How old is the Universe?
How big is the Universe?
What is the Universe made of?
What will happen to the Universe?
What is at the edge of the Universe?
Cosmology: a branch of metaphysics that deals with the nature of the universe

a theory or doctrine describing the natural order of the universe

a branch of astronomy that deals with the origin, structure and space-time relationships of the universe; also: a theory dealing with these matters

Cosmology (from Greek κοσμολογία - κόσμος, kosmos, "universe"; and -λογία, -logia, "study"), in strict usage, refers to the study of the Universe in its totality as it now is (or at least as it can be observed now), and by extension, humanity's place in it. Though the word cosmology is recent (first used in 1730 in Christian Wolff's Cosmologia Generalis), the study of the universe has a long history involving science, philosophy, esotericism, and religion.
Brief History of Cosmology

600 BC
- Thales

500 BC
- Anaximander
- Pythagoras
- Democritus

400 BC
- Aristotle

300 BC

200 BC

100 BC

0

100 AD
- Ptolemy
...the dark ages...
We live on Earth, which is one of 8 planets that make up the Solar system, together with the Sun, which is our nearest star.


Most planets in the Solar System have moons (only exceptions are Mercury & Venus).

Planets orbit around stars; moons (or satellites) orbit around planets. Stars emit lights, planets reflect light.

Other stars also can have planets, as of Aug 16, 2013, astronomers have detected a total of 914 extra-Solar planets, plus >18,000 candidates detected with the Kepler mission.

None of these planets have been `seen directly'. Astronomers infer their presence using the reflex motion of the star around which the planet orbits....
The Universe in a Nutshell

Most **planets** in the **Solar System** have **moons** (only exceptions are **Mercury** & **Venus**)

- **Moon** (Earth)
- **Io** (Jupiter)
- **Callisto** (Jupiter)
- **Titan** (Saturn)
- **Phobos** (Mars)

**Planets** orbit around **stars**; **moons** (or **satellites**) orbit around **planets**

**Stars** emit lights, **planets** reflect light

Other **stars** also can have **planets**, as of Aug 16, 2013, astronomers have detected a total of 914 extra-Solar **planets**, plus >18,000 candidates detected with the Kepler mission

....or using the periodic reduction in brightness from the **star** that arises from the **planet** transitting in front of the **star**.
The Universe in a Nutshell

All the stars that we can see with the naked eye on a clear night belong to our galaxy, the Milky Way.

The Milky Way is a spiral galaxy that consists of about 100 billion stars, most of which reside in a disk-like structure.

A galaxy is a large agglomeration of stars bound to each other by gravity.

Since our Solar System is located within disk, we see Milky Way as nebuluous band of star-light.

At the very center of the Milky Way, astronomers have discovered a supermassive black hole with a mass of 3 million Solar masses.
Our Milky Way is only one of billions of galaxies scattered throughout the Universe.

The Milky Way has several smaller satellite galaxies, which orbit around the Milky Way. The most famous of these are the Magellanic clouds, which are visible to the naked eye from the Southern hemisphere.

The nearest neighbor galaxy is the Andromeda galaxy, which also has two satellite galaxies. One can see this galaxy, located in the constellation Andromeda, with small binoculars.
Galaxies come in a variety of sizes, masses, and shapes...
Cosmic Collisions

Our nearest neighbour, the Andromeda galaxy, is on a collision course with the Milky Way. The collision, which will happen about 4 billion years from today, will cause the two disk galaxies to `merge' into a giant elliptical galaxy...

The images to the left show computer simulations of what this (slow) spectacle will look like from Earth... The first image is today, the last one corresponds to 7 billion years into the future...
Our nearest neighbour, the Andromeda galaxy, is on a collision course with the Milky Way. The collision, which will happen about 4 billion years from today, will cause the two disk galaxies to `merge’ into a giant elliptical galaxy...

The images to the left show computer simulations of what this (slow) spectacle will look like from Earth...The first image is today, the last one corresponds to 7 billion years into the future...
The Universe in a Nutshell

Some galaxies reveal energetic jets emerging from their center.

A ground-based image of the elliptical M87.

Other galaxies reveal immense lobes of emission at radio-wavelengths...

and a high-resolution image of the central region.
Astronomers believe these phenomena are due to a supermassive black hole in the nucleus of these galaxies that is accreting matter.

The Universe in a Nutshell

Artist Impressions

and the real deal...

Core of Galaxy NGC 4261

Hubble Space Telescope
Wide Field / Planetary Camera

Ground-Based Optical/Radio Image

HST Image of a Gas and Dust Disk

380 Arc Seconds
88,000 LIGHTYEARS

17 Arc Seconds
400 LIGHTYEARS
Galaxies like to live together; many galaxies reside in clusters, which are large groups of hundreds to thousands of galaxies. They are the largest bound structures known.
Astronomers have mapped the distribution of millions of galaxies with `redshift surveys' such as the Sloan Digital Sky Survey (SDSS).

Galaxies are found to be distributed in a filamentary, foamy structure, called "the cosmic web".

The redshift distribution of ~500,000 galaxies observed as part of the SDSS.

The 2.5m Sloan Survey Telescope at the Apache Point Observatory in New Mexico.
This is what the sky looks like if you had micro-wave sensitive eyes. What you see is **Milky Way** (foreground) and **Cosmic Microwave Background (CMB)**.
The Universe in a Nutshell

After careful removal of the foreground, this is what you see: tiny fluctuations in the CMB; reflecting the structure of the Universe when it was 300,000 yrs old.

The CMB is radiation left-over from the Big Bang. This is as ‘far’ as we can see; it indicates the edge of our detectable Universe.
The LCDM Concordance Cosmology

- Dark Energy: 73%
- Cold Dark Matter: 23%
- Atoms: 4%

Images of a galaxy and a cosmic microwave background map illustrating the distribution of dark matter and energy in the universe.
Looking Back in Time

Light has large, but finite & constant speed of $c = 300,000 \text{ km/s} = 186,000 \text{ miles/s}$.

To put this in perspective: a light wave travels 7.5 times around the Earth in 1 sec. The space shuttle takes 10.8 hours = 39,000 seconds to cover this distance.

Light is so fast that we do not notice its finite speed in everyday live. However, in cosmology the length scales are so big that the finite speed of light has a very important impact: we are looking back in time!

- Light from the Moon takes $1.3 \text{ seconds}$ to reach us
- Light from the Sun takes $8 \text{ minutes}$ to reach us
- Light from the star Sirius takes $8.6 \text{ years}$ to reach us
- Light from the center of the Milky Way takes $26,000 \text{ years}$ to reach us
- Light from the Andromeda Galaxy takes $2.5 \text{ million years}$ to reach us
Looking back in time

By looking at galaxies further away, we are looking further back in time.

