ASTR 610
Theory of Galaxy Formation

Lecture 10: Merger Trees & Halo Bias

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Yale University, Spring 2024

Press-Schechter Theory

In this lecture we will use the Extended Press-Schechter formalism to construct halo merger trees and study their statistical properties. We will also use the same formalism to explore the clustering properties of dark matter haloes, which we express in terms of a mass dependent halo bias.

Topics that will be covered include:

- Progenitor Mass Function
- EPS Merger Trees
- Mass Assembly Histories
- Halo Formation Times
- Halo Bias
- Assembly Bias

Recap: the halo mass function

In the excursion set formulation of PS theory, also called extended Press-Schechter, the halo mass function derives from first-upcrossing statistics of linear density field:

$$\boxed{n(M,t)\,\mathrm{M} = \frac{\bar{\rho}}{M}\, f_{\rm FU}(S,\delta_{\rm c}) \left|\frac{\mathrm{d}S}{\mathrm{d}M}\right| \mathrm{d}M}$$

Here $f_{\rm FU}(S,\delta_{\rm c})\,\mathrm{d}S$ is the fraction of trajectories that have their first upcrossing of barrier $\delta_{\rm c}(t)$ between S and $S+\mathrm{d}S$.

In the case of spherical collapse, the barrier $\delta_{\rm c}(t) \simeq 1.686/D(t)$ is independent of mass, and the upcrossing statistics are analytical:

$$\boxed{f_{\rm FU}(\nu) = \frac{1}{\sqrt{2\pi}}\,\frac{\delta_{\rm c}}{S^{3/2}} \exp\left[-\frac{\delta_{\rm c}^2}{2S}\right] = \frac{1}{2\,S} f_{\rm PS}(\nu)}$$

where $\nu = \delta_{\rm c}(t)/\sigma(M) = \delta_{\rm c}/\sqrt{S}$ and $f_{\rm PS}(\nu) = \sqrt{\frac{2}{\pi}}\,\nu\, e^{-\nu^2/2}$ is the multiplicity function

In the case of ellipsoidal collapse, Monte Carlo simulations of first-upcrossings with a moving barrier are well fit by

$$\boxed{f_{\rm FU}(\nu) = \frac{1}{2S}\, f_{\rm EC}(\nu) \quad \text{where} \quad f_{\rm EC}(\nu) = 0.322\left[1+\frac{1}{\tilde{\nu}^{0.6}}\right] f_{\rm PS}(\tilde{\nu}) \quad \text{with} \quad \tilde{\nu} = 0.84\,\nu}$$

Beyond a Halo Mass Function

An important advantage of EPS over PS is that the excursion set formalism provides a neat way to calculate the properties of the progenitors which give rise to a given class of objects (i.e., haloes of a given mass).

For example, one can calculate the mass function at $z=5$ of those haloes (progenitors) which by $z=0$ end up in a massive halo of 10^{15} solar masses.

These progenitor mass functions, in turn, can be used to describe how dark matter haloes assemble over time (in a statistical sense); in particular, they allow the construction of halo merger trees.

These merger trees are invaluable tools in galaxy formation studies...

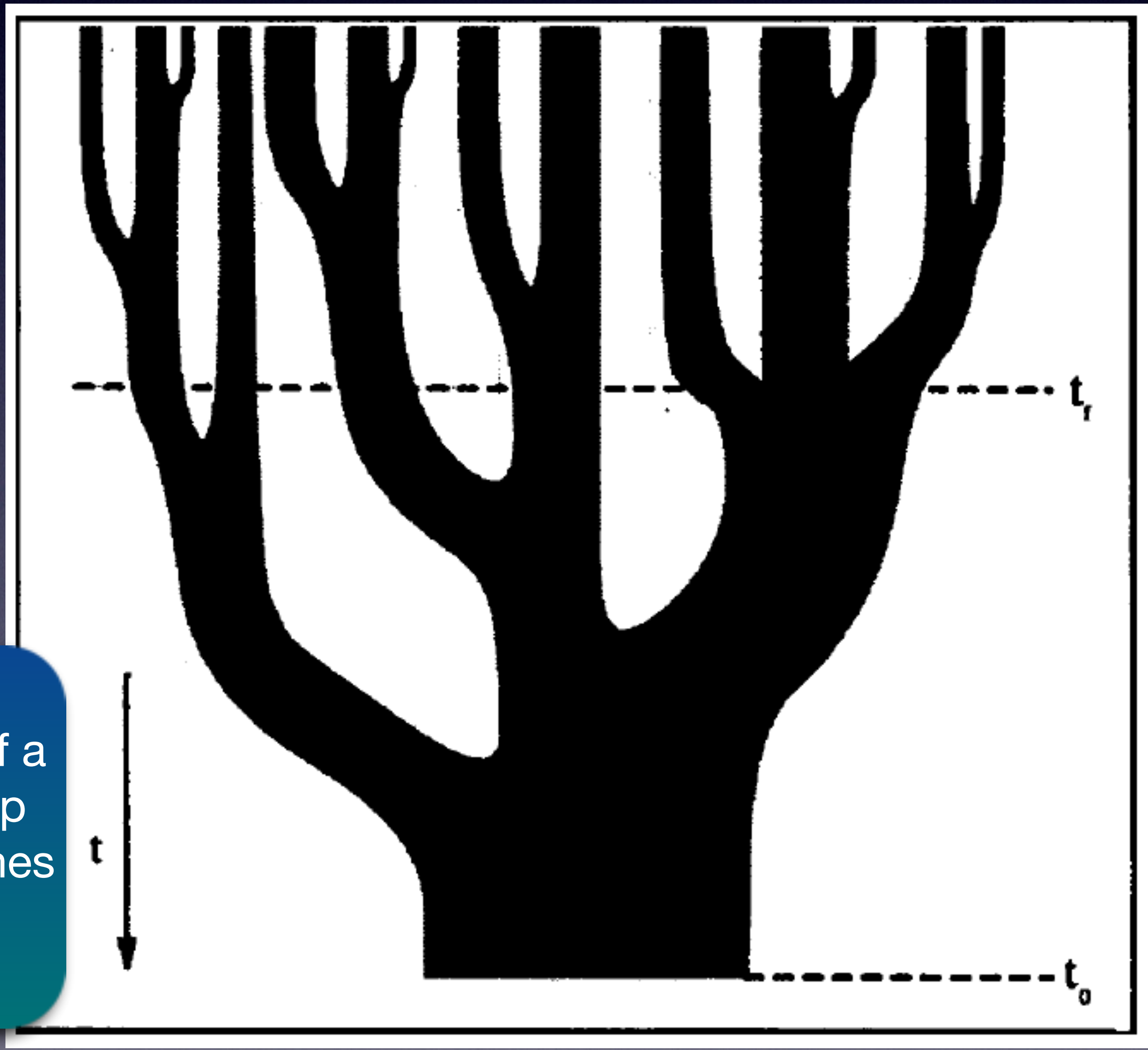


Illustration of a merger tree depicting the growth of a dark matter halo as a result of a series of mergers. Time increases from top to bottom and the width of the tree beaches represents the masses of the individual progenitors...

Source: Lacey & Cole, 1993, MNRAS, 262, 627

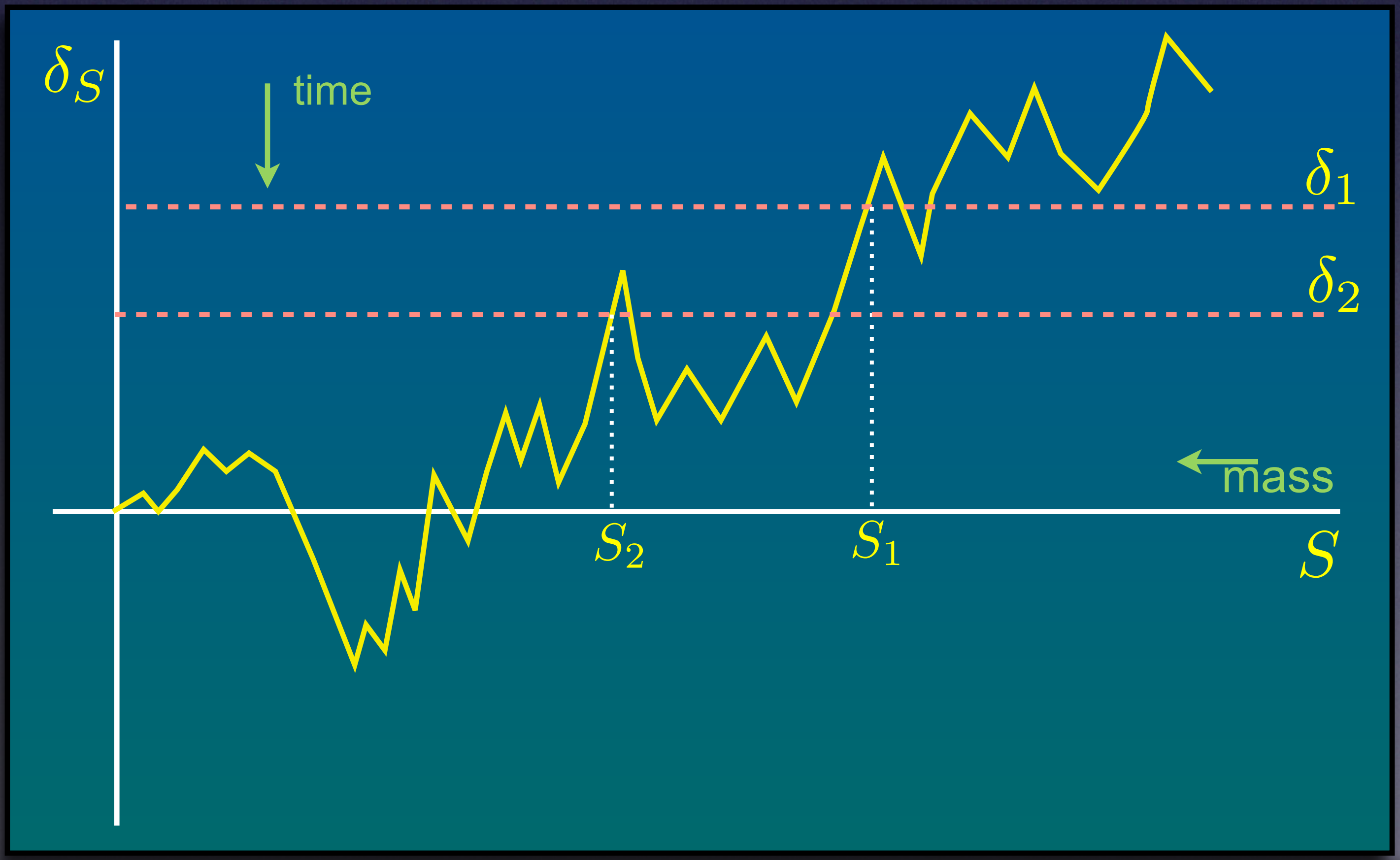
Progenitor Mass Function

Consider a spherical region (a patch) of mass M_2, corresponding to a mass variance $S_2 = \sigma^2(M_2)$ with linear overdensity $\delta_2 \equiv \delta_c(t_2) = \delta_c/D(t_2)$ so that it forms a collapsed object at time t_2.

We are interested in the fraction of M_2 that at some earlier time $t_1 < t_2$ was in a collapsed object of some mass M_1.

