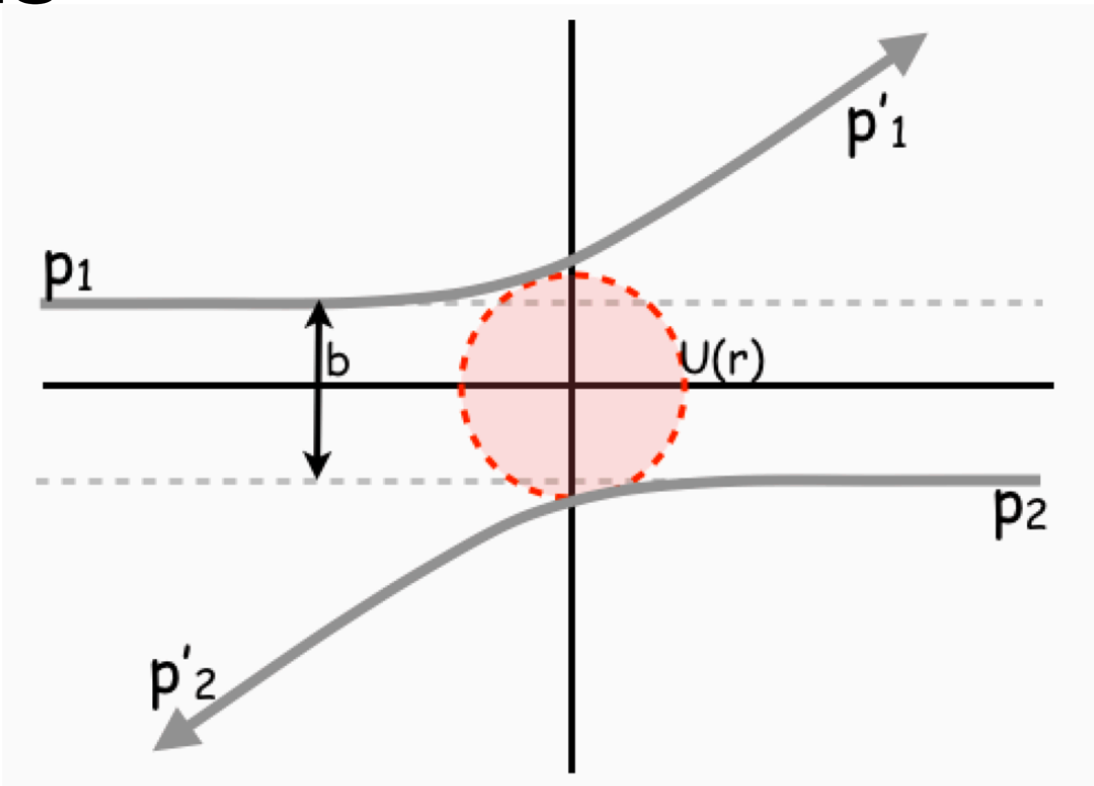
LECTURE 2

Collisions

Collisions are characterized by an impact parameter, b, incoming momenta, $\mathbf{p}_1$ & $\mathbf{p}_2$, and an interaction potential, $U(\mathbf{r})$



In ASTR501 we assume all collisions to be elastic ➡ Hamiltonian dynamics
We thus ignore radiative processes

collisional fluid:

interparticle force is short-range van der Waals force (r^{-7})

mean free path ≫ mean particle separation ($\lambda_{mfp} \gg \lambda_{int}$)

collisions are well separated in space and time

collisions drive system towards *local* thermal equilibrium
in which velocity distribution become Maxwell-Boltzmann (MB)

two-body relaxation time $\tau_{relax} = \tau_{coll} \simeq \lambda_{mfp} / \langle v \rangle$ is extremely short

Collisions

gravitational N-body system:

interparticle force is long-range gravitational force (r^{-2})

each particle `collides' simultaneous with all other particles

in limit $N \rightarrow \infty$, system become a continuum = no collisions

despite being extremely collisional, system behaves collisionless

Gravitational N-body system is only *approximately* collisionless.

impact of weak interactions ($b > b_{90}$) dominates over that of strong interactions ($b < b_{90}$)

The many weak interactions give rise to a net friction (slowing the particles down) plus a diffusion (broadening of velocity distribution)

These effects are described by the Fokker-Planck equation.

