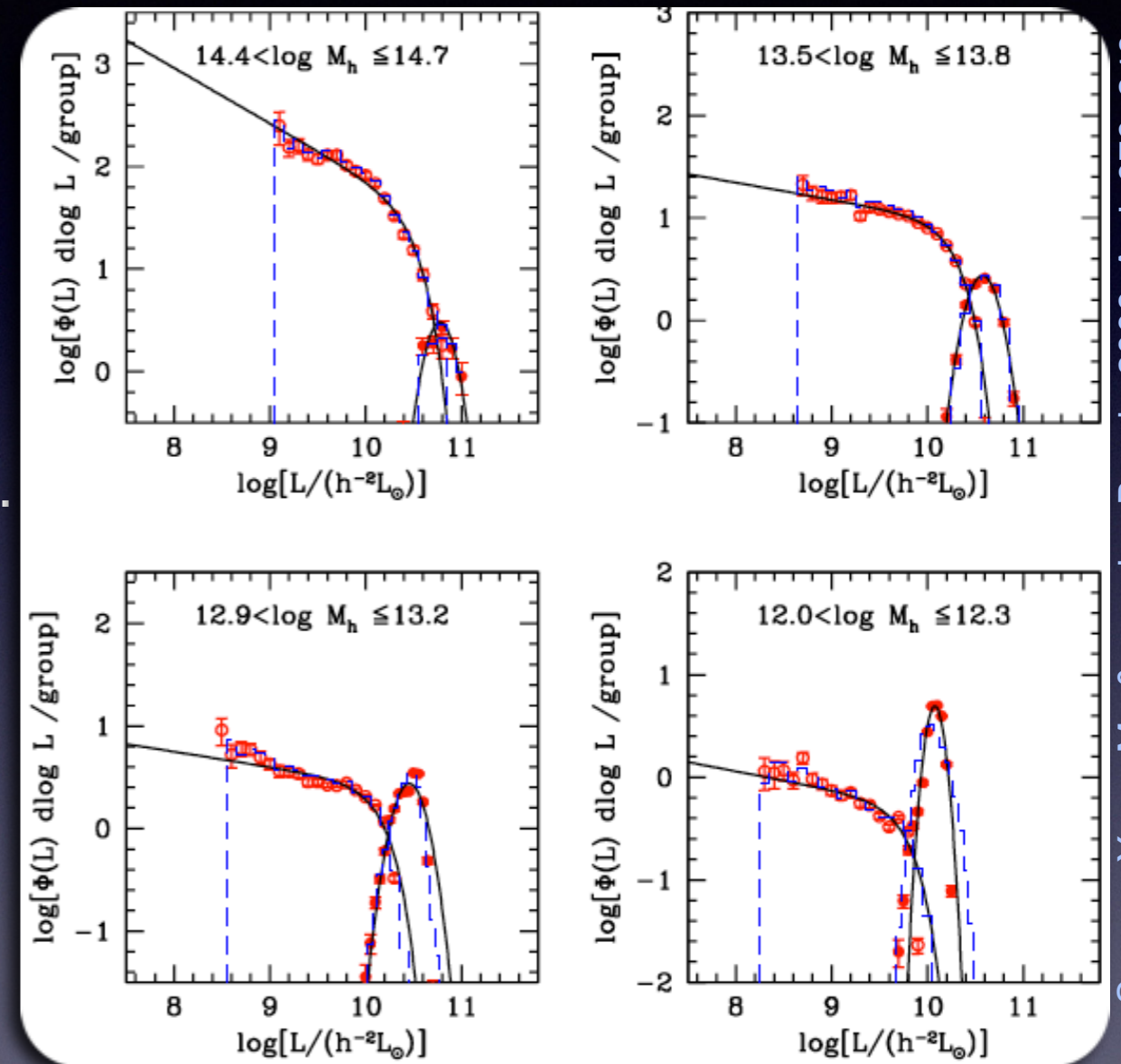
The **CLF** can be obtained from galaxy group catalogues. Yang, Mo & van den Bosch (2008) have shown that the **CLF** is well parameterized using the following functional form:

$$\Phi_c(L|M)dL = \frac{1}{\sqrt{2\pi}\sigma_c} \exp \left[- \left(\frac{\ln(L/L_c)}{\sqrt{2}\sigma_c} \right)^2 \right] \frac{dL}{L}$$

$$\Phi_s(L|M)dL = \frac{\phi_s}{L_s} \left(\frac{L}{L_s} \right)^{\alpha_s} \exp \left[- (L/L_s)^2 \right] dL$$

Note: $\{L_c, L_s, \sigma_c, \phi_s, \alpha_s\}$ all depend on halo mass. These dependencies are typically parameterized using ~ 10 free parameters.

Free parameters are constrained by the data, which can be galaxy group catalogs, galaxy clustering, galaxy-galaxy lensing, satellite kinematics, etc...



The CLFs inferred from a SDSS galaxy group catalog. Symbols are data, while the solid, black line is best-fit using the CLF parameterization indicated above...

Source: Yang, Mo & van den Bosch, 2008, ApJ, 676. 248

Halo Occupation Statistics

The **1-halo term** of the galaxy-galaxy correlation function requires the second moment

$$\begin{aligned}\langle N(N-1) \rangle_M &= \langle N_c^2 \rangle_M + 2\langle N_c N_s \rangle_M + \langle N_s^2 \rangle_M - \langle N_c \rangle_M - \langle N_s \rangle_M \\ &= \langle N_s(N_s-1) \rangle_M + 2\langle N_c \rangle_M \langle N_s \rangle_M\end{aligned}$$

where we assumed that occupation statistics of centrals and satellites are independent

Thus, we need to specify the second moment of the satellite occupation distribution:

$$\langle N_s(N_s-1) \rangle_M = \sum_{N_s=0}^{\infty} N_s(N_s-1) P(N_s|M) \equiv \beta(M) \langle N_s \rangle^2$$

where we have introduced the function $\beta(M)$

If the occupation statistics of satellite galaxies follow **Poisson statistics**, i.e.,

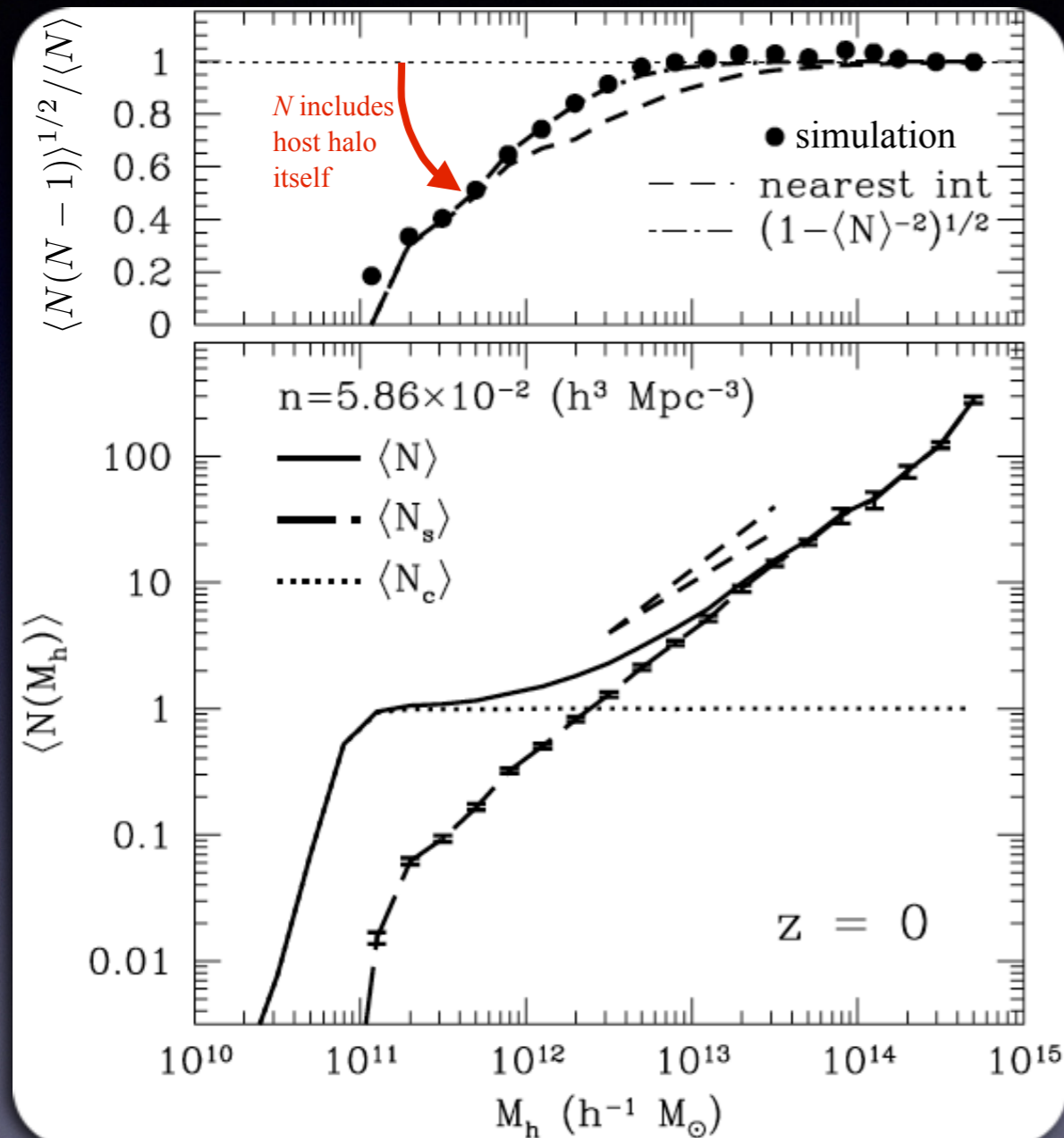
$$P(N_s|M) = \frac{\lambda^{N_s} e^{-\lambda}}{N_s!} \quad \text{with} \quad \lambda = \langle N_s \rangle_M$$

then $\beta(M) = 1$. Distributions with $\beta > 1$ ($\beta < 1$) are broader (narrower) than **Poisson**.

