Simulations of Structure Formation

...Status and Challenges...

a critic's view

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Outline

- what have we learned from N-body simulations[†]
- force softening
- spurious fragmentation
- collective relaxation
- subhalo disruption mechanisms
  - ★ discreteness-induced run-away instability
  - ★ force softening in the presence of a tidal field
- future perspectives

[†] I will focuss exclusively on N-body simulations of collisionless dark matter

Cosmological N-body Simulations

N-body simulations are used to study non-linear dynamical evolution:
Over the years, they have been used to

- study large scale structure (vindication of CDM model)
- probe evolution of non-linear, matter power spectrum
- establish a universal (NFW) density profile of CDM halos
- predict/quantify substructure of dark matter halos
- predict/quantify mass/velocity function of CDM halos

N-body simulations are routinely used as prime tool to address fundamental questions in astrophysics:

- nature of dark energy (determine growth rate of structures)
- nature of dark matter (determines small-scale structure)

It is crucial that we continue to scrutinize simulations

How to test numerical simulations?

- Ideal: simulate systems for which you have an anaytical solution

  very rare in non-linear structure formation...

- Compare simulations against simulations

  convergence; a necessary condition, but not sufficient...

  different quantities (halo mass function, matter power spectrum, halo density profile, subhalo mass function, etc) all converge differently.

  cross-correlation: $$K = \frac{\langle \delta_1 \, \delta_2 \rangle}{\sigma_1 \, \sigma_2}$$ (1 and 2 refer to different simulations with different numerical parameters)

- Look for signatures of collisional relaxation

  - mass segregation (use ≥2 particle species with different masses)
  - creation of isothermal cores in dark matter halos

Force Softening & Discreteness Effects

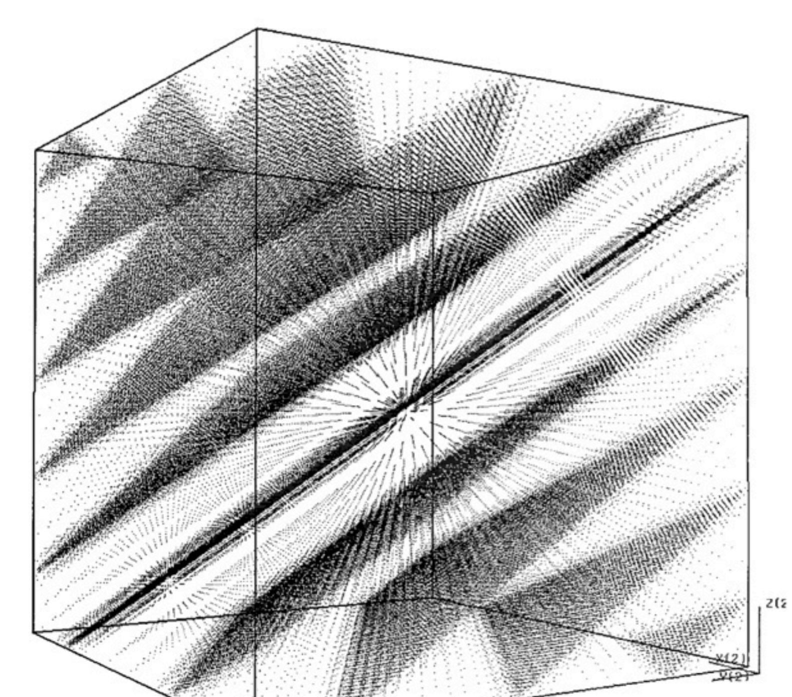
A `natural' choice for ε is the mean interparticle separation $d=L/N^{1/3}$
In what follows, ε is normalized to d (ε=ε/d)

Modern `high-res' codes (P^3M, tree-codes, AMR) typically use ε ~ 0.01-0.03

Efstathiou & Eastwood (1981):
P^3M simulations become collisional (i.e., reveal mass segregation) if ε<0.1

Peebles (1989): PM codes require ε ~1

Melott et al. (1997):
N-body codes in general require ε ~1

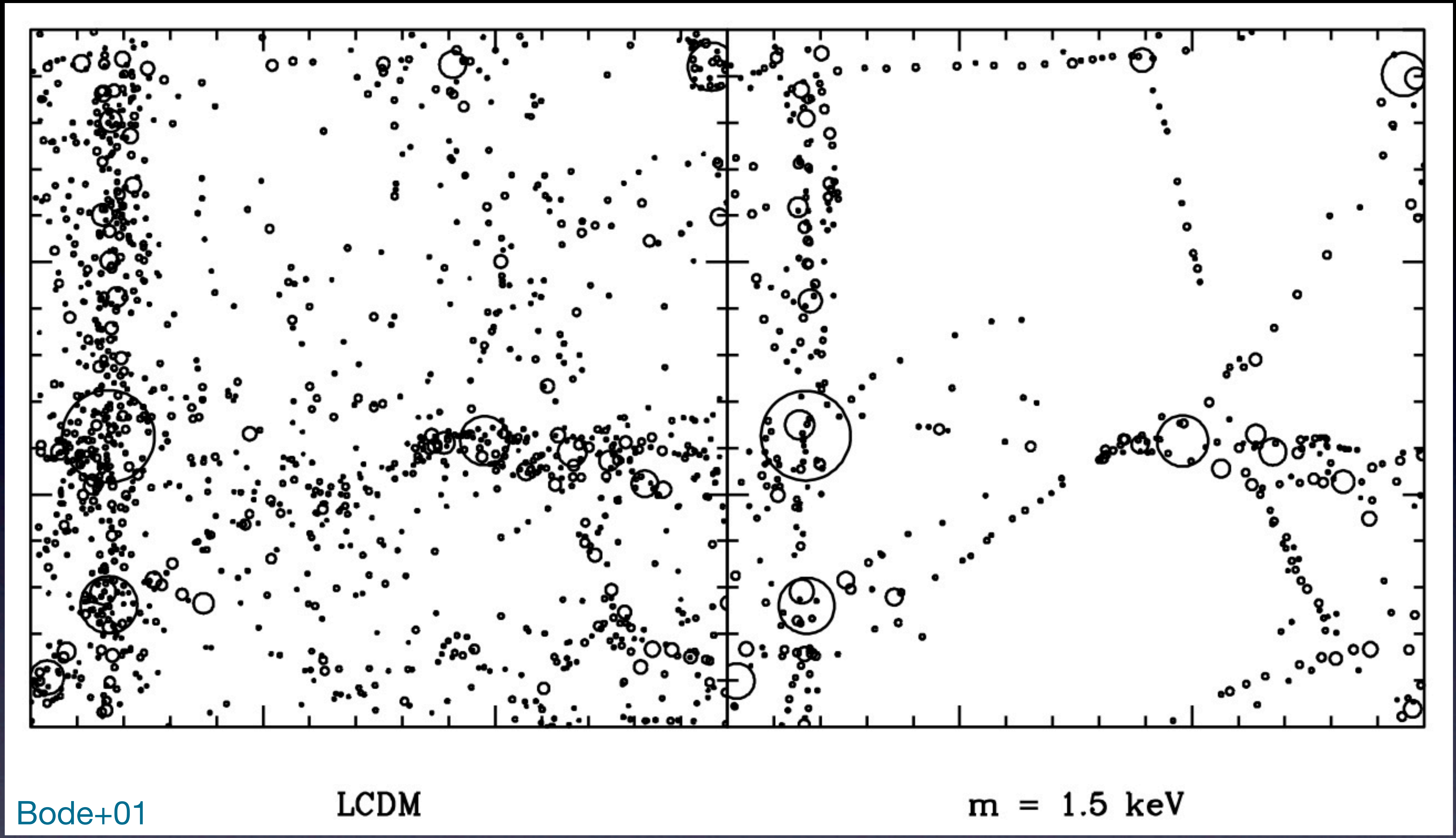

Melott+97

Lively, ongoing debate whether simulations with ε < 1 are reliable...
(Kuhlman+96; Splinter+98; Knebe+00; Melott 07; Romeo+08; Joyce+09; Benhaiem+16; Power+16)

What is the optimal softening length? (Athanassoula+00; Dehnen 01; Power+03)

At the very least, softening should be adaptive... (Iannuzzi & Dolag 11; Hobbs+15)

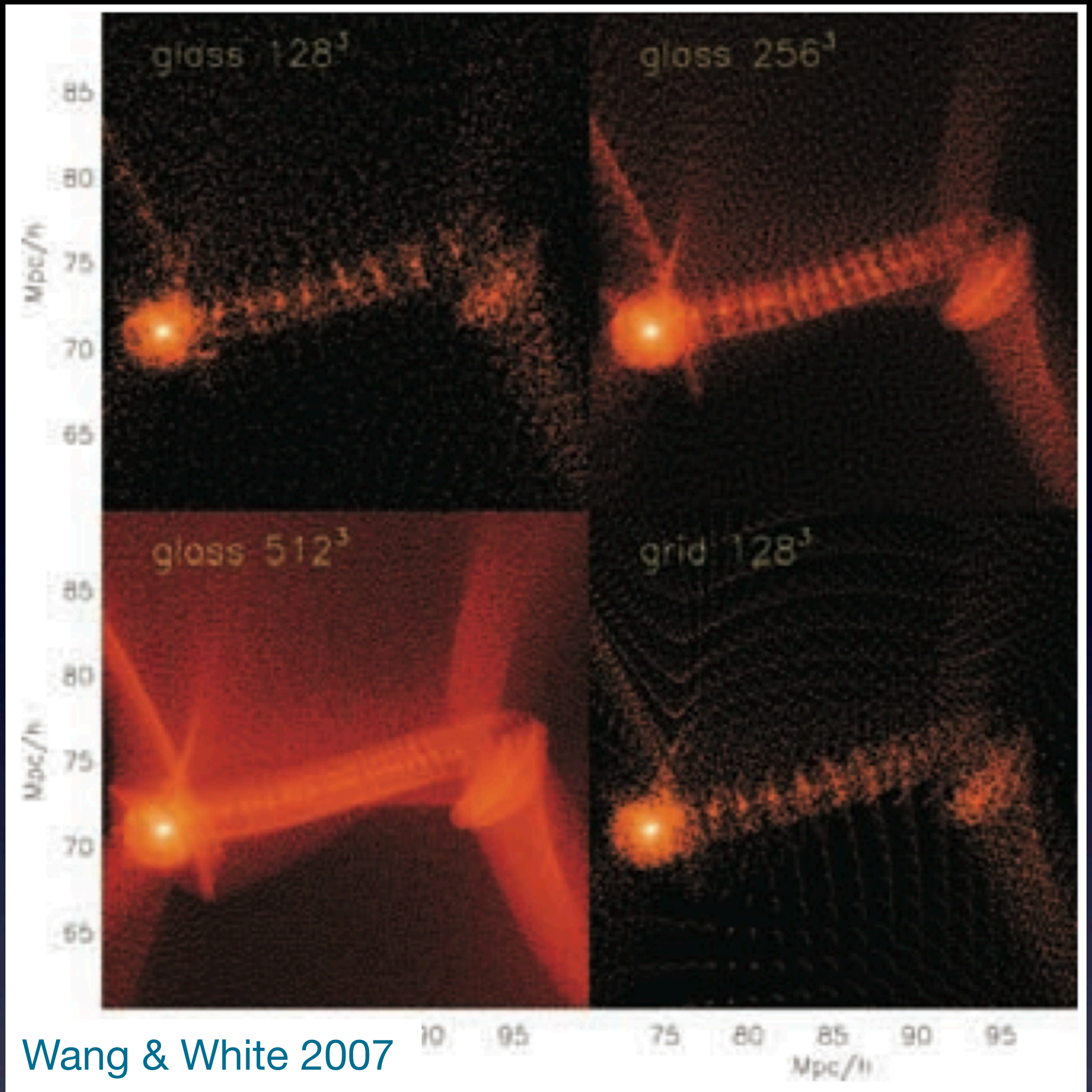
Spurious Fragmentation



Warm Dark Matter simulations show `beats-on-a-string' halos within filaments. These structures form on scales smaller than cut-off scale in power spectrum.
(Bode, Ostriker & Turok 2001; Knebe+02)

Initially interpreted as due to (physical) fragmentation (Knebe+03)

Spurious Fragmentation



Beats-on-a-string halos are manifestation of spurious fragmentation.