In that respect, a telescope is like a time-machine, and a deep image, is like an archeological dig.

In 2003, the Hubble Space Telescope imaged a tiny, apparently empty area close to the constellation Orion. The image was exposed for ~1 million seconds (11.5 days), resulting in the deepest image of the Universe ever taken: the Hubble Ultra Deep Field (HUDF)
Looking back in time

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Looking back in time

Movie: The Hubble Ultra Deep Field  http://www.youtube.com/watch?v=D-mwfUr-3Xc  2m 43s
**INTERMEZZO: Scientific Notation**

$10^n$ means a 1 with $n$ zeros after it

- $0.01 = 10^{-2}$
- $0.1 = 10^{-1}$
- $1 = 10^0$
- $10 = 10^1$
- $100 = 10^2$
- $1000 = 10^3$

$10^{-n}$ means $1/10^n$

In scientific notation any number is expressed as $ax10^n$ where $a$ is a number between 1 and 10 and $n$ is a positive or negative number

**Examples:**

- $2.43 \times 10^6 = 2,430,000$
- $2.43 \times 10^{-2} = 0.0243$
- 14 billion $= 14,000,000,000 = 1.4 \times 10^{10}$
- 1 year $= 365.25 \times 24 \times 60 \times 60$ sec $= 31,557,600$ sec $\approx 3 \times 10^7$ sec
- $9^{-1/2} = 1/\sqrt{9} = 1/3$
INTERMEZZO: Scientific Notation

Rules for doing arithmetic in scientific notation:

\[
a \times 10^n \times b \times 10^m = a \times b \times 10^{n+m}
\]
\[
\frac{a \times 10^n}{b \times 10^m} = (a/b) \times 10^{n-m}
\]
\[
[a \times 10^n]^m = a^m \times 10^{n \times m}
\]

Some examples of those rules in action:

\[
5.0 \times 10^4 \times 2.5 \times 10^{-2} = 12.5 \times 10^2 = 1.25 \times 10^3
\]
\[
\frac{5.0 \times 10^4}{2.5 \times 10^{-2}} = 2 \times 10^6
\]
\[
[4 \times 10^4]^{1/2} = 4^{1/2} \times 10^2 = \sqrt{4} \times 10^2 = 2 \times 10^2
\]
\[
[4 \times 10^3]^{1/2} = [4 \times 10 \times 10^2]^{1/2} = \sqrt{40} \times 10^1 = 6 \times 10^1
\]
**INTERMEZZO: Scientific Notation**

**Units:** throughout, whenever we need to use constants of nature (Newton’s gravitational constant, speed of light, proton mass), we use MKS units (Meters, Kilograms, Seconds).

In scientific notation, we regularly use a prefix to indicate big or small numbers. The most often encountered prefixes are listed here:

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>kilo</td>
<td>k</td>
<td>$10^3$</td>
</tr>
<tr>
<td>mega</td>
<td>M</td>
<td>$10^6$</td>
</tr>
<tr>
<td>giga</td>
<td>G</td>
<td>$10^9$</td>
</tr>
<tr>
<td>tera</td>
<td>T</td>
<td>$10^{12}$</td>
</tr>
<tr>
<td>milli</td>
<td>m</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>micro</td>
<td>µ</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>nano</td>
<td>n</td>
<td>$10^{-9}$</td>
</tr>
<tr>
<td>pico</td>
<td>p</td>
<td>$10^{-12}$</td>
</tr>
</tbody>
</table>

**Accuracy:** it suffices to express quantities with one significant digit (i.e., $3 \times 10^7$ rather than $3.1698219 \times 10^7$)

NO CALCULATORS NEEDED!!

$$3 = \pi = \sqrt{10}$$

Be Human, not Android.
In astronomy, it is customary to express distances in light-years. A light-year is the distance light travels in one year.

\[
c = 3 \times 10^5 \text{ km/s} \quad \text{1 lightyear} = 3 \times 10^5 \text{ km/s} \times 3 \times 10^7 \text{ s}
\]
\[
1 \text{ yr} = 3 \times 10^7 \text{ s} \quad \text{= 9} \times 10^{12} \text{ km} = 9 \times 10^9 \text{ m}
\]

Hence, a light-year (hereafter `ly') is nearly 10 trillion km, an extremely large distance by human standards. However, in cosmology it is still fairly small;
- The nearest star to the Sun is Proxima Centauri at a distance of 4.2 ly.
- The diameter of the Milky Way disk is approximately 100,000 ly.
- The distance to Andromeda galaxy is approximately 2.5 million ly.

In cosmology, one therefore typically expresses distances in Megaparsec (Mpc): A parsec is a distance measure equal to 3.26 ~ 3 light year.

\[
1 \text{ Mpc} = 10^6 \text{ pc} = 3 \times 10^6 \text{ ly} = 3 \times 10^{19} \text{ km}
\]

- The distance to Andromeda galaxy is approximately 0.8 Mpc.
Looking back in time

Question: A galaxy is located at a distance of 300 Mpc. How long ago was the light that we receive from this galaxy emitted?

[A] 300 years ($3 \times 10^2$ yr)
[B] 300 Mega years ($3 \times 10^8$ yr)
[C] $9 \times 10^8$ years
[D] $9 \times 10^{12}$ years

Answer: $300 \text{ Mpc} = 300 \times 3.26 \times 10^6 \text{ ly} = 3 \times 3.26 \times 10^8 \text{ ly} = 10^9 \text{ ly}$

or: $300 \text{ Mpc} = 3 \times 10^2 \times 3 \times 10^6 = 9 \times 10^8 \text{ ly}$

Hence, it takes about one billion years for the light from this galaxy to reach Earth; we see the galaxy as it was $\sim 10^9$ years ago...
A sense of scale: powers of ten

The objects we have covered so far cover a wide range in sizes:

- diameter of Earth is $\sim 1.3 \times 10^4$ km
- distance to Moon is $\sim 3.8 \times 10^5$ km
- diameter of Solar System is $\sim 5.9 \times 10^9$ km
- distance to Proxima Centauri is $\sim 4.0 \times 10^{13}$ km $\sim 4.2$ ly
- diameter of Milky Way is $\sim 9.5 \times 10^{17}$ km $\sim 100,000$ ly
- distance to Andromeda Galaxy is $\sim 2.3 \times 10^{19}$ km $\sim 0.8$ Mpc
- diameter of a cluster is $\sim 10^{20}$ km $\sim 3$ Mpc
- size of visible Universe is $\sim 4.3 \times 10^{23}$ km $\sim 14$ Gpc

To grasp the range of scales involved, the following movies are helpful

9 min  Movie: Powers of Ten (1968, by Ray & Charles Earnes)  
http://www.youtube.com/watch?v=0fKBhvDjuy0

6.5 min  Movie: The Known Universe (2009, by AMNH)  
http://www.youtube.com/watch?v=17jymDn0W6U&feature=player_embedded
A sense of scale

To get a true sense of scale, let’s consider a scale-model of the Sun and its neighboring stars.