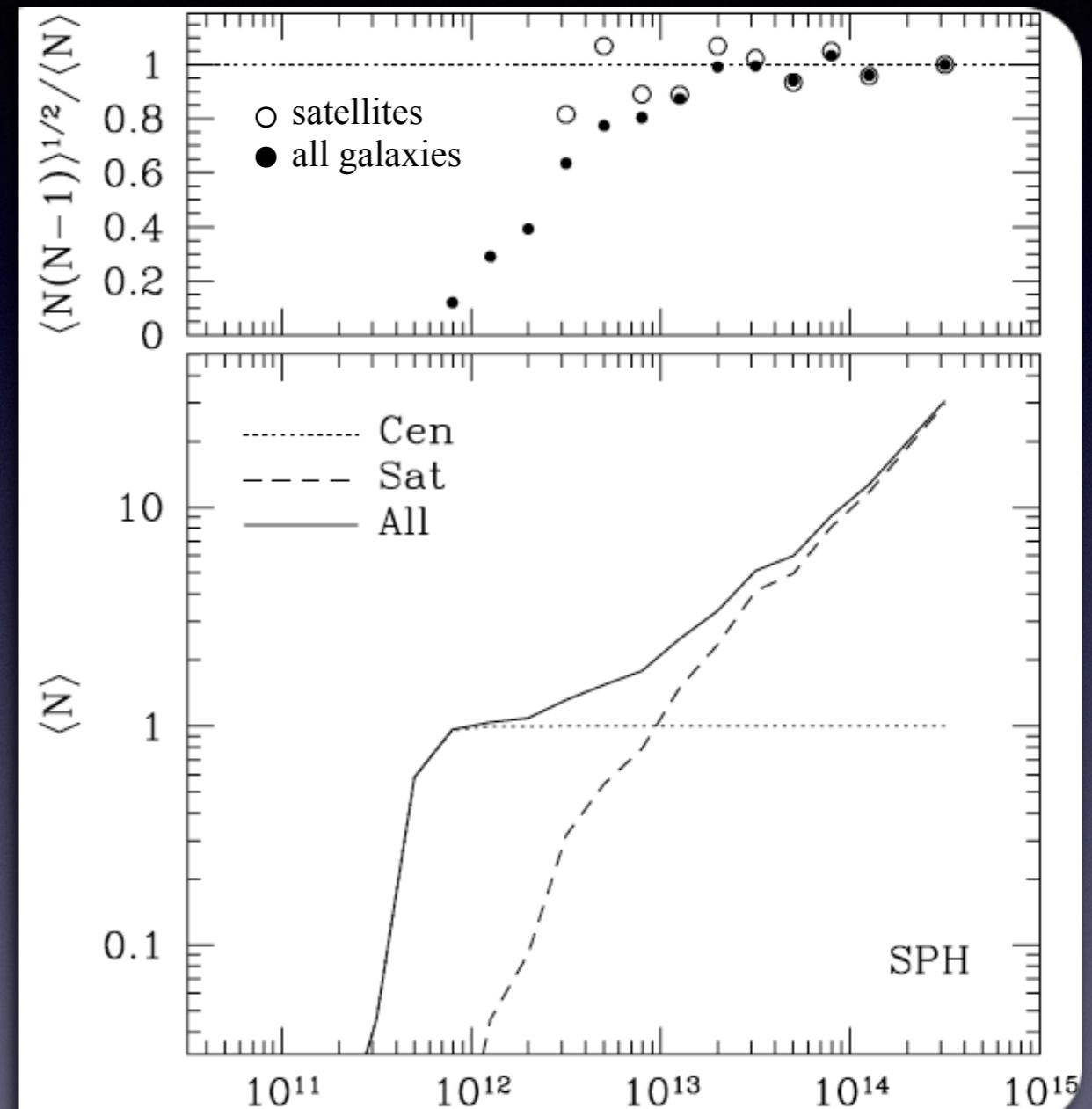
The second moment of the halo occupation statistics is completely described by $\beta(M)$

Halo Occupation Statistics

It is common practice to assume that satellites obey Poisson statistics. This is motivated by finding that dark matter subhaloes have occupation statistics that are (close to) Poissonian

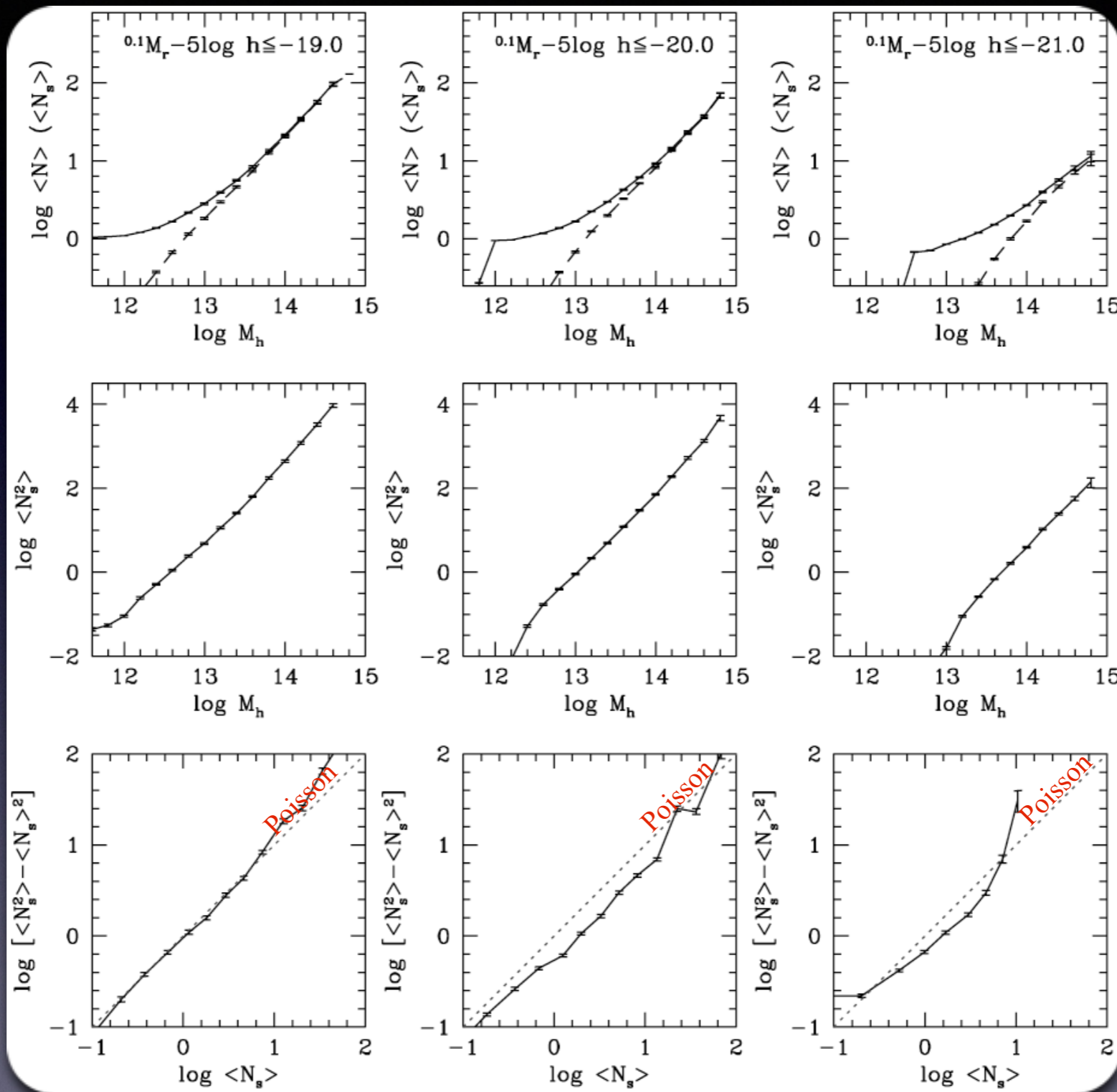


Occupation statistics of dark matter subhaloes in numerical (dark-matter-only) simulations. Subhaloes follow Poisson statistics...



Occupation statistics of simulated galaxies in hydrodynamical SPH simulations. Satellite galaxies follow Poisson statistics...

