CHAPTER 24

The Interaction of Light with Matter: II - Absorption

Absorption: The absorption of photons can have three effects:

- **heating of the absorbing medium** (heating of dust grains, or excitation of gas followed by collisional de-excitation)
- **acceleration of absorbing medium** (radiation pressure)
- **change of state of absorbing medium** (ionization, sublimation or dissociation)

Note that ionization (transition from neutral to ionized), sublimation (transition from solid to gas) and dissociation (transition from molecular to atomic) can also occur as a consequence of particle collisions. Therefore one often uses terms such as photo-ionization and collisional ionization to distinguish between these.

Photoionization: Photoionization is the process in which an atom is ionized by the absorption of a photon. For hydrogen, this is

$$\mathrm{HI} + \gamma \rightarrow \mathrm{p} + \mathrm{e}\,,$$

where HI denotes a neutral hydrogen atom. The photoionization rate, $\Gamma_{\gamma,\mathrm{H}}$, is proportional to the number density of ionizing photons and to the photoionization cross section, $\sigma_{\mathrm{pi}}(\nu)$, according to:

$$\Gamma_{\gamma,\mathrm{H}} = \int_{\nu_{\mathrm{t}}}^{\infty} c\,\sigma_{\mathrm{pi}}(\nu)\,\mathcal{N}_{\gamma}(\nu)\,\mathrm{d}\nu$$

where ν_{t} is the threshold frequency for ionization (corresponding to 13.6eV in the case of hydrogen). $\mathcal{N}_{\gamma}(\nu)\mathrm{d}\nu$ in the above equation is the number density of photons with frequencies in the range ν to $\nu + \mathrm{d}\nu$, and is related to the energy flux of the radiation field, $J(\nu)$, by

$$\mathcal{N}_{\gamma}(\nu) = \frac{4\,\pi\,J(\nu)}{c\,h\,\nu}\,.$$

The photoionization cross sections can be obtained from quantum electrodynamics by calculating the bound-free transition probability of an atom in a radiation field (see e.g., `Rybicki & Lightman 1979`).

Recombination: Recombination is the process by which an ion recombines with an electron. For hydrogen ions (i.e. protons), the process is

$$\mathrm{p} + \mathrm{e} \rightarrow \mathrm{HI} + \gamma \,.$$

For hydrogen (or a hydrogenic ion, i.e., an ion with a single electron), the **recombination cross section** to form an atom (or ion) at level n, $\sigma_{\rm rec}(v, n)$, is related to the corresponding **photoionization cross section** by the **Milne relation**:

$$\sigma_{\rm rec}(v, n) = \frac{g_n}{g_{n+1}} \left(\frac{h\,\nu}{m_{\rm e}\, c\, v} \right)^2 \sigma_{\rm pi}(\nu, n) \,,$$

where $g_n = 2n^2$ is the statistical weight of energy level n and ν and v are related by $m_{\rm e} v^2/2 = h(\nu - \nu_n)$, with $h\nu_n$ the threshold energy required to ionize an atom whose electron sits in energy state n. The **recombination coefficient** for a given level n is the product of the capture cross section and velocity, $\sigma_{\rm rec}(v, n)\, v$, averaged over the velocity distribution $f(v)$. For an optically thin gas where all photons produced by recombination can escape without being absorbed, the total recombination coefficient is the sum over all n:

$$\alpha_{\rm A} = \sum_{n=1}^{\infty} \alpha_n = \sum_{n=1}^{\infty} \int \sigma_{\rm rec}(v, n)\, v\, f(v)\, {\rm d}v$$

This is called the **Case A recombination coefficient**, to distinguish it from the **Case B** recombination in an optically thick gas. In Case B, recombinations to the ground level generate ionizing photons that are absorbed by the gas, so that they do not contribute to the overall ionization state of the gas. It is easy to see that the **Case B recombination coefficient** is $\alpha_{\rm B} = \alpha_{\rm A} - \alpha_1$.

Strömgren sphere: A sphere of ionized hydrogen (H II) around an ionizing source (e.g., AGN, O or B star, etc.). Ionization of hydrogen (from the ground state) requires a photon energy of at least 13.6eV, which implies UV photons. In a (partially) ionized medium, electrons and nuclei recombine to produce neutral atoms. The region around an ionizing source will ultimately establish **ionization equilibrium** in which the number of **ionizations** is equal to the number of **recombinations**.

Consider an ionizing source in a uniform medium of pure hydrogen. Let $\dot{N}_{\rm ion}$ be the number of ionizing photons produced per second. The corresponding recombination rate is given by

$$\dot{N}_{\rm rec} = n_{\rm e}\, n_{\rm p}\, \alpha_{\rm rec}\, V = n_{\rm e}^2\, \alpha_{\rm B}\, \frac{4}{3}\pi R_{\rm s}^3$$

where we have used that, for a pure hydrogen gas, $n_{\rm e} = n_{\rm p}$, and $R_{\rm s}$ is the radius of the **Strömgren sphere** (i.e., the radius of the sphere that is going to be ionized), which can be written as

$$R_{\rm s} = \left(\frac{3\,\dot{N}_{\rm ion}}{4\,\pi\,\alpha_{\rm B}\,n_{\rm e}^2}\right)^{1/3}$$

Using that the luminosity of the ionizing source, L_*, is related to its surface intensity, I_*, according to

$$L_* = 4\,\pi\,R_*^2\,F_* = 4\,\pi^2\,R_*^2\,I_*$$

where R_* is the radius of the ionizing source (i.e., an O-star) and we have used that $F_* = \pi\,I_*$ (see Chapter 20). Hence, we have that

$$\dot{N}_{\rm ion} = 4\,\pi^2\,R_*^2 \int_{\nu_{\rm t}}^{\infty} \frac{B_\nu(T)}{h\,\nu}\,{\rm d}\nu = \frac{\pi\,L_*}{\sigma_{\rm SB} T_{\rm eff}^4} \int_{\nu_{\rm t}}^{\infty} \frac{B_\nu(T)}{h\,\nu}\,{\rm d}\nu$$

where we have assumed that the ionizing source is a Black Body of temperature T, and, in the second part, that $L_* = 4\pi R_*^2 \sigma_{\rm SB} T_{\rm eff}^4$.

Thus, by measuring the luminosity and effective temperature of a star, and the radius of its Strömgren sphere, one can infer the (electron) density of its surroundings,