ASTR 501

Dynamics of Astrophysical Many-Body Systems

a consice treatment of fluid dynamics,
collisionless dynamics & plasma physics,
with an application to astrophysics

Instructor : Prof. Frank van den Bosch (Office: KT 649)
e-mail: frank.vandenbosch@yale.edu

Course Website : https://campuspress.yale.edu/astro501/
https://campuspress.yale.edu/astro501/

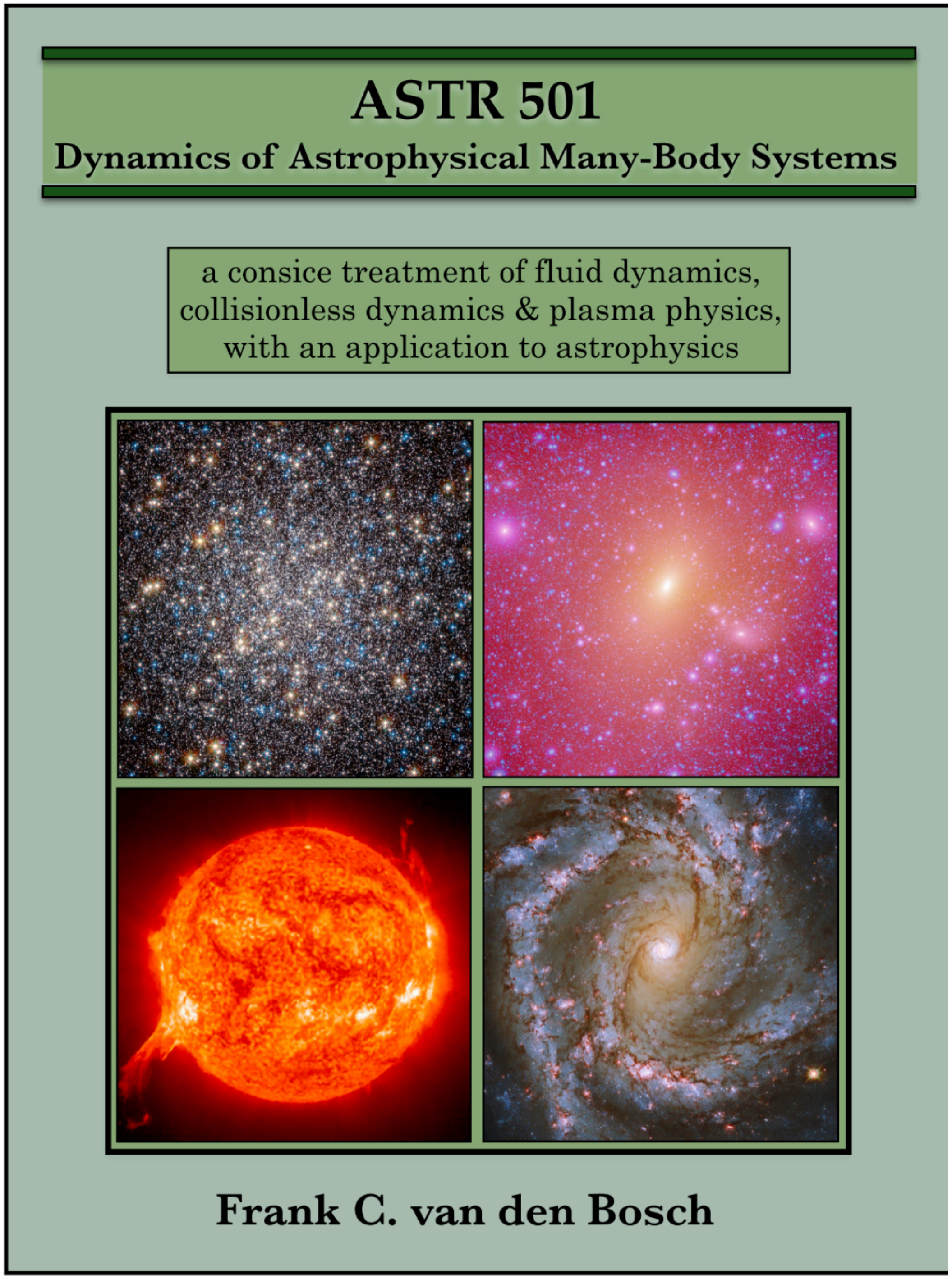
Lecture Hours : TTh 9.00-10.15am **Location** KT 221 (Kline Tower)

Office Hours : TBD **Location** KT 649 (Kline Tower)

