CHAPTER 25

The Interaction of Light with Matter: III - Extinction

As we have seen, there are numerous processes by which a photon can interact with matter. It is useful to define the **mean-free path**, l, for a photon and the related **opacity** and **optical depth**.

Opacity: a measure for the impenetrability to electro-magnetic radiation due to the combined effect of scattering and absorption. If the opacity is caused by *dust* we call it **extinction**.

Optical Depth: the dimensionless parameter, τ_{ν} , describing the opacity/extinction at frequency ν . In particular, the infinitesimal increase in optical depth along a line of sight, $d\tau_{\nu}$, is related to the infinitesimal path length dl according to

$$\mathrm{d}\tau_{\nu} = \sigma_{\nu} \, n \, \mathrm{d}l = \kappa_{\nu} \, \rho \, \mathrm{d}l = \alpha_{\nu} \, \mathrm{d}l$$

Here σ_{ν} is the effective cross section $([\sigma_{\nu}] = \text{cm}^2)$, κ_{ν} is the mass absorption coefficient $([\kappa_{\nu}] = \text{cm}^2 \text{g}^{-1})$, α_{ν} is the absorption coefficient $([\alpha_{\nu}] = \text{cm}^{-1})$, and n and ρ are the number and mass densities, respectively. The optical depth to a source at distance d is therefore

$$\tau_{\nu} = \int_0^d \mathrm{d}\tau_{\nu} = \int_0^d \kappa_{\nu}(l) \,\rho(l) \,\mathrm{d}l$$

The ISM/IGM between source and observer is said to be optically thick (thin) if $\tau_{\nu} > 1$ ($\tau_{\nu} < 1$).

Opacity/extinction reduces the intensity of a source according to

$$I_{\nu,\rm obs} = I_{\nu,0} e^{-\tau_{\rm i}}$$

where $I_{\nu,0}$ is the unextincted intensity (i.e., for $\tau_{\nu} = 0$).

Rosseland Mean Opacities: In the case of stars opacity is crucially important for understanding stellar structure. Opacities within stars are typically expressed in terms of the **Rosseland mean opacities**, $\overline{\kappa}$, which is a weighted average of κ_{ν} over frequency. Typically, one finds that $\overline{\kappa} \propto \rho T^{-3.5}$, which is known as **Kramer's opacity law**, and is a consequence of the fact that the opacity is dominated by bound-free and/or free-free absorption. A larger opacity implies stronger radiation pressure, which gives rise to the concept of the Eddington luminosity.

Eddington Luminosity: the maximum luminosity a star (or, more general, emitter) can achieve before the star's radiation pressure starts to exceed the force of gravity.

Consider an outer layer of a star with a thickness l such that $\tau_{\nu} \simeq \kappa_{\nu} \rho l = 1$. Then, a photon passing this layer will be absorbed and contribute to radiation pressure. The resulting force excerted on the matter is

$$F_{\rm rad} = \frac{L}{c}$$

This has to be compared to the gravitational force

$$F_{\rm grav} = \frac{G M_* m_{\rm layer}}{r^2}$$

Using that $m_{\text{layer}} = 4\pi r^2 l\rho = 4\pi r^2 / \kappa_{\nu}$, we find that $F_{\text{rad}} = F_{\text{grav}}$ if the luminosity is equal to

$$L_{\rm Edd} = \frac{4\,\pi\,G\,M_*\,c}{\kappa_{\nu}}$$

which is called the **Eddington luminosity**. Stars with $L > L_{Edd}$ cannot exist, as they would blow themselves appart ($F_{rad} > F_{grav}$). The most massive stars known have luminosities that are very close to their Eddington luminosity.

Since the luminosities of AGN (supermassive black holes with accretion disks) are set by their accretion rate, the same argument implies an upper limit to the accretion rate of AGN, known as the **Eddington limit**.



Figure 24: Empirical extinction laws, defined in terms of the ratio of color excesses, for the Milky Way (MW), the Large Magellanic Cloud (LMC) and the Small Magellanic Cloud (SMC). Note the strong feature around 2100 Å in the MW extinction law, believed to be due to graphite dust grains.

Extinction by Dust: Dust grains can scatter and absorb photons. Their ability to do so depends on (i) grain size, (ii) grain composition, and (iii) the presence of a magnetic field, which can cause grain alignment. Observationally, the extinction in the V-band is defined by

$$A_V \equiv -2.5 \log\left(\frac{f_V}{f_{V,0}}\right) = -2.5 \log\left(\frac{I_V}{I_{V,0}}\right)$$

where the subscript zero refers to the unextincted flux/intensity. Using that $I_V = I_{V,0} e^{-\tau_V}$ we have that

$$A_V = 1.086 \,\tau_V$$

More generally, $A_{\lambda} = 1.086 \tau_{\lambda}$; hence, an optical depth of unity roughly corresponds to an extinction of one magnitude.

Reddening: In addition to extinction, dust also causes reddening, due to the fact that dust extinction is more effective at shorter (bluer) wavelengths.

Color Excess: $E(B - V) \equiv A_B - A_V$, which can also be defined for any other wavebands.

Extinction law: conventionally, dust extinction is expressed in terms of an *empirical* **extinction law**:

$$k(\lambda) \equiv \frac{A_{\lambda}}{E(B-V)} \equiv R_V \frac{A_{\lambda}}{A_V}$$

where

$$R_V \equiv \frac{A_V}{E(B-V)}$$

is a quantity that is insensitive to the total amount of extinction; rather it expresses a property of the extinction law. Empirically, dust in the Milky Way seems to have $R_V \simeq 3.1$, while the dust in the Small Magellanic Cloud (SMC) is better characterized by $R_V \simeq 2.7$. Dust extinction law is not universal; rather, it is believed to depend on the local UV flux and the metallicity, among others (see Fig. 19).

Theoretical attempts to model the extinction curve have shown that dust comes in two varieties, graphites and silicates, while the grain-size distribution is well fitted by $dN/da \propto a^{-3.5}$ and covers the range from $\sim 0.005 \mu m$ to $\sim 0.25 \mu m$. Note that for radiation with $\lambda > a_{max} \simeq 2500$ Å dust mainly causes **Rayleigh scattering**.