Spacing of artificial halos equal to grid-spacing. Suggests link to regular, cubic lattice used for the initial particle load...

(Götz & Sommer-Larsen 2002, 2003)

But, spurious fragmentation also present with glass-like initial particle load, which has no preferred direction, and no long-range order.

(Wang & White 2007)

Spurious fragmentation now understood as arising from discreteness-induced velocity perturbations during early highly-anisotropic phase of structure formation

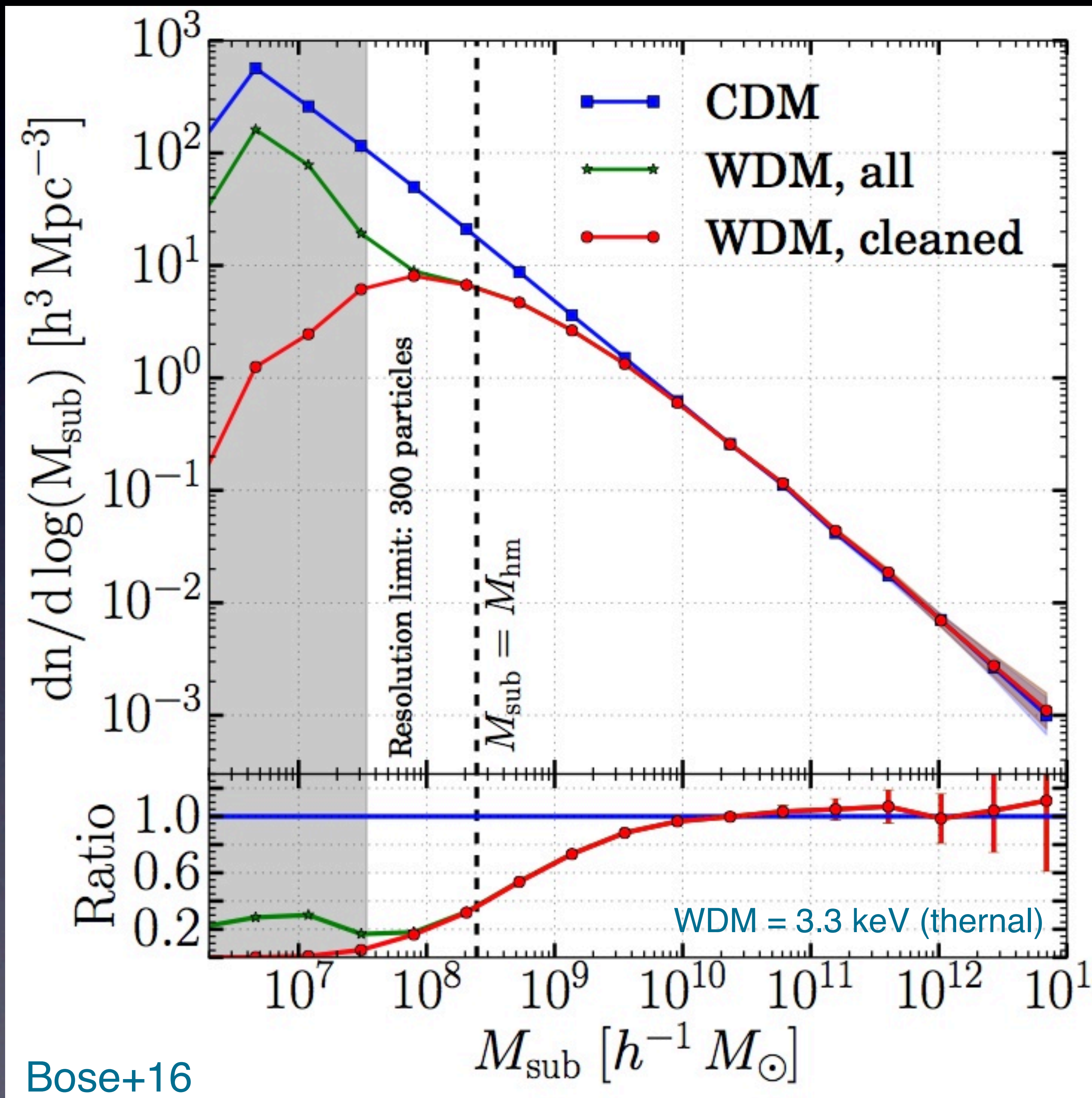
(Hahn & Angulo 2016; Power+16)

This is exactly the artefact that has been discussed again and again, since 1990, by Melott, Shandarin and collaborators!!

(see also Romeo+08; Joyce+09; Benhaiem+16; Power+16 and references therein)

Testing the Nature of Dark Matter

Abundance & demographics of dark matter substructure depends sensitively on nature of dark matter: CDM vs WDM vs SIDM



Different models mainly differ in abundance of low mass halos, where galaxy formation is expected to be supressed due to re-ionization.

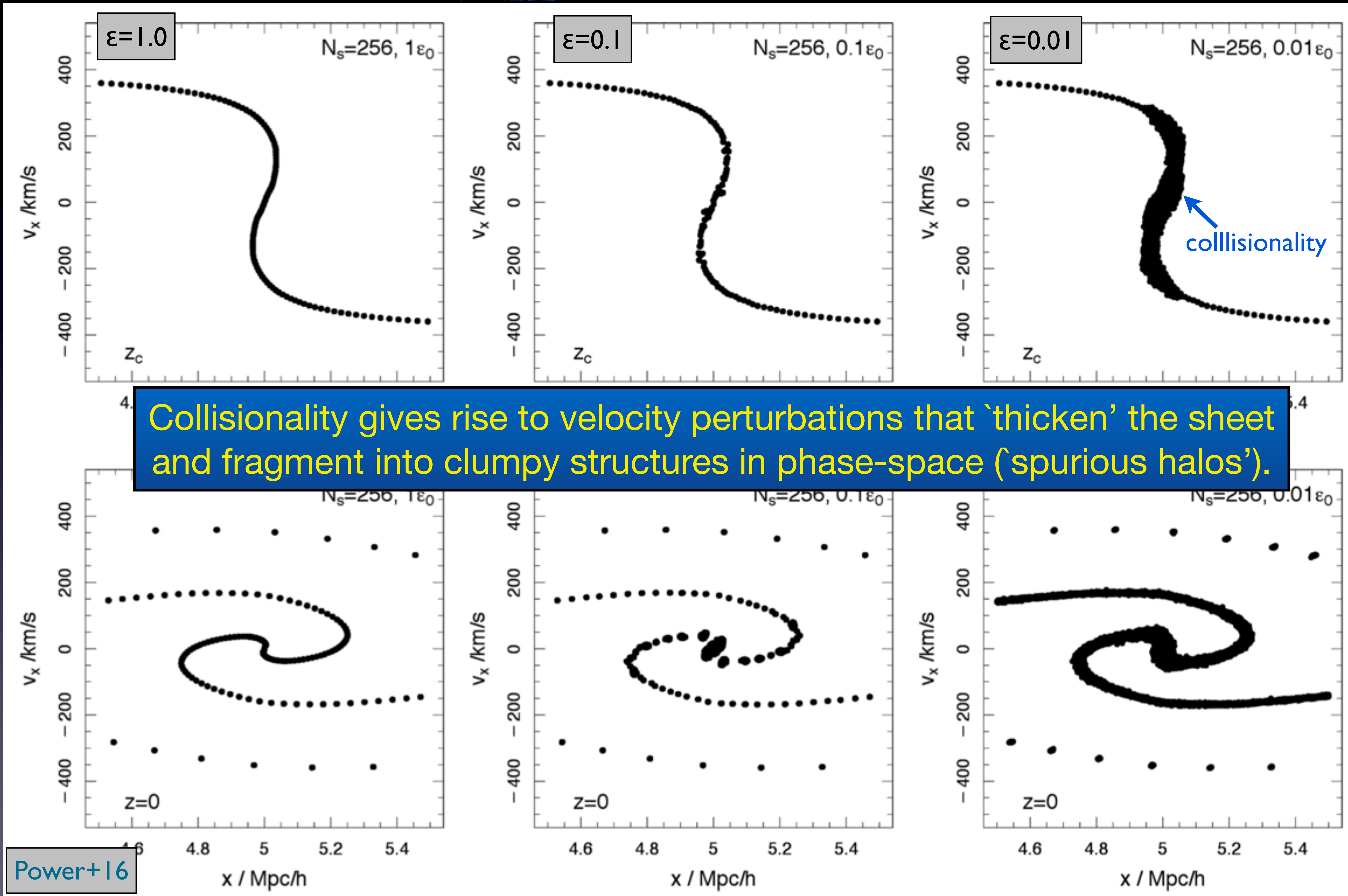
WDM simulations suffer from artificial fragmentation; cannot be avoided.