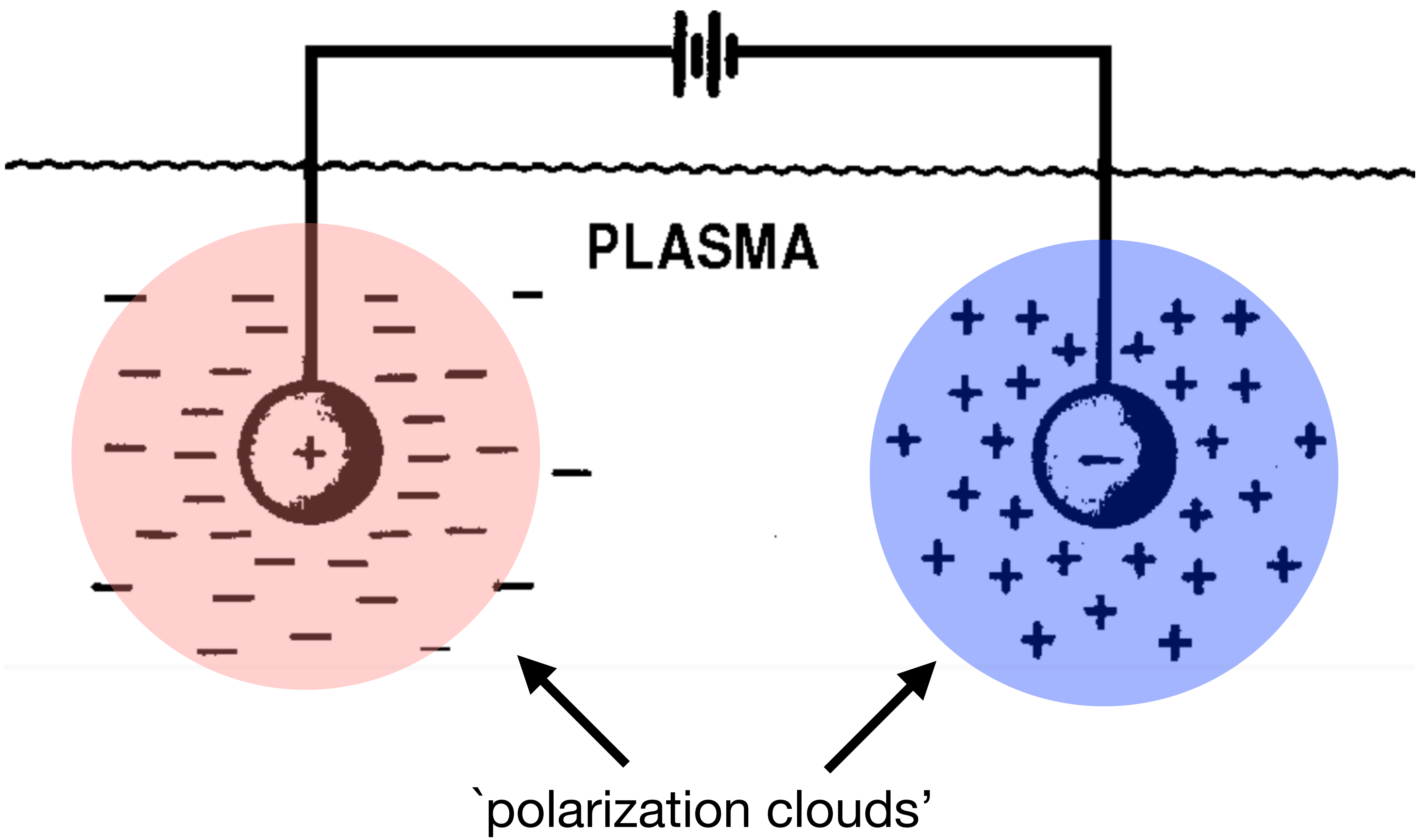
System relaxes to thermal equilibrium (MB distribution) on two-body relaxation time scale $\tau_{\rm relax} \sim (N/\ln N)\, \tau_{\rm cross}$