Grading : 35% Final Exam + 35% Problem Sets + 30% MidTerm Exam

Course Format : Blackboard presentations & in-class discussion

Lecture Notes



Recommended Textbooks

APPENDICES & WORKSHEETS

Astrophysical Fluids

A fluid is a substance that can flow
in which particles can flow freely past one another
with no fixed shape (takes on shape of its container)
shows little resistance to external shear

In ASTR 501 we study three different types of fluids:

[1] Collisional Fluids: interparticle force is short range
2-body collisions well separated in time & space
liquids and neutral gases

[2] Collisionless Fluids: interparticle force is long range
gravitational N-body systems
galaxies (stellar component), CDM halos

[3] Plasmas: interparticle force is long range
can be both collisional and collisionless
~90 percent of the baryonic Universe

Dynamical Models

A dynamical model consists of two components:

- a description of the state
- equations that describe the evolution of the state

There are different `levels' of dynamical models for fluids:

Level	**Description of state**	**Dynamical equations**
0: N quantum particles	$\psi(\vec{x}_1, \vec{x}_2, ..., \vec{x}_N)$	Schrödinger equation
1: N classical particles	$(\vec{x}_1, \vec{x}_2, ..., \vec{x}_N, \vec{v}_1, \vec{v}_2, ..., \vec{v}_N)$	Hamiltonian equations
2: Distribution function	$f(\vec{x}, \vec{v}, t)$	Boltzmann equation
3: Continuum model	$\rho(\vec{x})$, $\vec{u}(\vec{x})$, $P(\vec{x})$, $T(\vec{x})$	Hydrodynamic equations

Level	Description of state	Dynamical equations
0: N quantum particles	$\psi(\vec{x}_1, \vec{x}_2, ..., \vec{x}_N)$	Schrödinger equation
1: N classical particles	$(\vec{x}_1, \vec{x}_2, ..., \vec{x}_N, \vec{v}_1, \vec{v}_2, ..., \vec{v}_N)$	Hamiltonian equations
2: Distribution function	$f(\vec{x}, \vec{v}, t)$	Boltzmann equation
3: Continuum model	$\rho(\vec{x}),\ \vec{u}(\vec{x}),\ P(\vec{x}),\ T(\vec{x})$	Hydrodynamic equations

Level 0: unfeasible (if N large), but also unneccesary as long as $\lambda_{int} >> \lambda_{dB}$
(the interparticle separation is larger than the de Broglie wavelength)

exceptions: white dwarfs, neutron stars, ultralight axions

Level 1: state described by the 6N phase-space parameters of N particles

Dynamics described by Hamiltonian $\mathcal{H}(\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_N,\mathbf{p}_1,\mathbf{p}_2\ldots\mathbf{p}_N)$

This is the approach used in N-body simulations, albeit with N significantly downsampled

Plasma physics makes use of Particle-in-Cell (PIC) simulations where each N-body particle represents a single charged particle

Level	Description of state	Dynamical equations
0: N quantum particles	$\psi(\vec{x}_1, \vec{x}_2, ..., \vec{x}_N)$	Schrödinger equation
1: N classical particles	$(\vec{x}_1, \vec{x}_2, ..., \vec{x}_N, \vec{v}_1, \vec{v}_2, ..., \vec{v}_N)$	Hamiltonian equations
2: Distribution function	$f(\vec{x}, \vec{v}, t)$	Boltzmann equation
3: Continuum model	$\rho(\vec{x}),\ \vec{u}(\vec{x}),\ P(\vec{x}),\ T(\vec{x})$	Hydrodynamic equations

Level 2: Kinetic Theory; state is described by 1-particle distribution function

$$f(\mathbf{x},\mathbf{v},t) = \frac{d^6N}{d^3\mathbf{x}\, d^3\mathbf{v}}$$

Equation that describes evolution of $f(\mathbf{x},\mathbf{v},t)$ depends on type of fluid:

collisionless fluid: collisionless Boltzmann equation (CBE) aka Vlasov equation — $df/dt = 0$

collisional fluid:

short-range forces: Boltzmann equation — $df/dt = I[f]$

long-range forces: (weak only) Fokker-Planck equation — $df/dt = (\partial f/\partial t)_c$

plasma:

collisionless: Vlasov eq. — $df/dt = 0$

collisional: Lenard-Balescu eq. — $\partial f/\partial t = F(f)$

Level	Description of state	Dynamical equations
0: N quantum particles	$\psi(\vec{x}_1, \vec{x}_2, ..., \vec{x}_N)$	Schrödinger equation
1: N classical particles	$(\vec{x}_1, \vec{x}_2, ..., \vec{x}_N, \vec{v}_1, \vec{v}_2, ..., \vec{v}_N)$	Hamiltonian equations
2: Distribution function	$f(\vec{x}, \vec{v}, t)$	Boltzmann equation
3: Continuum model	$\rho(\vec{x}),\ \vec{u}(\vec{x}),\ P(\vec{x}),\ T(\vec{x})$	Hydrodynamic equations