State-of-the-Art `solution':
remove spurious halos `by hand'

(e.g., Schneider+13; Lovell+14; Bose+16)

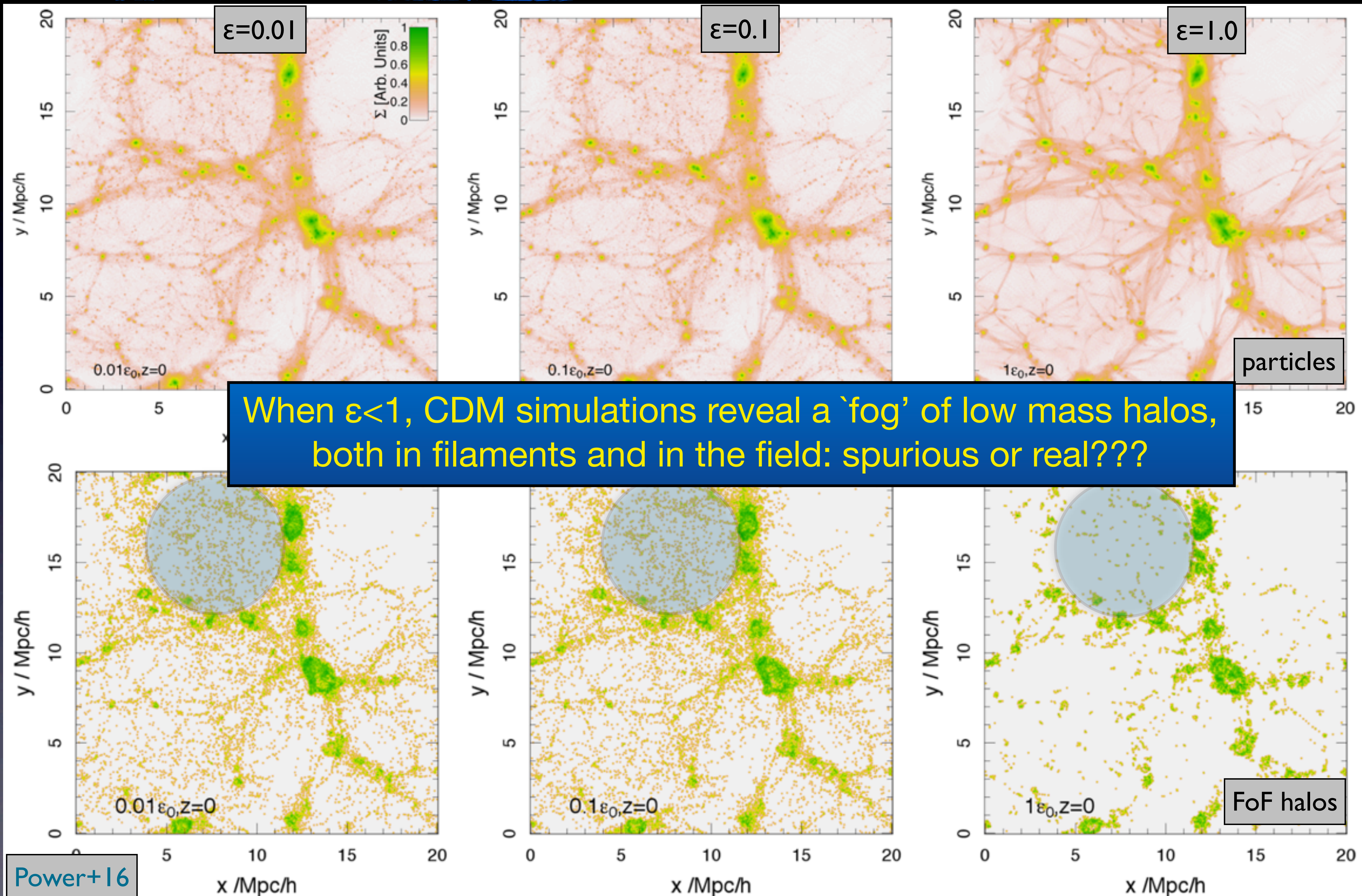
If spurious fragmentation is an outcome of discreteness relaxation, shouldn't it also be present in CDM simulations?

Plane-Symmetric Collapse



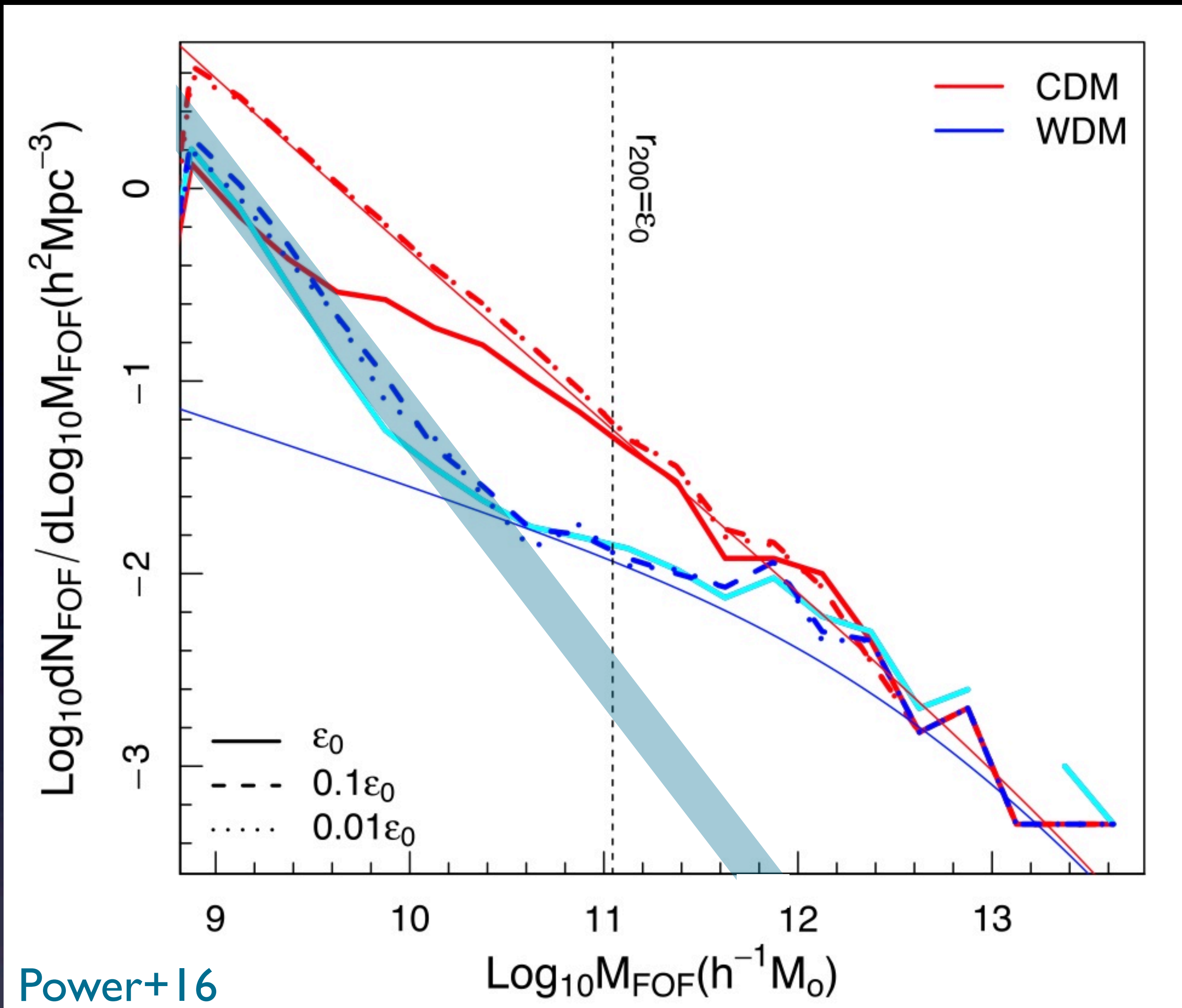
Collisionality gives rise to velocity perturbations that `thicken' the sheet and fragment into clumpy structures in phase-space (`spurious halos').

Spurious Fragmentation in CDM



When ε<1, CDM simulations reveal a `fog' of low mass halos, both in filaments and in the field: spurious or real???

Spurious Fragmentation in CDM



optimist's view

CDM does NOT suffer from spurious fragmentation, because
- no upturn, as for WDM
- agreement with EPS-predictions
- convergence; running at higher force resolution yields consistent results...

reality check...

BUT: WDM results have also converged.
But converged to garbage....

Why would spurious fragmentation not occur, or not matter, for CDM?

My View

- It does happen, with similar abundance of spurious halos as for WDM
- Real halos dominate ➢ no significant impact on CDM mass function
- But what about internal structure of halos? Is NFW reliable?

Two-Body Relaxation

Chandrasekhar (1943) derived change in orbit by summing over all (independent) two-body scatterings with all other particles in homogeneous system.

Roughly equal contribution from every decade in impact parameter:

$$\Gamma_{\rm relax} \propto \frac{\ln \Lambda}{N} \qquad t_{\rm relax} \simeq \frac{N}{8 \ln \Lambda} t_{\rm cross} \qquad \Lambda = \frac{b_{\rm max}}{b_{\rm min}} \qquad b_{\rm min} = b_{90^{\rm o}} \simeq \frac{2 G m_{\rm p}}{\sigma^2} \qquad b_{\rm max} = L$$

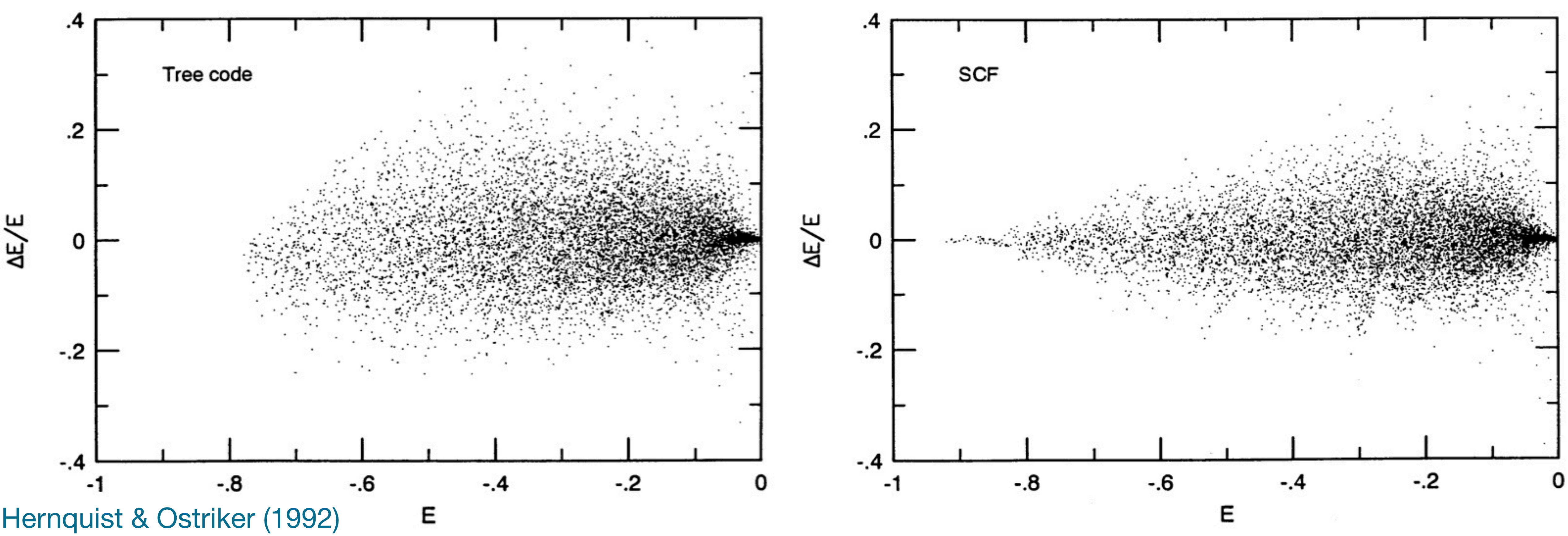
Two-body relaxation generally deemed unimportant, since $t_{\rm relax} > t_H$, for $N \gtrsim 100$

This `standard' treatment of relaxation ignores three important points:

- each halo starts out small, when it is subject to severe relaxation
- orbits are quasi-periodic, giving rise to resonant effects
- self-gravity of large-scale fluctuations [responsible for spurios fragmentation]

Discreteness-Driven Relaxation

Self-Consistent Field (SCF) method, which uses basis-functions to compute gravitational potential (no softening required) suffer from same amount of relaxation as regular N-body codes (e.g, tree-code)!!