Collisions

plasma:

interparticle force is long-range Coulomb force (r^{-2})

presence of both positive and negative charges causes shielding; charges shielded beyond Debye length, λ_D, due to polarization cloud



plasma:

Collisions

interparticle force is long-range Coulomb force (r^{-2})

presence of both positive and negative charges causes shielding; charges shielded beyond Debye length, λ_D, due to polarization cloud

each particle undergoes simultaneous collisions with $N_D \sim n\,\lambda_D^3$ particles inside its Debye volume

$N_D < 1$: collisions well separated in time and space; plasma behaves like neutral fluid (this is not considered a plasma)

$N_D \gg 1$: plasma limit. Plasma reveals collective effects and behaves like a collisionless system (on short enough time-scales)

Two-body relaxation time is $\tau_{relax} \sim (N_D/\ln N_D)\,\tau_p$

Here τ_p is the inverse of the plasma frequency, ω_p, which is the natural frequency with which electrons oscillate inside plasma

Although τ_{relax} is much longer than τ_p, it is short on astrophysical time scales (typically a few hours) → plasma is schizophrenic

collisionless on small time and length scales

collisional on large time and length scales

Collective Behavior

Collective behavior is present when individual particles affect many other particles simultaneously

- ✓ Individual star simultaneously affects all other N stars in a galaxy (due to long-range gravitational force)
- ✓ Individual charge simultaneously affects all N_D charges inside its Debye volume (due to long-range Coulomb force)
- ✗ Individual atom in neutral gas at most affects one other particle (that with which it has short-range vdWaals interaction)

Examples of collective behavior:

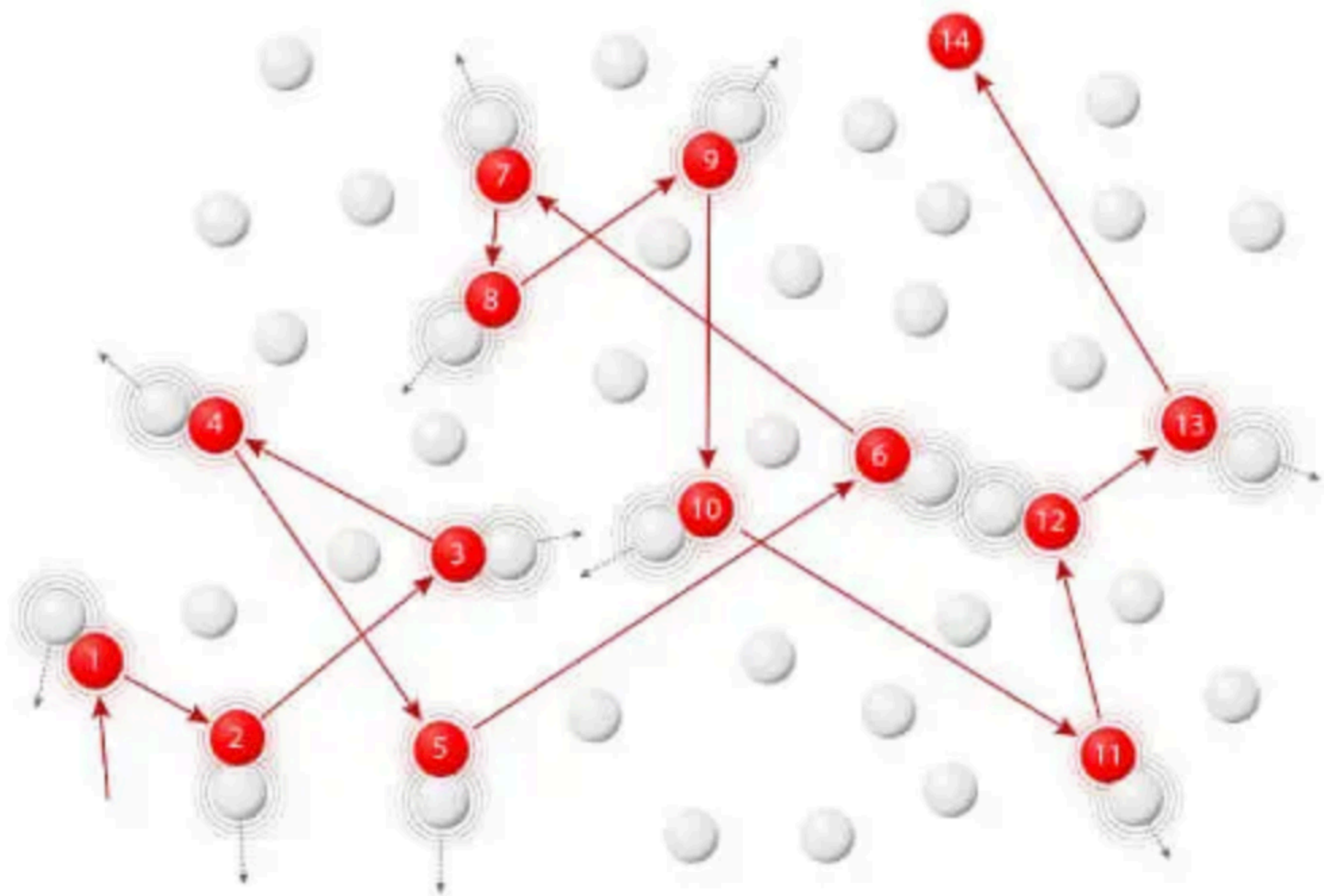
- Plasma oscillation & Langmuir waves
- Landau damping
- Violent relaxation
- Warps, bars, spiral arms