Level 3: Continuum model; state is described in terms of continuous fields, rather than discrete particles

state described by: $\rho(\mathbf{x})$, $\mathbf{u}(\mathbf{x})$, $P(\mathbf{x})$, $T(\mathbf{x})$

evolution described by: velocity moments of the (collisionless) Boltzmann equation

$\int d\mathbf{v}\ v^0\ (df/ft)$ → continuity equation

$\int d\mathbf{v}\ v^1\ (df/ft)$ → momentum equations

$\int d\mathbf{v}\ v^2\ (df/ft)$ → energy equations

Level	Description of state	Dynamical equations
0: N quantum particles	$\psi(\vec{x}_1, \vec{x}_2, ..., \vec{x}_N)$	Schrödinger equation
1: N classical particles	$(\vec{x}_1, \vec{x}_2, ..., \vec{x}_N, \vec{v}_1, \vec{v}_2, ..., \vec{v}_N)$	Hamiltonian equations
2: Distribution function	$f(\vec{x}, \vec{v}, t)$	Boltzmann equation
3: Continuum model	$\rho(\vec{x})$, $\vec{u}(\vec{x})$, $P(\vec{x})$, $T(\vec{x})$	Hydrodynamic equations

Level 3: Continuum model; state is described in terms of continuous fields, rather than discrete particles

collisional fluid:

momentum equations are called Navier-Stoker equations

in case of ideal fluid, these become the Euler equations

(no viscosity or conductivity)

set of hydrodynamic equations can be closed by using using an equation of state (EOS) $P(\rho,T)$

Level	**Description of state**	**Dynamical equations**
0: N quantum particles	$\psi(\vec{x}_1, \vec{x}_2, ..., \vec{x}_N)$	Schrödinger equation
1: N classical particles	$(\vec{x}_1, \vec{x}_2, ..., \vec{x}_N, \vec{v}_1, \vec{v}_2, ..., \vec{v}_N)$	Hamiltonian equations
2: Distribution function	$f(\vec{x}, \vec{v}, t)$	Boltzmann equation
3: Continuum model	$\rho(\vec{x})$, $\vec{u}(\vec{x})$, $P(\vec{x})$, $T(\vec{x})$	Hydrodynamic equations

Level 3: Continuum model; state is described in terms of continuous fields, rather than discrete particles

collisionless fluid:

momentum equations are called Jeans equations

collisionless systems lack equation of state: no closure

Jeans equations can only be solved imposing symmetries (e.g., system is spherical and/or isotropic)

Level	Description of state	Dynamical equations
0: N quantum particles	$\psi(\vec{x}_1, \vec{x}_2, ..., \vec{x}_N)$	Schrödinger equation
1: N classical particles	$(\vec{x}_1, \vec{x}_2, ..., \vec{x}_N, \vec{v}_1, \vec{v}_2, ..., \vec{v}_N)$	Hamiltonian equations
2: Distribution function	$f(\vec{x}, \vec{v}, t)$	Boltzmann equation
3: Continuum model	$\rho(\vec{x})$, $\vec{u}(\vec{x})$, $P(\vec{x})$, $T(\vec{x})$	Hydrodynamic equations

Level 3: Continuum model; state is described in terms of continuous fields, rather than discrete particles

plasma:

collisionless regime: take moments of Vlasov equation

two-fluid model (electrons + ions)

collisional regime: hydrodynamic equations + Maxwell equations

one-fluid model (MHD)

