CHAPTER 23

The Interaction of Light with Matter: I - Scattering

Understanding radiative processes, and the interaction of photons with matter, it is important to realize that *all* photon emission mechanisms arise from **accelerating electrical charge**.

The interactions of light with matter can be split in two categories:

- **scattering** (photon + matter $\rightarrow$ photon + matter)
- **absorption** (photon + matter $\rightarrow$ matter)

We first discuss **scattering**, which gives rise to a number of astrophysical phenomena:

- reflection nebulae (similar to looking at street-light through fog)
- light echos
- polarization
- Ly-α forest in quasar spectra

The **scattering cross-section**, $\sigma_{\rm s}$, is a hypothetical area ($[\sigma_{\rm s}] = {\rm cm}^2$) which describes the likelihood of a photon being scattered by a target (typically an electron or atom). In general, the scattering cross-section is different from the geometrical cross-section of the particle, and it depends upon the frequency of the photon, and on the details of the interaction (see below).

It is useful to split scattering in **elastic (coherent) scattering**, in which the photon energy is unchanged by the scattering event, and **inelastic (incoherent) scattering**, in which the photon energy changes.

Elastic scattering comes in three forms:

- **Thomson scattering** $\gamma + \mathrm{e} \rightarrow \gamma + \mathrm{e}$
- **Resonant scattering** $\gamma + \mathrm{X} \rightarrow X^{+} \rightarrow \gamma + \mathrm{X}$
- **Rayleigh scattering** $\gamma + \mathrm{X} \rightarrow \gamma + \mathrm{X}$

Here γ indicates a photon, e a free electron, X an atom or ion, and X^{+} an excited state of X.

Inelastic scattering comes in two forms:

- **Compton scattering** $\gamma + \mathrm{e} \rightarrow \gamma' + \mathrm{e}'$
- **Fluorescence** $\gamma + \mathrm{X} \rightarrow X^{++} \rightarrow \gamma' + X^{+} \rightarrow \gamma' + \gamma'' + \mathrm{X}$

Here accents indicate that the particle has a different energy (i.e., γ' is a photon with a different energy than γ), and X^{++} indicates a higher-excited state of X than X^{+}.

In what follows we discuss each of these five processes in more detail.

Thomson scattering: is the elastic (coherent) scattering of electromagnetic radiation by a free charged particle, as described by classical electromagnetism. It is the low-energy limit of **Compton scattering** in which the particle kinetic energy and photon frequency are the same before and after the scattering. In Thomson scattering the electric field of the incident wave (photon) accelerates the charged particle, causing it, in turn, to emit radiation at the same frequency as the incident wave, and thus the wave is scattered. The particle will move in the direction of the oscillating electric field, resulting in <u>electromagnetic dipole radiation</u> that appears **polarized** unless viewed in the forward or backward scattered directions (see Fig. 14).

The cross-section for Thomson scattering is the **Thomson cross section:**

$$\sigma_{\mathrm{s}} = \sigma_{\mathrm{T}} = \frac{8\pi}{3}\, r_{\mathrm{e}}^{2} = \frac{8\pi e^{4}}{3 m_{\mathrm{e}}^{2} c^{4}} \simeq 6.65 \times 10^{-25} \mathrm{cm}^{2}$$

Note that this cross section is independent of wavelength!

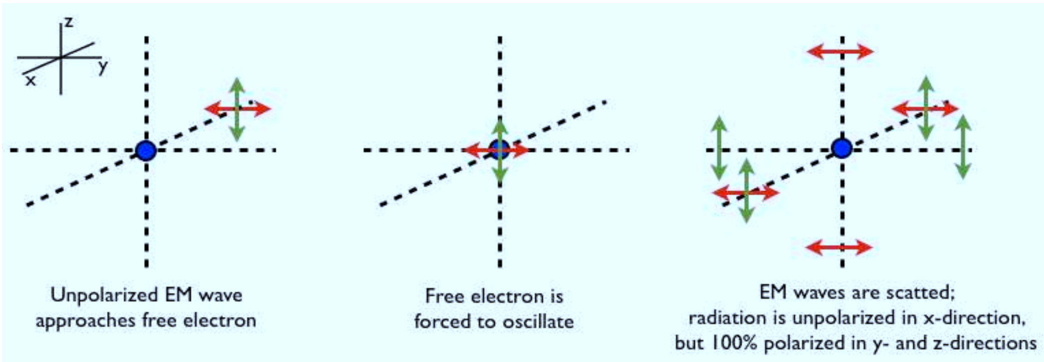

Figure 19: *Illustration of how Thomson scattering causes polarization in the directions perpendicular to that of the incoming EM radiation. The incoming EM wave causes the electron to oscillate in the direction of the oscillation of the $\vec{E}$-field. This acceleration of the electrical charge results in the emission of dipolar EM radiation.*

In the quantum mechanical view of radiation, electromagnetic waves are made up of photons which carry both energy ($h\,\nu$) and momentum ($h\,\nu/c$). This implies that during scattering the photon exchanges momentum with the electron, causing the latter to recoil. This **recoil** is negligble, until the energy of the incident photon becomes comparable to the rest-mass energy of the electron, in which case Thomson scattering becomes Compton scattering.