Hernquist & Ostriker (1992)

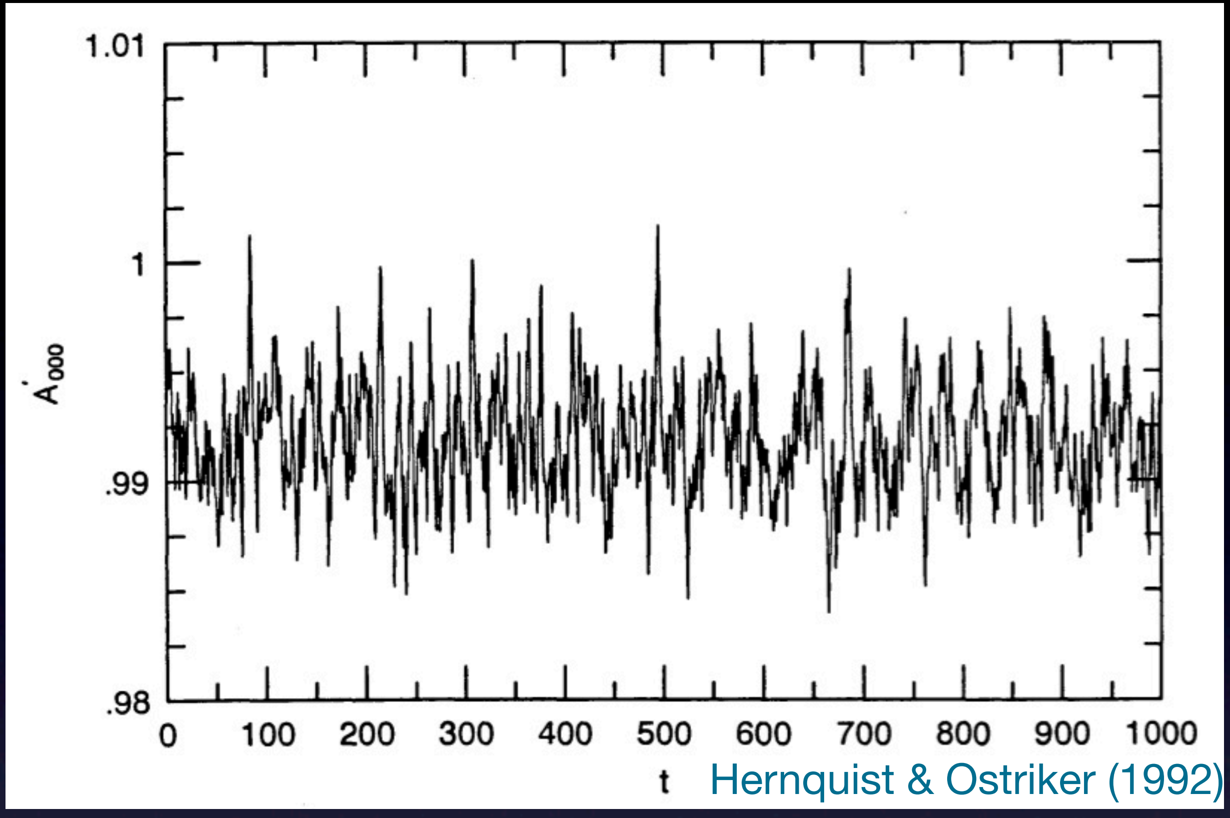
Dominant contribution to relaxation arises from non-local, collective modes of order the size of system in question.

(Weinberg 1993)

Discreteness-Driven Relaxation

Poisson fluctuations cause fluctuations in large-scale potential, which drives relaxation (akin to violent relaxation).

In SCF method, this is evident from rapid fluctuations in the amplitude of zero-th expansion coefficient.



Hernquist & Ostriker (1992)

Weinberg (1993): during the initial collapse phase, Poisson fluctuations may contribute relaxation that is factor 10-100 larger than what is predicted by local (i.e., Chandrasekhar) theory.

- Softening only suppresses impact of large-angle scattering events; not impact of small-angle scattering (large-impact parameters)
- Softening has little to no impact on these large-scale relaxation processes
- Only way to suppress these is by increasing number of particles

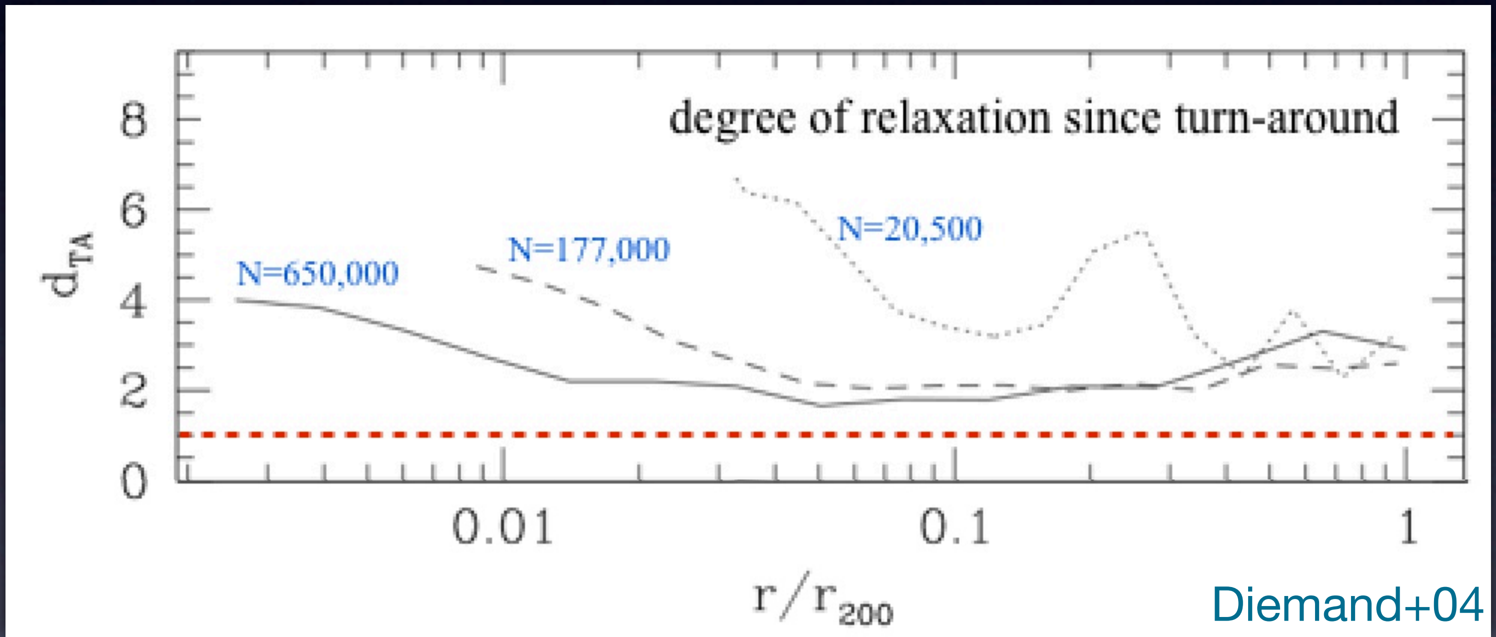
Discreteness-Driven Relaxation

Progenitors of every halo start out small, and thus experience periods during which relaxation rate is large...

Fokker-Planck estimate

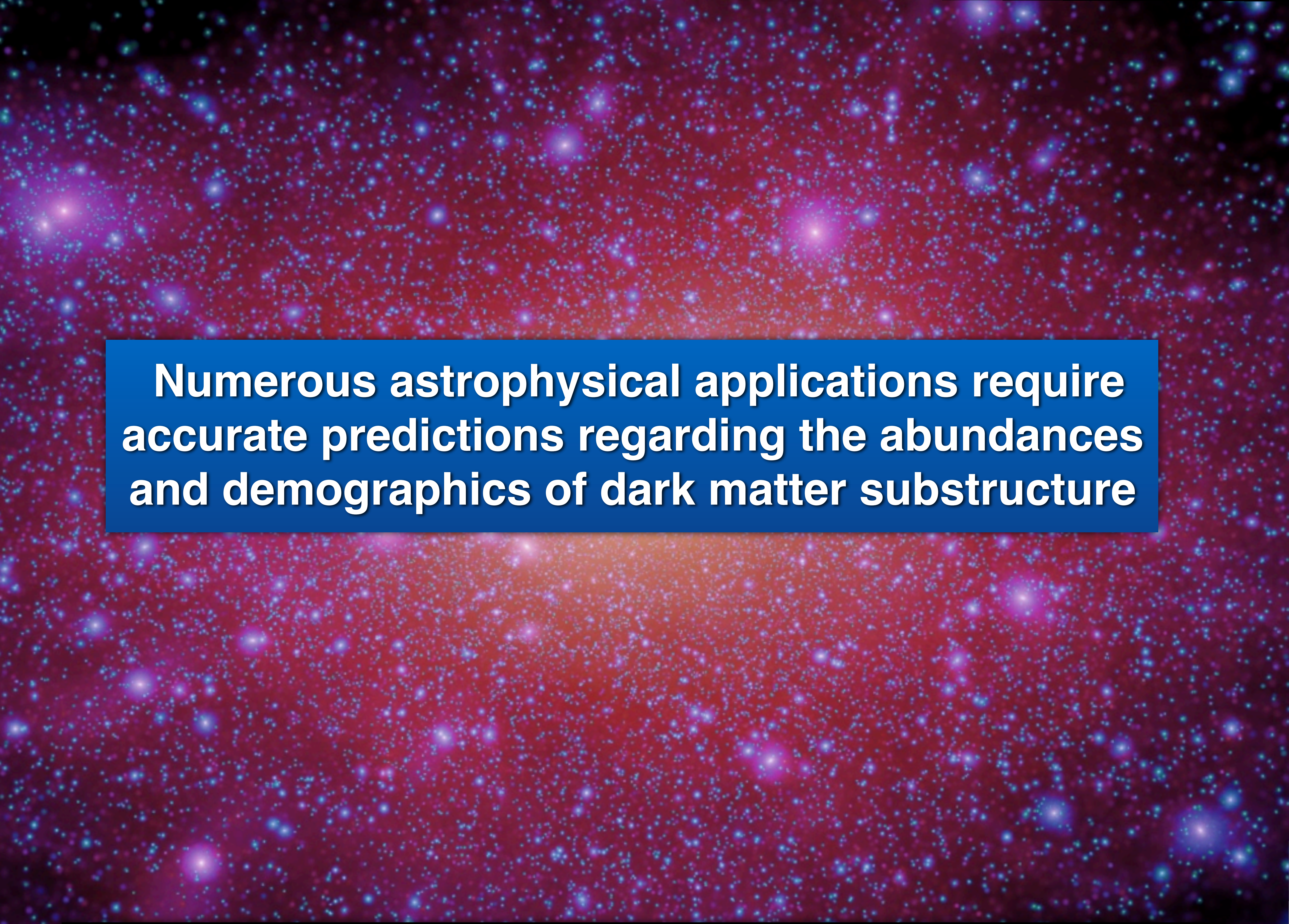
$$\Gamma_{\rm relax} = \frac{G^2\, m_{\rm p}\, \rho\, \ln\Lambda}{\sigma^3} \qquad d(t) \equiv \frac{\Delta E^2(t)}{E^2} = \int \Gamma_{\rm relax}(t)\, {\rm d}t = \sum_{i=1}^{N_{\rm step}} \Gamma_{\rm relax}(t_i)\, \Delta t$$

Diemand+04 integrated cumulative impact of relaxation for individual particles in cosmological N-body simulation.



