ASTR 320: Problem Set 1

This problem set consists of 3 problems for a total of 38 points.
Due date: tuesday February 7

In this problem set, you will make order-of-magnitude estimates for the densities, masses, and velocities encountered in astrophysical fluids. This will give you some appreciation for the huge dynamic range of fluids we will be dealing with.

Problem 1: Astrophysical Densities

The average, present-day density of the Universe is roughly $2.76 \times 10^{-30} \mathrm{g\,cm^{-3}}$. Only about 15 percent of this is in the form of normal (astronomers say 'baryonic') matter; the remaining 85 percent is dark matter of unknown make-up.

a) [`1 point`] What is the average, baryonic matter density in the present-day Universe, in units of Solar masses per kubic Mpc?

b) [`3 points`] What is the average density of the Sun in $\mathrm{g\,cm^{-3}}$? How does this compare to the density of water and air (both at room temperature).

c) [`3 points`] If the Sun where to form by contraction of a spherical region with a density equal to the average, baryonic density of the present-day Universe. By what factor does that sphere have to shrink to reach the average density of the Sun?

d) [`3 points`] We believe that our Milky Way (MW) galaxy resides inside an (assumed spherical) dark matter halo with a radius of ~ 0.25 Mpc and a mass of $\sim 10^{12}\,\mathrm{M_\odot}$. What is the corresponding, average density of this system. How much denser is this than the average density of the Universe?

e) [`3 points`] The warm phase of the interstellar medium (ISM) has a typical density of about 1 particle per cubic centimeter ($n \sim 1\mathrm{cm}^{-3}$) and a typical temperature of 10^4 K. Under the assumption that the gas mainly consists of hydrogen, what is the average density of the warm ISM in units of the average density of the Universe.

Problem 2: The mean-free path of astrophysical fluids

The mean free path is defined as $\lambda_{\rm mfp} = (n\sigma)^{-1}$. Here n is the number density of particles, and σ is the collisional cross section of the particles. In what follows, you may ignore the fact that hydrogen will be ionized above a temperature of $\sim 10^4\,$K.

a) [`4 points`] The collisional cross section for hydrogen is of order $\sigma = \pi r_{\rm B}^2$, where $r_{\rm B} = 5.29 \times 10^{-9}\,$cm is called the Bohr radius. Compute the mean free path of baryonic matter in the Universe (at its average, present day density). Express your answer in both cm and Mpc.

b) [`4 points`] What is the mean-free path for hydrogren atoms in the warm phase of the ISM. Express your answer in cm and in pc.

c) [`3 points`] What is the mean-free path for a hydrogen atom in the Solar interior (assume that the Sun is made purely of hydrogen, ignore the fact that all hydrogren inside the Sun will be ionized, and simply use the average density of the Sun). Express your answer in units of the Solar radius.

Problem 3: Ideal Gas

An ideal gas has an equation of state $P = n\,k_{\rm B}\,T$, and the microscopic velocities (see end of Chapter 4 in lecture notes) of the particles follow a Maxwell-Boltzmann distribution:

$$f(\vec{v}){\rm d}^3\vec{v} = \left(\frac{m}{2\pi k_{\rm B}T}\right)^{3/2} \exp\left[-\frac{mv^2}{2k_{\rm B}T}\right] {\rm d}^3\vec{v}$$

a) [`3 points`] What is the typical pressure of the warm phase of the ISM (i.e., the phase at $T \sim 10^4\,$K). Express your answer in cgs units as well as in atmospheres (atm).

b) [`3 points`] The mean velocity corresponding to a Maxwell-Boltzmann distribution of temperature T is $\langle v \rangle = \sqrt{8k_{\rm B}T/\pi m}$, where m is the mean mass per particle. What is the mean microscopic velocity of HI atoms at $T = 10^4$ K? Express your answer in km/s.

c) [`4 points`] What is the kinetic energy of an HI atom moving at this mean speed? Express your answer in eV.

d) [`4 points`] What is the typical time between collisions for an individual, neutral HI atom at $T = 10^4$ K and the average (baryonic) density of the Universe. How does this compare to the free-fall time of gas at this density, which is defined as $t_{\rm ff} = (3\pi/32G\rho)^{1/2}$. Express both time scales in years.

(Potentially) Useful Constants

Gravitional constant	G	$=$	$6.674 \times 10^{-8}\ {\rm cm^3\,g^{-1}\,s^{-2}}$
		$=$	$4.299 \times 10^{-9}\ {\rm Mpc}\, M_\odot^{-1}\,({\rm km/s})^2$
Proton mass	$m_{\rm p}$	$=$	$1.673 \times^{-24}$ g
Electron mass	$m_{\rm e}$	$=$	9.109×10^{-28}g
Boltzmann constant	$k_{\rm B}$	$=$	$1.38 \times 10^{-16}\ {\rm erg\,K^{-1}}$
electron volt	eV	$=$	1.602×10^{-12} erg
parsec	pc	$=$	3.086×10^{18} cm
Solar mass	$M_\odot$	$=$	2×10^{33}g
Solar Radius	$R_\odot$	$=$	6.960×10^{10} cm
atmospheric pressure	atm	$=$	$1.01 \times 10^6\ {\rm g\,cm^{-1}s^{-2}}$
average density of Universe	$\bar{\rho}$	$=$	$3 \times 10^{-30}\ {\rm g\,cm^{-3}}$
baryon fraction of Universe	$f_{\rm bar}$	$=$	0.17