Halo Occupation Statistics

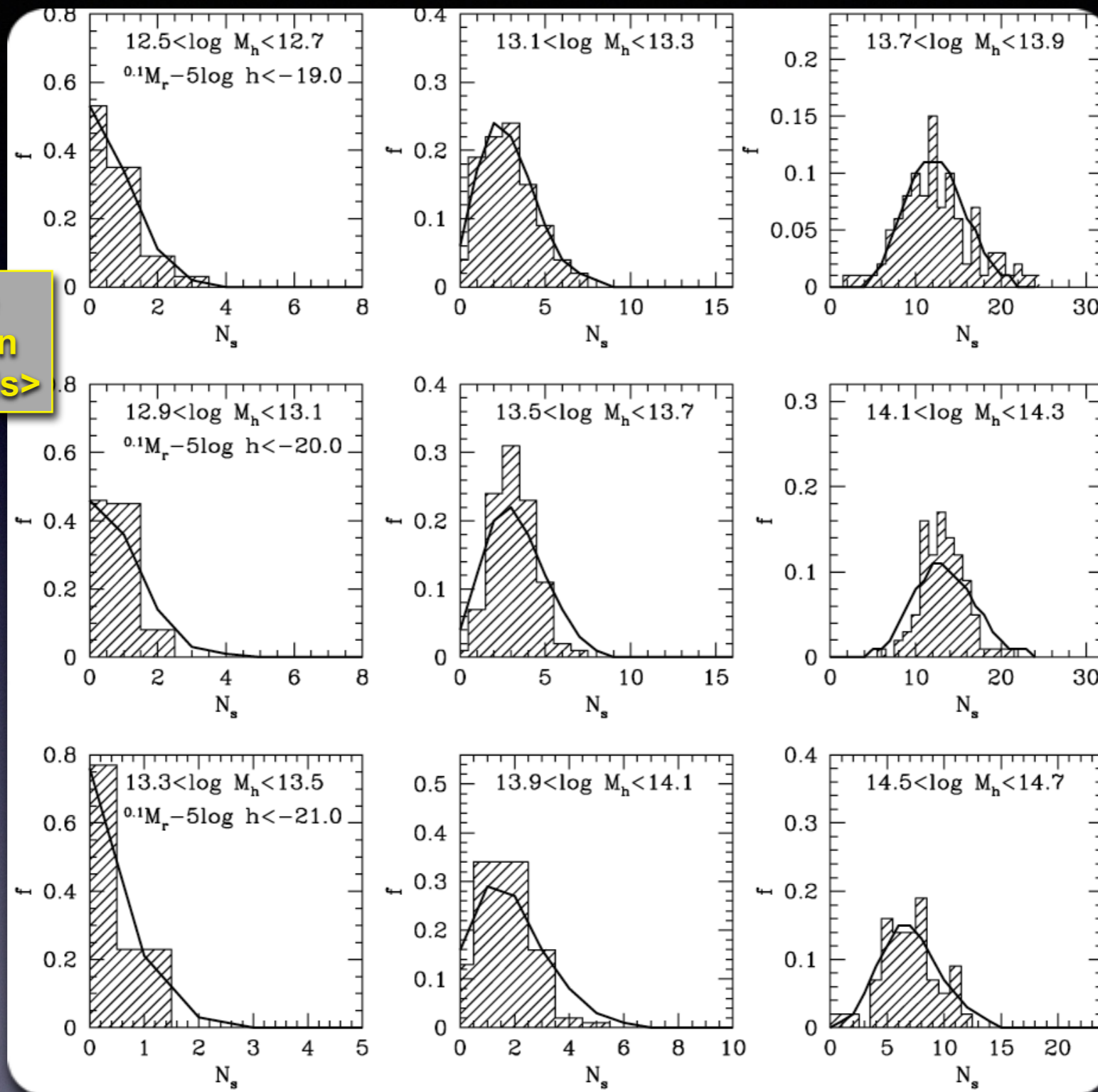


Source: Yang, Mo & van den Bosch, 2008, ApJ, 676. 248

Even real data shows that the occupation statistics of satellites are (close to) **Poissonian**.

Halo Occupation Statistics

Solid lines are the Poisson distribution corresponding to $\langle N_s \rangle$



Source: Yang, Mo & van den Bosch, 2008, ApJ, 676, 248

Even real data shows that the occupation statistics of satellites are (close to) **Poissonian**.

Radial Number Density Profile of Satellites

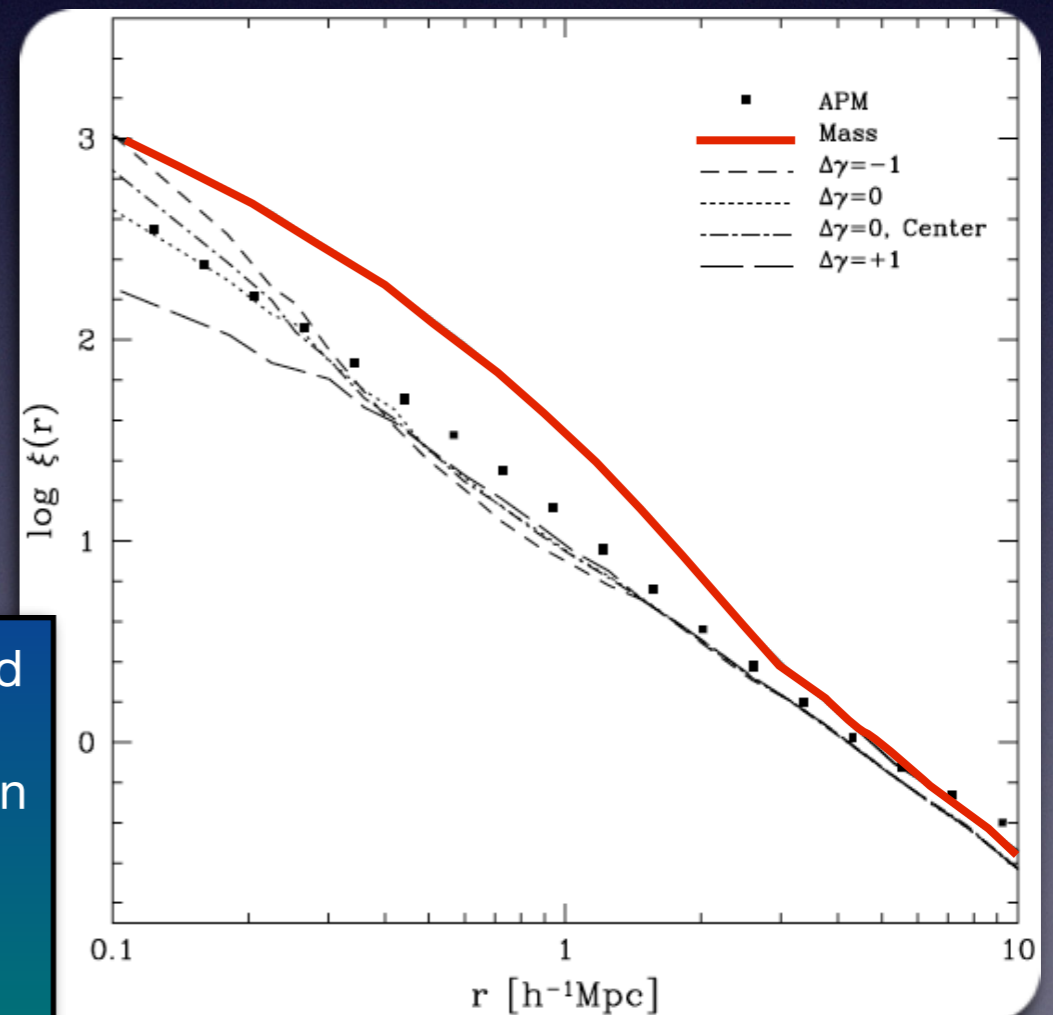
The radial number density profile of satellite galaxies is typically modelled as a 'generalized NFW profile':

$$u_s(r|M) \propto \left(\frac{r}{\mathcal{R} r_s} \right)^{-\gamma} \left[1 + \frac{r}{\mathcal{R} r_s} \right]^{\gamma-3}$$

Here γ is a parameter that controls the central cusp slope, and $\mathcal{R} = c_{\text{sat}}/c_{\text{dm}}$ sets the ratio between the concentration parameter of the satellites and that of the dark matter. For $\gamma = \mathcal{R} = 1$ satellites are an unbiased tracer of the mass distribution (within individual haloes)

The radial number density profile of satellites controls the clustering on small scales (only has significant effect on 1-halo term).

