CHAPTER 21

Radiation Essentials

Spectral Energy Distribution: the radiation from a source may be characterized by its spectral energy distribution (SED), $L_\nu \, \mathrm{d}\nu$, or, equivalently, $L_\lambda \, \mathrm{d}\lambda$. Some texts refer to the SEDs as the *spectral luminosity* or the *spectral power*. The SED is the total energy emitted by photons in the frequency interval $[\nu, \nu + \mathrm{d}\nu]$, and is related to the total **luminosity**, $L \equiv \mathrm{d}E/\mathrm{d}t$, according to

$$L = \int L_\nu \, \mathrm{d}\nu = \int L_\lambda \, \mathrm{d}\lambda$$

Note that $[L_\nu] = \mathrm{erg\,s^{-1}\,Hz^{-1}}$, while $[L] = \mathrm{erg\,s^{-1}}$.

Flux: The flux, f, of a source is the radiation energy per unit time passing through a unit area

$$\mathrm{d}L = f \, \mathrm{d}A \qquad\qquad [f] = \mathrm{erg\,s^{-1}\,cm^{-2}}$$

where A is the area. Similarly, we can also define the **spectral flux density** (or simply 'flux density'), as the flux per unit spectral bandwidth:

$$\mathrm{d}L_\nu = f_\nu \, \mathrm{d}A \qquad\qquad [f_\nu] = \mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}}$$

In radio astronomy, one typically expresses f_ν in **Jansky**, where $1\mathrm{Jy} = 10^{-23}\,\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}}$. As with the SEDs, one may also express spectral flux densities as f_λ. Using that $\lambda = c/\nu$, and using that $f_\nu \mathrm{d}\nu = f_\lambda \mathrm{d}\lambda$ one has that

$$f_\nu = \frac{\lambda^2}{c} f_\lambda \,, \qquad\qquad f_\lambda = \frac{\nu^2}{c} f_\nu$$

Luminosity and **flux** are related according to

$$L = 4\,\pi\,r^2\,f$$

where r is the distance from the source.

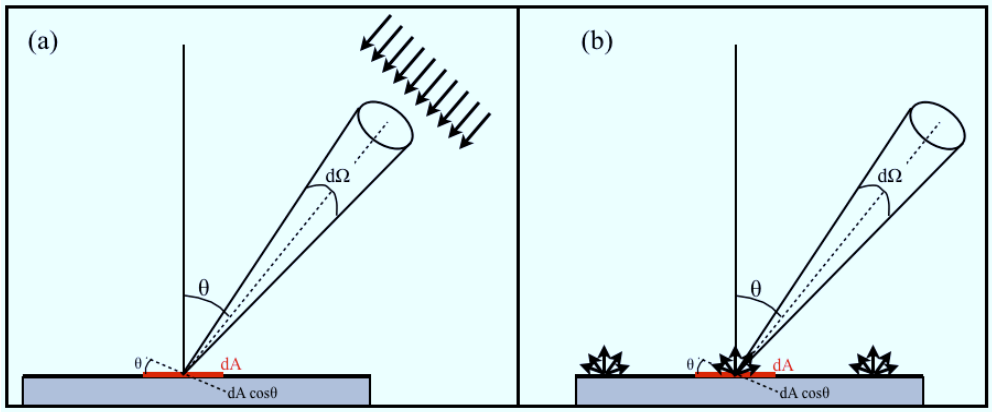

Figure 16: *Diagrams showing intensity and its dependence on direction and solid angle. Fig. (a) depicts the 'observational view', where* dA *represents an element of a detector. The arrows show incoming rays from the* center *of the source. Fig. (b) depicts the 'emission view', where* dA *represents the surface of a star. At each point on the surface, photons leave in all directions away from the surface.*

Intensity: The intensity, I, also called **surface brightness** is the flux emitted in, or observed from, a solid angle dΩ. The intensity is related to the flux via

$$\mathrm{d}f = I\,\cos\theta\,\mathrm{d}\Omega$$

where θ is the angle between the normal of the surface area through which the flux is measured and the direction of the solid angle. The unit of intensity is $[I] = \mathrm{erg\,s^{-1}\,cm^{-2}\,sr^{-1}}$. Here 'sr' is a **steradian**, which is the unit of solid angle measure (there are 4π steradians in a complete sphere). As with the flux and luminosity, one can also define a **specific intensity**, I_ν, which is the intensity per unit spectral bandwidth ($[I_\nu] = \mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}\,sr^{-1}}$).