Question: are sound waves in a neutral gas collective behavior?

Particle Trajectories

collisional fluid:

individual particles in liquid or neutral gas execute Brownian motion

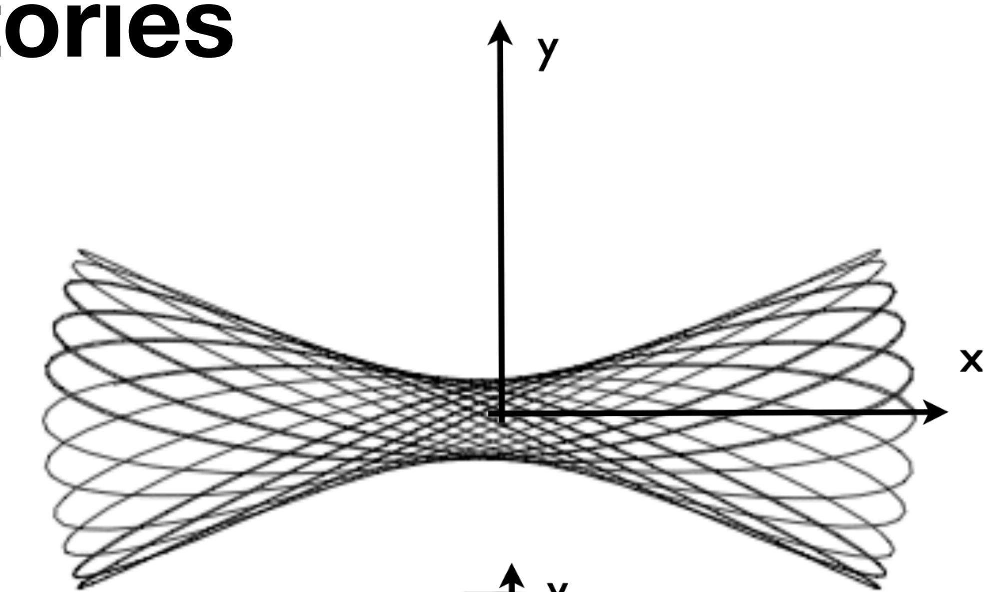


Rapid two-body relaxation assures *local* thermal equilibrium

This assures that velocity distribution is Maxwell-Boltzmann distribution

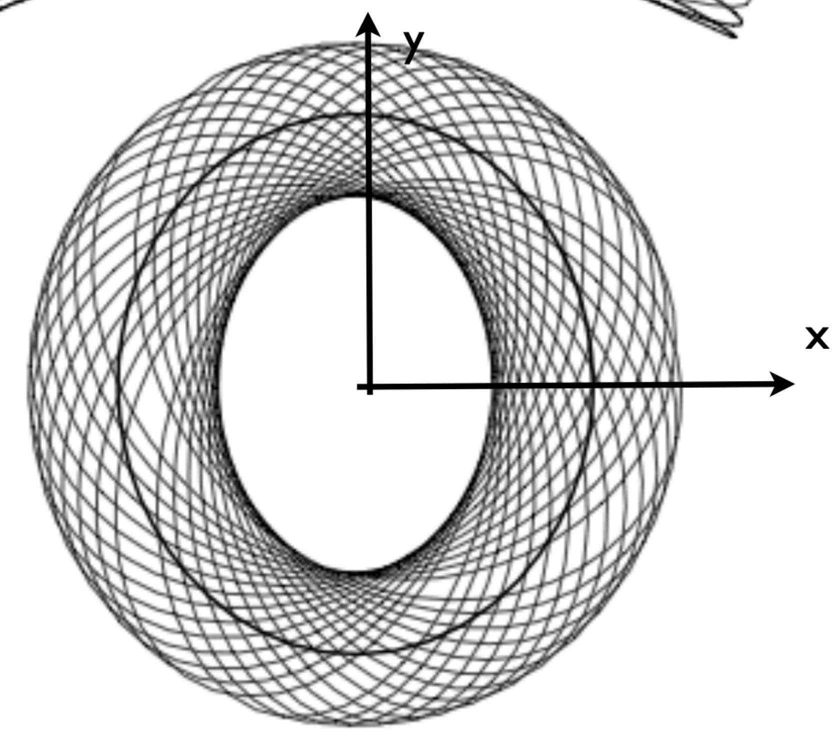
This gives rise to an equation of state (EOS) $P(\rho,T)$