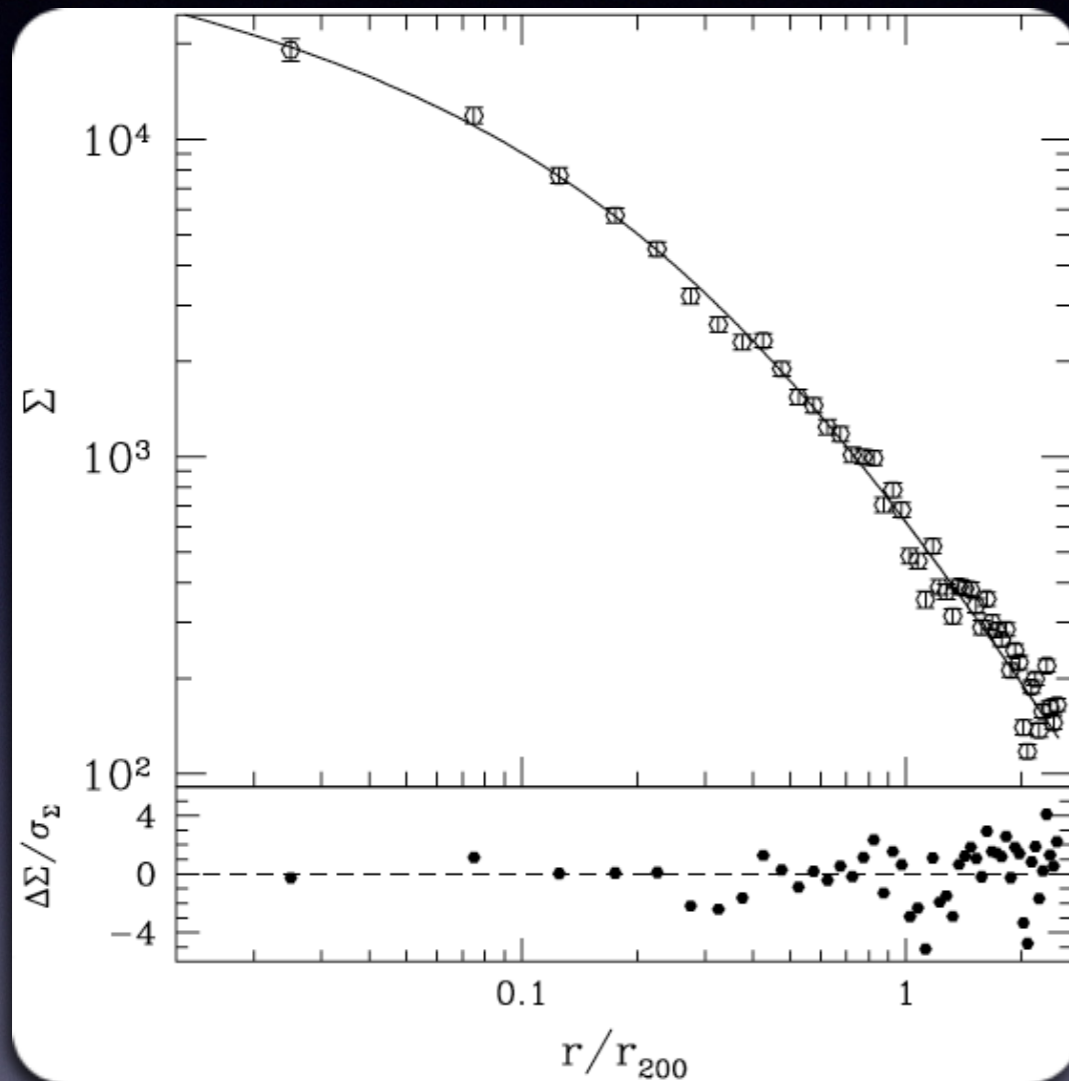
The two-point correlation function of galaxies, calculated using the halo model. Solid dots are data from the APM catalogue. The solid line is the model's matter correlation function, and the other lines are galaxy correlation functions in which the number density profile of satellite galaxies is varied.



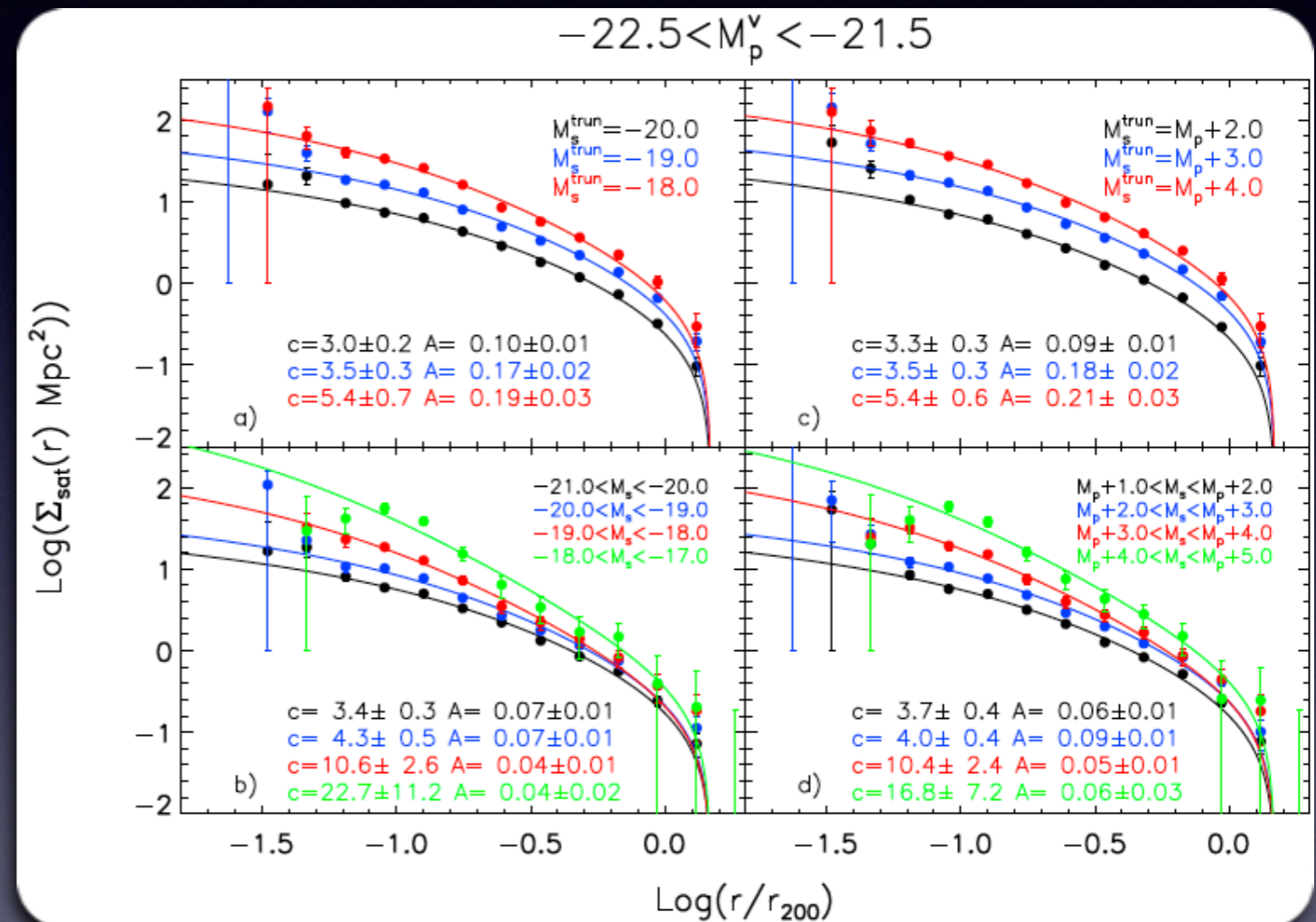
Source: Berlind & Weinberg, 2002, ApJ, 575, 587

Radial Number Density Profile of Satellites

The radial number density profile of satellite galaxies can be constrained using the **clustering** data itself, or by directly measuring the (projected) profiles of satellite galaxies in groups/clusters, or around isolated centrals...



The surface density profile of satellite galaxies in clusters. Solid line is the best-fit **NFW** profile.

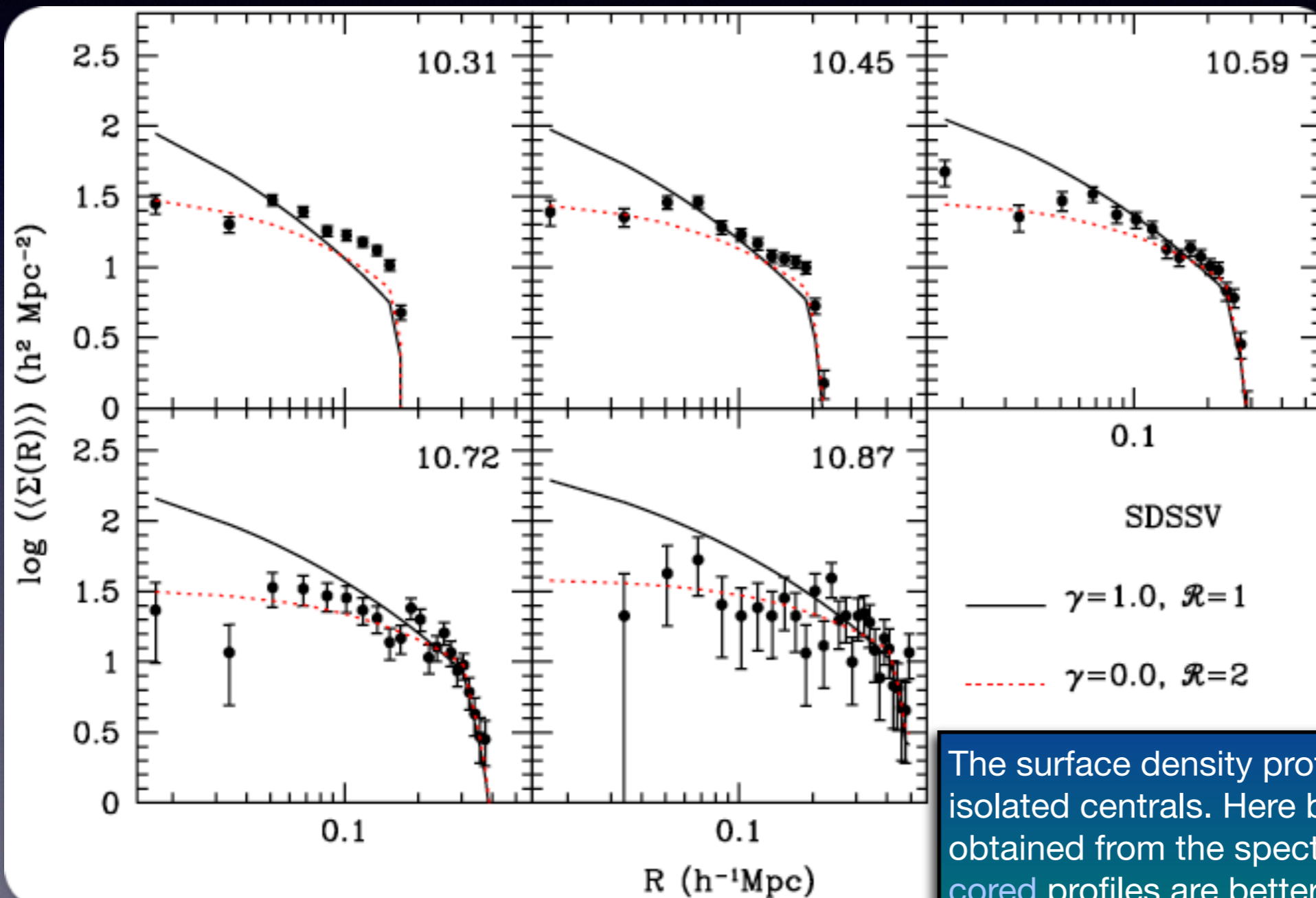


The surface density profiles of satellite galaxies around isolated centrals in the SDSS. Satellites are identified in photometric catalogue using statistical background subtraction. Lines are best-fit **NFW** profiles.

