ASTR 501: Problem Set 1

This problem set consists of 4 problems for a total of 44 points.
Due date: tuesday Sep 12

In this problem set, you will make order-of-magnitude estimates for the densities, masses, velocities, and various time and length scales encountered in astrophysical fluids. Contemplate your answers!!

Problem 1: Astrophysical fluids

a) [`3 points`] Assuming that the typical radius of a hydrogen atom is given by the Bohr radius $r_{\rm B} = 5.29 \times 10^{-9}$ cm, compute the mean free path (assuming direct collisions) of baryonic matter in the Universe at its average, present day density ($\bar{\rho}_{\rm bar} \sim 5 \times 10^{-31}$ g cm^{-3}). Assume that the baryonic matter is entirely made up of neutral hydrogen and express your answer in both cm and Mpc. What is the typical mass (in Solar masses) enclosed by a sphere of radius $\lambda_{\rm mfp}$.

b) [`4 points`] Consider a plasma made of electrons and protons only. Assume that the electrons and protons have thermalized to the same temperature. What is the ratio of the Larmor radii for the electrons and the protons?

Problem 2: The Interstellar medium (ISM)

The warm phase of the interstellar medium (ISM) has a typical density of about 0.1 particles per cubic centimeter and a typical temperature of 10^4 K.

a) [`3 points`] Under the assumption that the ISM mainly consists of hydrogen, what is the average density of the warm ISM in units of the average density of the Universe.

b) [`3 points`] What is the mean-free path for neutral hydrogren atoms in the warm phase of the ISM. Express your answer in cm and in pc. What is the typical mass (in Solar masses) enclosed by a sphere of radius $\lambda_{\rm mfp}$.

c) [`5 points`] Now assume that all the hydrogen is ionized (which is a more realistic assumption at the assumed temperature). Compute the Debye length, plasma frequency, and plasma two-body relaxation time of of the ISM.

d) [`4 points`] The typical magnetic field strength in the ISM is of the order of a microGauss. Compute the corresponding cyclotron frequency and Larmor radius.

Problem 3: The Sun

a) [`3 points`] What is the average mean-free path for a hydrogen atom in the Solar interior under the (unrealistic) assumption that all hydrogen in the Sun is neutral. Express your answer in units of the Solar radius.

b) [`6 points`] The core of the Sun has an electron density of $n_{\rm e} \sim 10^{26}\,{\rm cm}^{-3}$ and a temperature of $T \sim 10^7$ K. Compute the corresponding Debye length and plasma frequency. How many particles are there, on average, per Debye volume? Can we approximate the plasma in the core of the Sun as collisionless?

Problem 4: Gravitational N-body systems

a) [`5 points`] Compute a rough estimate of the mean free path for director collisions of stars (stars actually touch each other) in a galaxy similar to the Milky Way galaxy (i.e., a galaxy consisting of $N \sim 10^{10}$ stars within in radius of $R \sim 10\,{\rm kpc}$ that move with an average speed of $200\,{\rm km\,s^{-1}}$). You may assume that all stars are Sun-like. Express your answer in both cm and in R (the size the galaxy). How many direct collisions does an average star experience per Hubble time?

b) [`5 points`] The typical gravitational impact parameter for which the deflection angle is 90 degrees is given by $b_{90} = 2Gm/v^2$. Here m and v are the typical masses and velocities of the particles experiencing the gravitational encounter. Compute b_{90}, and the corresponding mean free path for largeangle deflection (express both in cm and in R). How many strong collisions does an average star experience per crossing time? How does b_{90} compare to the mean particle separation (order of magnitude)?

c) [`3 points`] Dark matter halos are often defined to have a radius that encloses a fixed density (of the order of 200 times the background density). Using that definition, what is the ratio of the dynamical time in a massive cluster ($M \sim 10^{15}\mathrm{M}_\odot$) to that in a low mass dwarf galaxy halo ($M \sim 10^{9}\mathrm{M}_\odot$). Motivate your answer.

(Potentially) Useful Constants

Gravitional constant	G	$=$	$6.674 \times 10^{-8}\,\mathrm{cm}^3\,\mathrm{g}^{-1}\,\mathrm{s}^{-2}$
		$=$	$4.299 \times 10^{-9}\,\mathrm{Mpc}\,M_\odot^{-1}\,(\mathrm{km/s})^2$
Proton mass	m_p	$=$	$1.673\,\times^{-24}\,\mathrm{g}$
Electron mass	m_e	$=$	$9.109 \times 10^{-28}\mathrm{g}$
Electron charge	e	$=$	$4.8 \times 10^{-10}\mathrm{esu}$
Boltzmann constant	k_B	$=$	$1.38 \times 10^{-16}\,\mathrm{erg}\,\mathrm{K}^{-1}$
electron volt	eV	$=$	$1.602 \times 10^{-12}\,\mathrm{erg}$
parsec	pc	$=$	$3.086 \times 10^{18}\,\mathrm{cm}$
Solar mass	$M_\odot$	$=$	$2 \times 10^{33}\mathrm{g}$
Solar Radius	$R_\odot$	$=$	$6.960 \times 10^{10}\,\mathrm{cm}$