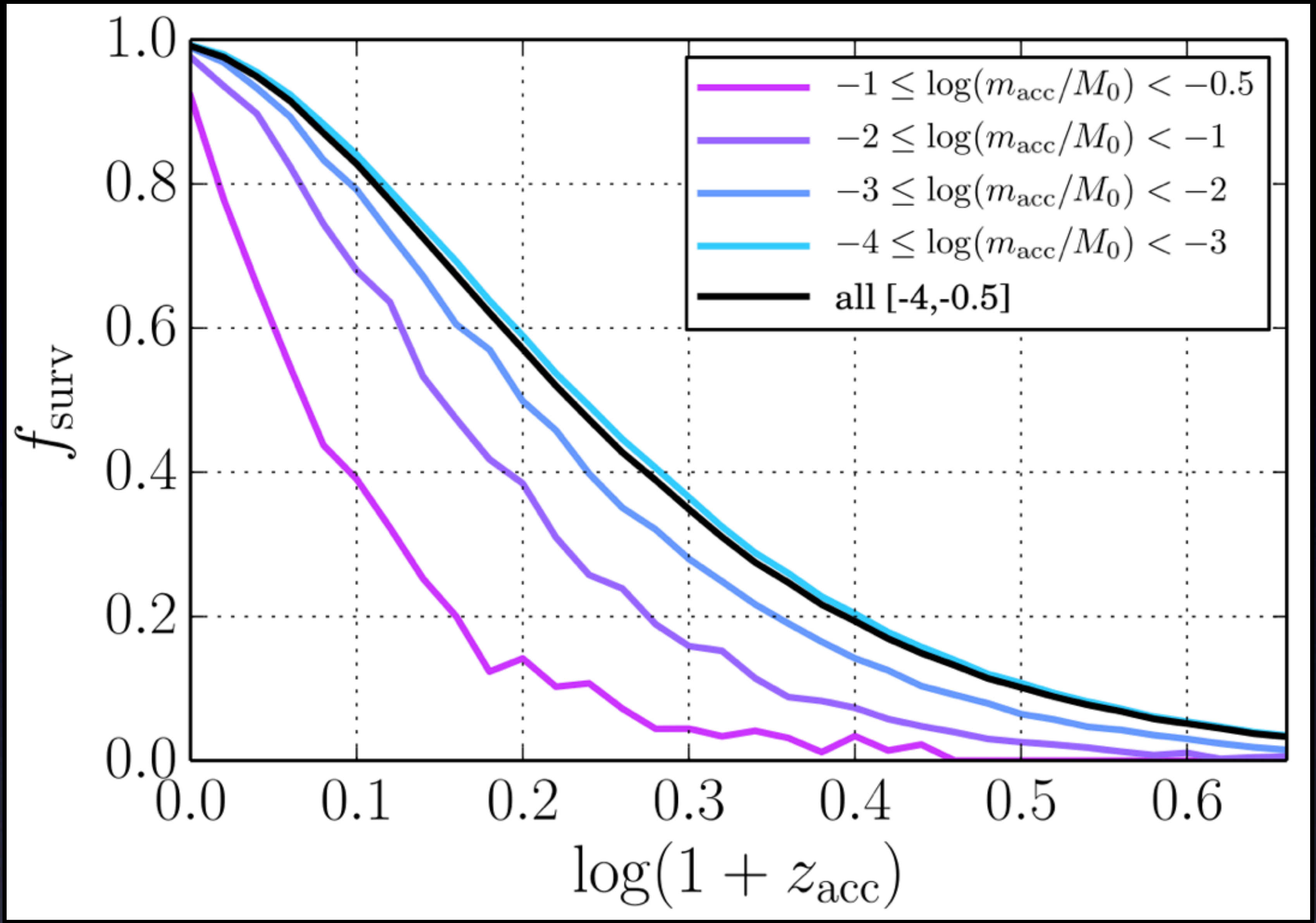
Even in a halo with N=650,000 (at z=0), the cumulative relaxation is such that $|\Delta E| > E$

All knowledge about initial conditions has been erased due to two-body relaxation!!
And this doesn't even acount for collective relaxation, which may well dominate...



Numerous astrophysical applications require accurate predictions regarding the abundances and demographics of dark matter substructure

Subhalo Disruption in Bolshoi



Jiang & vdB, 2016

- Fractional Disruption Rate ≈13 percent per Gyr
- Only ~35 percent of subhaloes accreted at z=1 survive to z=0
- Is tidal disruption real or numerical artifact?
  If real, what are the physical conditions for disruption?

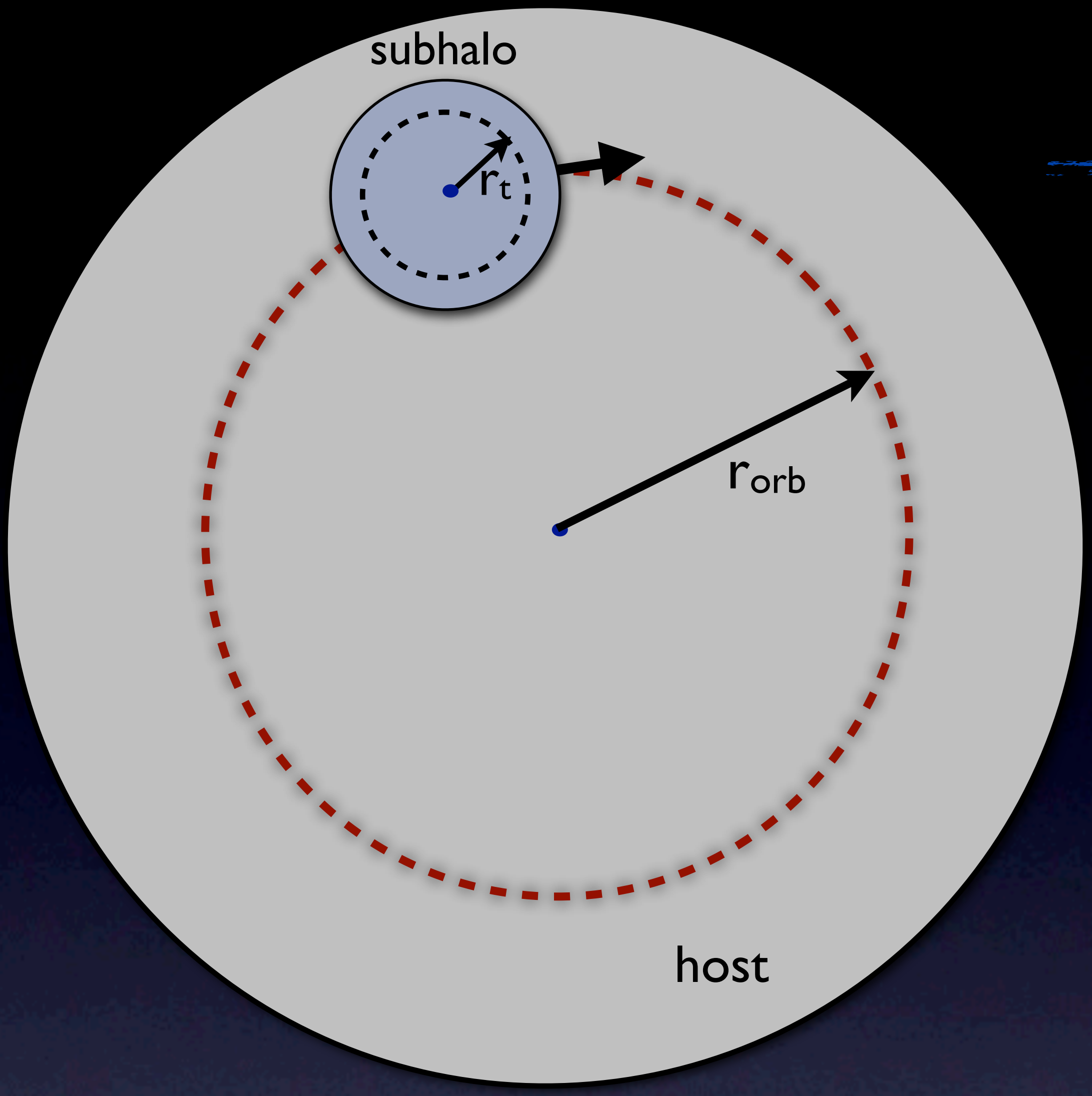
Subhalo Disruption Mechanisms

- Tidal Stripping

- Tidal Heating
  - Pericentric Passage
  - Subhalo-Subhalo Encounter

- Dynamical Friction

- Numerical overmerging



Numerical Simulations

Simulate NFW halo orbiting on circular orbit inside static potential of host halo.