Radial Number Density Profile of Satellites

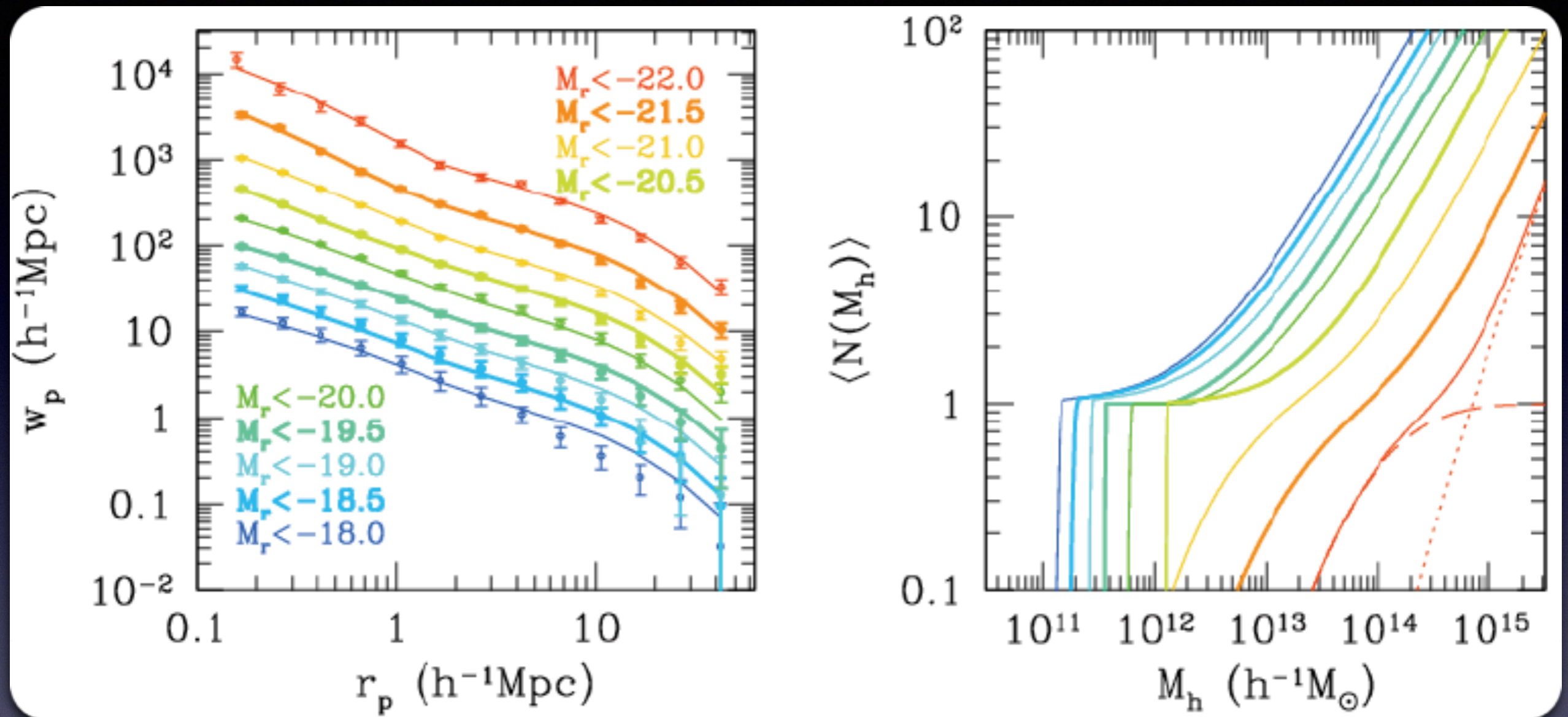
Although several studies have suggested that satellite galaxies follow a radial number density profile that is well fitted by **NFW** profile, others find that $u_s(r|M)$ has a core and is less centrally concentrated than the dark matter.

This is consistent with distribution of **subhaloes** in dark-matter-only simulations....



The surface density profile of satellite galaxies around isolated centrals. Here both centrals & satellites are obtained from the spectroscopic **SDSS**. Note that cored profiles are better fit than **NFW** profile.

Constraints on Halo Occupation Statistics

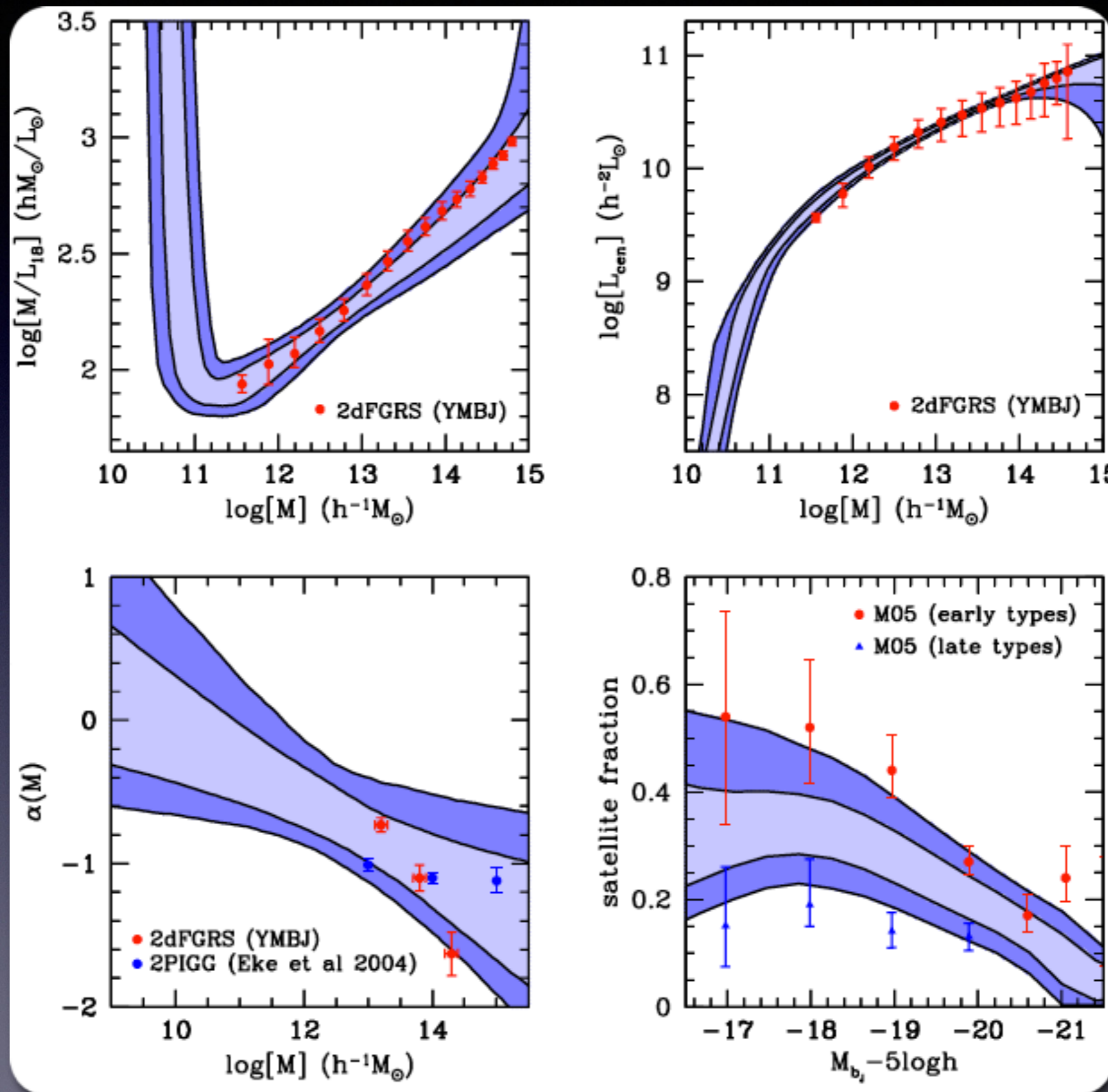


Source: Zehavi et al. 2011, ApJ, 736, 59

Zehavi et al. 2011 used halo occupation models to fit the projected correlation functions obtained from the SDSS for 9 different luminosity threshold samples.