Within the excursion set formalism this means we want to calculate the probability that a trajectory that upcrosses barrier δ_2 at S_2 has its first upcrossing of barrier $\delta_1 = \delta_c(t_1)$ at $S_1 > S_2$ (see illustration).

This is the same problem as before, except for a translation of the origin in the (S, δ_S)-plane .



Progenitor Mass Function

$$f_{\rm FU}(S,\delta_{\rm c}) = \frac{1}{\sqrt{2\pi}}\,\frac{\delta_{\rm c}}{S^{3/2}}\,\exp\left[-\frac{\delta_{\rm c}^2}{2S}\right]$$

translation

$$f_{\rm FU}(S_1,\delta_1|S_2,\delta_2) = \frac{1}{\sqrt{2\pi}}\,\frac{\delta_1-\delta_2}{(S_1-S_2)^{3/2}}\,\exp\left[-\frac{(\delta_1-\delta_2)^2}{2\,(S_1-S_2)}\right]$$

Converting from mass- to number-weighting

$$n(M_1,t_1|M_2,t_2)\,{\rm d}M_1 = \frac{M_2}{M_1} f_{\rm FU}(S_1,\delta_1|S_2,\delta_2)\,\left|\frac{{\rm d}S_1}{{\rm d}M_1}\right|\,{\rm d}M_1$$

$n(M_1,t_1|M_2,t_2)\,{\rm d}M_1$ is the progenitor mass function; it gives the average number of progenitor haloes at time t_1 in the mass range $(M_1, M_1+{\rm d}M_1)$ that at time $t_2 > t_1$ have merged to form a halo of mass M_2 .

Merger Trees

The progenitor mass function allows one to construct halo merger trees using the following algorithm:

For a given host halo mass, M_0, and a given time step, Δt, draw a set of progenitor masses from the progenitor mass function $n(M_{\rm p}, t_0 + \Delta t | M_0, t_0)$

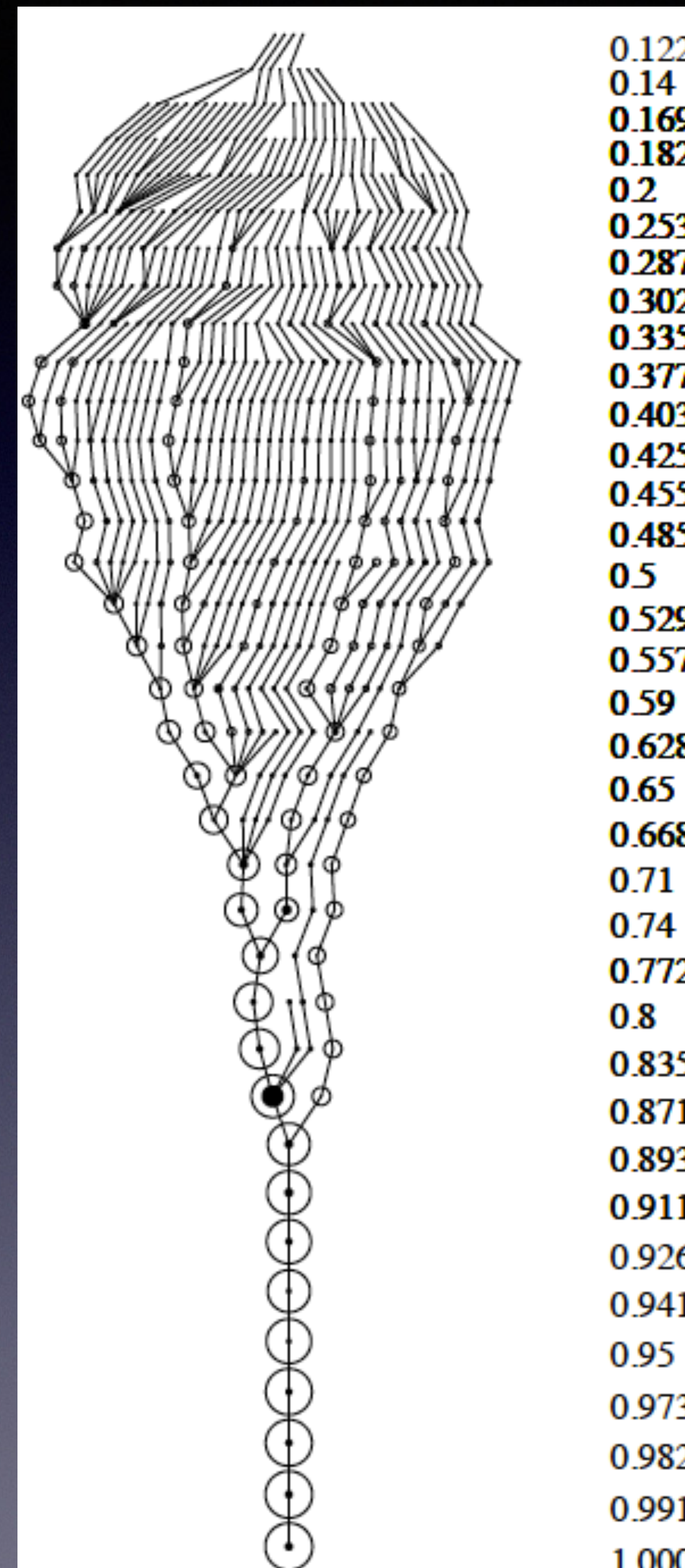
The progenitors must obey the following two conditions:

- accurately sample the progenitor mass function
- mass conservation: $\sum_i M_{{\rm p},i} = M_0$

For each progenitor, repeat above procedure, thus stepping back in time.

Sounds easy.....is not...

Several different methods have been suggested to contruct halo merger trees; none of them is perfect......



Source: Wechsler et al. 2002, ApJ, 568, 52

How to Plant a healthy merger tree...

The problem with how to construct a merger tree can be summarized as follows:

once I have drawn the first progenitor mass, $M_{\rm p,1}$, from the progenitor mass function, $n(M_{\rm p}, t_0 + \Delta t | M_0, t_0)$ mass conservation now implies a constraint on the second progenitor mass: $M_{\rm p,2} \leq M_0 - M_{\rm p,1}$. Unfortunately, there is no analytical method to include this `condition' in the progenitor mass function, i.e., it is not clear how to specify $n(M_{\rm p}, t_0 + \Delta t | M_0, t_0, M_{\rm p,1})$. Different methods for constructing halo merger trees mainly differ in how to deal with this issue...

In what follows we briefly discuss the pros and cons of two of the earlier (and least accurate) methods:

- The Binary Method (Lacey & Cole 1993)
- The N-branch Method with Accretion (Somerville & Kolatt 1999)

These two methods serve to highlight the difficulties in constructing accurate halo merger trees. We emphasize though, that numerous other methods have been discussed in the literature. The following is an incomplete list of relevant papers:

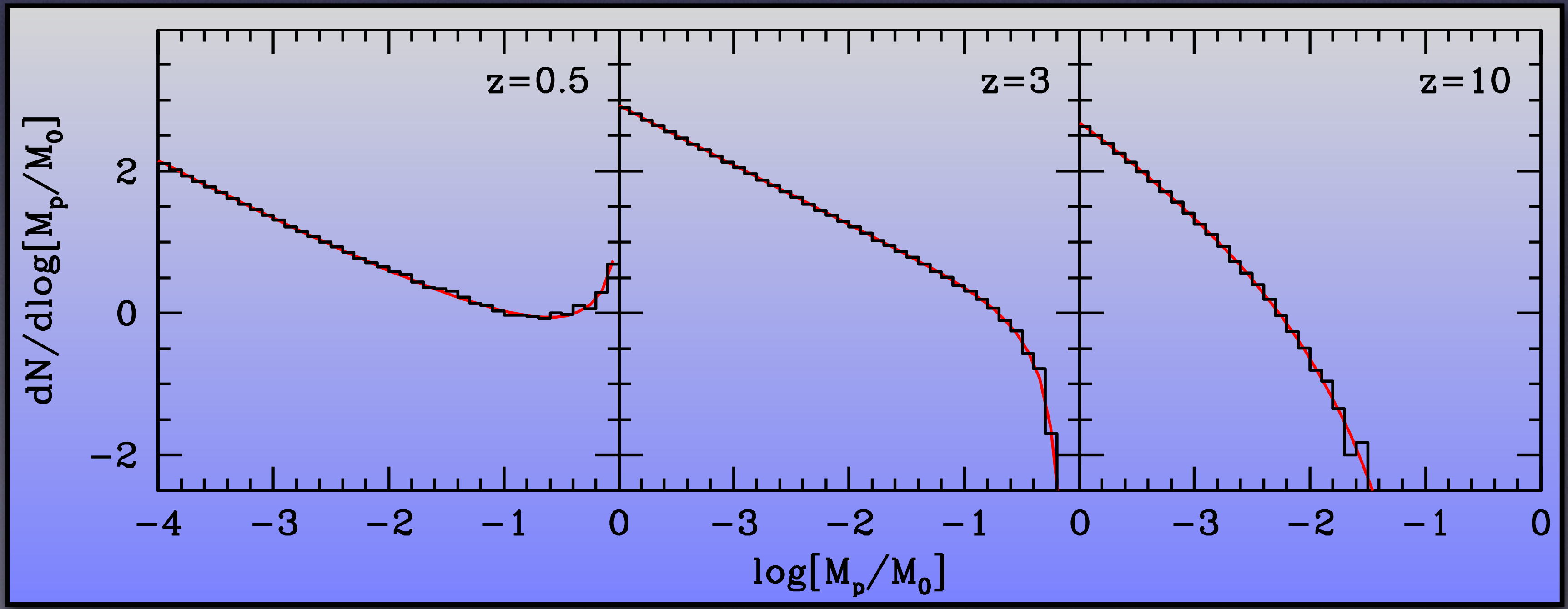
Kauffmann & White (1993)
Sheth & Lemson (1999)
Cole et al. (2000)
Parkinson, Cole & Helly (2008)
Zhang, Fakhouri & Ma (2008)
Jiang & van den Bosch 2014

How to Plant a healthy merger tree...

There are two tests that one can perform to test the accuracy of a merger tree:

1: The Self-Consistency Test

- Construct a larger number of merger trees (using small time steps) for a host halo of a given mass, and compute the average mass function of all progenitors at different redshifts.
- Compare these directly to the EPS progenitor mass functions at those redshifts.
- These need to be in agreement with each other....



Example of a method that successfully passes the Self-Consistency Test: Black histograms are the progenitor mass functions for a halo of 10^{12} Msun obtained from 2000 merger trees constructed using Method B of Zhang, Fakhouri & Ma (2008). The red lines are the direct EPS predictions, and are in excellent agreement with the merger tree results...