In our scale model the Sun is represented by a tennis-ball, which has a radius of about 3 cm. At what distance do we have to place our model for Proxima Centauri?

[A] roughly 3 meters  
[B] roughly 10 meters  
[C] Calhoun College (roughly 50 meters)  
[D] The New Haven Green (roughly 250 meters)  
[E] The Yale Bowl (about 2000 meters)
A sense of scale

In our scale model the Sun is represented by a tennis-ball, which has a diameter of about 6.7cm. At what distance do we have to place our model for Proxima Centauri?

Diameter of Sun: \( 1.39 \times 10^9 \text{ m} \rightarrow 6.7 \times 10^{-2} \text{ m} \)

\[
\text{Scale factor} = \frac{14 \times 10^8}{7 \times 10^{-2}} = 2 \times 10^{10}
\]

Distance to Proxima Centauri: \( 4.2 \text{ ly} = 4.2 \times 9 \times 10^{15} \text{ m} = 4 \times 10^{16} \text{ m} \)

In our scale model this becomes: \( \frac{4 \times 10^{16} \text{ m}}{2 \times 10^{10}} = 2 \times 10^6 \text{ m} = 2000 \text{ km} \)
In our scale model, Proxima Centauri will be located in New Orleans, ~2000 km away from Harkness Hall room 208.
The location of Pluto
Along similar lines, now consider another scale model, focusing on galaxies.

In this scale model the Milky Way is represented by a frisbee, which has a diameter of about 21 cm. At what distance do we have to place our model for the Andromeda Galaxy?

Answer: ~5 meter
Science
Science (from the Latin scientia, meaning "knowledge") is an enterprise that builds and organizes knowledge in the form of testable explanations and predictions about the world.

The key-word here is testable; the claims of a theory ("knowledge") are subject to observational or experimental testing!

Science is a process by which we come up with possible explanations or theories for what we observe in nature and then use observation and experiment to filter out theories that do not work.

Science works on the basis of empirical falsification (Karl Popper): a theory cannot be proven correct, only demonstrated to be wrong.
How does science operate?

A **theory** is a proposed explanation for how some aspect of the universe behaves*.  
- must make **predictions** that are **falsifiable**  
- when theory is first proposed it is called a **hypothesis**

A theory that is **simpler** and **explains more** is often considered a **better** theory.  
Here `simpler` means having fewer **free parameters**, not necessarily that the **math** is simpler!!

The three tooth-fairies (dark matter, dark energy & inflation) make our LCDM concordance cosmology `complicated` and `ugly`

A theory that has made many predictions that have been tested successfully is often called a **physical law**. We believe physical laws to hold (under certain conditions), but we can never prove this to be the case.

An **experiment** is an observation made under conditions arranged so as to isolate and test a particular prediction made by a theory.  
- must be **repeatable**  
- must carefully **control** external/environmental variables

Prior to publication, scientific results (new theory, experimental data, etc.) are subjected to **peer review**.

---

*NOTE: scientists often use the words **model** and **theory** interchangeably. Strictly speaking, though, a theory must be testable, while a model may simply be used for convenience.*
The Scientific Method

Principles of Scientific Method

- Disagreement is settled by observation & experiment
- A theory is never proven correct, but supporting evidence does make a theory stronger
- Better theories explain more with fewer “parameters”
- Observation & experiment need to be repeatable
- Publications are subjected to peer review

See also Sections 1.2 & 1.3 of textbook!!!
History of Cosmology
Thales of Miletus (624 BC - 546 BC)

Often regarded as the first philosopher in the Greek tradition.

Influenced by travels to Egypt & Babylonia

Thales tried to explain phenomena without reference to mythology.

Used Geometry: he famously measured the height of the pyramids by measuring the length of their shadow at the time of the day when his own shadow was as long as he himself.

Cosmological Thesis:

- Earth floats on water; earthquakes occur when Earth is rocked by waves.

- The world started from water; earth is result of silting, just as he had observed at the river Nile...
Anaximander (610 BC - 546 BC)

Also from Miletus. Pupil of Thales, teacher of Anaximenes & Pythagoras

Considered “Father of Cosmology”

Nature is ruled by laws

First proponent of evolution: animals originated from fish, humans evolved from simpler forms...
[these claims were based on the fact that fossils were known to exist]

Cosmological Thesis:

Introduced the abstract **apeiron** (indefinite, infinite, boundless, unlimited) as an origin of the Universe; he argued that Earth, water, fire & air formed out of **apeiron** during a period of primal chaos. World originates in separation of opposites in the **apeiron**: (hot vs. cold; wet vs. dry)
Anaximander (610 BC - 546 BC)

Also from Miletus. Pupil of Thales, teacher of Anaximenes & Pythagoras

Considered “Father of Cosmology”

Anaximander’s Cosmology

Earth is cylindrical drum. Flat top forms inhabited world, surrounded by circular oceanic mass. Earth floats very still in center of infinite (revolutionary idea, allowing for bodies to pass under the earth). This world is surrounded by concentric wheels with holes in it through which we see Fire --> stars, planets, Sun & Moon. In case of Sun & Moon wheels, holes could change shape to explain moon-phases and lunar & solar eclipses.
Empedocles (ca 490 BC - 430 BC)

Lived in Agrigentum (Greek city in Sicily)

Empedocles demonstrated existence of air, using a water thief.

Originator of the cosmogenetic theory of four classical elements

- **Air** is primarily wet and secondarily hot.
- **Fire** is primarily hot and secondarily dry.
- **Earth** is primarily dry and secondarily cold.
- **Water** is primarily cold and secondarily wet.

Everything is made out of (some combination of) these elements. This `dogma' survived for ~2000 yrs.
Pre-Socratic Philosophers

Pythagoras (ca 570 BC - 495 BC)

Born on island of Samos.