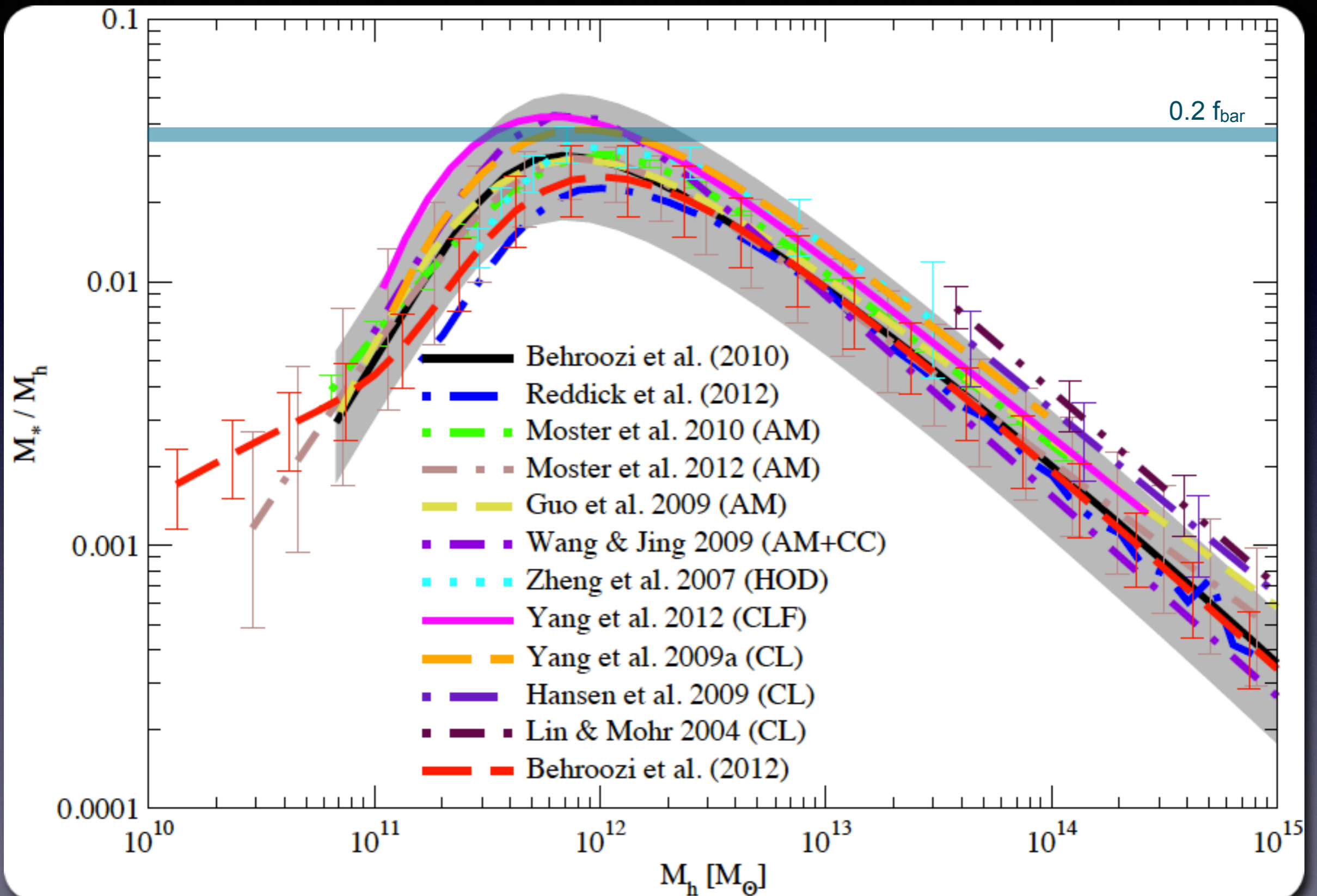
- The left-hand panel shows data+fits (offset vertically for clarity).
- The right-hand panel shows first moments of best-fit halo occupation distributions.

The Galaxy Halo Connection



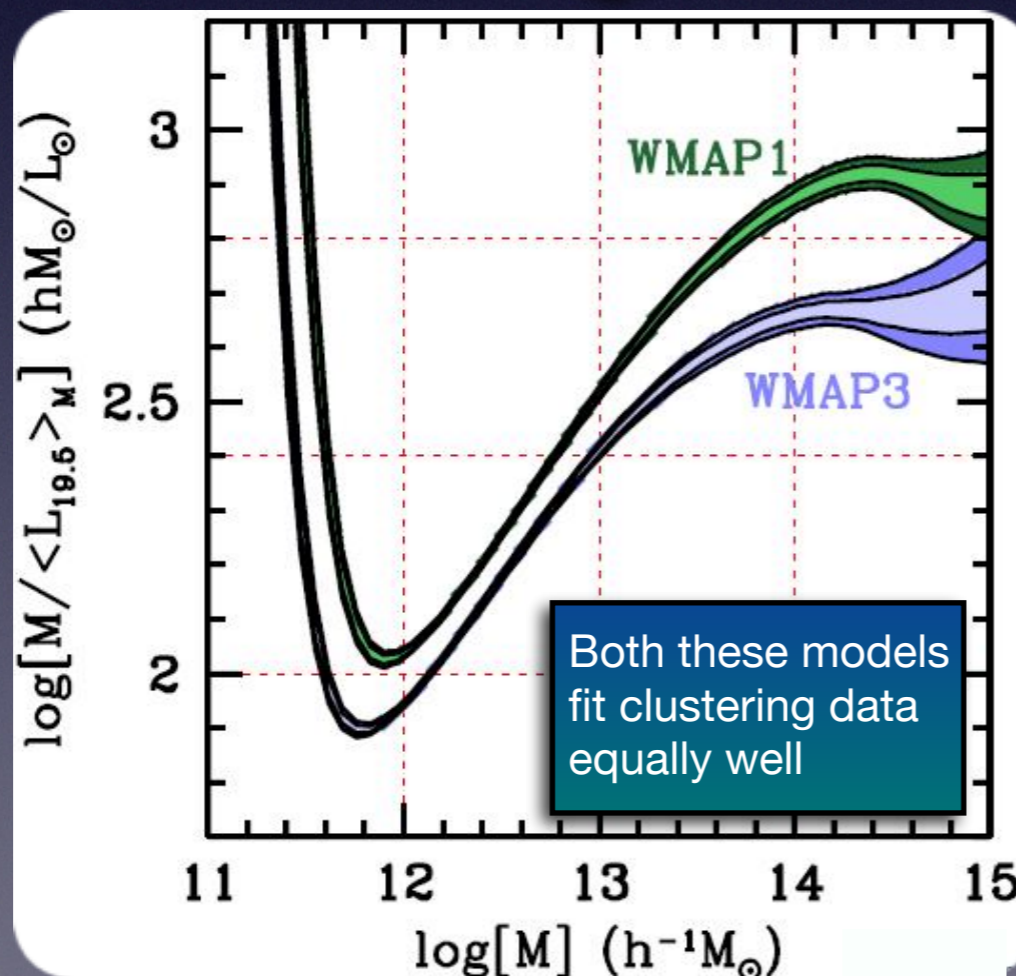
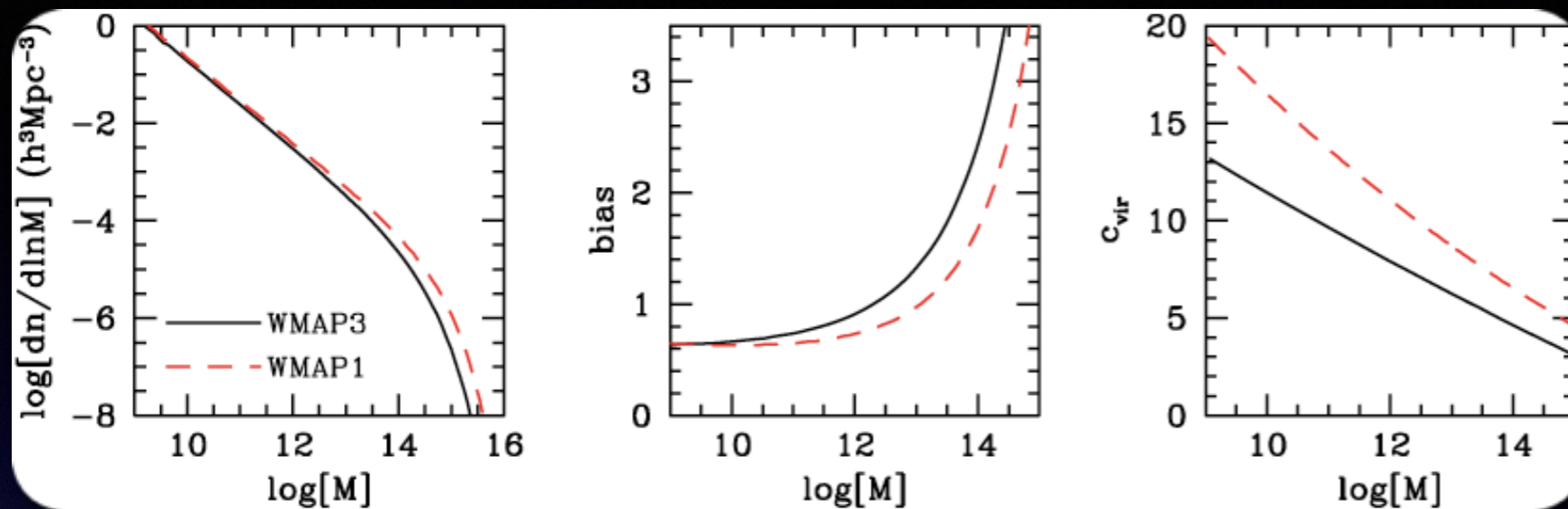
Source: van den Bosch et al. 2007, MNRAS, 376, 841