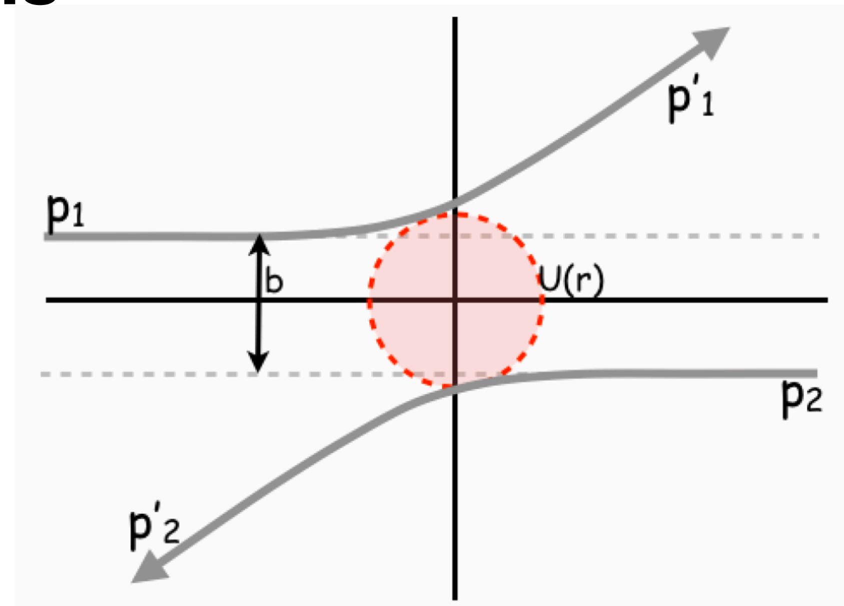
Preliminary Schedule

week	Date	Topic
1	Thu 08/31	**Theoretical Foundations:** Introduction
2	Tue 09/05	**Theoretical Foundations:** Classical Dynamics; a primer
2	Thu 09/07	**Theoretical Foundations:** Hamilton-Jacobi Theory
3	Tue 09/12	**Theoretical Foundations:** Action-Angle Variables
3	Thu 09/14	**Kinetic Theory:** from Liouville to Boltzmann
4	Tue 09/19	**Kinetic Theory:** from Boltzmann to Navier-Stokes
4	Thu 09/21	**Kinetic Theory:** stochasticity & Langevin equation
5	Tue 09/26	**Kinetic Theory:** the Fokker-Planck equation
5	Thu 09/28	**Fluid Dynamics:** Introduction to hydrodynamics
6	Tue 10/03	**Fluid Dynamics:** Transport Mechanisms & Constitutive equations
6	Thu 10/05	**Fluid Dynamics:** hydrodynamic equations
7	Tue 10/10	Fluid Dynamics: vorticity & circulation
7	Thu 10/12	**Fluid Dynamics:** hydrostatics and steady flows
8	Tue 10/17	**Fluid Dynamics:** viscous flow and accretion flow
8	Thu 10/19	**NO CLASS** **[OCTOBER RECESS]**

9	Tue 10/24	**MIDTERM EXAM**
9	Thu 10/26	**Fluid Dynamics:** turbulence
10	Tue 10/31	**Fluid Dynamics:** sound waves & shocks
10	Thu 11/02	**Fluid Dynamics:** fluid instabilities
11	Tue 11/07	**Collisionless Dynamics:** Jeans equations & dynamical modeling
11	Thu 11/09	**Collisionless Dynamics:** virial theorem & gravothermal catastrophe
12	Tue 11/14	**Collisionless Dynamics:** collisions & encounters
12	Thu 11/16	**Plasma Physics:** introduction
13	Tue 11/21	**NO CLASS** **[THANKSGIVING BREAK]**
13	Thu 11/23	**NO CLASS** **[THANKSGIVING BREAK]**
14	Tue 11/28	**Plasma Physics:** plasma orbit theory
14	Thu 11/30	**Plasma Physics:** Plasma kinetic theory
15	Tue 12/05	**Plasma Physics:** Vlasov equation & the two-fluid model
15	Thu 12/07	**Plasma Physics:** magnetohydrodynamics (MHD)

Collisions

Collisions are characterized by an impact parameter, b, incoming momenta, $\mathbf{p}_1$ & $\mathbf{p}_2$, and an interaction potential, $U(\mathbf{r})$



In ASTR501 we assume all collisions to be elastic ➡ Hamiltonian dynamics
We thus ignore radiative processes

collisional fluid:

interparticle force is short-range van der Waals force (r^{-7})

mean free path ≫ mean particle separation ($\lambda_{mfp} \gg \lambda_{int}$)

collisions are well separated in space and time

collisions drive system towards *local* thermal equilibrium
in which velocity distribution become Maxwell-Boltzmann (MB)

two-body relaxation time $\tau_{relax} = \tau_{coll} \simeq \lambda_{mfp} / \langle v \rangle$ is extremely short