Moved to Croton in Southern Italy where he did most of his `work' (no writings of him have ever been found)

Credited with Pythagorean Theorem, though this was known & utilized before by Babylonians and Indians. Pythagoras (or one of his followers) may have been first to proof theorem (controversial)

$\sqrt{c^2 = a^2 + b^2}$

Introduced idea that mathematics & numbers are `fundamental'

“The so-called Pythagoreans, who were the first to take up mathematics, not only advanced this subject, but saturated it. They fancied that the principles of mathematics were the principles of all things.” \hspace{1cm} (Aristotle, in Metaphysics, ca 350 BC)

Founded a very secretive, religious sect in Croton...

Movie: Three-Minute Philosophy \hspace{1cm} http://www.youtube.com/watch?v=HlBA9_3zj9w
Pre-Socratic Philosophers

Democritus (460 BC - 370 BC)

Born in Abdera, Thrace, Greece.
Pupil of Leucippus
By many considered the “father of modern science”.

Extreme believer in physical laws (mechanical explanations):
no room for mysticism, randomness, purpose or prime mover as in
the teleological views of Plato & Aristotle

Because of this, Plato disliked him so much that he wished all his books burned.

Build on Leucippus’ atomic theory for the cosmos:
Everything is composed of atoms, which are
- indivisible
- indestructible
- always in motion
- and between which lies empty space
Properties of material (iron, water, air, rock, etc) depend the properties
of their atoms: very different from Empedocles’ four elements
Aristotle’s Universe

Aristotle (384 BC - 322 BC)

Born in Stageira, Chalcidice
Student of Plato, Teacher of Alexander the Great
Writings cover, among other, physics, metaphysics, poetry, music, logic, rhetoric, politics, ethics & biology

The Five Elements:

Aristotle accepted the four elements of Empedocles, but added a fifth element, “aether”, which he considered the divine substance that makes up the heavenly spheres & bodies.

Motion:

Each of four earthly elements has its natural place. Earth is at center of the Universe, then water, then air, then fire. When they are NOT at their natural place --> motion: bodies sink in water, air bubbles rise up, rain falls in air, fire rises in air. Heavenly element “aether” has perpetual circular motion.
Earth is spherical

Based on the following observations, Aristotle considered a spherical Earth:
- during lunar eclipse, Earth’s shadow on moon is round
- when travelling North-South, new constellations appear
- when boat sails off in any direction, its hull dissapears before its sails

Note: this was not a new idea; Pythagoras (~500 BC) already `knew’ that Earth was spherical...
Aristotle’s Universe

Universe is composed of 55 concentric spheres to which the celestial bodies are attached. Earth is unmovable at the center, while the spheres rotate at different but constant velocities. Outside of spheres was the Prime Mover, who imparted motion from the outside in.

Order of Spheres: Earth, Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn, Stars

- Ancient Greek philosophers believed spheres and/or circles to be the most perfect shapes
- Spheres fitted tightly together. No empty space or vacua, unlike in Democritus’s atomic world view: nothing is `floating’
- Rotation speed of each sphere was `free parameter’ adjusted to best fit observations.
Aristotle’s Universe consisted of two separate regions:

**Sublunary region:**
Here all substances were made of four elements: earth, water, air & fire.
This region is ‘corrupt’ (not perfect) and subject to chance and decay.
- mountains on earth = deviation from perfect sphericity
- things come into being, mature, decay & die
- transmutations (i.e. air --> water, etc) occur constantly

**Heavens:**
Here substance is made out of aether.
The heavens are perfect (=spherical), unchanging and uncorruptable.

The moon, although part of the heavens, changes phase and shows colored spots (signs of imperfection). This was explained as due to fact that lunar sphere was touching and rubbing against the corrupt, sublunar region.

**Movie:** Three-Minute Philosophy  [http://www.youtube.com/watch?v=Tm0Uq08xXhY](http://www.youtube.com/watch?v=Tm0Uq08xXhY)
Eratosthenes (ca 276 BC - 195 BC)

Born in Cyrene, (modern day Libya)

Third chief librarian of the
Great Library of Alexandria, Egypt

Eratosthenes was mathematician, poet, athlete, geographer, astronomer, and music theorist

“Invented” discipline of geography

“Invented” system of longitude & latitude

“Invented” leap day

Measured circumference of the Earth with a stick & brains and with amazing accuracy

Movie: Carl Sagan explains....  http://www.youtube.com/watch?v=G8cblWMyoRl
Aristotle’s Cosmology faced three important problems:

- Retrograde Motion of planets
- Brightness variation of planets
- Variable speed of planets
To solve these problems, Ptolemy introduced three new constructions:

**Epicycles**: circles on top of circles, allows for retrograde motion

**Eccentric**: offset earth from center, yields variability in speed & brightness

**Equant**: point, opposite center from Earth, around which motion of planets (or epicycle centers) is taken to be uniform.

These constructs were needed to accurately match data on planetary motion.

This Ptolemaic cosmology is the first complete system of mathematical constructions that successfully account for motion of all heavenly bodies.

- its predictive power made it an important astronomical `tool' for centuries!!!
The Antikythera Mechanism

The Greeks not only developed a model to compute/predict the motion of the planets, they also build sophisticated computers to perform the actual computations!!

Discovered in 1901 by sponge-divers off the coast of the Greek island `Antikythera` as part of treasures on board of large Roman vessel...
The Antikythera Mechanism

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Discovered in 1901 by sponge-divers off the coast of the Greek island `Antikythera' as part of treasures on board of large Roman vessel...

Robustly dated at first century BC!!

First studied in detail in 1950s by Yale prof. Derek de Solla Price, who, using X-ray imaging revealed the presence of multiple gear-wheels.

Believed to originate from the city of Syracuse, a colony of Corinth in present-day Sicily, and speculated to be made by Archimedes or his disciples...
Researchers brought a specially developed CT scanning machine to the museum in Greece to image (tomography) the mechanism......this is what the data revealed

http://www.youtube.com/watch?v=6Wp3wL8g2Eg&feature=youtu.be

If you want to know even more, search for the 1hr long BBC documentary entitled: “The Two-Thousand-Year Old Computer”

http://www.hpdst.gr/events/exhibitions/antikythera-mechanism [7.5 min]
http://www.nature.com/nature/videoarchive/antikythera/ [14 min]
The worldview/cosmology laid out by Aristotle and Ptolemy contains the following main elements:

- **Aristotle’s cosmology** which placed Earth at center of a spherical Universe. The terrestrial and celestial regions were made up of different elements with had different kinds of natural movement.
  - The terrestrial region, consisting of concentric spheres of the four elements—earth, water, air, and fire. All bodies naturally moved in straight lines until they reached the sphere appropriate for their elemental composition—their natural place.
  - The celestial region was made up of the fifth element, Aether, which was unchanging and moved naturally with circular motion. The observed irregular motion of celestial objects is considered the combined effects of multiple uniform circular motion.