The Galaxy Halo Connection

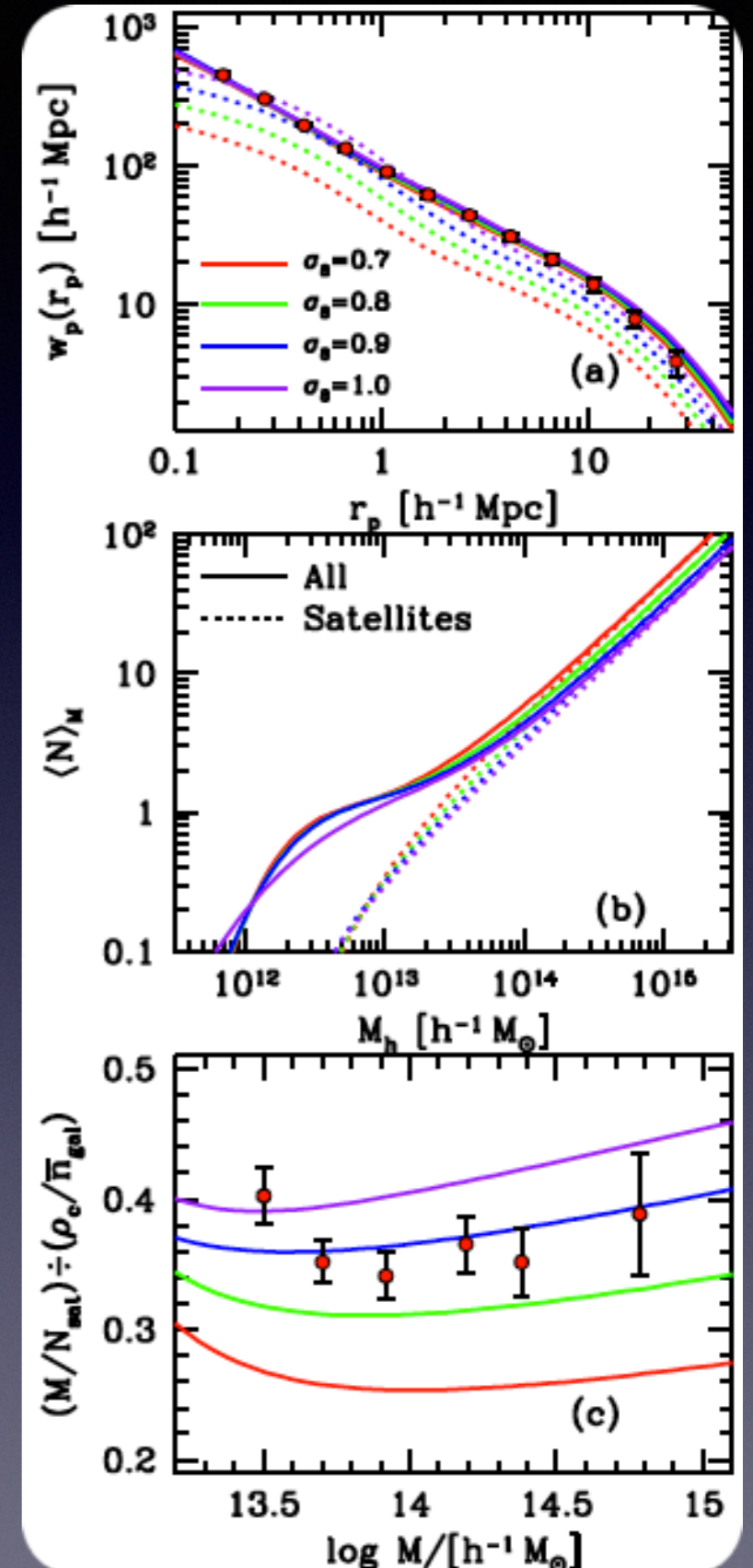


Source: Behroozi, Wechsler & Conroy, 2012, arXiv:1207.6105

Cosmology Dependence



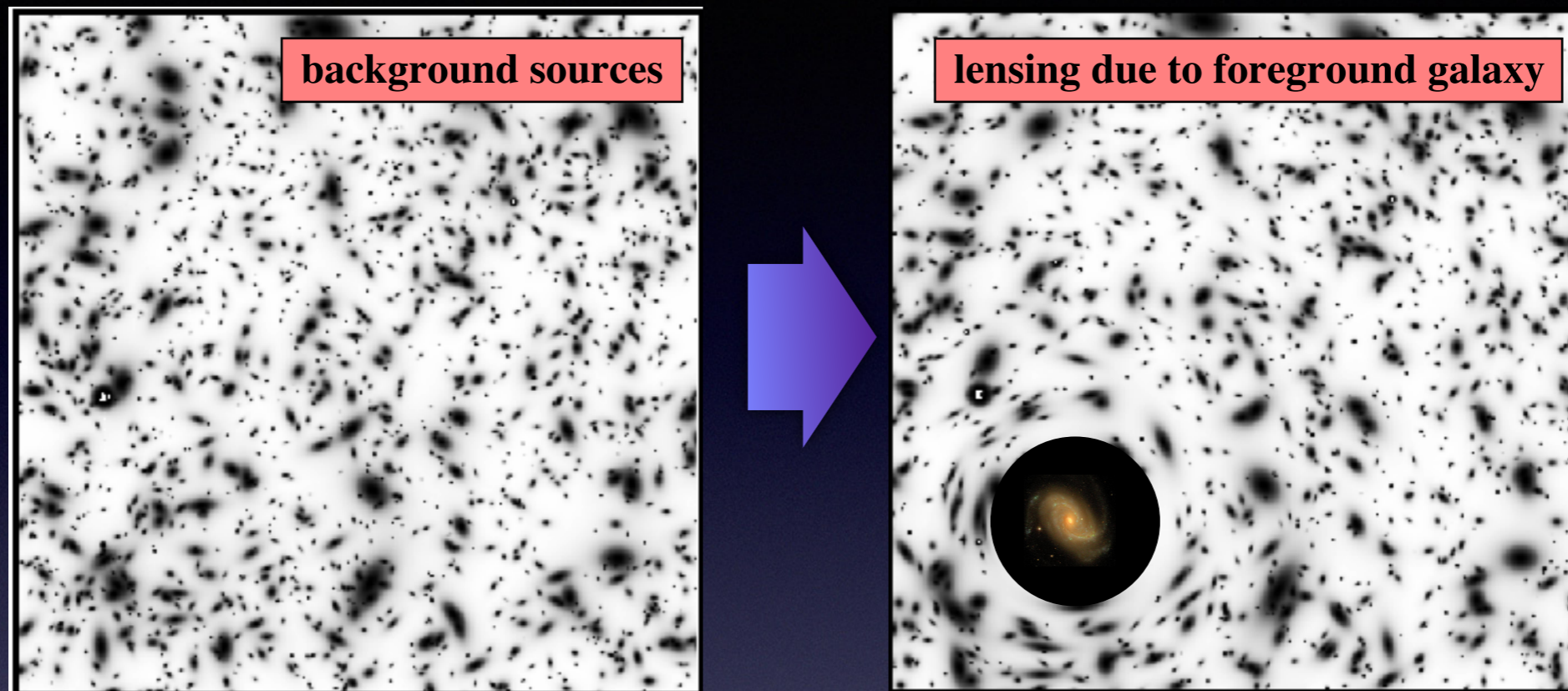
Source: Cacciato et al., 2009, MNRAS, 394, 929



Source: Tinker et al., 2012, ApJ, 745, 16

Galaxy-Galaxy Lensing

The mass associated with galaxies lenses background galaxies



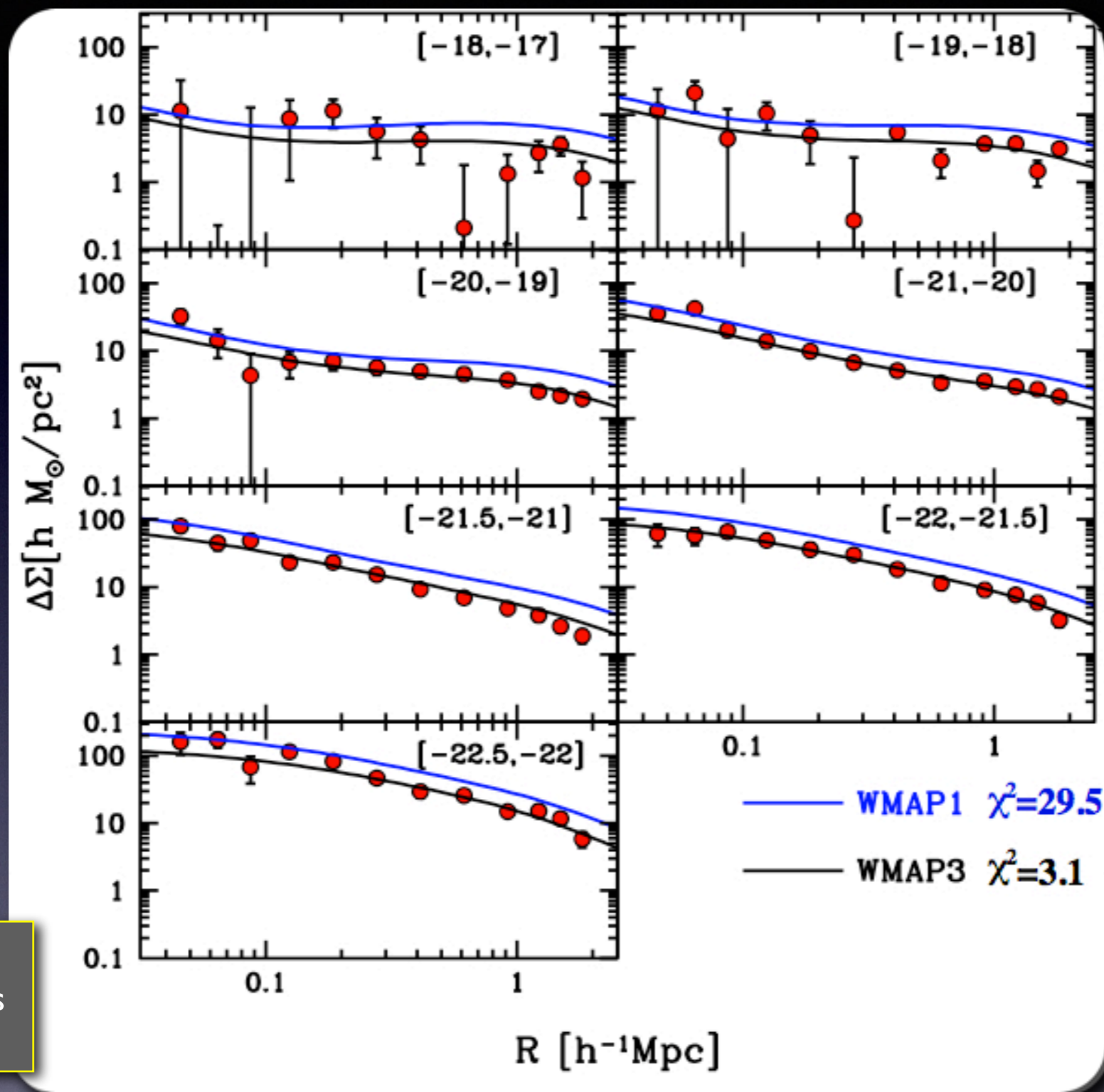
Lensing causes correlated ellipticities, the tangential shear, γ_t which is related to the excess surface density, $\Delta\Sigma$, according to