The flux emerging from the surface of a star with luminosity L and radius R_* is

$$F \equiv \frac{L}{4\pi R_*^2} = \int_{\text{half sphere}} I\,\cos\theta\,\mathrm{d}\Omega = \int_0^{2\pi}\mathrm{d}\phi \int_0^{\pi/2}\mathrm{d}\theta\, I\,\cos\theta\,\sin\theta = \pi\, I$$

where we have used that $\mathrm{d}\Omega = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$, and the fact that the integration over the solid angle Ω is only to be performed over half a sphere. Note that an observer can only measure the surface brightness of *resolved* objects; if unresolved, the observer can only measure the objects flux.

Consider a resolved object (i.e., a galaxy), whose surface brightness distribution on the sky is given by $I(\Omega)$. If the objects extents a solid angle Ω_S on the sky, its flux is given by

$$f = \int_{\Omega_S} I(\Omega)\,\cos\theta\,\mathrm{d}\Omega \simeq \int I(\Omega)\,\mathrm{d}\Omega \equiv \langle I\rangle\,\Omega_S$$

where we have assumed that Ω_S is small, so that variations of $\cos\theta$ across the object can be neglected. Since both $f \propto r^{-2}$ and $\Omega_S \propto r^{-2}$, where r is the object's distance, we see that the average surface brightness $\langle I\rangle$ is independent of distance.

Energy density: the energy density, u, is a measure of the radiative energy per unit volume (i.e., $[u] = \mathrm{erg\,cm^{-3}}$). If the radiation intensity as seen from some specific location in space is given by $I(\Omega)$, then the energy density at that location is

$$u = \frac{1}{c}\int I\,\mathrm{d}\Omega \equiv \frac{4\pi}{c}\,J$$

where

$$J \equiv \frac{1}{4\pi}\int I\,\mathrm{d}\Omega$$

is the **mean intensity** (i.e., average over 4π sterradian). If the radiation is isotropic (i.e., the center of a star, or, to good approximation, a random location in the early Universe), then $J = I$. If the radiation intensity is due to the summed intensity from a number of individual sources, then $u = \frac{1}{c}\sum_i f_i$, where f_i is the flux due to source i.

Recall from Chapter 13 that the number density of photons emerging from a **Black Body** of temperature T is given by

$$n_\gamma(\nu,T)\,\mathrm{d}\nu = \frac{8\pi\,\nu^2}{c^3}\,\frac{\mathrm{d}\nu}{\mathrm{e}^{h\nu/k_{\mathrm{B}}T}-1}$$

Hence, we have that

$$u(\nu,T)\,\mathrm{d}\nu = n_\gamma(\nu,T)\,h\nu\,\mathrm{d}\nu = \frac{8\pi\,h\,\nu^3}{c^3}\,\frac{\mathrm{d}\nu}{\mathrm{e}^{h\nu/k_\mathrm{B}T}-1}$$

Using that $u(\nu,T) = (4\pi/c)J_\nu(T)$ we have that the **mean specific intensity** from a black body [for which one typically uses the symbol $B_\nu(T)$] is given by

$$B_\nu(T)\,\mathrm{d}\nu = \frac{2\,h\,\nu^3}{c^2}\,\frac{\mathrm{d}\nu}{\mathrm{e}^{h\nu/k_\mathrm{B}T}-1}$$

which is called the **Planck curve** (or 'formula'). Integrating over frequency yields the total, mean intensity emitted from the surface of a Black Body

$$J = J(T) = \int_0^\infty B_\nu(T)\,\mathrm{d}\nu = \frac{\sigma_\mathrm{SB}}{\pi}\,T^4$$

where σ_SB is the **Stefan-Boltzmann constant**. This implies an energy density

$$u = u(T) = \frac{4\pi}{c}\,J = \frac{4\sigma_\mathrm{SB}}{c}T^4 \equiv a_\mathrm{r}\,T^4$$

where $a_\mathrm{r} \simeq 7.6\times10^{-15}\,\mathrm{erg\,cm^{-3}\,K^{-4}}$ is called the **radiation constant** (see also Chapter 13).

Wien's Displacement Law: When the temperature of a Black Body emitter increases, the overall radiated energy increases and the peak of the radiation curve moves to shorter wavelengths. It is straightforward to show that the product of the temperature and the wavelength at which the Planck curve peaks is a constant, given by

$$\lambda_\mathrm{max}T = 0.29$$

where T is the *absolute temperature*, expressed in degrees Kelvin, and λ_max is expressed in cm. This relation is called **Wien's Displacement Law**.

Stefan-Boltzmann Law: The flux emitted by a Black Body is

$$F_\mathrm{BB} = \pi\,I(T) = \sigma_\mathrm{SB}T^4$$

which is known as the **Stefan-Boltzmann law**. This law is used to define the effective temperature of an emitter.