- **Ptolemaic model of planetary motion**: Ptolemy’s Almagest demonstrated that geometrical calculations could compute the exact positions of the Sun, Moon, stars, and planets in the future and in the past, and showed how these computational models were derived from astronomical observations. The physical basis for the Ptolemaic model invoked layers of spherical shells, though the most complex models (with eccentrics, epicycles and equants) were inconsistent with this physical explanation.

These ideas were adopted and considered main-stream ingredients of Roman/Byzantine science, mediaval Islamic science and the teachings at schools & universities in medieval Europe, until the onset of the scientific revolution during the renaissance in the 16-17th century AD.
The Scientific Revolution

Nicolaus Copernicus (1473 - 1543AD)

Born in Torun, Royal Prussia, Kingdom of Poland
Wrote De Revolutionibus Orbium Coelestium

At time of Copernicus, the geocentric worldviews of Aristotle & Ptolemy were firmly grounded in main-stream thinking; standard curriculum at universities...

Copernicus postulated a heliocentric model, in which the Sun is located at the center of the Universe, the planets (in correct order!) rotate around the Sun on circles, and the moon rotates around the Earth.

Motivation: Ptolemy’s model was becoming insufficiently accurate and Copernicus wanted to “rid heavenly motion from monstrous equant”
An important advantage of Copernicus’ heliocentric model is that it naturally explains the retrograde motion of the outer planets.

Since Copernicus adopted circular orbits, his model was only marginally more accurate than the Ptolemaic model (but did not require equants).

Copernicus never got in trouble with church. He sold his work as astronomical tool to predict orbits; he never stated his model reflects the truth.
The Scientific Revolution

Nicolaus Copernicus (1473 - 1543AD)

Born in Torun, Royal Prussia, Kingdom of Poland
Wrote *De Revolutionibus Orbium Coelestium*

At time of publication, *De Revolutionibus* received little attention since the notion of a moving/rotating Earth was considered ‘absurd’:

- why don’t things fall off
- why would Earth be only planet with a moon?
- lack of stellar *parallax* implies that stars are at much larger distances than planets...seems “unnatural”

Contrary to standard lore, Copernicus’ model was NOT the first heliocentric model. That honor goes to Aristarchus of Samus, 18 centuries earlier!!! Copernicus knew, but didn’t cite Aristarchus

Distance Earth-Sun is called an Astronomical Unit (AU)

This angle is called the (annual) parallax of star A
Aristarchus (ca 310 BC - 230 BC)

Born on island of Samos

Originator of first heliocentric cosmology

Used notion (knowledge?) that moon is illuminated by Sun to estimate distance to Sun using simple geometry.

He found Sun to be 18x further away than moon (actual ratio is ~400), and since both appear equally big on the sky, concluded that Sun is therefore 18x bigger. Since he deemed it unnatural to have such a big object orbit a much smaller object, he placed the Sun at the center of the Universe.

Aristarchus was charged with impiety for “putting in motion the hearth of the Universe”
Aristarchus (ca 310 BC - 230 BC)

Born on island of Samos

Originator of first heliocentric cosmology

But how did Aristarchus determine the size of the Moon relative to the Earth?

Aristarchus measured time from beginning of lunar eclipse to total eclipse, and found it to be half the time of total eclipse. Assuming the above drawing is correct he concluded that Earth is twice as large as Moon. Hence, the size of the Sun is $18/2 = 9\times$ the size of Earth...

Unknown to Aristarchus, the true situation looks more like this: Aristarchus therefore overestimated the Lunar size by about a factor two.
Intermezzo: trigonometry

Small angle approximation:
If the angle $\alpha$ is small ($a \ll b$), then the following approximations hold:

\[
\begin{align*}
\sin \alpha & = \tan \alpha = \alpha \\
\cos \alpha & = \sqrt{1 - \alpha^2} = 1 \\
\end{align*}
\]

Note that these equations ONLY hold if $\alpha$ is measured in radians.
A full circle is 360 degrees and is $2\pi$ radians. Hence,

\[
1 \text{ rad} = \frac{360^\circ}{2\pi} \approx 60^\circ
\]

1 degree = 60 arcmin
1 arcmin = 60 arcsec

See small_angle_apm.pdf on Classes*v2 website for more information
Combined geometrical benefits of Copernican system with philosophical benefits of Ptolemaic system in a geo-heliocentric system:

**Tycho Brahe (1546 - 1601)**

Danish astronomer

Lost part of nose in duel with another student, wore metal insert over missing part

Is credited with most accurate astronomical observations of his time.

Refuted theory of celestial spheres by demonstrating that (super)novae and comets are superlunar objects, rather than atmospheric phenomena: `celestial spheres' are not in immutable, unchangeable state of perfection

Combined geometrical benefits of Copernican system with philosophical benefits of Ptolemaic system in a geo-heliocentric system:

Sun & Moon orbit around central Earth, while planets orbit around Sun. This **Tychonic model** for the Universe was popular in early 17th century.
In 1609 he build his first telescope (3x magnification). One year later he build an improved version (30x magnification) and published *Sidereus Nuncius* ("The Starry Messenger"), the first scientific treatise based on telescopic observations. It reported:

- discovery of Galilean moons of Jupiter
- mountains & valleys on Moon's surface
- that Milky Way band consists of many stars
- planets appear as disks, stars don't

These first two discoveries both refuted the Aristotelian/Ptolemaic worldview...
Galileo Galilei (1564 - 1642)
Born in Pisa, Italy
Central Figure in Scientific Revolution
Father of “Modern Science”

Mountains & Valleys on Moon’s surface

Galileo observed that line separating lunar day from lunar night (=terminator) is smooth across dark regions, but irregular where it crosses brighter areas. He correctly deduced that darker regions are flat, low-lying areas, while brighter regions are rough & covered with mountains. He estimated mountaintops to be at least 4 miles in height.

This contradicts Aristotelean cosmology according to which all heavenly bodies are perfect (=smooth) spheres!!!
Galileo Galilei (1564 - 1642)
Born in Pisa, Italy
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Father of “Modern Science”

Discovery of Galilean Moons of Jupiter

On Jan 7, 1610 Galileo observed Jupiter and noted “three stars strung on a line through the planet”. In the following weeks he noted these `stars’ never leave Jupiter, appear at varying distances and sides of Jupiter, and are four rather than three. He soon realized they must be moons orbiting Jupiter....