Particle Trajectories

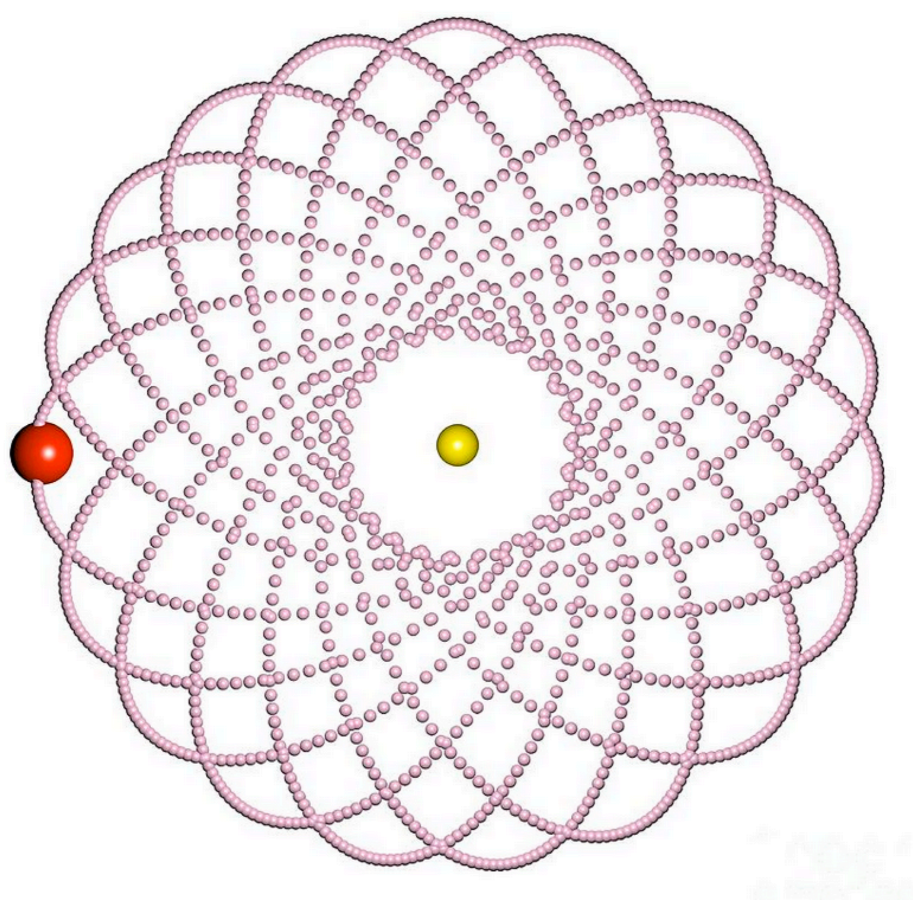


gravitational N-body system:

particle trajectories are smooth orbits in a gravitational potential, conserving integrals of motion



orbits are the `building blocks’ of galaxies



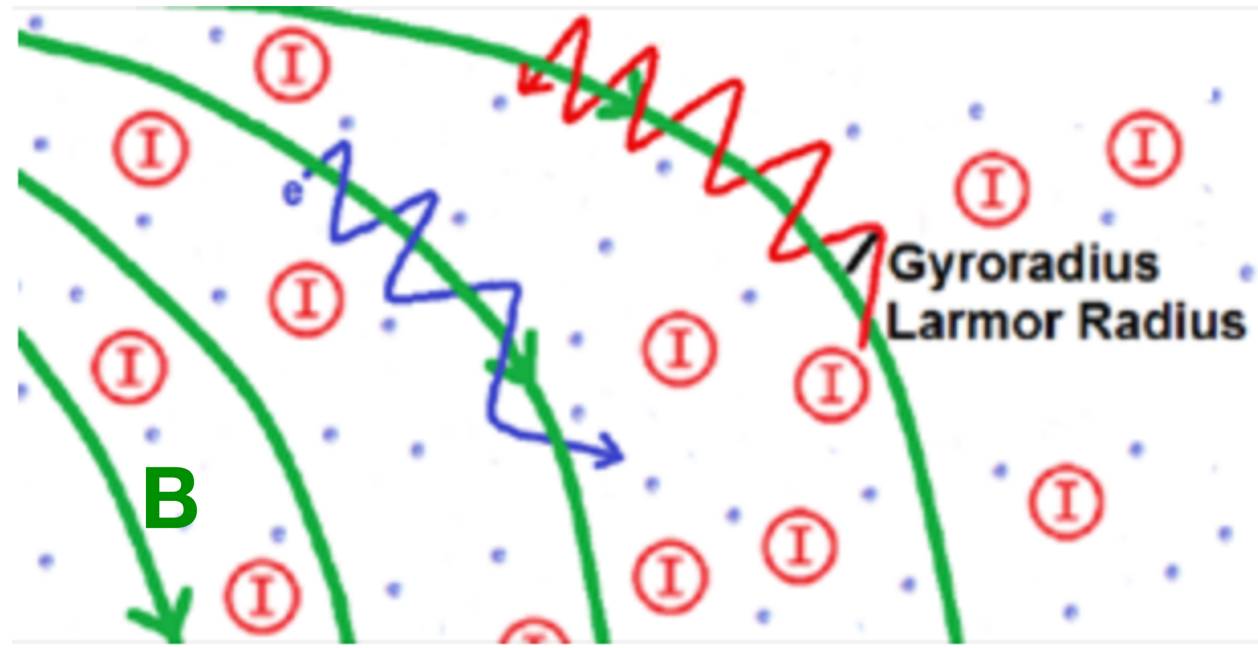
Weak collisionality (due to finite N) gives rise to diffusion in action-angle space (scattering onto `nearby’ orbits in 6D phase space)

Particle Trajectories

plasma:

in presence of magnetic fields, charges experience Lorentz force

$$\mathbf{F} = q\,(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$



This causes charged particles to gyrate magnetic field lines

Motion is combination of gyration plus translatory motion of guiding center.