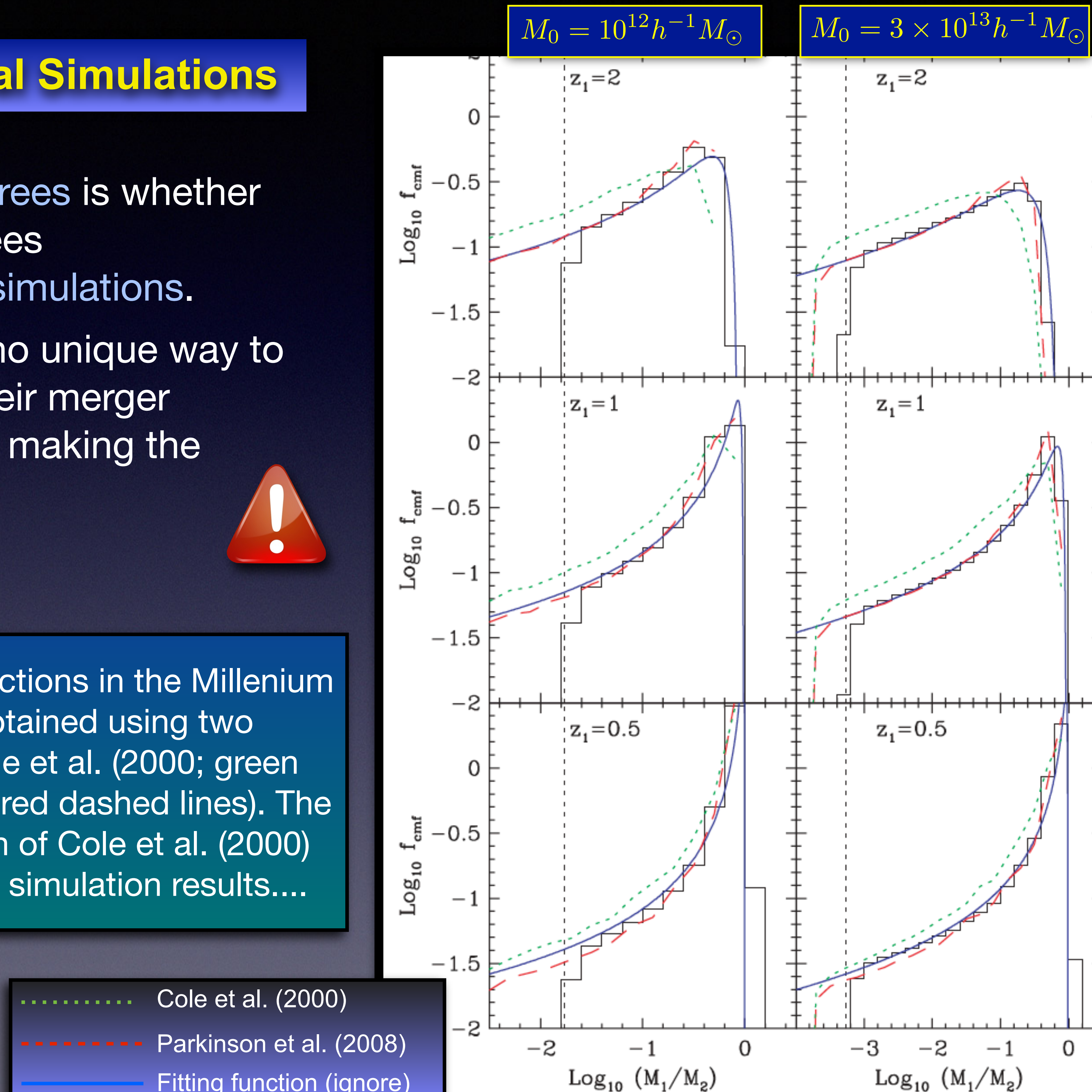
How to Plant a healthy merger tree...

2: Comparison with Numerical Simulations

An important test of EPS merger trees is whether they can reproduce the merger trees obtained from numerical N-body simulations.

We caution, though, that there is no unique way to identify dark matter haloes and their merger histories in numerical simulations, making the comparison non-trivial....

The figure compares progenitor mass fractions in the Millenium simulation (black histograms) to those obtained using two different EPS merger tree algorithms: Cole et al. (2000; green dotted lines), and Parkinson et al. (2008; red dashed lines). The latter is an empirical, ad-hoc modification of Cole et al. (2000) tuned towards better agreement with the simulation results....



Source: Parkinson et al. 2008, MNRAS, 383, 557

The Binary Method

Merger rates in hierarchical models of galaxy formation

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ABSTRACT

We present an analytical description of the merging of virialized haloes which is applicable to any hierarchical model in which structure grows via gravitational instability. The formulae are an extension of the Press–Schechter model. The dependence of the merger rate on halo mass, epoch, the spectrum of initial density fluctuations and the density parameter Ω_0 is explicitly quantified. We calculate the distribution of halo formation times and survival times. We also describe a Monte Carlo method for constructing representative histories of merger events leading to formation of haloes of a prescribed mass.

Applying these results to the age distribution of rich clusters of galaxies, we infer that a high value of the density parameter ($\Omega_0 \gtrsim 0.5$) is required to reproduce the substantial fraction of rich clusters that exhibit significant substructure, if such substructure only persists for a time $0.2\,t_0$ after a merger, where t_0 is the present age of the universe. We also investigate the rate of infall of satellite galaxies into galactic discs, by combining our Monte Carlo technique for halo mergers with an estimate of the time required for dynamical friction to erode the orbits of the baryonic cores of the accreted galaxies. We find that, even for $\Omega_0 = 1$, the infall rate is low (provided that the satellite orbits are not too eccentric), and that we would expect only a modest fraction of stellar discs to be thickened or disrupted by this process.

Key words: galaxies: clustering – galaxies: evolution – galaxies: formation – galaxies: interactions – cosmology: theory – dark matter.

The Binary Method

The Binary method makes the assumption that during each time step, the parent halo splits in two (and only two) progenitors...It's algorithm is extremely simple:

1) draw a progenitor mass $M_{\rm p,1} \in [M_0/2, M_0]$ from progenitor mass function

2) set the mass of the second progenitor to be $M_{\rm p,2} = M_0 - M_{\rm p,1}$

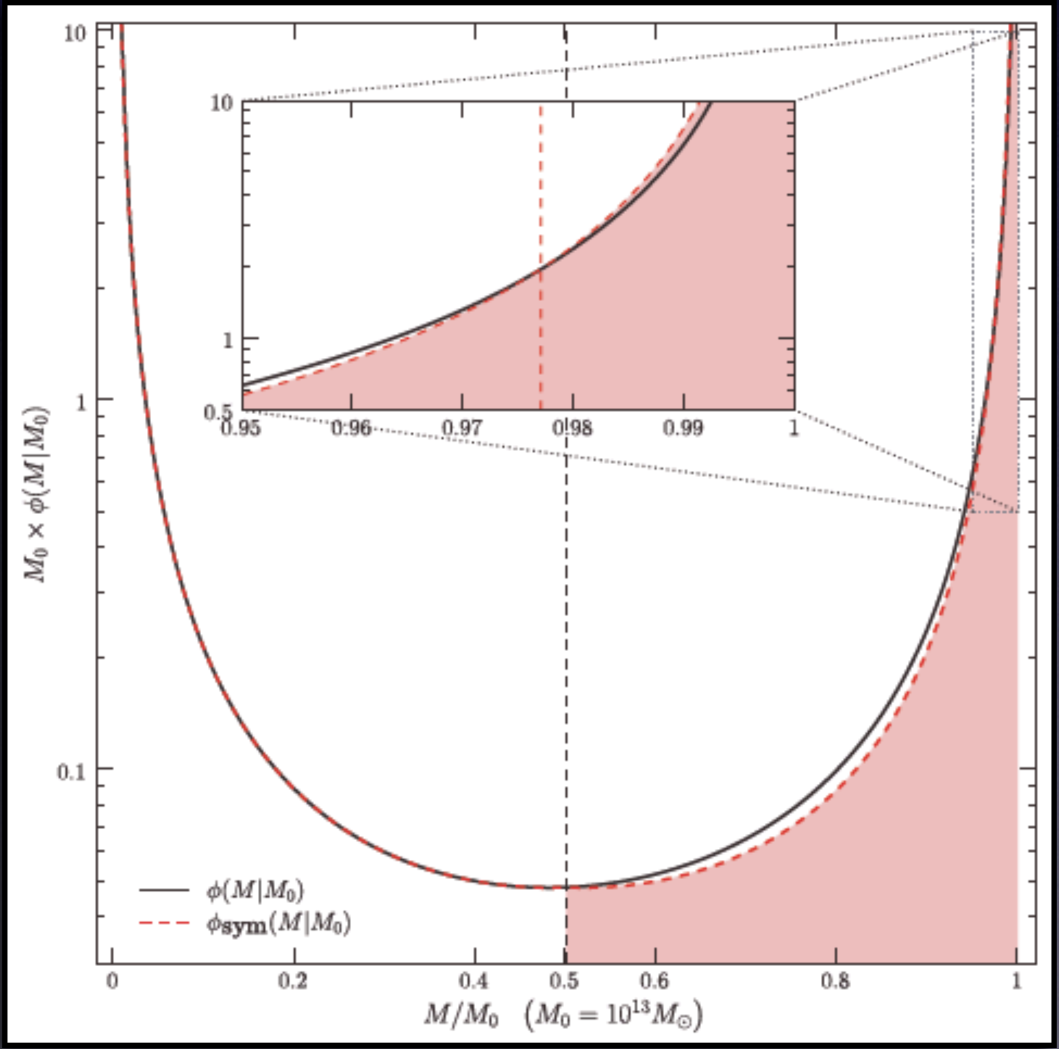
3) repeat steps 1-2 for each of these progenitors

Pros:

- obeys mass conservation (by construction)
- easy to implement & fast to execute

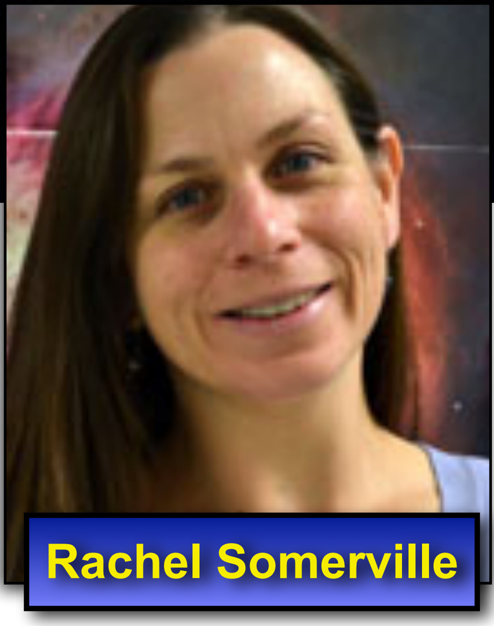
Cons:

- assumption of binarity is oversimplification, even for very small time steps
- makes implicit assumption that progenitor mass function is symmetric around $M_{\rm p} = M_0/2$, which is not the case (see illustration)
- it fails the self-consistency test



Source: Zhang, Fakhouri & Ma, 2008, MNRAS, 389. 1521

The N-Branch Method with Accretion



How to plant a merger tree

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ABSTRACT

We investigate several approaches for constructing Monte Carlo realizations of the merging history of virialized dark matter haloes ('merger trees') using the extended Press–Schechter formalism. We describe several unsuccessful methods in order to illustrate some of the difficult aspects of this problem. We develop a practical method that leads to the reconstruction of the mean quantities that can be derived from the Press–Schechter model. This method is convenient, computationally efficient, and works for any power spectrum or background cosmology. In addition, we investigate statistics that describe the distribution of the number of progenitors and their masses as a function of redshift.

Key words: galaxies: clusters: general – galaxies: formation – cosmology: theory – dark matter.

The N-Branch Method with Accretion

The N-Branch Method with Accretion does not make assumption that all branches are binary. It continues to draw progenitors as allowed by mass budget. Progenitors below the (user-defined) mass limit are assumed to be accreted `smoothly'...