This contradicts Aristotelean cosmology according to which all heavenly bodies orbit Earth!!!

Galilean moons of Jupiter (Io, Europe, Ganymede, Callisto) seen through small telescope
The Scientific Revolution

Galileo Galilei (1564 - 1642)

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Father of “Modern Science”

Discovery of Phases of Venus

In Sept 1610 Galileo observed that Venus reveals phases like the moon. In 1613 he published his results, showing that Venus goes through a full set of phases.

This was last nail in coffin of Ptolemaic cosmology, in which Venus can never be more than half-lit, because Venus and Sun are never observed to be far apart.

Most astronomers quickly abandoned Ptolemaic model, but most converted to Tychonic model, rather than Copernican model.
Pope Urban VIII, who was a friend and admirer of Galileo, assured him he could write about Copernican theory as long as he treated it as a mathematical proposition. However, with the printing of his book "Dialogue Concerning the Two Chief World Systems", Galileo was called to Rome in 1633 to face the Inquisition. He was found "vehemently suspect of heresy" and placed under house arrest (in Florence) for the remainder of his life.

**Movie:** Three-Minute Philosophy
http://www.youtube.com/watch?v=wlawvC117mM
Intermezzo: Galileo’s observations of Saturn

Galileo also used his telescope to observe Saturn. On July 30, 1610 he wrote to his Medici patron:

I discovered another very strange wonder, which I should like to make known to their Highnesses . . . , keeping it secret, however, until the time when my work is published . . . . the star of Saturn is not a single star, but is a composite of three, which almost touch each other, never change or move relative to each other, and are arranged in a row along the zodiac, the middle one being three times larger than the lateral ones, and they are situated in this form: oOo.

In order to avoid being scooped, while he waited to publish these results in his new book, Galileo “announced” his discovery in the form of an anagram (fairly common prior to scientific journals!!)

altissimum planetam tergemimum observavi

“I have observed the highest planet tri-form”

It wasn’t until 1659 that the Dutch astronomer Christiaan Huygens published his theory that Saturn is surrounded by “a thin flat ring that nowhere touches it”
Galileo Galilei (1564 - 1642)

Born in Pisa, Italy

Central Figure in Scientific Revolution

Father of “Modern Science”

Galileo’s revision of the concept of motion

Galileo also made important contributions to physics. He disproved the Aristotelian idea that heavier objects fall faster through a medium (air/water) than lighter ones. Using various experiments Galileo asserted that all objects, regardless of their mass, fall at the same rate (in a vacuum).

He also discovered that pendulums are isochronic: their period is independent of the amplitude (the arc of the swing).
Johannes Kepler (1571 - 1630)

Born in Weil der Stadt (near Stuttgart), Germany
Key Figure (astronomer) in Scientific Revolution
Worked under Tycho Brahe, using his accurate data

Kepler's three laws of planetary motion:

- The orbit of every planet is an ellipse with the Sun at one of the two foci
- A line joining a planet and the Sun sweeps out equal area during equal time intervals
- The square of the orbital period, \( P \), of a planet is directly proportional to the cube of the semi-major axis, \( a \), of its orbit

\[ P^2 \propto a^3 \]
Kepler’s First Law

The orbit of every planet is an ellipse with the Sun at one of the two foci.

Orbital eccentricity

\[ e = \frac{r_a - r_p}{r_a + r_p} \]

where

- \( r_a \) = aphelion distance
- \( r_p \) = perihelion distance

The eccentricity of the Earth’s orbit is \( e = 0.017 \).
A circle has \( e = 0.0 \).
Kepler's Second Law

A line joining a planet and the Sun sweeps out equal area during equal time intervals.

Orbital motion is fastest at **perihelion** (closest to the Sun), and slowest at **aphelion** (farthest from the Sun).
Kepler's Third Law

The square of the orbital period, $P$, of a planet is directly proportional to the cube of the semi-major axis, $a$, of its orbit.

$$\frac{P_{\text{planet}}^2}{P_{\text{Earth}}^2} = \frac{a_{\text{planet}}^3}{a_{\text{Earth}}^3}$$

**Example:**

- The semi-major axis of Earth is 1 AU (eccentricity is negligible).
- The period of Earth is 1 year.
- The period of Jupiter is observed to be 11.85 years.

$$a_{\text{Jupiter}} = \left( \frac{P_{\text{Jupiter}}}{P_{\text{Earth}}} \right)^{2/3}$$
$$a_{\text{Earth}} = (11.85)^{2/3} \text{ AU} = 5.2 \text{ AU}$$
Isaac Newton (1642 - 1727)

Born in England, the year Galileo died.
Revolutionized physics & mathematics
One of most influential people in human history

Among his many achievements are:
- invention of reflecting telescope
- invention of calculus
- development of law of universal gravity
- elucidation of three laws of motion

Was named Lucasian Professor of Mathematics at Cambridge in his mid-twenties (recently held by Stephen Hawking)

Many of his great ideas came in 1665-66, when he spent time back at home (Woolsthorpe) while his Cambridge was closed because of the plague (would eventually kill 1 out of every 6 in London). He made himself a study, began reading (Euclid, Descartes, Galileo, Copernicus, Kepler), taking notes, and thinking (mainly about concept of motion and infinity). Since he was poor, he invented his own shorthand (tiny script) to save on paper costs.
Isaac Newton (1642 - 1727)

Born in England, the year Galileo died.
Revolutionized physics & mathematics
One of most influential people in human history

Newton as the inventor of Calculus

Calculus: branch of mathematics that allows one to deal with infinitesimals & infinities, including limits, differentials & integrals

Calculus offers solution to Zeno’s paradox (ca. 440 BC) (infinite series can be finite; \( \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots = 1 \))

Largely invented by Newton as a tool to mathematically deal with his ideas about motion.

Important contributions and additions due to the German mathematician Gottfried Leibniz (1646 - 1716).
Isaac Newton (1642 - 1727)

Born in England, the year Galileo died.