Larmor radius of ions much larger than that of electrons

Gyrofrequency of ions much lower than that of electrons

Motion of guiding center subject to drift & reflection (magnetic mirror)

Collisional effects cause diffusion across neighboring field lines

Characteristic Length Scales

Hubble Radius	$\lambda_H \simeq c/H_0 \simeq 3000\ h^{-1}$ Mpc	size of Universe
System Size	R	size of many-body system
Jeans Length	$\lambda_J = c_s\,(\pi/G\rho)^{1/2}$	distance sound wave can travel in a dynamical time
Debye Length	$\lambda_D = (k_B T\ /\ 4\pi n_e e^2)^{1/2}$	size of polarization cloud
Mean Free Path	$\lambda_{mfp} \simeq (n\ \sigma)^{-1}$	mean distance between collisions
Mean Interparticle Separation	$\lambda_{int} \simeq n^{-1/3} = R/N^{1/3}$	mean distance between particles
de Broglie wavelength	$\lambda_{dB} = h/p \simeq h/(mk_B T)^{1/2}$	size of quantum wavepacket
Larmor Radius	$\lambda_L = (m\ v_\perp\ c)\ /\ (\lvert q \rvert\ B)$	radius of gyration for charge q in magnetic field of strength B

Characteristic Time Scales

$\tau = \lambda / \langle v \rangle$

Hubble Time	$\tau_H \simeq \lambda_H/c \simeq 9.78\ h^{-1}$ Gyr	roughly the age of the Universe
Crossing Time	$\tau_c = R/\langle v \rangle$	time to cross system
Sound Crossing Time	$\tau_s = R/c_s$	time for sound wave to cross system
Collision Time	$\tau_{coll} = \lambda_{mfp} / \langle v \rangle \simeq (n\sigma\langle v \rangle)^{-1}$	time *between* collisions
Plasma Oscillation Time	$\tau_P = (\pi m_e/n_e e^2)^{1/2} \simeq \lambda_D/\langle v \rangle$	characteristic time for plasma to reach to charge imbalance
Gyration Period	$\tau_{gyro} = 2\pi\lambda_L/v_\perp$	period of gyration around magnetic field line

Characteristic Length & Time Scales

- $\lambda_{int} \gg \lambda_{dB}$ quantum effects can largely be ignored

- $\lambda_{mfp} \gg \lambda_{int}$ collisions well separated in time and space liquids & neutral gases

- A collisionless system that has undergone gravitational collapse has $R \simeq \lambda_J$

proof for collisionless system $\lambda_J = c_s\,(\pi/G\rho)^{1/2} \rightarrow \lambda_J = \sigma\,(\pi/G\rho)^{1/2}$

after gravitational collapse, virialization ensures $2K + W = 0$

$$M\,\sigma^2 - G\,M^2/R = 0$$

$$R = G\,M\,/\,\sigma^2$$

$$R \sim G\,\rho\,R^3\,/\,\sigma^2$$

$$R \sim \sigma\,(1/G\rho)^{1/2}$$

- For a collisionless system, the collision time is ill-defined.

If we approximate collisional cross section as $\sigma = \pi\,b_{90}^2$

then it is easy to show that $\lambda_{mfp} \simeq NR$ and thus $\tau_{coll} \simeq N\,\tau_C \simeq \ln N\;\;\tau_{relax}$

Strong collisions ($b<b_{90}$) are less important than weak collisions ($b>b_{90}$) **!**

Characteristic Length & Time Scales

- The crossing time for a virialized system scales as $\tau_c \simeq (G\rho)^{-1/2}$

> **proof** a virialized system obeys $2K + W = 0 \;\longrightarrow\; M\sigma^2 - G M^2/R = 0$
>
> Hence $\sigma = \langle v \rangle = (GM/R)^{1/2} = (G\rho R^2)^{1/2} \;\longrightarrow\; \tau_c = R / \langle v \rangle \simeq (G\rho)^{-1/2}$