Compton scattering: is an inelastic scattering of a photon by a free charged particle, usually an electron. It results in a decrease of the photon's energy/momentum (increase in wavelength), called the Compton effect. Part of the energy/momentum of the photon is transferred to the scattering electron ('recoil'). In the case of scattering off of *electrons at rest*, the Compton effect is only important for high-energy photons with $E_\gamma > m_{\rm e}c^2 \sim 0.511\,{\rm MeV}$ (X-ray and/or gamma ray photons).

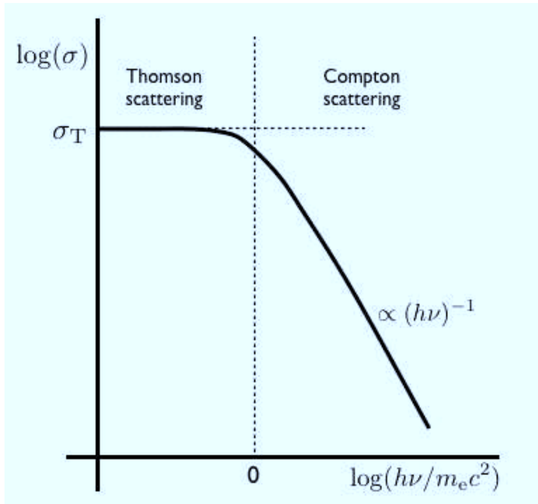

Figure 20: *The Klein-Nishina cross section for Compton scattering. As long as $h\nu \ll m_e c^2$ one is in the Thomson scattering regime, and $\sigma_s = \sigma_T$. However, once the photon energy becomes comparable to the rest-mass energy of the electron, Compton scattering takes over, and the cross-section (now called the Klein-Nishina cross-section), starts to drop asν^{-1}.*

Because of the recoil effect, the energy of the outgoing photon is

$$E'_\gamma = \frac{E_\gamma}{1 + \frac{E_\gamma}{m_e c^2}(1 - \cos\theta)}$$

where θ is the angle between in incident and outgoing photon. This can also be written as

$$\lambda' - \lambda = \lambda_C \,(1 - \cos\theta)$$

which expresses that Compton scatter increases the wavelength of the photon by of order the **Compton wavelength** $\lambda_C = h/(m_e c) \sim 2.43 \times 10^{-10}$cm. If $\lambda \gg \lambda_C$ such a shift is negligble, and we are in the regime that is well described by **Thomson scattering**.

Compton scattering is a quantum-mechanical process. The quantum aspect also influences the actual cross section, which changes from the Thomson cross section, σ_T, at the low-frequency end, to the **Klein-Nishina cross section**, $\sigma_{KN}(\nu)$, for $h\nu > m_e c^2$ (see Fig. 15). Note how scattering becomes less efficient for more energetic photons.

So far we have considered the scattering of photons off of *electrons at rest*. A more realistic treatment takes into account that electrons are also moving, and may do so relativistically. This adds the possibility of the electron giving some of its kinetic energy to the photon, which results in **Inverse Compton (IC) scattering**.

Whether the photon looses (Compton scattering) or gains (IC scattering) energy depends on the energies of the photon and electron. Without derivation, the average energy change of the photon *per Compton scattering* against electrons of temperature $T_{\rm e} = m_{\rm e}\langle v^2\rangle/(3\,k_{\rm B})$ is

$$\left\langle \frac{\Delta E_\gamma}{E_\gamma} \right\rangle = \frac{4\,k_{\rm B}\,T_{\rm e} - h\,\nu}{m_{\rm e}\,c^2}$$

Hence, we have that

$h\,\nu > 4\,k_{\rm B}\,T_{\rm e}$: Compton effect; photon looses energy to electron
$h\,\nu = 4\,k_{\rm B}\,T_{\rm e}$: No energy exchange
$h\,\nu < 4\,k_{\rm B}\,T_{\rm e}$: Inverse Compton effect; electron looses energy to photon

As an example, consider ultra-relativistic electrons with $4\,k_{\rm B}\,T_{\rm e} \gg h\,\nu$. In that case it can be shown that IC Compton scattering causes the photons to increase their frequency according to $\nu_{\rm out} \approx \gamma^2\,\nu_{\rm in}$, where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

is the **Lorentz factor**. Hence, for ultra-relativistic electrons, which have a large Lorentz factor, the frequency boost of a single IC scattering event can be enormous. It is believed that this processes, upscattering of low energy photons by the IC effect, is at work in Active Galactic Nuclei.

