ASTR 320: Problem Set 5

This problem set consists of 4 problems for a total of 44 points.
Due date: Thursday Dec 9

Problem 1: The Lyα transition

When an electron in the first excited state of the Hydrogren atom falls back to the ground level, it emits a Lyα photon with a wavelength of $\lambda = 121.567$nm. The cross section for a Lyα photon to excite a neutral hydrogen atom (at $T = 10^4$K) is $\sigma_{\rm Ly\alpha} = 5.0 \times 10^{-14}\,{\rm cm}^2$, while the time scale for spontaneous decay of an electron in the excited Lyα state to the ground state is $\tau_{2,1} = 1.6 \times 10^{-9}$ s. For comparison, at $T = 10^4$ K the collision rate for an electron to de-excite a Hydrogen atom from the Lyα state is $\gamma_{\rm Ly\alpha} = \langle v \rangle \sigma_{\rm eff} = 6.8 \times 10^{-9}\,{\rm cm}^3\,{\rm s}^{-1}$. Here $\sigma_{\rm eff}$ is the effective cross section, and $\langle v \rangle$ is the typical velocity of a free electron at $T = 10^4$ K.

a) [`5 points`] Consider two ionized clouds of pure hydrogen, each with the same fractional ionization of 99.9 per cent, density of 0.01 cm^{-3} and temperature of $T = 10^4$ K. Cloud 1 has a line of sight thickness of $l = 1$ pc and Cloud 2 has $l = 100$ pc. Compare (by computing) the optical depth of Cloud 1 at a wavelength of $\lambda = 121.567$nm to the optical depth of Cloud 2 at a wavelength of $\lambda = 200$nm. Assume that only scattering processes are important and ignore scattering into the line of sight.

b) [`4 points`] A Lyα photon travels through a pure hydrogen gas of density $n = 10^3\,{\rm cm}^{-3}$ and temperature $T = 10^4$ K. At this temperature, the gas is almost entirely ionized. If the photon excites an upwards Lyα transition in one of the small fraction of particles that are neutral and in the ground state, will the net outcome be scattering or absorption. Explain your answer.

c) [`4 points`] Suppose the pure hydrogen gas described under (b) is in the form of a homogeneous spherical cloud. What is the upper mass of the cloud, in Solar masses, such that it is optically thin for Lyα photons?

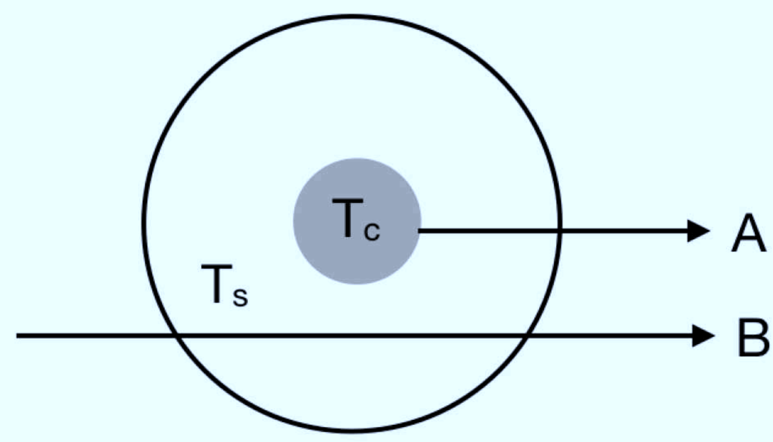

Figure 1:

Problem 2: Radiative Transfer
A spherical, opaque object emits as a blackbody at temperature T_c. Surrounding this central object is a spherical shell of material, thermally emitting at a temperature T_s where $T_s < T_c$. The shell absorbs in a narrow spectral line, i.e., its absoprtion coefficient becomes large at the frequency ν_0 and is negligibly small at other frequencies, such as ν_1. The object is observed at frequencies ν_0 and ν_1 (which is a little bit larger than ν_0) and along two rays, A and B (see Fig. 1). Assume that the Planck function does not vary appreciably from ν_0 to ν_1.

a) [`3 points`] At which frequency, ν_0 or ν_1, will the observed brightness be largest along ray A. Explain you answer and make a small drawing of what the brightness as function of frequency looks like over the range from a bit below ν_0 to a bit larger than ν_1.

b) [`3 points`] Same question as under (a), but now for ray B.

c) [`3 points`] Same question as under (a), but now when $T_s > T_c$.

d) [`3 points`] Same question as under (c), but now for ray B.

Problem 3: More Radiative Transfer [`8 points`]
Consider two gas clouds of identical temperature that are emitting black-body radiation. Cloud 1 is located at a distance of 100 parsec, and has an optical depth of 50 (at frequency ν). Cloud 2, which is located at a distance of 50 parsec, has an optical depth of $\tau_\nu = 0.2$. If both clouds are observed at the same frequency ν, what will the ratio of observed fluxes be?

Problem 4: Cylclotron & Synchrotron emission
Consider an interstellar medium (ISM) with a magnetic field strength of $B = 3\mu\text{G}$.

a) [`4 points`]Determine the radius of gyration (in km), period and gyrofrequency of a typical electron within the warm ($T \sim 10^4$K) component of the ISM. You may assume that the electrons move perpendicular to the magnetic field lines.

b) [`3 points`]Determine the same as under (a), but now for an ultra-relativistic electron with a Lorentz boost of $\gamma = 10^4$.

c) [`4 points`] Assuming that the relativistic electron under (b) has a pitch angle of $\phi = 45^o$, determine the opening angle of the emission cone (in degreees), the critical frequency, and the spacing of the harmonics for this particle.

(Potentially) Useful Constants

m_p	=	1.673×10^{-24}g
m_e	=	9.109×10^{-28}g
e	=	-4.803×10^{-10} esu
k_B	=	1.38×10^{-16} erg K^{-1}
σ_T	=	6.65×10^{-25} cm^2
$M_\odot$	=	2×10^{33}g
pc	=	3.086×10^{18} cm