Revolutionized physics & mathematics

One of most influential people in human history

The Scientific Revolution

Newton's laws of motion:

- Objects will remain at rest or in uniform motion in a straight line unless acted upon by an unbalanced force. "law of inertia"

- \( F = m \cdot a \); Force = mass \times acceleration

- \( m = \) inertial mass

- To every action there is always opposed an equal and opposite reaction

  table pushes back on book
Newton’s law of Gravity

\[ F = \frac{GM_1 M_2}{r^2} \]

F is the gravitational force. \([F] = \text{N (Newton)} = \text{kg m s}^{-2}\)

G is the gravitation constant. \(G = 6.7 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{s}^{-2}\)

M is the gravitational mass. \([M] = \text{kg}\)

r is the distance between the objects. \([r] = \text{m}\)

Newton realized that his law applies to both falling apples on Earth, as well as to planets orbiting the Sun, or any other celestial body: his law is universal !!!
Kepler empirically found that \( P_{\text{planet}} \propto R_{\text{planet}}^3 \)

However, he did not understand the origin of this scaling relation.

Using his universal law of gravity, Newton derived:

\[
P^2 = \frac{4\pi^2}{G(M_1 + M_2)} R^3
\]

Newton’s version applies to all objects orbiting each other, not just planets orbiting around the Sun: Newton’s version is **universal**

In case of solar system, \( M_1 \) is mass of Sun, and \( M_2 \ll M_1 \) is the mass of a planet. Hence, in the Solar system one has that \( P^2 \propto R^3 \) in agreement with the empirical scaling relation of Kepler.
Isaac Newton (1642 - 1727)

Born in England, the year Galileo died.
Revolutionized physics & mathematics
One of most influential people in human history

Kepler empirically found that \( P_{\text{planet}}^2 \propto R_{\text{planet}}^3 \)

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Nature and nature’s laws lay hid in night;
God said, Let Newton be! and all was light

Alexander Pope (1688-1744)
Summary: The Scientific Revolution

The following new ideas contributed to what is called the scientific revolution:

- The replacement of the Earth by the Sun as the center of the solar system.
- The replacement of the Aristotelian theory that matter was continuous and made up of the elements Earth, Water, Air, Fire, and Aether by rival ideas that matter was atomistic or corpuscular or that its chemical composition was even more complex.
- The replacement of the Aristotelian idea that heavy bodies, by their nature, moved straight down toward their natural places; that light bodies, by their nature, moved straight up toward their natural place; and that ethereal bodies, by their nature, moved in unchanging circular motions with the idea that all bodies are ‘heavy’ (inert) and move according to Newton’s three laws of motion.
- The replacement of the Aristotelian concept that all motions require the continued action of a cause (prime mover) by the inertial concept that motion is a state that, once started, continues indefinitely without further cause.
- The introduction of the universal law gravity, which applies to all (both Earthly and Heavenly) bodies.

But probably the most innovative idea is the scientific method itself; as Galileo put it: "Philosophy [i.e., physics] is written in this grand book—I mean the universe—which stands continually open to our gaze, but it cannot be understood unless one first learns to comprehend the language and interpret the characters in which it is written. It is written in the language of mathematics, and its characters are triangles, circles, and other geometrical figures, without which it is humanly impossible to understand a single word of it; without these, one is wandering around in a dark labyrinth."

Key Players: Copernicus, Brahe, Galileo, Kepler, Newton, Bacon, Descartes
Newtonian Gravity
Newton’s Law of Gravity

Any object (of non-zero mass) will exert a gravitational force on any other object (of non-zero mass). The strength of this force is proportional to the masses involved, and inversely proportional to the square of the distance between them.

\[ F = \frac{GM_1M_2}{r^2} \]

**Question:** Which of the following exerts a stronger gravitational force on you?

a) the supermassive black hole at the center of the Milky Way

b) a pingpong ball 2m away from you

- Distance to galactic center = 26,000 ly
- SMBH at galactic center has mass of 3 million suns
- Sun weighs 2x10^{30} kg
- A pingpong ball weighs about 2.4 grams
Black Hole or Ping-Pong Ball?

Answer: The ratio of the two forces is given by

\[
\frac{F_{\text{pp}}}{F_{\text{BH}}} = \frac{M_{\text{pp}}}{M_{\text{BH}}} \left(\frac{r_{\text{BH}}}{r_{\text{pp}}}\right)^2
\]

\(M_{\text{BH}} = 3 \times 10^6 \times 2 \times 10^{30} \text{ kg} = 6 \times 10^{36} \text{ kg}\)
\(M_{\text{pp}} = 2.4 \times 10^{-3} \text{ kg}\)
\(r_{\text{BH}} = 2.6 \times 10^4 \times 9 \times 10^{12} \text{ km} = 2 \times 10^{17} \text{ km}\)
\(r_{\text{pp}} = 2 \times 10^{-3} \text{ km}\)

Substituting these values gives

\[
\frac{F_{\text{pp}}}{F_{\text{BH}}} = \frac{2.4}{6} \times 10^{-39} \times 10^{40} = \frac{24}{6} = 4
\]

Hence, the gravitational force due to the ping-pong ball is four times stronger than that due to the supermassive black hole at the center of the Milky Way.
Gravity is an extremely weak force

Gravitational Constant: \( G = 6.7 \times 10^{-11} \text{ m}^3\text{ kg}^{-1}\text{ s}^{-2} \)

For comparison, lifting a suitcase of 20 kg requires a force of \( \sim 200 \text{ N} \)

Gravitational Acceleration of the Earth: \( g = 9.8 \text{ m s}^{-2} \)

Newton’s law of gravity: \( F = \frac{G m M_\oplus}{r^2} \)

Newton’s 2nd law of motion: \( F = m \cdot g \)

\( M_\oplus = 6.0 \times 10^{24} \text{ kg} \)

\( r_\oplus = 6.4 \times 10^6 \text{ m} \)
Intermezzo: Scalars versus Vectors

**SCALARS**

A scalar quantity, $S$, is a quantity that only has a magnitude.

- temperature
- mass
- length
- area
- volume
- energy
- pressure
- density

Amplitude of a vector $v = |\vec{v}|$

**VECTORS**

A vector quantity, $\vec{v}$, has both a magnitude and a direction.

- velocity
- acceleration
- force
- momentum
- gradient (of a scalar)
- electrical field
- magnetic field
- gravitational field
In physics and mathematics, we often use Greek letters to indicate certain constants or variables.

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Uniform Circular Motion
If I swing a bucket with a ball as shown in the picture, the ball moves outwards...

Question: what force causes this movement?

ANSWER: the centrifugal force

CORRECT ANSWER: no force; this is just manifestation of Newton's first law of motion
Consider a small, green ball glued to the top of a black ball. If you swing fast enough, the green ball will be 'pulled off' the black ball by the fictitious centrifugal force...

Q: In what direction will the green ball fly off?