The algorithm for single time-step is as follows:

$N_{\rm p} = 0 \qquad M_{\rm left} = M_0$

DO WHILE ($M_{\rm left} \geq M_{\rm min}$)

1) draw a progenitor mass $M_{\rm p}$

2) IF $M_{\rm p} > M_{\rm left}$ GOTO 1

3) IF $M_{\rm p} \geq M_{\rm min}$ THEN $N_{\rm p} = N_{\rm p} + 1$

4) $M_{\rm left} = M_{\rm left} - M_{\rm p}$

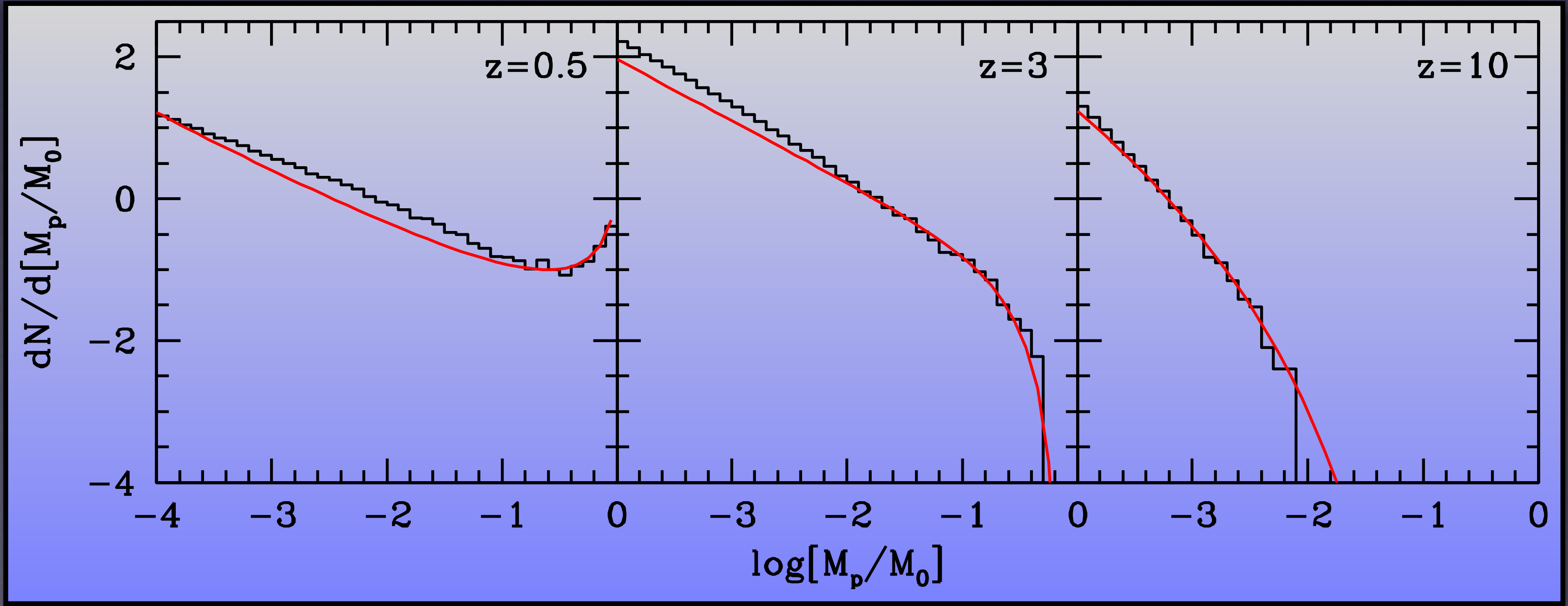
END DO

Pros:

- obeys mass conservation (up to mass resolution)
- easy to implement & fast to execute
- allows for arbitrarily many branches

Cons:

- step 2 implies that accepted progenitors do not properly sample progenitor mass function
- it fails the self-consistency test (see figure)



Progenitor mass distributions at different redshifts: Black histograms are the progenitor masses obtained using the N-Branch method with accretion. Red lines are the EPS progenitor mass functions.

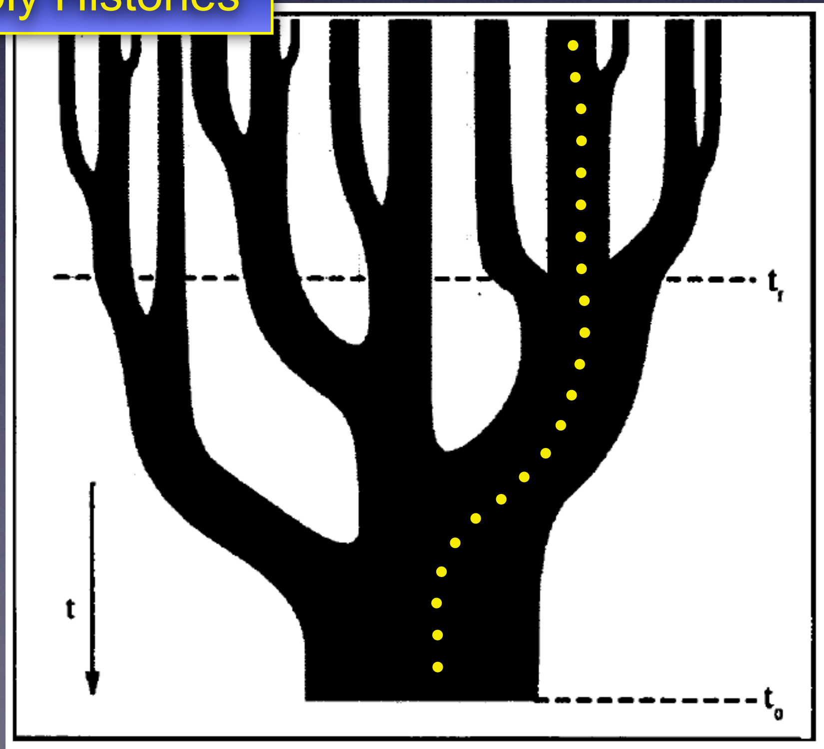
Mass Assembly Histories

A very useful, reduced characterization of a merger tree is its Mass Assembly History (MAH), also called Mass Accretion History or Main Progenitor History.

The MAH $M(z)$ gives the mass of the main progenitor as a function of redshift; at each time step one associates $M(z)$ with the most massive progenitor, and one follows that progenitor, and that progenitor only, further back in time....

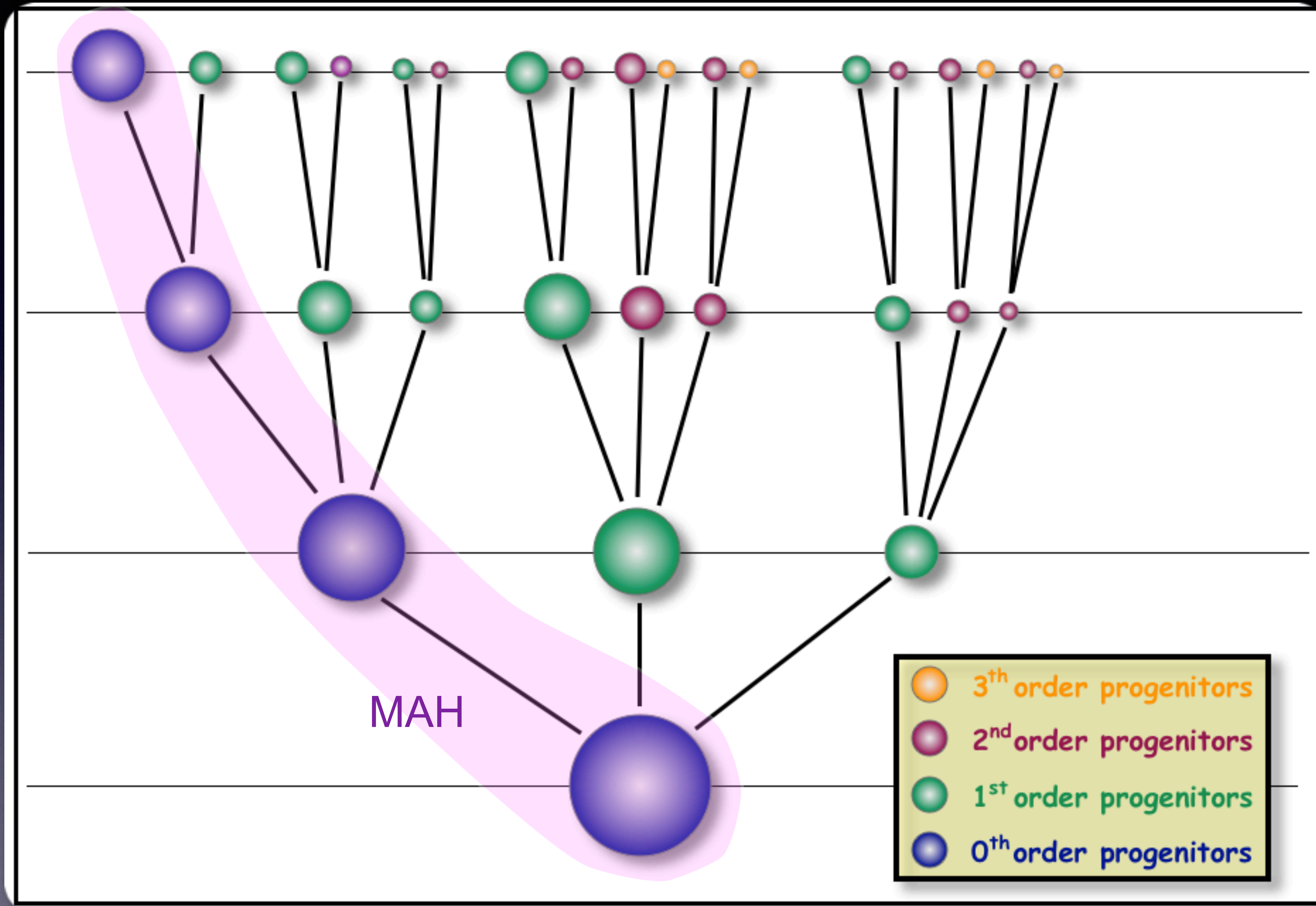
NOTE: the main progenitor is not necessarily also the most massive of all progenitors at a give redshift...