- No impulsive (tidal) heating
- No dynamical friction

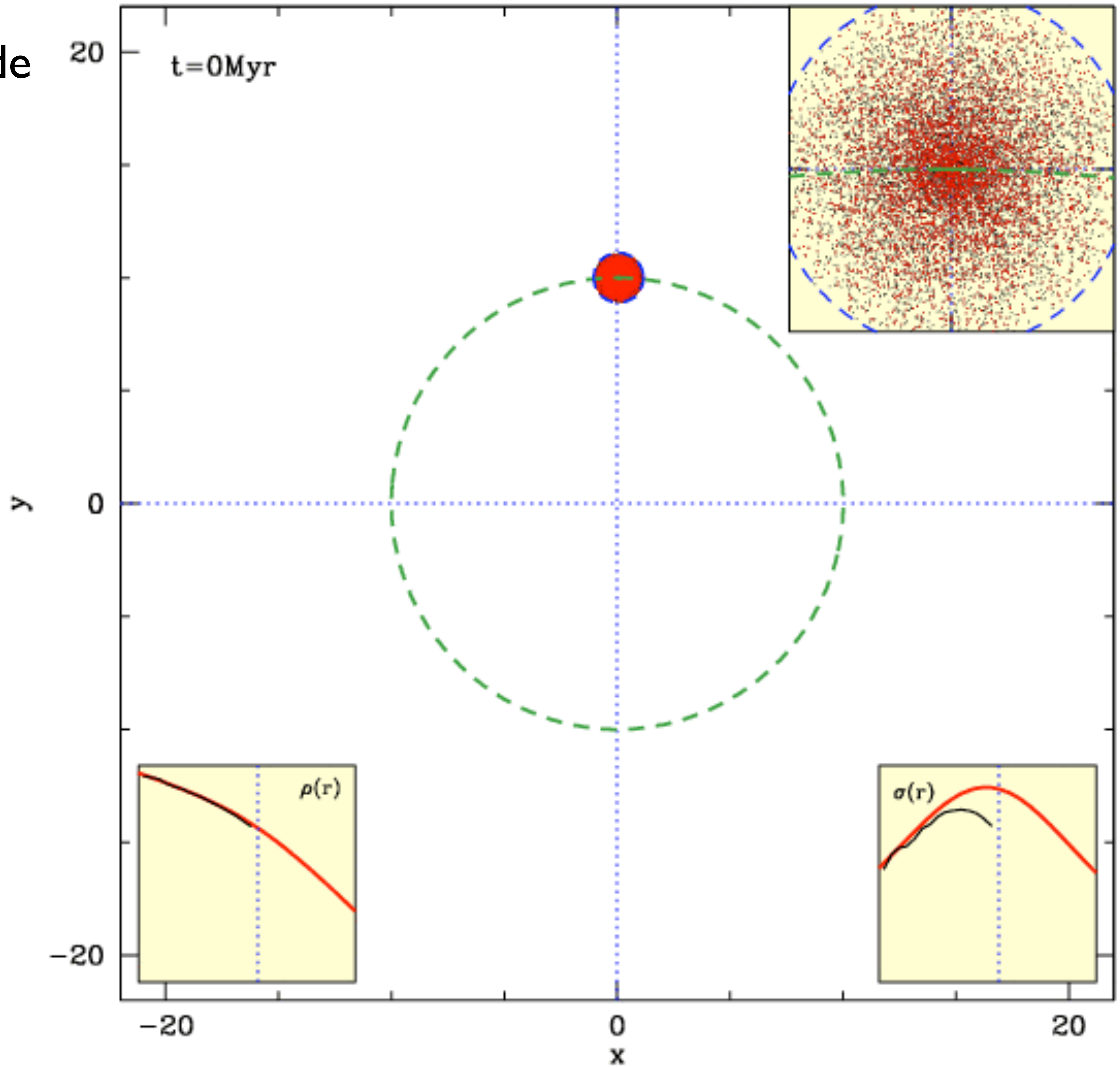
Naive Prediction:

all matter outside of tidal radius will be stripped of over time...

More `Sophisticated' Prediction:

all matter with an apocenter $r_{apo} > r_t$ will be stripped of over time...