Another astrophysical example of IC scattering is the **Sunyaev-Zel'dovic (SZ) effect** in clusters; the hot (but non-relativistic) electrons of the intracluster gas (with a typical electron temperature of $T_{\rm e} \sim 10^8\,{\rm K}$) upscatter Cosmic Microwave Background (CMB) photons by a small, but non-negligble amount. The result is a **comptonization** of the energy spectrum of the photons; while Compton scattering maintains photon numbers, it uncreases their energies, so that they no longer can be fit by a Planck curve. The strength of this Comptonization is measure for the electron pressure $P_{\rm e} \propto n_{\rm e}\,T_{\rm e}$ along the line-of-sight through the cluster. Observations of the SZ effect provide a nearly redshift-independent means of detecting galaxy clusters.

Resonant scattering: Resonant scattering, also known as line scattering or bound-bound scattering is the scattering of photons off electrons bound to nuclei in atoms or ions. Before we present the quantum mechanical view of this process, it is useful to consider the classical one, in which the electron is viewed as being bound to the nucleus via a spring with a **natural, angular frequency**, $\omega_0 = 2\pi\nu_0$. If the electron is perturbed, it will oscillate at this natural frequency, which will result in the emission of photons of energy $E_\gamma = h\nu_0$. This in turn implies energy loss; hence, the bound electron is an example of a **damped, harmonic oscillator**. The classical *damping constant* is given by $\Gamma_{\rm cl} = \omega_0^2\,\tau_{\rm e}$, where $\tau_{\rm e} = 2e^2/(3m_{\rm e}c^2) \sim r_{\rm e}/c \sim 6.3\times10^{-24}$s. This damping, and the corresponding emission of EM radiation, is the classical analog of **spontaneous emission**.

Now consider the case of an EM wave of angular frequency, ω, interacting with the atom/ion. The result is a **forced, damped, harmonic oscillator**, whose effective cross section is given by

$$\sigma_{\rm s}(\omega) = \sigma_{\rm T}\,\frac{\omega^4}{(\omega^2-\omega_0^2)^2 + (\omega_0^3\,\tau_{\rm e})^2}$$

(see `Rybicki & Lightmann 1979` for a derivation). We can distinguish three regimes:

$\boxed{\omega \gg \omega_0}$ In this case $\sigma_{\rm s}(\omega) = \sigma_{\rm T}$ and we are in the regime of regular Thomson scattering. The oscillator responds to the high-frequency forcing by adopting the forced frequency; hence, the system behaves as if the electron is free.

$\boxed{\omega \simeq \omega_0}$ In this case

$$\sigma_{\rm s}(\omega) \simeq \frac{\sigma_{\rm T}}{2\tau_{\rm e}}\,\frac{(\Gamma_{\rm cl}/2)}{(\omega-\omega_0)^2 + (\Gamma_{\rm cl}/2)^2}$$

which corresponds to **resonant scattering**, in which the cross section is hugely boosted wrt the Thomson case. NOTE: for resonant scattering to be important, it is crucial that *spontaneous* de-excitation occurs before *collisional* excitation or de-excitation (otherwise the photon energy is lost, and we are in the realm of absorption, rather than scattering). Typically, this requires sufficiently low densities.

$\boxed{\omega \ll \omega_0}$ In this case

$$\sigma_{\rm s}(\omega) \simeq \sigma_{\rm T} \left(\frac{\omega}{\omega_0}\right)^4$$

which corresponds to **Rayleigh scattering**, which is characterized by a strong wavelength dependence for the effective cross section of the form $\sigma_{\rm s} \propto \sigma_{\rm T}\, \lambda^{-4}$.

Rayleigh scattering results from the electric polarizability of the particles. The oscillating electric field of a light wave acts on the charges within a particle, causing them to move at the same frequency (recall, the forcing frequency in this case is much smaller than the natural frequency). The particle therefore becomes a small radiating dipole whose radiation we see as scattered light.

Rayleigh scattering, and its strong wavelength dependence of $\sigma_{\rm s}$, is responsible for the fact that the sky appears blue during the day, and for the fact that sunsets turn the sky red (see Fig. 16).