Examples of Mass Assembly Histories



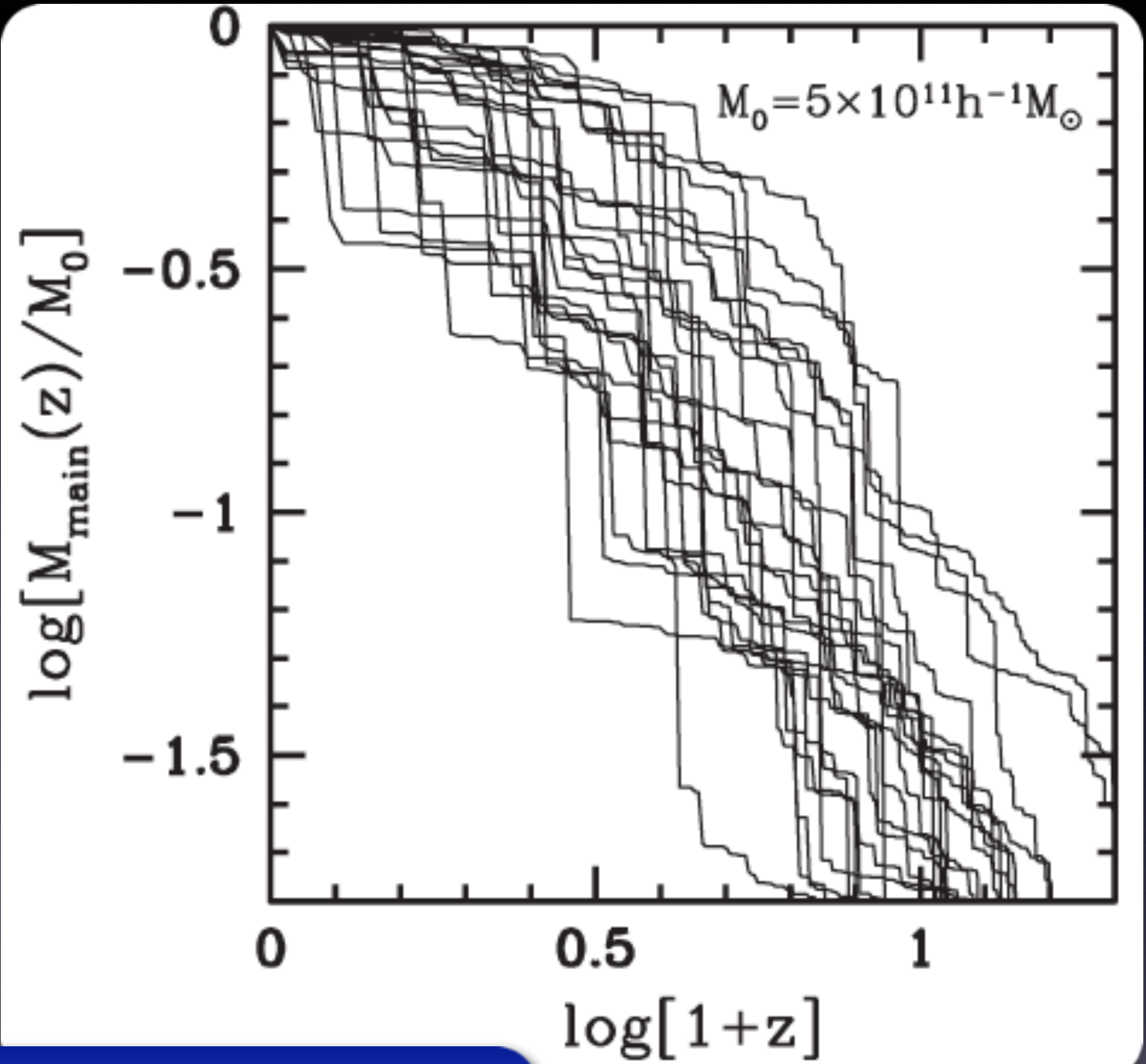
At each branching point in the tree, the MAH follows the most massive branch. Hence, the MAH is sometimes called the main trunk of the merger tree...

Anatomy of a Merger Tree



The MAH is the mass history of the 0th order progenitor...

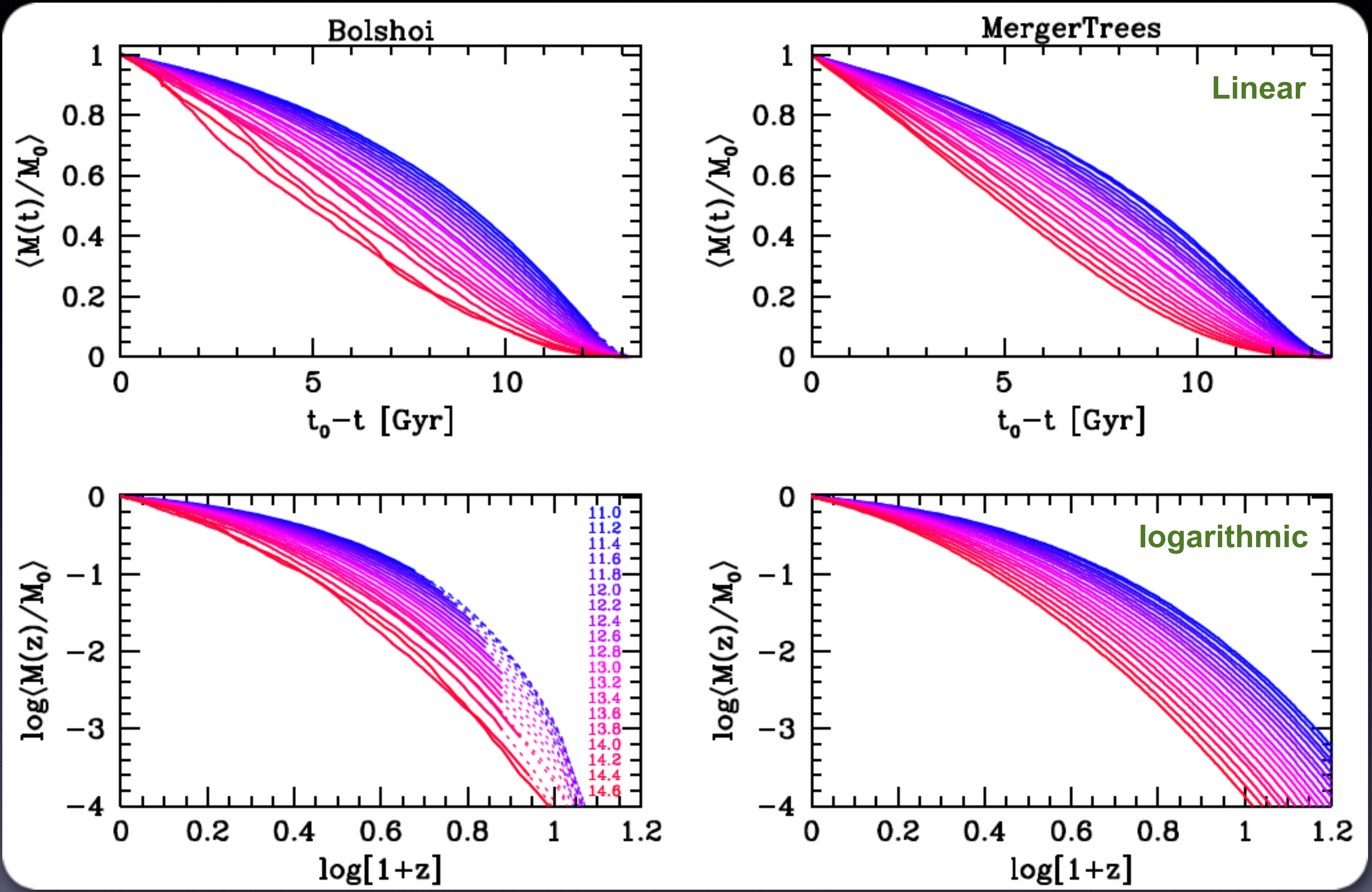
Mass Assembly Histories



A random subset of MAHs for a halo of mass $M_0 = 5 \times 10^{11} h^{-1} M_\odot$ in an EdS Universe. Note the large halo-to-halo variance.

Source: van den Bosch, 2002, MNRAS, 331, 98

Mass Assembly Histories



Source: van den Bosch et al., 2014, MNRAS, 445, 1713

Average MAHs for haloes of different masses. Panels on left and right show results from Bolshoi simulation (Klypin et al. 2011) and from EPS MergerTrees constructed using Parkinson+08 formalism, respectively. Note that more massive haloes assemble later; a clear manifestation of hierarchical structure formation...

Halo Formation Times

Because of the complex structure of merger trees, it is not obvious how to define the formation time of a halo. The most often used definition, is that of the half-mass time of the main progenitor, hereafter called the halo assembly time...

The assembly time, $t_{\rm a}$, of a halo of mass M_0 , with MAH $M_{\rm main}(t)$ is defined by

$$\boxed{M_{\rm main}(t_{\rm a}) = M_0/2}$$

The probability distribution of $t_{\rm a}$ can be obtained as follows:

Any halo can have at most one progenitor with mass in the range $M_0/2 \leq M_1 \leq M_0$
Hence, the probability that it has a progenitor in that mass range at time t_1 , and thus an assembly time $t_{\rm a} < t_1$, is given by

$$\begin{aligned}\mathcal{P}(< t_{\rm a}|M_0, t_0) &= \int_{M_0/2}^{M_0} n(M_1, t_1|M_0, t_0)\, {\rm d}M_1 = \int_{M_0/2}^{M_0} \frac{M_0}{M_1}\, f_{\rm FU}(S_1, \delta_1|S_0, \delta_0) \left|\frac{{\rm d}S_1}{{\rm d}M_1}\right| {\rm d}M_1 \\ &= \int_{S_0}^{S_{1/2}} \frac{M(S_0)}{M(S_1)}\, f_{\rm FU}(S_1, \delta_1|S_0, \delta_0)\, {\rm d}S_1\end{aligned}$$

where $S_{1/2} = S(M_0/2)$ and $t_1 = t_{\rm a}$.

Halo Formation Times

Upon introducing the variables

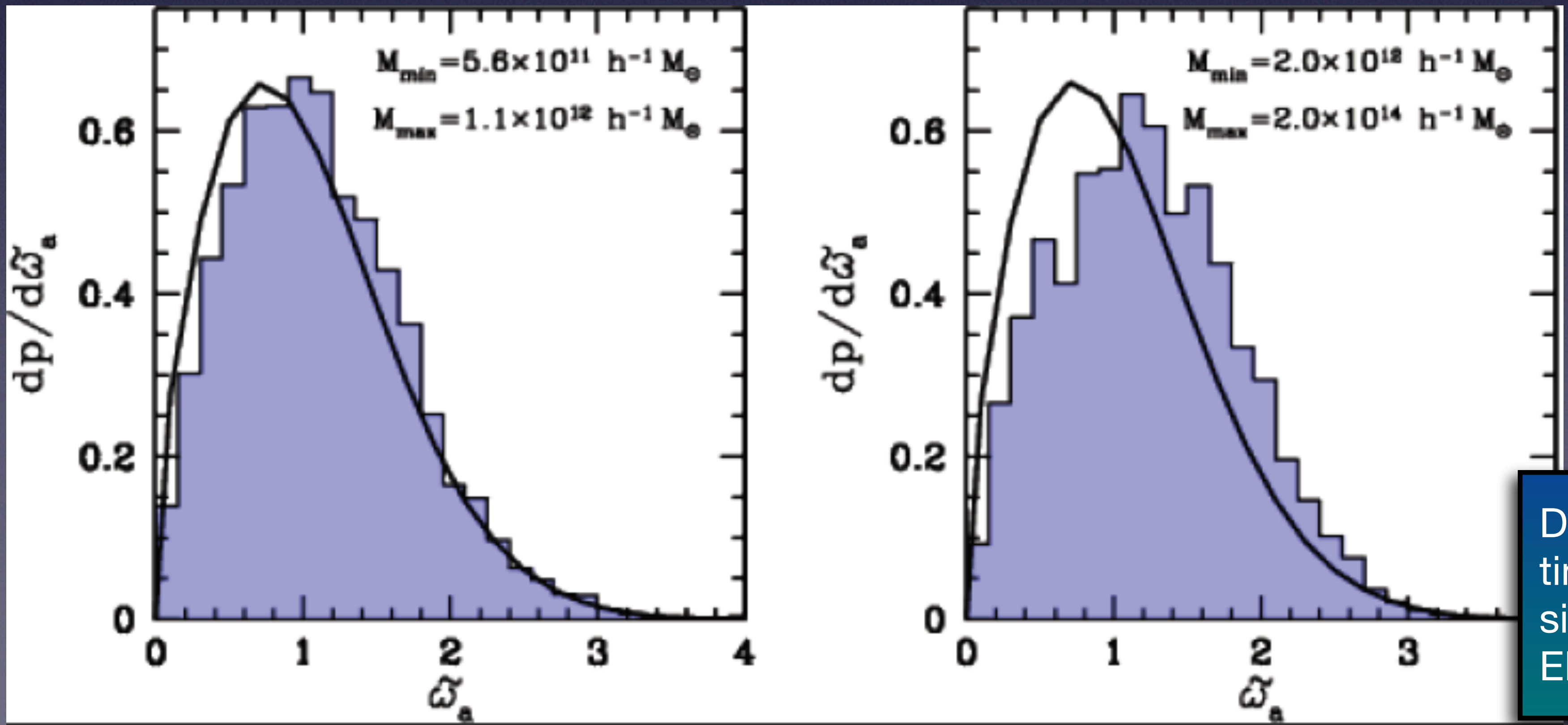
$$\tilde{S} = \frac{S_1 - S_0}{S_{1/2} - S_0} \qquad \tilde{w}(t_1) = \frac{\delta_1 - \delta_0}{\sqrt{S_{1/2} - S_0}}$$

