ASTR 320: Problem Set 3

This problem set consists of 3 problems for a total of 34 points.
Due date: tuesday March 4

In this problem set, you will study the Venturi meter, the lift on an airfoil, and examine a rotating disk without circulation.

Problem 1: The Venturi Meter

The venturi meter is used to measure the flow speed in a pipe. An example is shown in Fig. 1, where the venturi meter (indicated by the dashed lines) is placed in a pipe of diameter A_1. The venturi meter itself consists of a pipe of diameter A_2, as indicated. The pipe transports an incompressible fluid of density ρ with a flow velocity u_1 (this is the quantity to be meaured). The flow velocity in the narrow pipe is u_2. The main and narrow pipes are connected via a U-shaped pipe that is filled with a fluid of density $\tilde{\rho} > \rho$. The quantity to be measured is h, the difference in the heights of the colums of the dense fluid. Throughout you may assume that the fluid flowing through the venturi meter is ideal.

a) [`3 points`] Show that, for an ideal fluid, the specific enthalpy $h \equiv \varepsilon + P/\rho$ takes on the following form:

$$h = \frac{\gamma}{\gamma - 1} \frac{P}{\rho}$$

with γ the adiabatic index.

b) [`5 points`] Assume that the venturi meter of Fig. 1 is located in the Earth's gravitational field, with the direction of gravitational acceleration perpendicular to the flow direction. Express the pressure difference at points 1 and 2 in terms of u_1, u_2, and ρ. You can assume that the flow is laminar.

c) [`6 points`] Derive an expression for u_1 as function of h, the densities ρ and $\tilde{\rho}$, and the areas A_1 and A_2.

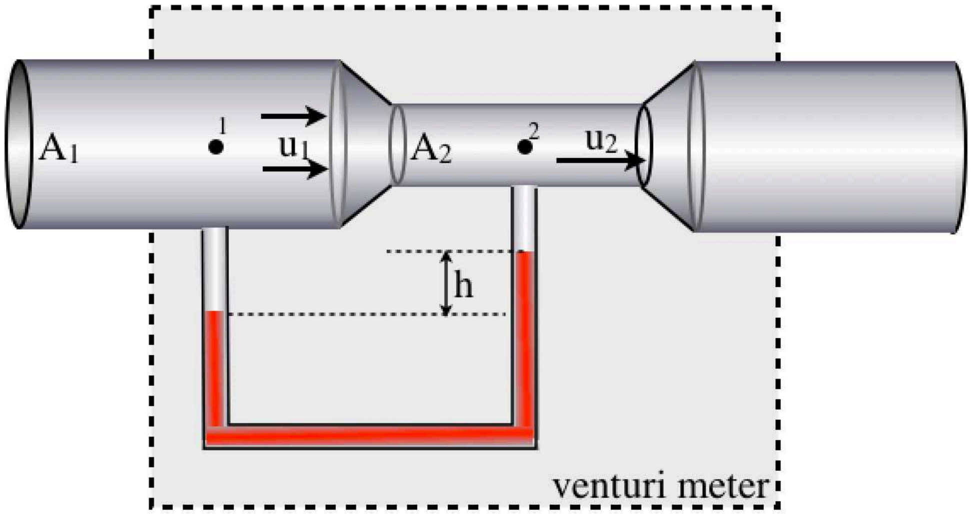

Figure 1: Illustration of a venturi meter.

Problem 2: Vorticity in a thin disk [`10 points`]
Consider an infinitessimally thin disk of fluid in rotation around the disk's (vertical) symmetry axis. It is given that the flow is symmetric around the same symmetry axis, and that the circulation around any curve C on the disk is zero, as long as the curve does not enclose the symmetry point of the disk, $R = 0$. Derive an expression for the velocity field $\vec{u}(\vec{x})$, i.e., what are the various components of $\vec{u}$ as function of location in the disk? Clearly explain your steps in this derivation.

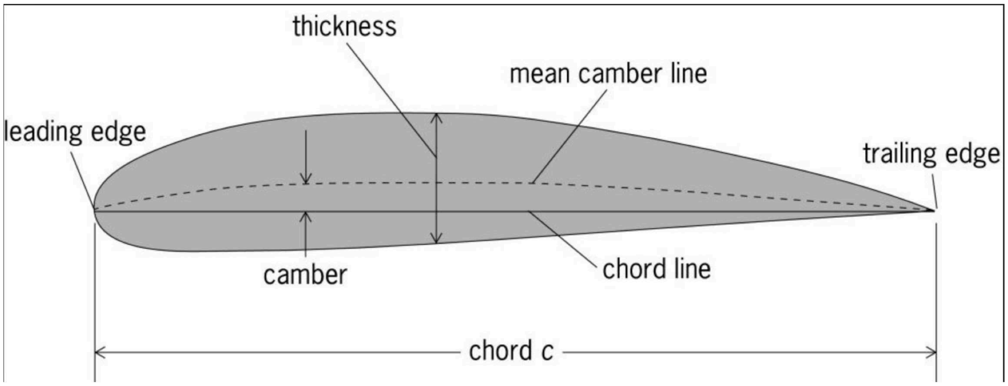

Figure 2: Illustration of an airfoil, indicating the chord and chamber.

Problem 3: Lift on an airfoil

Consider the wing of airplane, a cross section of which is depicted below. Let c be the chord of the wing (see Fig. 2 for the definition of the chord). Because of the shape of the airfoil, air flows *over* the wing with a speed $u_\infty + \Delta u_\uparrow$, and *under* the wing with a speed $u_\infty - \Delta u_\downarrow$. Here u_∞ is the speed of the airplane with respect to the air far from the wing. In general we have that $\Delta u \equiv \Delta u_\uparrow + \Delta u_\downarrow \ll u_\infty$.

a) [`4 points`] Give an expression for the circulation, Γ, *around* the airfoil in terms of c, $\Delta u_\uparrow$ and $\Delta u_\downarrow$. In doing so, you may ignore the thickness of the wing.

b) [`6 points`] Use Bernoulli's theorem to calculate the lift (=force) per unit wing-length on the airfoil. You may treat the air as an ideal fluid with an adiabatic index $\gamma = 5/3$, and assume that the plane is flying sub-sonic, so that the air may be treated as incompressible. Express your answer in terms of u_∞, Γ, and the density of the air, ρ. You may once again ignore the thickness of the wing.