We now turn our attention to a quantum-mechanical view of **resonant scattering**. The main difference between the classical view (above) and the quantum view (below), is that in the latter there is not one, but many 'natural frequencies', ν_{ij}, corresponding to all the possible energy-level-transitions $\Delta E_{ij} = h\nu_{ij}$ that correspond to the atom/ion in question.

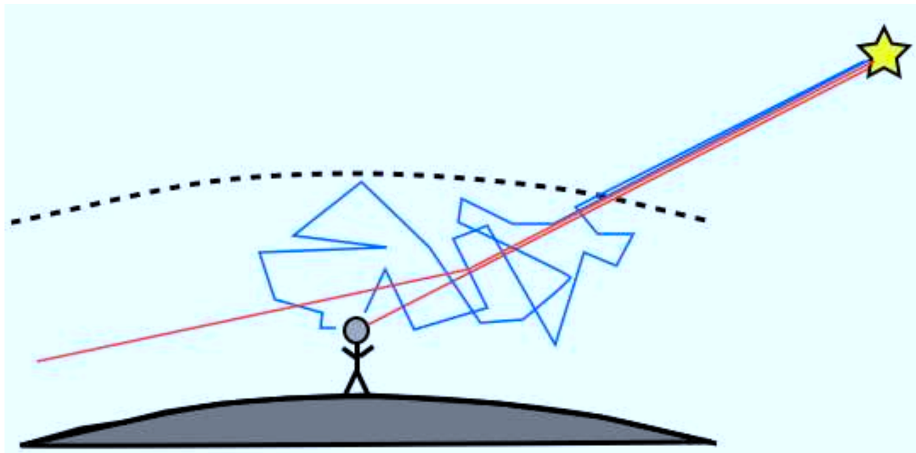

Figure 21: *Illustration of how Rayleigh scattering causes the sky to be blue. Because of its strong (λ^{-4}) wave-length dependence, blue light is much more scattered than red light. This causes the Sun light to appear redder than it really is, an effect that strenghtens when the path length through the atmosphere is larger (i.e., at sunrise and sunset). The blue light is typically scattered multiple times before hitting the observer, so that it appears to come from random directions on the sky.*

Oscillator strength: With each transition corresponds an oscillator strenght, f_{ij}, which is a dimensionless quantity that expresses the 'strength' of the $i \leftrightarrow j$ transition. It expresses the quantum mechanical probability that transition $i \rightarrow j$ occurs under the incidence of a ν_{ij} photon given the quantum-mechanical selection rules, which state the degree to which a certain transition between degrees is allowed. You can think of f_{ij} as being proportional to the probability that the incidence of a ν_{ij} photon results in the corresponding electronic transition.

In the quantum-mechanical view, the three regimes of bound-bound scattering have effective cross sections:

$\sigma_{\rm s} = \sigma_{\rm T}$	$\nu \gg \nu_{ij}$	**Thomson scattering**
$\sigma_{\rm s}(\nu) = \frac{\pi e^2}{m_{\rm e} c} f_{ij} \, \phi_{\mathcal{L}}(\nu)$	$\nu \simeq \nu_{ij}$	**Resonant scattering**
$\sigma_{\rm s}(\nu) = \sigma_{\rm T} f_{ij} \left(\frac{\nu}{\nu_{ij}}\right)^4$	$\nu \ll \nu_{ij}$	**Rayleigh scattering**