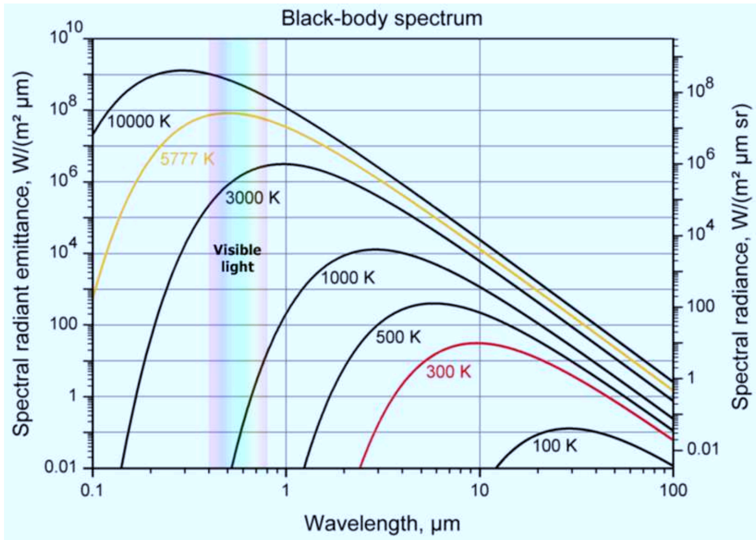

Figure 17: *Various Planck curves for different temperatures, illustrating Wien's displacement law. Note how the Planck curve for a black body with the temperature of the Sun peaks at the visible wavelengths, where the sensitivity of our eyes is maximal*

Effective Temperature: The temperature an emitter of flux F would have if it where a Black Body; using the Stefan-Boltzmann law we have that $T_{\rm eff} = (F/\sigma_{\rm SB})^{1/4}$. We can also use the effective temperature to express the emitter's luminosity;

$$L = 4\,\pi\,R^2\,\sigma_{\rm SB}\,T_{\rm eff}^4$$

where R is the emitter's radius. The effective temperature is sometimes also called the **radiation temperature**, as a measure for the temperature associated with the radiation field.

Brightness Temperature: the brightness temperature, $T_{\rm B}(\nu)$, of a source at frequency ν is defined as the temperature which, when put into the Planck formula, yields the specific intensity actually measured at that frequency. Hence, for a Black Body $T_{\rm B}(\nu)$ is simply equal to the temperature of the Black Body. If $T_{\rm B}(\nu)$ depends on frequency, then the emitter is not a Black Body. The brightness temperature is a frequency-dependent version of the effective, or radiation, temperature.

Wavebands: Astronomers typically measure an object's flux through some filter (waveband). The measured flux in 'band' X is, f_X, is related to the spectral flux density, f_λ, of the object according to

$$f_X = \int f_\lambda \, F_X(\lambda) \, R(\lambda) \, T(\lambda) \, \mathrm{d}\lambda$$

Here $F_X(\lambda)$ describes the transmission of the filter that defines waveband X, $R(\lambda)$ is the transmission efficiency of the telescope + instrument, and $T(\lambda)$ describes the transmission of the atmosphere. The combined effect of F_X, R, and T is typically 'calibrated' using standard stars with known f_λ.

Magnitudes: For historical reasons, the flux of an astronomical object in waveband X is usually quoted in terms of **apparent magnitude**:

$$m_X = -2.5 \, \log \left(\frac{f_X}{f_{X,0}} \right)$$

where the flux zero-point $f_{X,0}$ has traditionally been taken as the flux in the X band of the bright star Vega. In recent years it has become more common to use 'AB-magnitudes', for which

$$f_{X,0} = 3.6308 \times 10^{-20} \, \mathrm{erg\, s^{-1} cm^{-2} Hz^{-1}} \int F_X(c/\nu) \, \mathrm{d}\nu$$

Similarly, the luminosities of objects (in waveband X) are often quoted as an **absolute magnitude:**

$$M_X = -2.5 \log(L_X) + C_X$$

where C_X is a zero point. It is usually convenient to write L_X in units of the solar luminosity in the same band, $L_{\odot X}$, so that

$$M_X = -2.5 \log \left(\frac{L_X}{L_{\odot X}} \right) + M_{\odot X} \,,$$

where $M_{\odot X}$ is the absolute magnitude of the Sun in the waveband in consideration. Using the relation between luminosity and flux we have that

$$m_X - M_X = 5 \log(r/r_0)$$

where r_0 is a fiducial distance at which m_X and M_X are defined to have the same value. Conventionally, r_0 is chosen to be 10 pc.

Distance modulus: the distance modulus of an object is defined as $m_X - M_X$.