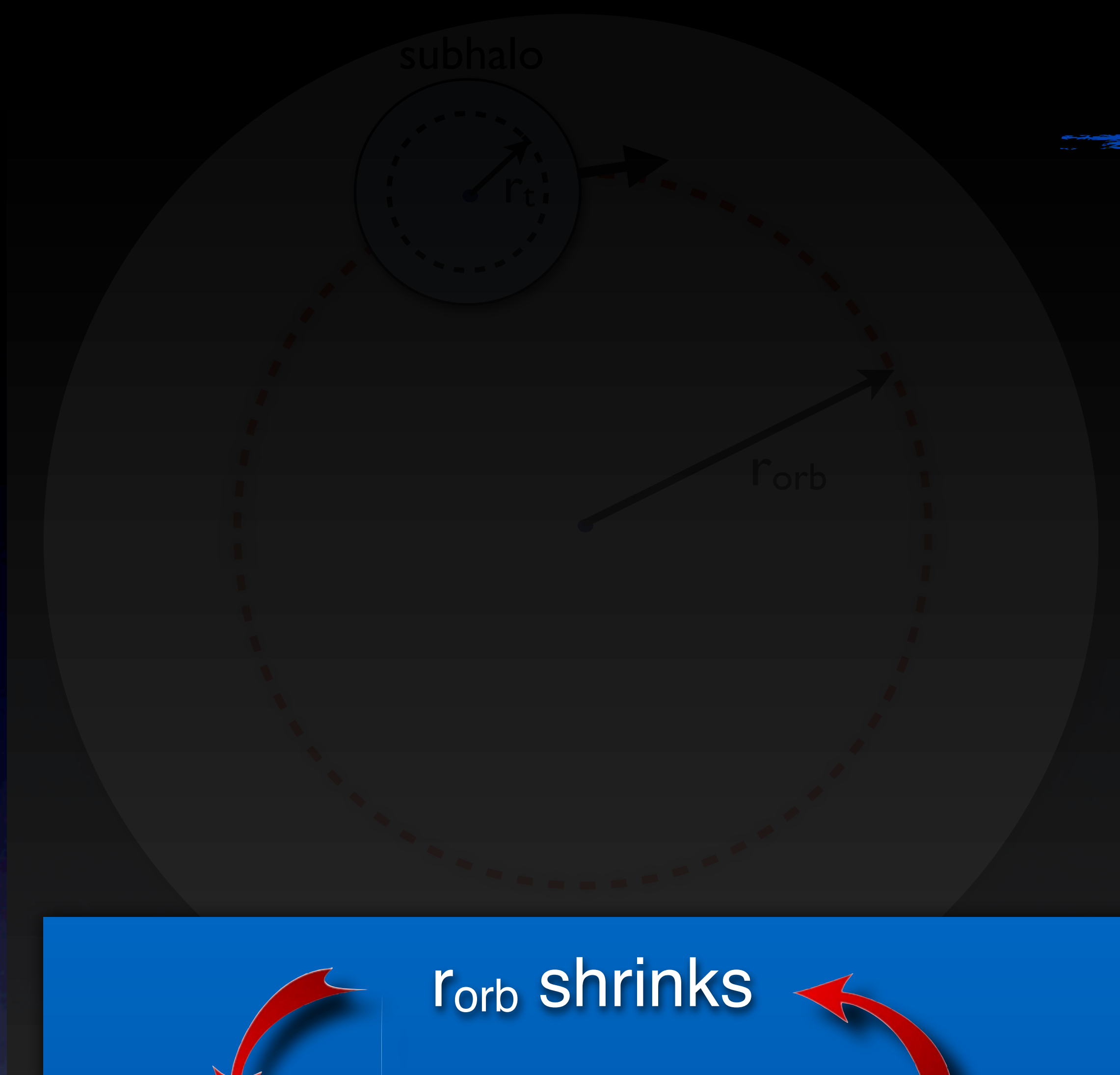
$N=10^5$
tree-code



$r_{orb}=0.1\ r_{vir,h}$

$r_t=0.11\ r_s$

Numerical Simulations



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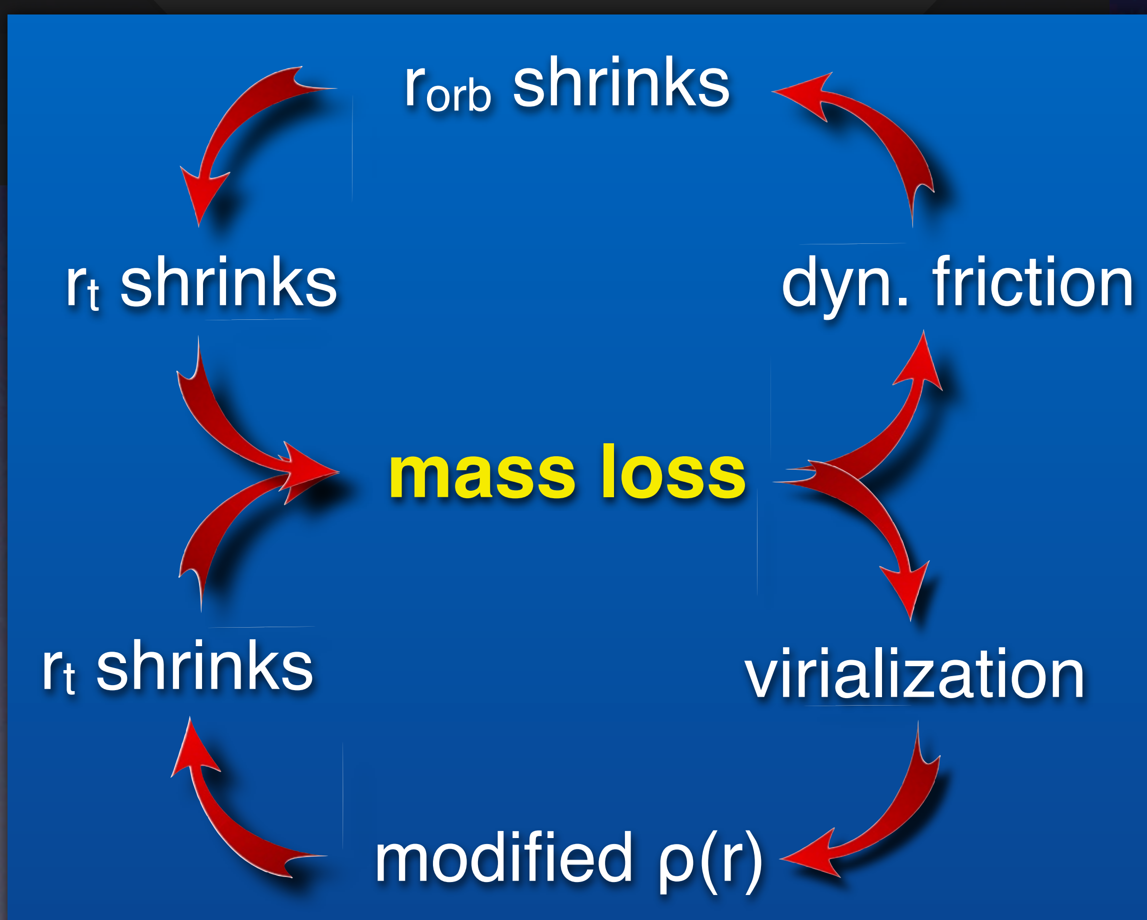
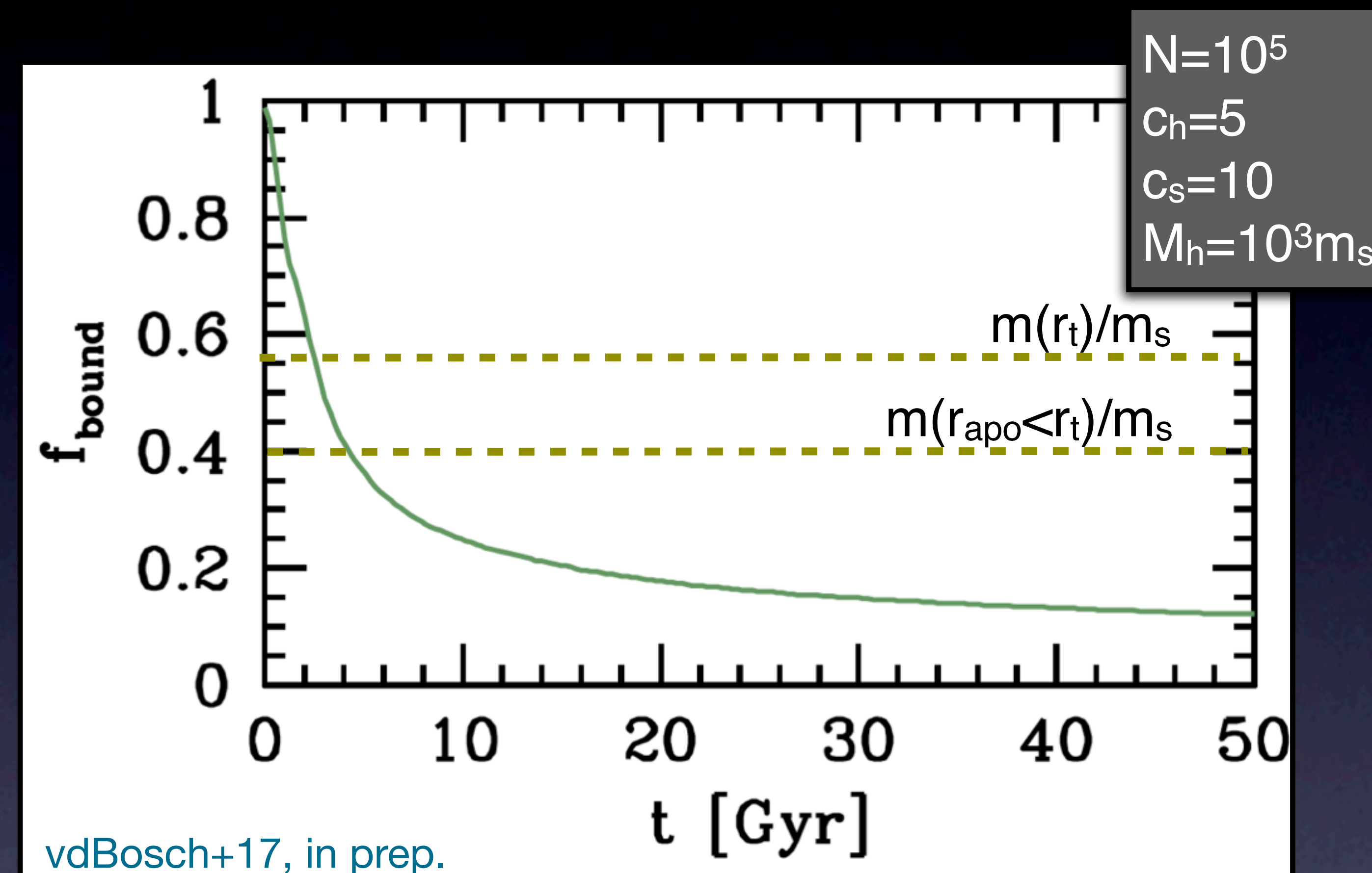
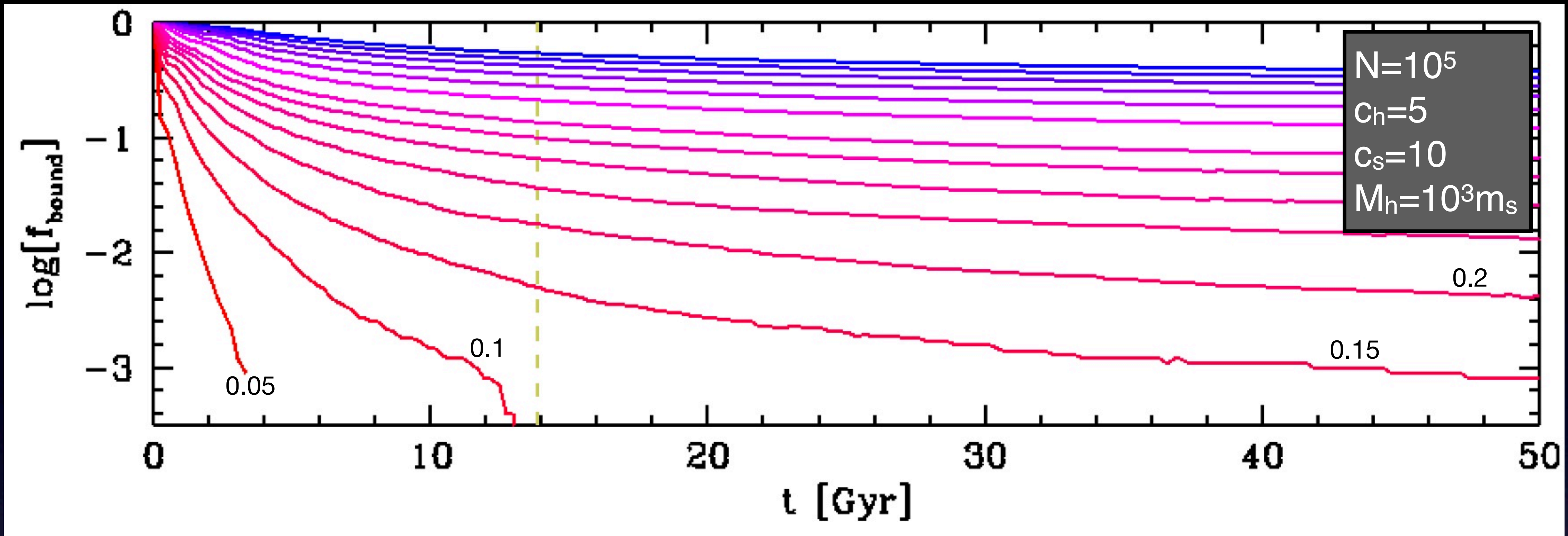


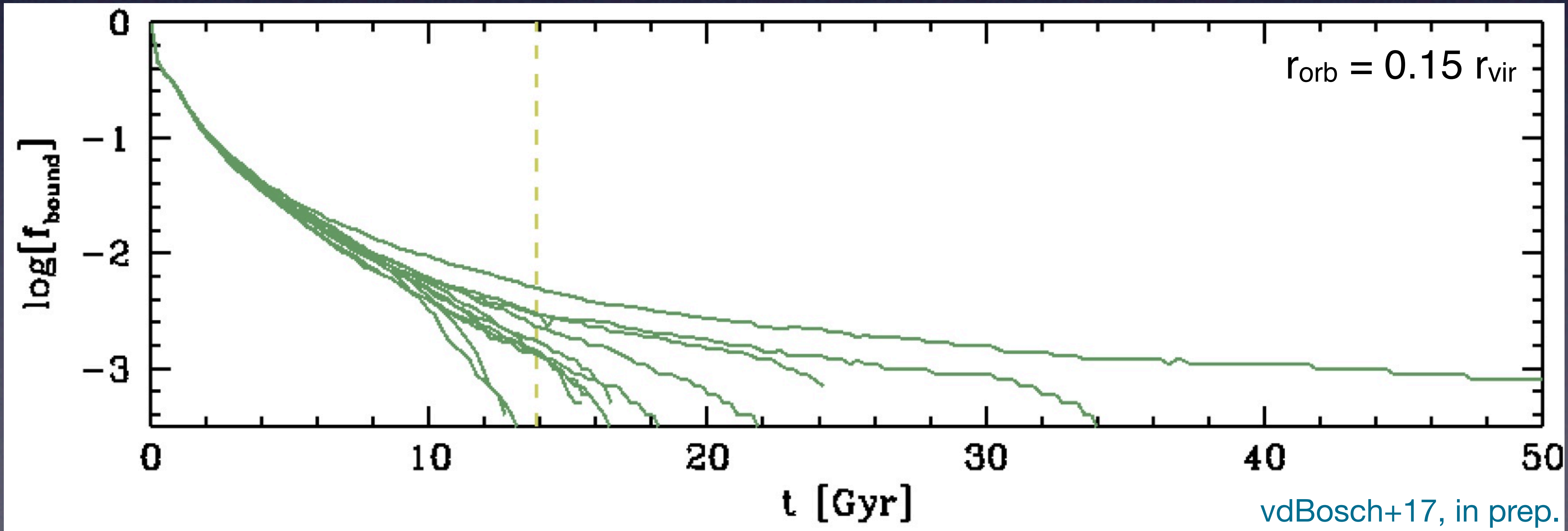


- Analytical predictions fail to predict amount of mass stripped
- Mass loss continues for >50 Gyr

Tidal Stripping on Circular Orbits



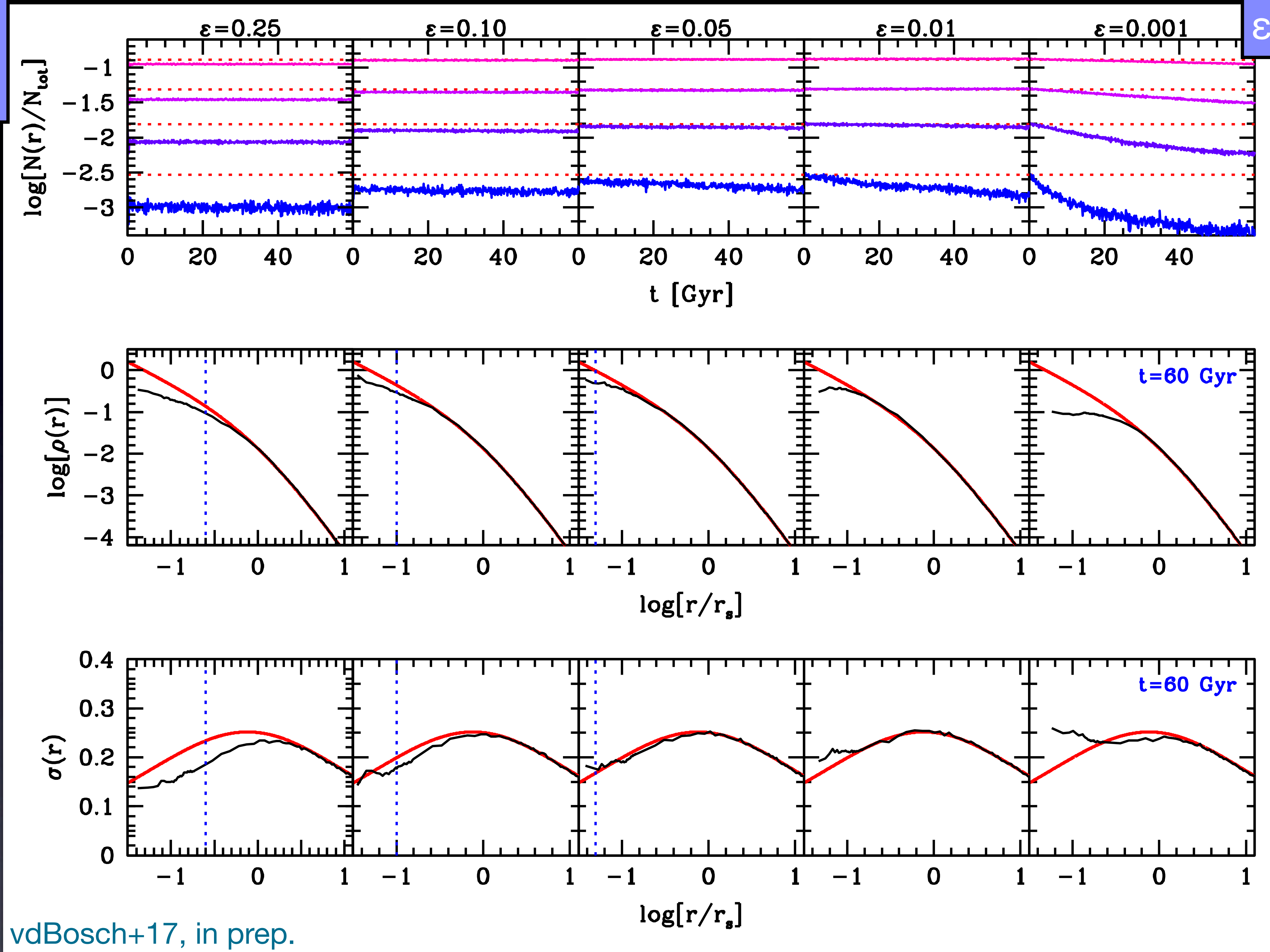
Disruption for $r_{orb} < 0.15\ r_{vir}$......or numerical artefacts?