one can cast this in the form

$$\mathcal{P}(< t_a|M_0, t_0) = \mathcal{P}(> \tilde{w}_{\rm a}|M_0, t_0) = \frac{1}{\sqrt{2\pi}} \int_0^1 \frac{M(S_0)}{M(S_1)} \frac{\tilde{w}_{\rm a}}{\tilde{S}^{3/2}} \exp\left[-\frac{\tilde{w}_{\rm a}^2}{2\tilde{S}}\right] {\rm d}\tilde{S}$$

Differentiating yields the PDF for halo assembly times according to the EPS formalism.

A comparison with the assembly times obtained from numerical simulations shows that EPS predicts assembly times that are too high (i.e., $\tilde{w}_{\rm a}$ too low)....



Source: van den Bosch, (2002)

As shown by Giocoli et al. (2007), this problem is alleviated when using ellipsoidal collapse conditions within the EPS formalism...

Differential distributions of halo assembly times obtained from a ΛCDM N-body simulation (histograms) compared to the EPS predictions (solid lines)

more to read...

- For alternative definitions of `halo formation time', which are likely to be more closely related to formation times of stars that form in the halo,

  see Navarro, Frenk & White et al.1996
  Neistein, van den Bosch & Dekel (2006)
  Giocoli, Tormen & Sheth (2012)

- For a more indepth discussion of halo formation times and merger statistics,

  see Lacey & Cole (1993, 1994)
  Li, Mo, van den Bosch & Lin (2007)
  Cole et al. (2008)
  Jiang & van den Bosch (2014)

- For a more indepth discussion of halo assembly histories,

  see van den Bosch (2002)
  Wechsler et al. (2002)
  Zhao et al. (2009)
  van den Bosch et al. (2014)

Halo Bias

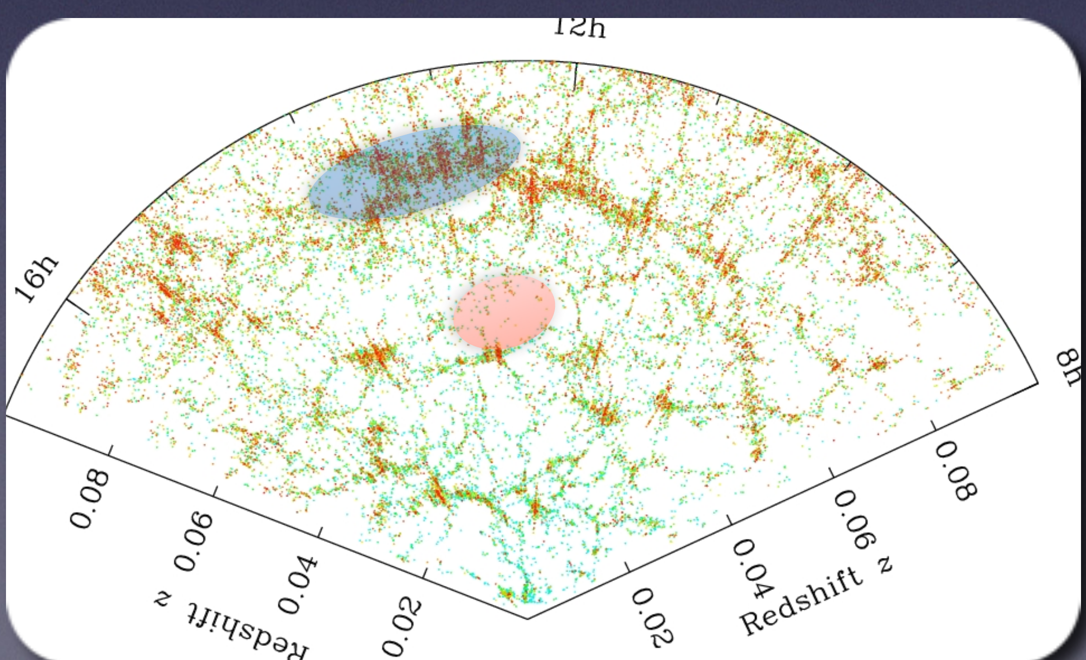
Spatial Distribution of Dark Matter Halos

Having discussed the collapse and assembly history of dark matter haloes, we now turn to their spatial distribution.

Since dark matter haloes are made out of (dark) matter, a naive (and wrong) expectation might be that DM haloes sample the DM mass distribution; i.e., the number density of haloes is simply proportional to the matter density....

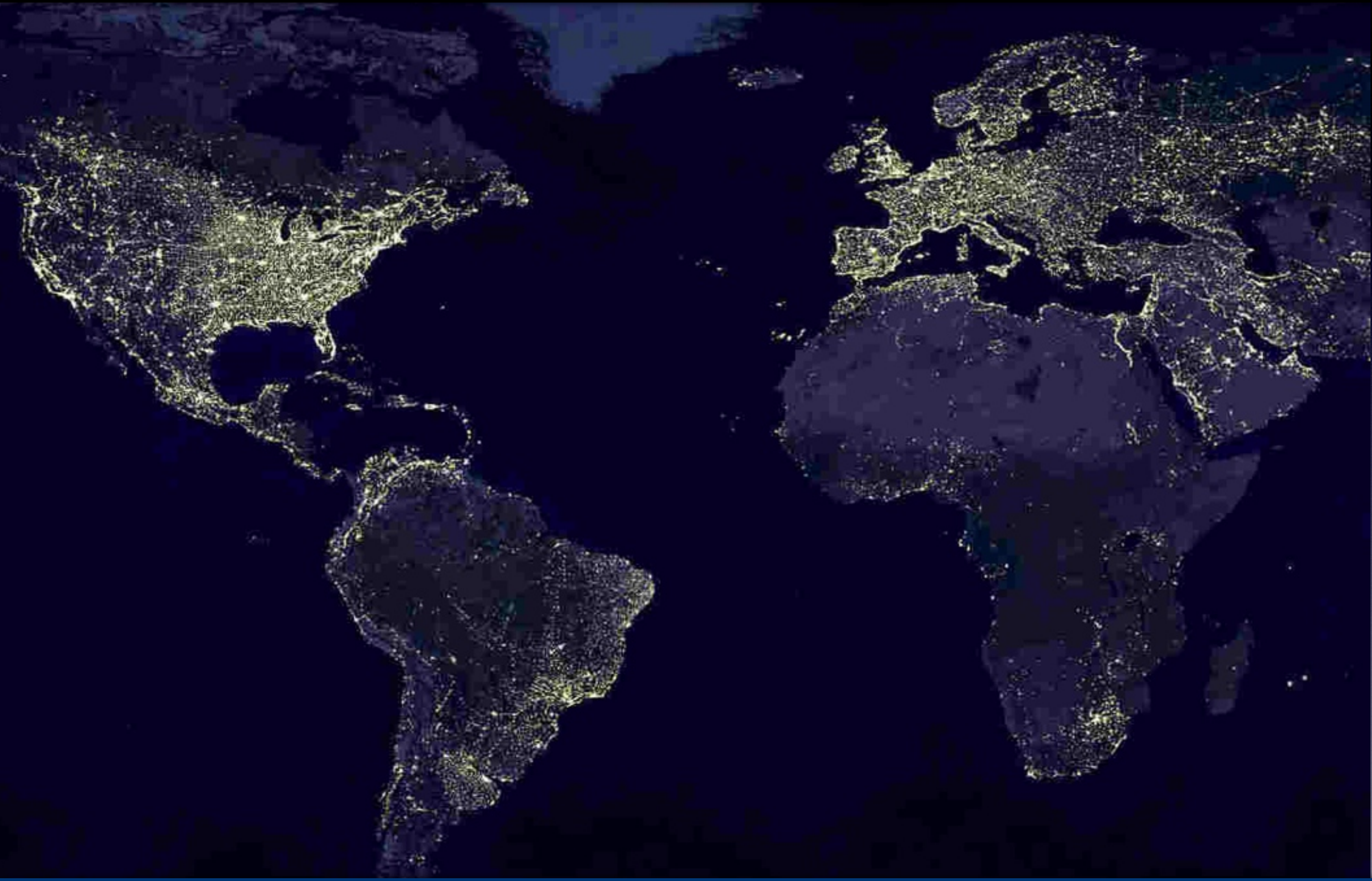
$$N_{\rm h}(V) \propto M(V) \quad \Rightarrow \quad \delta_{\rm h} = \frac{\delta n_{\rm h}}{\bar{n}_{\rm h}} = \frac{\delta \rho_{\rm m}}{\bar{\rho}_{\rm m}} = \delta_{\rm m}$$

If this were the case, and galaxies form and reside in dark matter haloes, then the number density of galaxies would be an unbiased estimator of the local mass density, i.e., light traces matter....



However, large galaxy redshift surveys show huge variations in the number densities of galaxies on very large scales (supercluster vs. void). If light follows mass, this implies that even on those very large scales the matter distribution is highly non-linear....