$$\gamma_t(R) \Sigma_{\text{crit}} = \Delta\Sigma(R) = \bar{\Sigma}(< R) - \Sigma(R)$$

$\Delta\Sigma$ is line-of-sight projection of galaxy-matter cross correlation

$$\Sigma(R) = \bar{\rho} \int_0^{D_s} [1 + \xi_{g, \text{dm}}(r)] d\chi$$

Galaxy-Galaxy Lensing

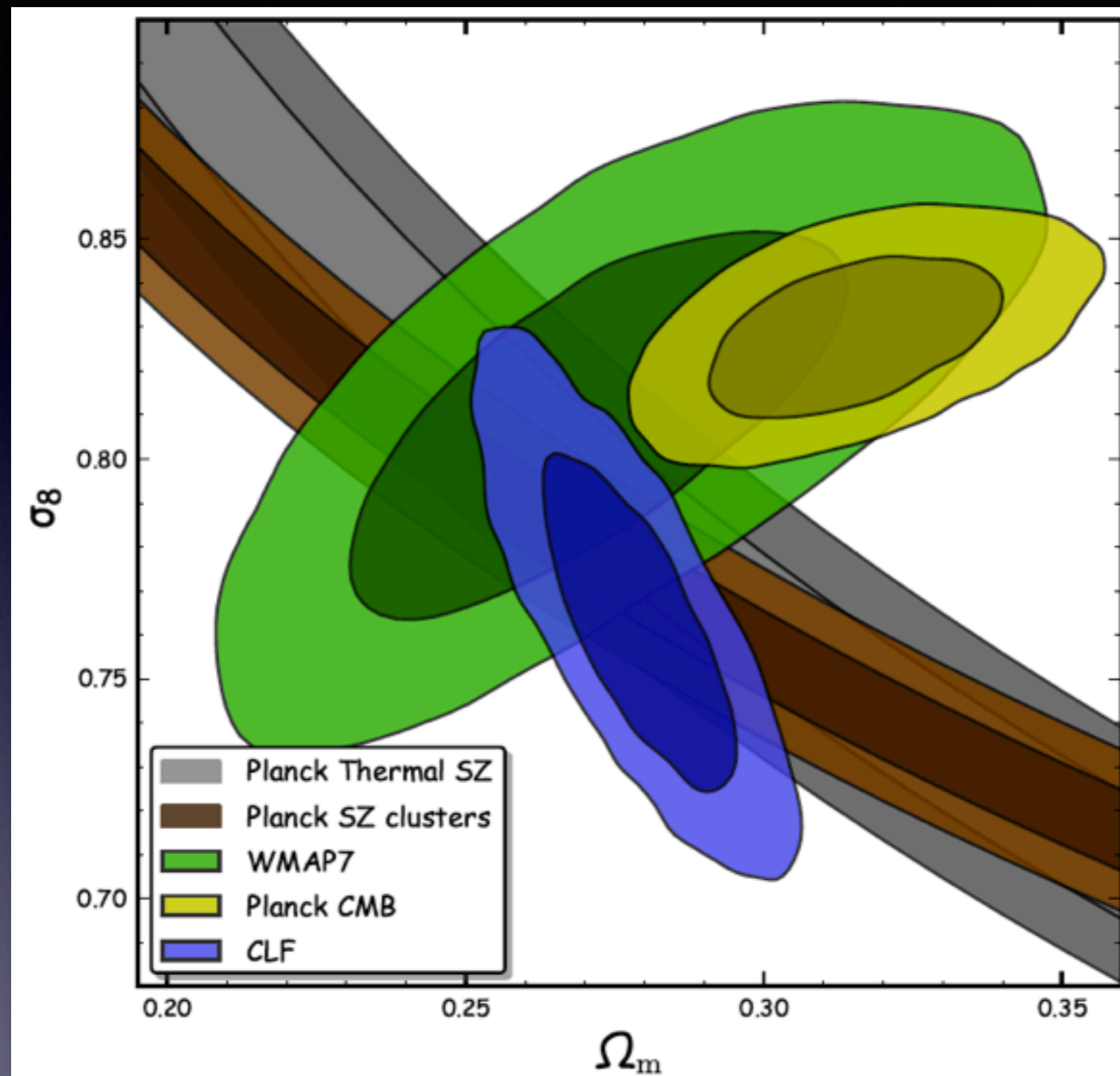


Data:
SDSS measurements
by Mandelbaum+06

Cacciato, vdB et al. (2009)

Combination of clustering & lensing can constrain cosmology!!!

Cosmological Constraints



Cacciato, vdB et al. (2013)

New physics beyond the “vanilla” Λ CDM cosmology or systematic errors?

Lecture 13

SUMMARY

Summary: key words & important facts

Key words

Halo model	1-halo & 2-halo terms
halo exclusion	Halo Occupation Distribution (HOD)
galaxy-galaxy lensing	Conditional Luminosity Function (CLF)

- The **Halo model** is an analytical model that describes dark matter density distribution in terms of its **halo building blocks**, under **ansatz** that all dark matter is partitioned over haloes.
- In combination with a **halo occupation model** (HOD or CLF), the **halo model** can be used to compute galaxy-galaxy correlation function and galaxy-matter cross-correlation function. The latter is related to the **excess surface density** measured with galaxy-galaxy lensing.
- **HOD** is mainly used to model clustering of luminosity threshold samples. **CLF** can be used to model clustering of galaxies of any luminosity (bin).
- It is common to assume that **satellite galaxies** obey **Poisson statistics**, such that $\langle N_s(N_s-1)|M \rangle = \langle N_s \rangle^2$, and only the first moment of $P(N_s|M)$ is required. This is not exact and may cause significant errors in the predicted clustering.

Summary: key equations & expressions

halo model

$$P(k) = P^{1h}(k) + P^{2h}(k)$$

$$P^{1h}(k) = \frac{1}{\bar{\rho}^2} \int dM M^2 n(M) |\tilde{u}(k|M)|^2$$

$$P^{2h}(k) = P^{lin}(k) \left[\frac{1}{\bar{\rho}} \int dM M b(M) n(M) \tilde{u}(k|M) \right]^2$$

Galaxy-Galaxy lensing: tangential shear, excess surface density and galaxy-matter cross correlation

$$\gamma_t(R) \Sigma_{crit} = \Delta \Sigma(R) = \bar{\Sigma}(< R) - \Sigma(R)$$

$$\Sigma(R) = \bar{\rho} \int_0^{D_s} [1 + \xi_{g, dm}(r)] d\chi$$

CLF: the link between light and mass

$$\Phi(L) = \int_0^\infty \Phi(L|M) n(M) dM$$

$$\langle L \rangle_M = \int_0^\infty \Phi(L|M) L dL$$

$$\langle N_x \rangle_M = \int_{L_1}^{L_2} \Phi_x(L|M) dL$$

Characteristic examples of CLF and HOD for both centrals and satellites

$$\Phi_c(L|M) dL = \frac{1}{\sqrt{2\pi}\sigma_c} \exp \left[- \left(\frac{\ln(L/L_c)}{\sqrt{2}\sigma_c} \right)^2 \right] \frac{dL}{L}$$

$$\langle N_c \rangle_M = \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{\log M - \log M_{\min}}{\sigma_{\log M}} \right) \right]$$

$$\Phi_s(L|M) dL = \frac{\phi_s}{L_s} \left(\frac{L}{L_s} \right)^{\alpha_s} \exp \left[- (L/L_s)^2 \right] dL$$

$$\langle N_s \rangle_M = \begin{cases} \left(\frac{M}{M_1} \right)^\alpha & \text{if } M > M_{\text{cut}} \\ 0 & \text{if } M < M_{\text{cut}} \end{cases}$$