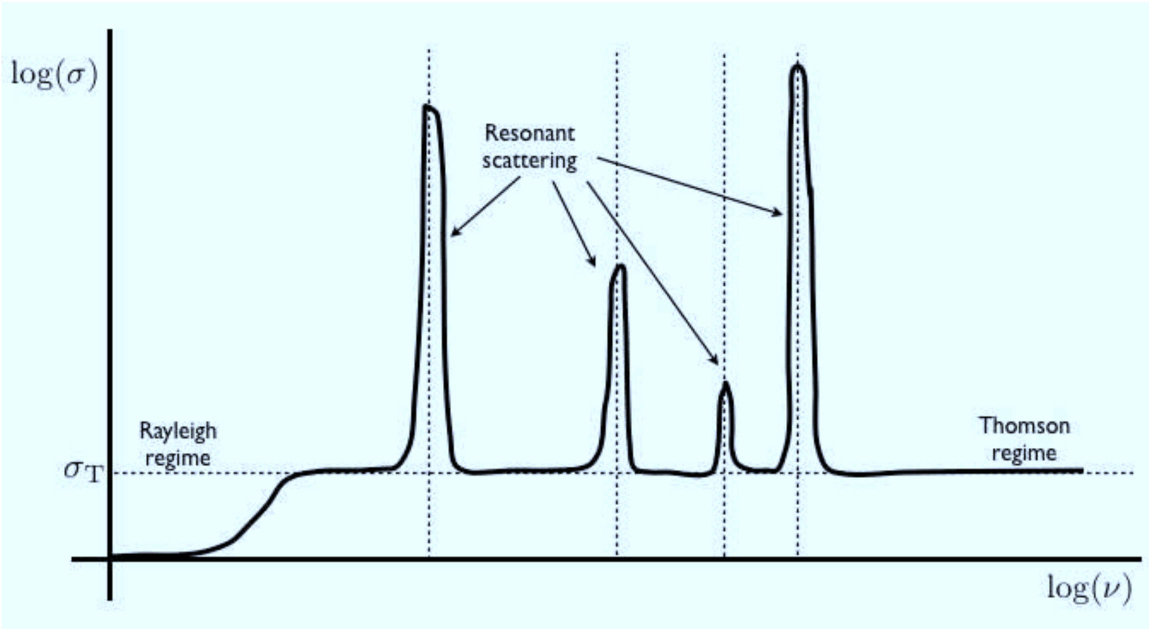

Figure 22: *Illustration of the scattering cross section of an atom or ion with at least one bound electron. At high (low) frequency, scattering is in the Thomson (Rayleigh) regime; at specific, intermediate frequencies, set by the transition energies of the atom/ion, resonant scattering dominates; the profiles are Lorentz profiles, and reflect the natural line broadening. The relative heights of the peaks are set by their oscillator strengths. NOTE: figure is not to scale; typically the cross section for resonant scattering is orders of magnitude larger than the Thomson cross section.*

Here $\phi_{\mathcal{L}}(\nu)$ is the **Lorentz profile**, which describes the **natural line broadening** associated with the transition in question. The non-zero width of this Lorentz profile implies that resonant scattering is not perfectly coherent; typically the energy of the outgoing photon will be slightly different from that of the incident photon. The probability distribution for this energy shift is described by $\phi_{\mathcal{L}}(\nu)$, and originates from the **Heisenberg Uncertainty Principle**, according to which $\Delta E\,\Delta t \geq \hbar/2$; hence, the uncertainty related to the time it takes for the electron to spontaneously de-excite results in a related 'uncertainty' in energy.

Fig. 17 shows the frequency dependence of a (quantum-mechanical) atom/ion. It shows the **Rayleigh regime** at small ν, the **resonant scattering** peaks

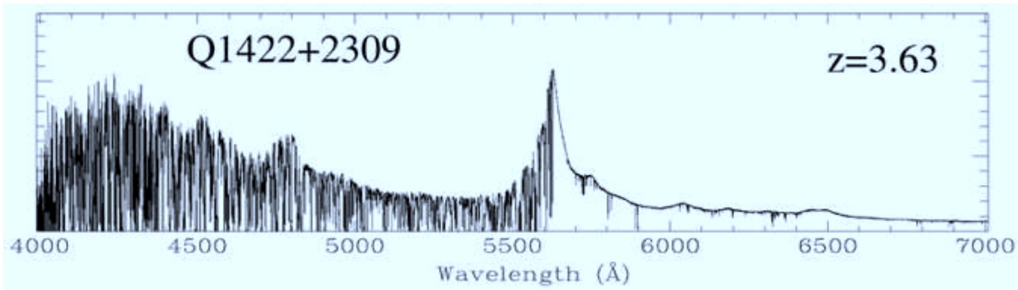

Figure 23: *Example of a quasar spectrum revealing the Ly-α forest due to resonant scattering of Ly-α photons by neutral hydrogen along the line-of-sight from quasar to observer.*

at around a few transition frequencies, and the **Thomson regime** at large ν. Note that the height of the various peaks are set by their respective **oscillator strenghts**.

An important, astrophysical example of resonant scattering is the **Ly-α forest** in the spectra of (high-redshift) quasars. Redward of the quasar's Ly-α emission line one typically observes a 'forest' of 'absorption lines', called the Ly-α forest (see Fig. 18). These arise from resonant scattering in the Ly-α line of neutral hydrogen in gas clouds along the line-of-sight between the quasar and observer. NOTE: although these are called 'absorption lines' they really are a manifestation of (resonant) scattering.

Fluorescence: fluorescence is an inelastic (incoherent) scattering mechanism, in which a photon excites an electron by at least two energy states, and the spontaneous de-excitation occurs to one or more of the intermediate energy levels. Consequently, the photon that is 'scattered' (i.e., absorbed and re-emitted) has changed its energy.