Light as a Biased Tracer of Land Mass

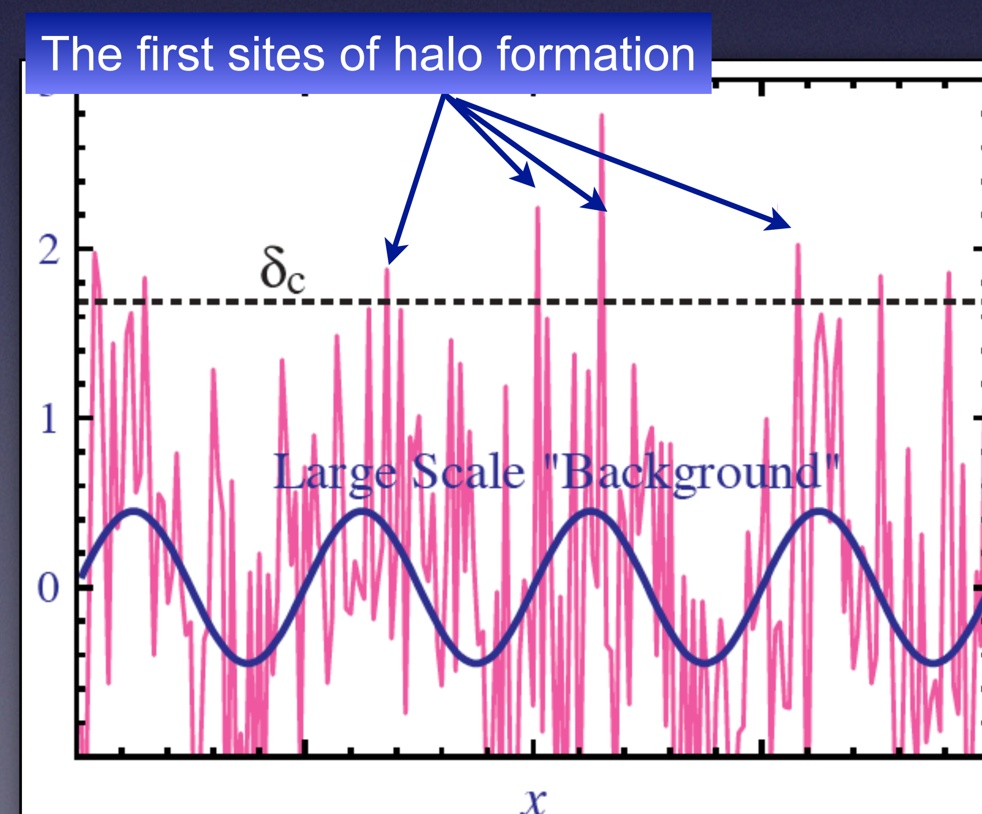


Halo Bias

The solution is that halo formation is not a random process; haloes are not a Poisson sampling of the matter field. Rather, they only form where the (smoothed) density field has a sufficiently high value: the critical overdensity for collapse.

This `threshold' causes haloes to be biased tracers of the mass distribution.

Because of the modulation of the small-scale density field by the long-wavelength modes, overdense regions (on large scales) contain enhanced abundance of dark matter haloes, so that these haloes display enhanced clustering...



Halo Bias

We can use the EPS formalism to derive an expression for this halo bias:

The following derivation is due to Mo & White (1996); a different approach, based on the "peak-background split" can be found in Cole & Kaiser (1989)

Recall that the average number of haloes of mass M_1 at t_1 that will merge into a larger halo with mass M_2 at time $t_2 > t_1$ is given by

$$\boxed{N(M_1, t_1|M_2, t_2)\, \mathrm{d}M_1 = \frac{M_2}{M_1} f_{\mathrm{FU}}(S_1, \delta_1|S_2, \delta_2) \left|\frac{\mathrm{d}S_1}{\mathrm{d}M_1}\right| \mathrm{d}M_1}$$

Note, though, that it is not necessary that M_2 is a halo: the above equation holds equally well even if M_2 is an uncollapsed spherical region whose overdensity, linearly extrapolated to the present, is equal to δ_2 .

Consider a region of mass M_0 that at $z = 0$ has a linearly extrapolated overdensity δ_0.

The proper volume of this region at redshift z_1 is

$$\boxed{V(M_0, z_1|\delta_0) = \frac{M_0}{\bar{\rho}(z_1)\,[1 + \delta(z_1)]}}$$

where $\delta(z_1) = \delta_0\, D(z_1)$

Halo Bias

The average number of haloes of mass M_1 that one finds in an average region of this volume, at redshift z_1, is $\bar{N}(M_1, z_1|M_0) = n(M_1, z_1)\, V(M_0, z_1|\delta_0)$ where

$$n(M_1, z_1) = \frac{\bar{\rho}(z_1)}{M_1} f_{\rm FU}(S_1, \delta_1) \left|\frac{{\rm d}S_1}{{\rm d}M_1}\right| {\rm d}M_1$$

Here $S_1 = \sigma^2(M_1)$ and $\delta_1 = \delta_c(z_1) \simeq 1.686/D(z_1)$. Note that n is the halo number density in proper (not comoving) coordinates....

$$\bar{N}(M_1, z_1|M_0)\, {\rm d}M_1 = \frac{1}{1+\delta(z_1)} \frac{M_0}{M_1} f_{\rm FU}(S_1, \delta_1) \left|\frac{{\rm d}S_1}{{\rm d}M_1}\right| {\rm d}M_1$$

However, our region is not average, in that it has a specific overdensity $\delta(z_1) = \delta_0\, D(z_1)$
Hence, the average number of haloes will be different from the above number....

We can use EPS to write the average number of haloes of mass M_1, in a region of mass M_0 and linearly extrapolated overdensity δ_0 as

$$N(M_1, z_1|M_0, \delta_0)\, {\rm d}M_1 = \frac{M_0}{M_1} f_{\rm FU}(S_1, \delta_1|S_0, \delta_0) \left|\frac{{\rm d}S_1}{{\rm d}M_1}\right| {\rm d}M_1$$

Halo Bias

Thus, the overdensity of haloes of mass M_1 at z_1 that are in a region that has a linearly extrapolated overdensity δ_0 is

$$\boxed{\delta_{\rm h}(M_1, z_1|M_0, \delta_0) = \frac{N(M_1, z_1|M_0, \delta_0)}{\bar{N}(M_1, z_1|M_0)} - 1 = [1 + \delta(z_1)]\, \frac{f_{\rm FU}(S_1, \delta_1|S_0, \delta_0)}{f_{\rm FU}(S_1, \delta_1)} - 1}$$

This expression is particularly simple when $M_0 \gg M_1$, so that $S_0 \ll S_1$, and $|\delta_0| \ll \delta_1$:

$$\boxed{\delta_{\rm h}(M_1, z_1|M_0, \delta_0) = \delta(z_1) + \frac{\nu_1^2 - 1}{\delta_1}\delta_0 + \frac{\nu_1^2 - 1}{\delta_1}\delta_0\delta(z_1)} \qquad \nu_1 = \delta_1/\sqrt{S_1}$$

In the linear regime, where $\delta(z_1) = \delta_0\, D(z_1) \ll 1$, the last term is negligible and we have that

$$\boxed{\delta_{\rm h}(M_1, z_1|\delta_0) \equiv b_{\rm h}(M_1, z_1)\delta(z_1)}$$

Here we have introduced the linear halo bias term

$$\boxed{b_{\rm h}(M, z) = 1 + \left(\frac{\nu^2 - 1}{\delta_{\rm c}}\right)}$$

where $\nu = \delta_{\rm c}(z)/\sqrt{S(M)}$ and $\delta_{\rm c} = \delta_{\rm c}(0) \simeq 1.686$.

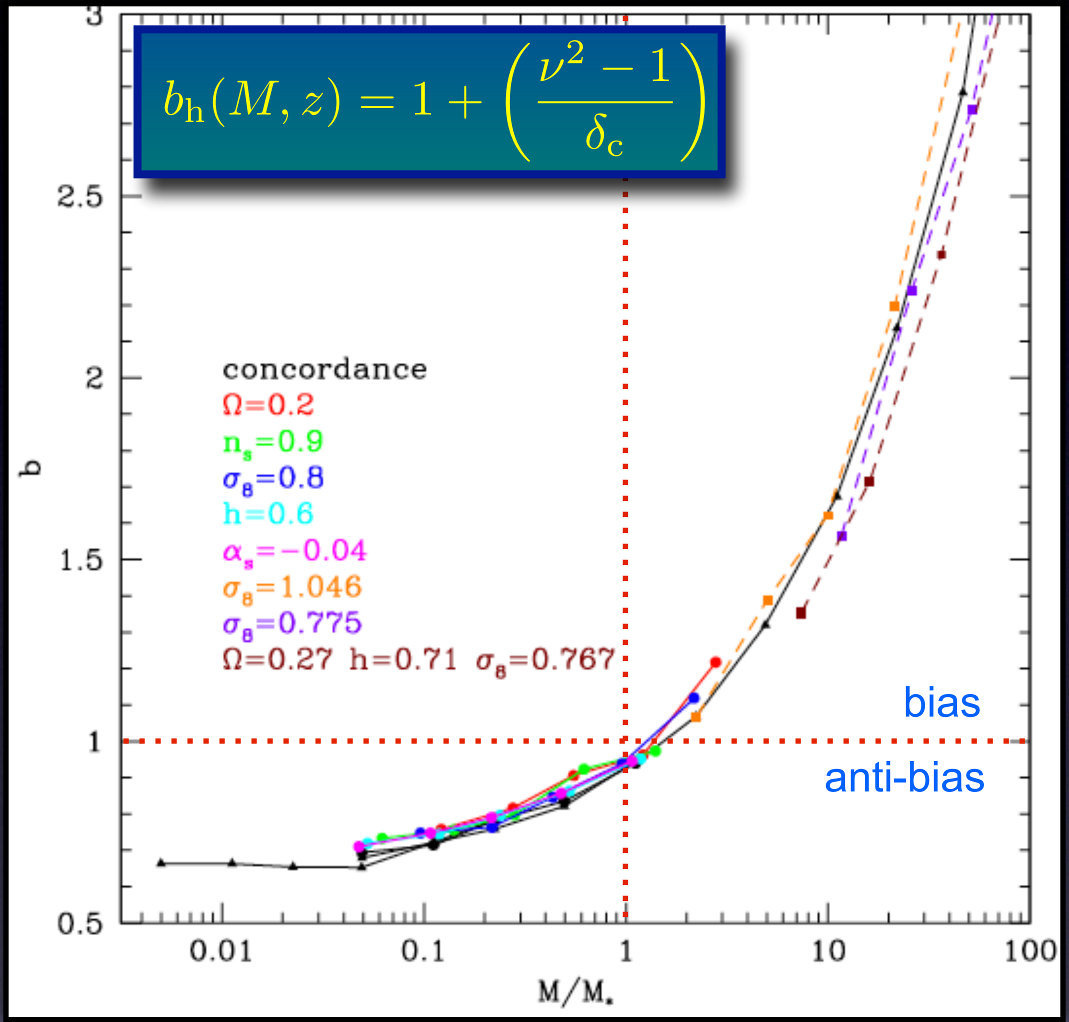
The Mass Dependence of Halo Bias

Upon inspection it is immediately clear that massive haloes with $M > M^*$ (i.e., $\nu > 1$) are positively biased $b_{\rm h} > 1$, while the opposite is true for low mass haloes ($M < M^*$)

This is confirmed by simulations (see next slide for how this is done), as shown in the figure to the right.

Symbols are results obtained from N-body simulations covering a variety of cosmologies.

More massive haloes, are more strongly clustered.