Tuning the Softening Length

NFW halo
N=10^5

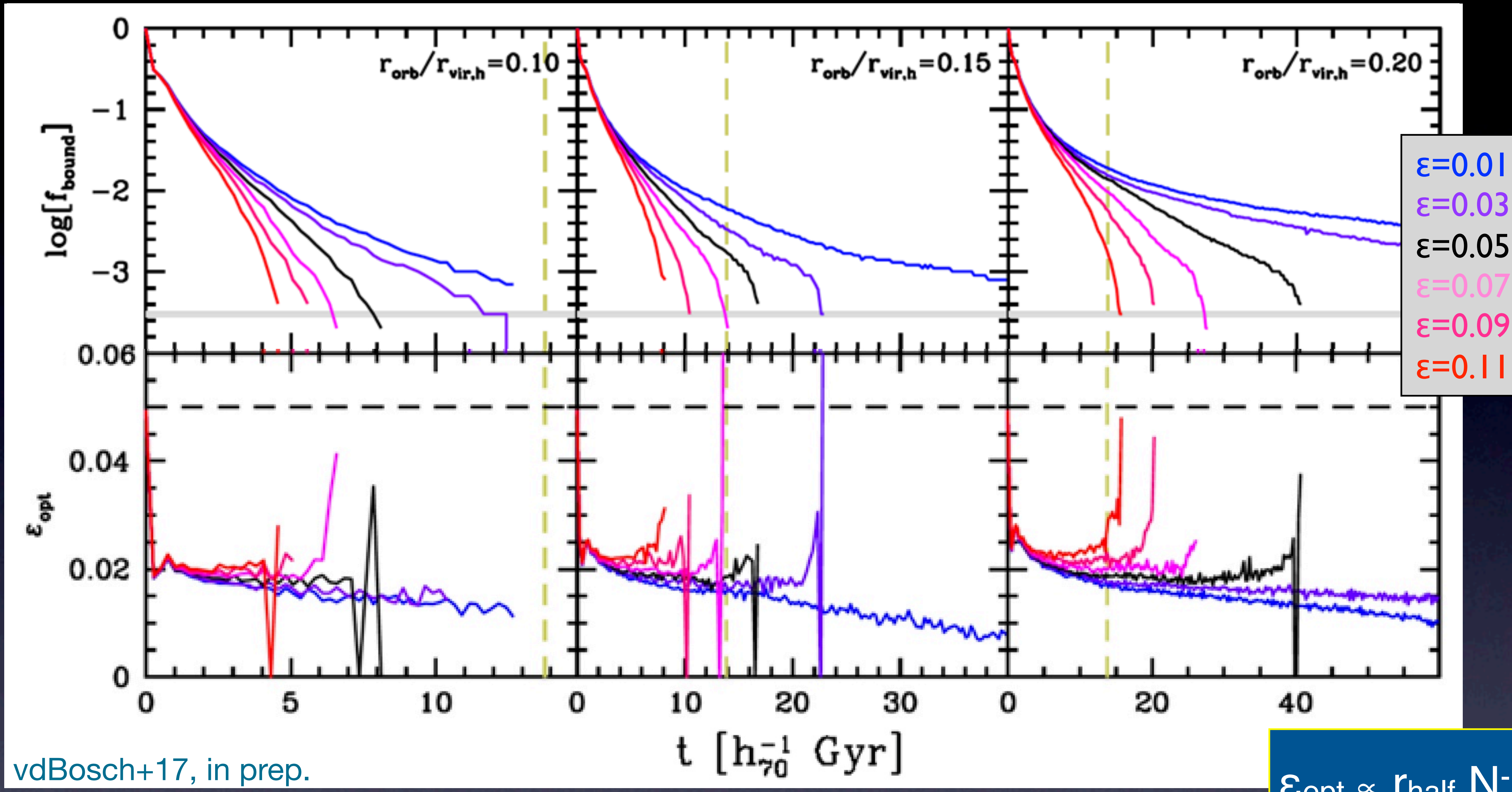
$\varepsilon_{opt} \simeq 0.05$



ε too large ➢ force bias ➢ central cusp unresolved

ε too small ➢ force noise ➢ artificial large-angle deflections ➢ isothermal core

Force Softening



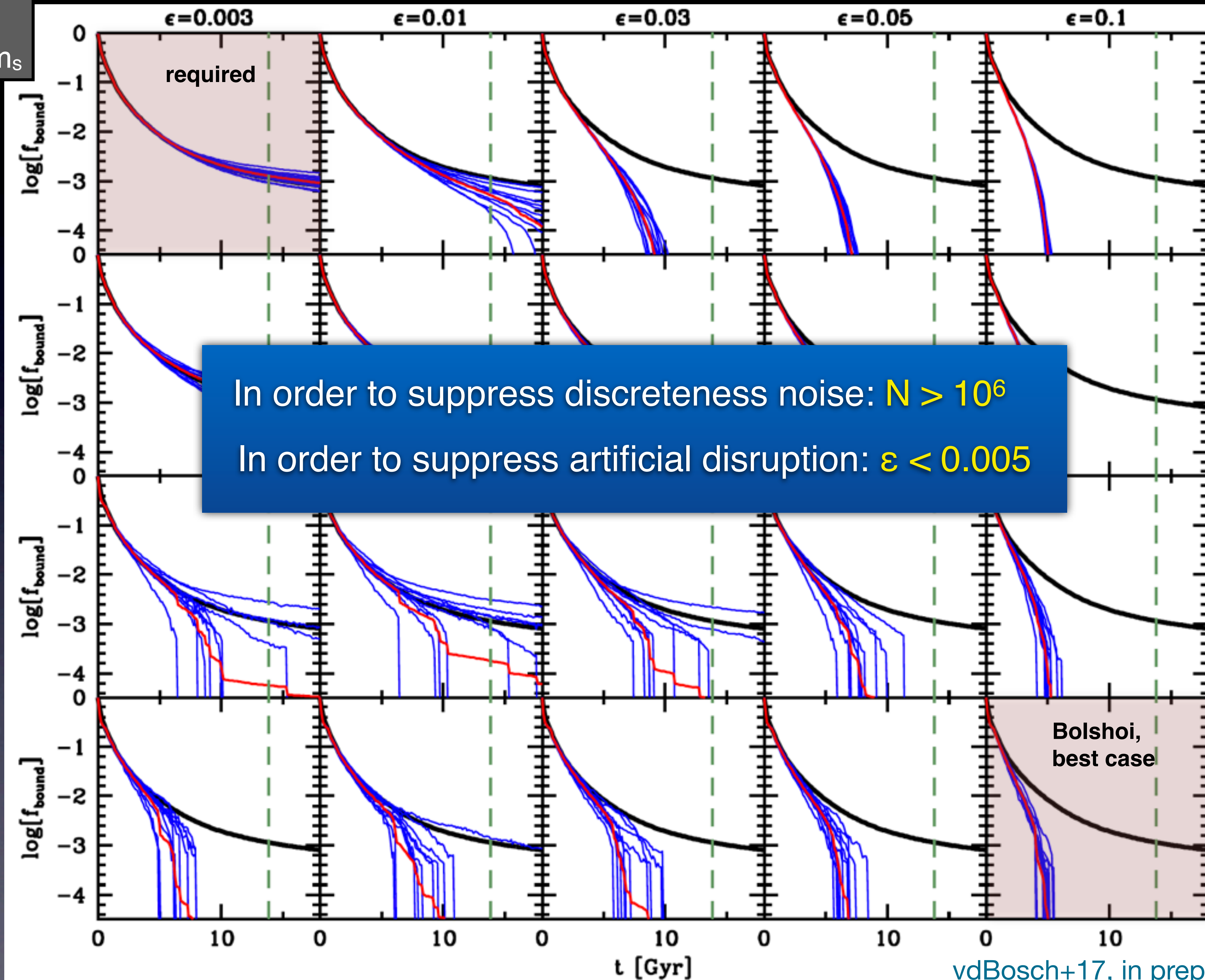
$\varepsilon_{opt} \propto r_{half}\, N^{-1/3}$

(Dehnen+01; Power+03)

- Mass evolution and disruption extremely sensitive to softening length
- As subhalo looses mass, its optimal softening length decreases
- Adaptive, individual softening is required (e.g., Iannuzzi & Dolag 11; Hobbs+15)

Towards Numerical Convergence

$r_{orb}=0.1$
$c_h=5$
$c_s=10$
$M_h=10^3 m_s$



In order to suppress discreteness noise: $N > 10^6$

In order to suppress artificial disruption: $\varepsilon < 0.005$

N=1,000,000

N=300,000

N=100,000

N=30,000

Conclusions

- ★ N-body simulations suffer far more from discreteness-induced relaxation effects than generally acknowledged
  - causes spurious fragmentation and relaxation
  - overall impact of these effects remains unclear (ignored)
  - can we trust (universal) density profiles of CDM halos?
- ★ Current generation of cosmological simulations still suffers from severe overmerging.
  - serious road-block for small-scale cosmology program
  - serious road-block for understanding galaxy formation
- ★ How to proceed?
  - use adaptive softening (has to adapt to local tidal field)
  - develop new phase-space based techniques
  - characterize shortcomings of N-body simulations, and complement simulation results with semi-analytical model.


Well, what do you think of simulations now?
I think they are very relaxing