This scaling holds for all dynamical times (free-fall time, orbital time, dynamical time) **!**

- From the definitions of the time scales we have that $R / \lambda_J = \tau_S / \tau_{ff}$

Jeans stability criterion

If $R > \lambda_J$, the system cannot respond hydrodynamically to changes in gravity, and the system is unstable to gravitational collapse

- **Plasma frequency** $\omega_P = 2\pi / \tau_P = (4\pi n_e e^2 / m_e)^{1/2}$

the *natural* frequency with which a plasma responds to charge separation

Plasmas are opaque to EM waves with $\omega < \omega_P$

Earth's ionosphere reflects radio waves with $\nu \lesssim 10$ MHz

ISM is opaque to cyclotron emission ($\omega_C < \omega_P$, unless $B >$ several Tesla)

Conducting metals (~plasma) opaque to visual, but transparent to UV

Relaxation Time

the time scale on which a system that is perturbed returns to equilibrium

Different mechanisms can cause virialization:

Two-body relaxation: due to two-body interactions/collisions

drives system towards (local) thermal equilibruim with MB distribution

collisional fluid	$\tau_{relax} \sim \tau_{coll}$
collisionless fluid	$\tau_{relax} \sim (N/\ln N)\, \tau_{cross}$
plasma	$\tau_{relax} \sim (N_D/\ln N_D)\, \tau_p$

Collective effects: Violent Relaxation

Landau Damping

Bars/Spiral arms

$\tau_{collective} \ll \tau_{relax}$ **!**

all these processes involve some form of wave-particle interaction, in which waves/oscillations lose energy to the particles, thereby damping away (=relaxation)

Phase-mixing

loss of coherence in response due to different phases of the particles involved.

Local vs. Global Thermal Equilibrium

In collisional fluid, two-body interactions drive system towards *local* thermal equilibrium on two-body relaxation time $\tau_{\rm relax} \sim \tau_{\rm coll}$

Collisions also act to erase gradients in number density, velocity (shear), temperature etc, which thus tries to establish *global* thermal equilibrium.

Time-scale for this is governed by transport coefficients

diffusion coefficient	(transport of number density)
viscosity	(transport of momentum)
conduction	(transport of energy/heat)

All these mechanisms are diffusive in nature

flux = transport coefficient x gradient in property

In absence of gravity, global thermal equilibrium is the highest entropy state

Formally, global TE is only achievable if it involves the entire Universe…

Local vs. Global Thermal Equilibrium

In collisionless fluids, collective effects drive system towards a semi-global equilibrium on a time-scale much shorter than two-body relaxation time.

These equilibria are only quasi-equilibria (evolution on two-body relaxation time)

These quasi-equilibria are typically poorly understood;

Galaxies and dark matter halos appear relaxed, often with characteristic density profiles (i.e., universal NFW profile of CDM halos). It is not understood why nature selects these quasi-equilibrium configurations

On two-body relaxation time, gravitational system evolves to *global* thermal equilibrium (*global* rather than *local* due to long-range interparticle force).

However, Maxwell-Boltzmann distribution has tail to velocities $v > v_{esc}$

Hence, two-body relaxation causes evaporation, which implies that thermal equilibrium is unachievable.

Time scale for evaporation $\tau_{evap} \sim 100\ \tau_{relax}$

Local vs. Global Thermal Equilibrium

Typically, velocity dispersion of gravitational N-body system is higher in its center than in its outskirts

Two-body collisions cause conduction of heat from inside to outside

Gravitational systems have negative heat capacity $C = dE/dT = -3/2\, N\, k_B$

adding heat, causes cooling
extracting heat, causes heating

Conduction causes center to become hotter (velocity dispersion increases)

This increases temperature gradient, which increases conductive transport

→ gravo-thermal catastrophe → core collapse

Another indication that thermal equilibrium is unattainable

These processes play a role in globular clusters and in SIDM halos