Source: Seljak & Warren, 2004, 355, 129

One can also apply the same formalism as above using ellipsoidal collapse conditions. This yields the following, modified halo bias function:

$$b_{\rm h}(M,z) = 1 + \frac{1}{\delta_{\rm c}}\left[\nu'^2 + 0.5\nu'^{0.8} - \frac{1.19\,\nu'^{1.2}}{\nu'^{1.2} + 0.14}\right] \quad \text{where} \quad \nu' = 0.84\,\nu$$

Halo Bias: comparison with simulations

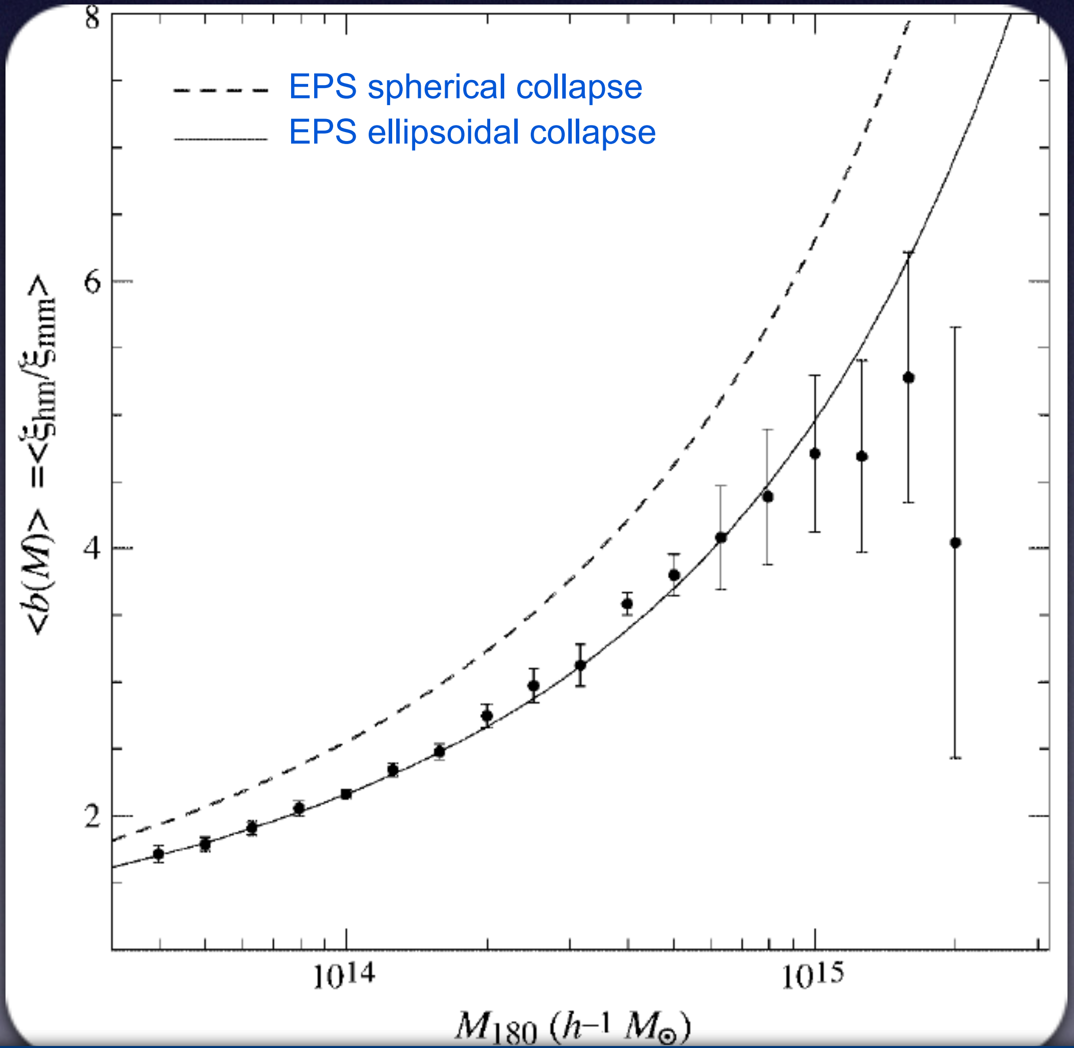
Similar to the halo mass function, the EPS prediction for the halo bias function has to be tested against numerical simulations. Using that the cross-correlation function between dark matter haloes of mass M and dark matter (particles) can be written as

$$\boxed{\xi_{\rm hm}(r|M) = \langle \delta_{\rm h}(\vec{x}|M)\, \delta_{\rm m}(\vec{x}+\vec{r})\rangle = b(M)\langle \delta_{\rm m}(\vec{x})\, \delta_{\rm m}(\vec{x}+\vec{r})\rangle}$$

we see that $b(M) = \langle \xi_{\rm hm}/\xi_{\rm mm}\rangle$ where $\xi_{\rm mm}(r)$ is the two-point correlation function of the dark matter, and $\langle\cdot\rangle$ indicates an averaging over large (linear) radii)

Thus, we can measure the halo bias in simulations by comparing the halo-matter cross correlation function with the auto-correlation function of the dark matter particles...

As with halo mass function, the ellipsoidal collapse predictions are in much better agreement with simulation results than predictions based on spherical collapse

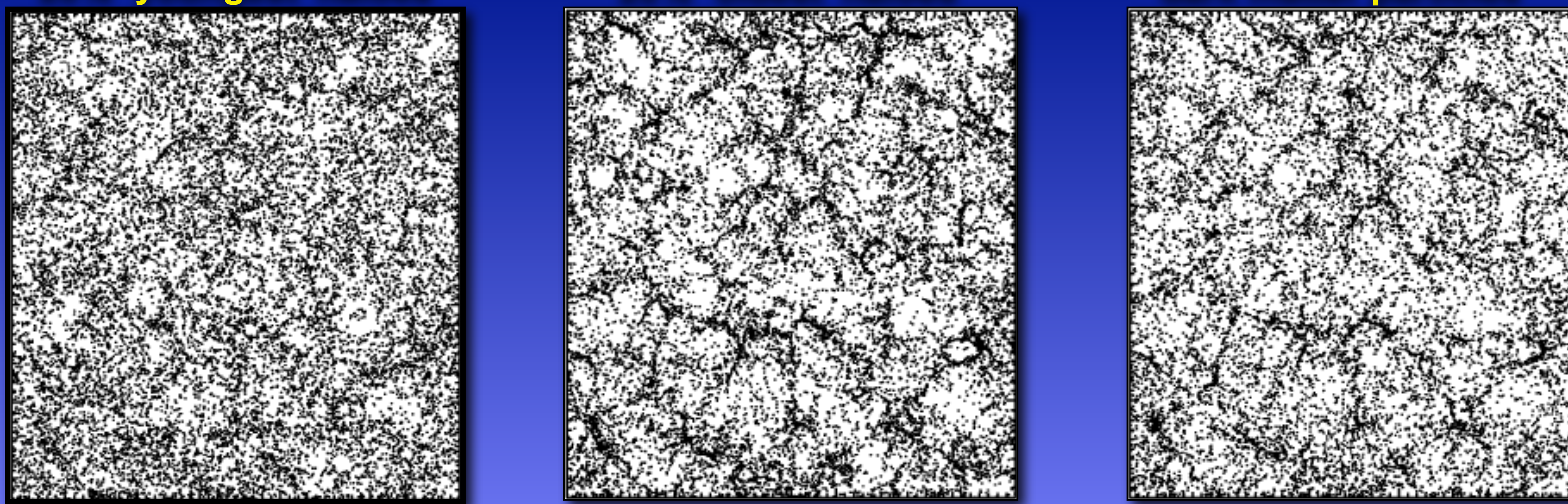


Source: Hu & Kravtsov, 2003, ApJ, 584, 702

Assembly Bias

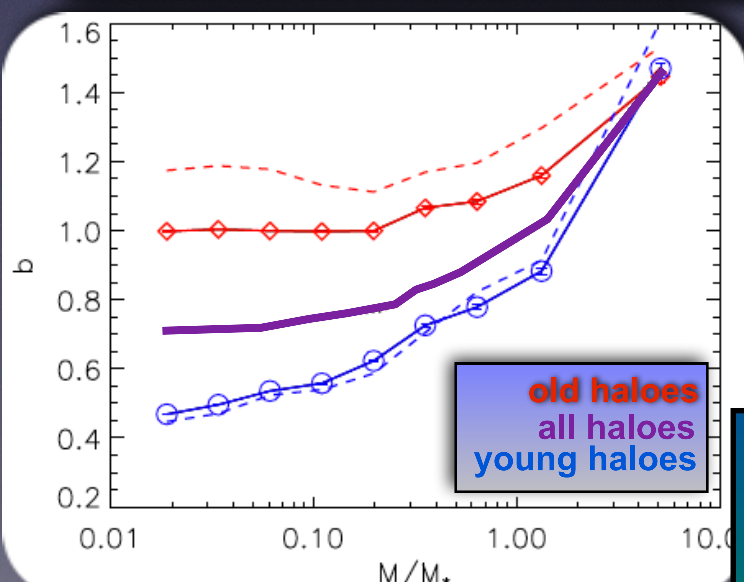
According to EPS formalism, halo bias depends only on halo mass (for given cosmology). However, N-body simulations show that halo bias also depends on halo assembly time....

20% `youngest' haloes** | **20% `oldest' haloes | **dark matter particles**



Source: Gao et al. 2005, MNRAS, 363, L66

A 30 Mpc/h slice through the Millenium simulation. Left and middle panels show distribution of halos with between 100 and 200 particles. Right-hand panel shows random subsample of dark matter particles



The linear bias parameter of dark matter haloes in the Millenium simulation. Red and blue curves show the 20 percentiles based on halo assembly time.

Haloes that assemble earlier (=are ‘old’) are more strongly clustered than haloes of the same mass that assemble late (=are `young’). This phenomenon is called assembly bias

Lecture 10

SUMMARY

Summary: key words & important facts

Key words	
Merger Tree	Halo Formation time
Progenitor Mass Function	Halo Bias
Mass Assembly History	Assembly Bias

- Construction of halo merger tree is subject to two conditions
  accurately samples progenitor mass function at all times (self-consistency)
  mass conservation (sum of progenitor masses = descendent mass)
  Different methods for constructing EPS merger trees differ in handling corresponding subtleties..

- Even in the limit of infinitesimally small time-step there is a non-zero probability of having more than two progenitors → binary merger tree method fails

- Mass assembly histories of dark matter halos are universal, if scaled appropriately.

- More massive halos assemble later, and are more strongly clustered (i.e., $db_h/dM > 0$)

- Halos that assemble earlier are more strongly clustered than halos of the same mass that assemble later (= halo assembly bias)

Summary: key equations & expressions

Progenitor Mass Function:

$$\boxed{n(M_1,t_1|M_2,t_2)\,\mathrm{d}M_1 = \frac{M_2}{M_1} f_{\rm FU}(S_1,\delta_1|S_2,\delta_2)\left|\frac{\mathrm{d}S_1}{\mathrm{d}M_1}\right| \mathrm{d}M_1}$$

Halo Bias

$$\boxed{\delta_{\rm h}(M_1,z_1|M_0,\delta_0) = \delta(z_1) + \frac{\nu_1^2-1}{\delta_1}\delta_0 + \frac{\nu_1^2-1}{\delta_1}\delta_0\delta(z_1)}$$

in linear regime $\delta_{\rm h}(M_1,z_1|\delta_0) \equiv b_{\rm h}(M_1,z_1)\delta(z_1)$ with $b_{\rm h}(M,z) = 1 + \left(\frac{\nu^2-1}{\delta_{\rm c}}\right)$

$$\delta_{\rm h}(\vec{x}|M) = b_{\rm h}(M)\,\delta(\vec{x})$$

$$\xi_{\rm hh}(r|M_1,M_2) = b_{\rm h}(M_1)\,b_{\rm h}(M_2)\,\xi_{\rm mm}(r)$$