A: In the direction of the circular motion, not in the direction of the fictitious force!!!

What really happens is that the ball obeys Newton's first law of motion, and wants to continue its linear, uniform motion. From the perspective of the rotating bucket, the ball now moves with respect to the bucket, as if a force is acting on it. In reality, a force acts on the bucket, not on the ball!!!
Uniform Circular Motion

Consider a mass \( m \) in uniform circular motion around a point \( O \). The word uniform indicates that the speed \( v = |\vec{v}| \) is constant.

Note, though, that the velocity, which is a vector, constantly changes its direction.

The period of rotation is equal to the circumference divided by the speed:

\[
P = \frac{2\pi r}{v}
\]

We can also define an angular speed, as the number of radians covered per unit time:

\[
\omega = \frac{2\pi}{P} = \frac{v}{r}
\]

The angular speed, \( \omega \), which has the units rad/s, can be written as \( \omega = \frac{v}{r} \).

Based on Newton’s first law, there must be a force acting on \( m \). This force is called the centripetal force, which causes a centripetal acceleration, \( \vec{a}_c \), in the radial direction, always pointing towards the center of rotation, \( O \).

Without derivation, which requires calculus:

\[
a_c = |\vec{a}_c| = \frac{v^2}{r} = \omega^2 r
\]
Centripetal Force

The centripetal force can be due to different things:

The tension in your arm: The tension in a rope:

**Tension:** is the magnitude of the pulling force exerted by a string, cable, chain, etc. on some object. The direction of the tension force is always parallel to the string, cable, chain. The tension is due to the electromagnetic forces between the molecules in the string, cable, chain.
The centripetal force can be due to different things:

**The tension in your arm:**  **The tension in a rope:**  **The normal force:**

**Normal Force:** is the component of the contact force that is perpendicular to the surface of contact.

**Contact Force:** a force between objects in direct contact. This force is electromagnetic in origin, and is exerted between the molecules of the objects in contact.
**Centripetal Force**

The centripetal force can be due to different things:

- The tension in your arm: 
- The tension in a rope: 
- The normal force: 
- Friction:

**Friction:** is the component of the contact force that acts parallel to the contacting surfaces, opposing sliding.

**NOTE:** The friction force has the same magnitude as the component of the gravitational force acting in the direction of the slide, as long as the object remains at rest. If it is moving, friction is no longer able to resist the opposing force (but it is not absent).

Similarly, the normal force has the same magnitude as the component of the gravitational force acting in direction perpendicular to the slide.
The centripetal force can be due to different things:

- The tension in your arm:
- The tension in a rope:
- The normal force:
- Friction:

And most importantly for ASTR 170: **GRAVITY**

\[ a_c = \frac{F_g}{m} = \frac{GM}{r^2} \]
According to Newton’s third law of motion, “action = reaction”, there must be a force opposing the centripetal force.

This force is called the **reactive centrifugal force**, not to be confused with the **fictitious centrifugal force**. The latter is not a real force: it is not part of an interaction (but mere consequence of rotation), and has no reaction-force component: many textbooks do NOT distinguish these two forces, causing LOTS of confusion.

Consider a small, **green** ball glued to top of black ball. If you swing fast enough, the **green** ball will be `pulled off` the black ball by the fictitious centrifugal force...

**Q**: in what direction will **green** ball fly off?

**A**: in direction of circular motion. not in direction of fictitious force!!!

Fictitious centrifugal force consequence of inertia and Newton’s first law of motion....
How do you like my centrifuge, Mister Bond? When I throw this lever, you will feel centrifugal force crush every bone in your body.

You mean centripetal force. There's no such thing as centrifugal force.

A laughable claim, Mister Bond. Perpetuated by overzealous teachers of science. Simply construct Newton's laws in a rotating system and you will see a centrifugal force term appear as plain as day.

Come now, do you really expect me to do coordinate substitution in my head while strapped to a centrifuge?

No, Mister Bond. I expect you to die.
**Centripetal vs. Centrifugal Force: Definitions**

**Centripetal Force:** the force that makes a body follow a curved path. It points from the center of the body to the center of motion, always orthogonal to the motion of the body. It is exerted on the body by the object causing the centripetal acceleration.

**Reactive Centrifugal Force:** the reactive force that is paired with the centripetal force. It is the force directed away from the center of motion and is exerted by the rotating mass on the object that is responsible for the centripetal acceleration.

**Fictitious Centrifugal Force:** is most commonly introduced as an outward force apparent in a rotating frame of reference. It is apparent (fictitious) in the sense that it is not part of an interaction, but is a result of rotation — with no reaction-force counterpart. It is a manifestation of inertia and Newton’s first law of motion, and therefore often called an `inertia force`.
Newton's Orbital Cannon

What has a falling apple got to do with orbits?

In his *Principia*, Newton described a thought experiment: imagine a cannon placed on top of a very high mountain (above atmosphere --> no friction). If projectile is fired with sufficient speed, it will fall at the same rate as the Earth's surface curves away --> projectile continues to fall forever = circular orbit.

Astronauts in space shuttle or ISS, in orbit around the Earth, are continuously falling. Although Earth's gravitational force continuously pulls on astronauts, they experience `weightlessness' because they are falling (cf. you in falling elevator).

To gain insight, play with the following animation:  
http://waowen.screaming.net/revision/force&motion/ncananim.htm
Newton's Orbital Cannon

What has a falling apple got to do with orbits?

In his *Principia*, Newton described a thought experiment: imagine a cannon placed on top of a very high mountain (above atmosphere --> no friction). If projectile is fired with sufficient speed, it will fall at the same rate as the Earth's surface curves away --> projectile continues to fall forever = circular orbit.

Weight

Unlike mass, weight is not an intrinsic property of an object. In fact, weight is a force! An object's weight is the gravitational force that it experiences. We normally express weight in kg (though not in physics!!!), because on the surface of Earth the gravitational force on a body of mass m, i.e., its weight, is

\[ \vec{w} = \vec{F} = mg \]

Since the gravitational acceleration, \( \vec{g} \), is a constant, approximately equal to 9.8 m s\(^{-2}\), an object's weight is directly proportional to its mass. You experience weight (gravitational force) if it is balanced by a normal force (the earth, your chair, the scale...). If unbalanced, you experience 'weightlessness'.

Astronauts in space shuttle or ISS, in orbit around the Earth, are continuously falling. Although Earth's gravitational force continuously pulls on astronauts, they experience 'weightlessness' because they are falling (cf. you in falling elevator)

To gain insight, play with the following animation:
http://waowen.screaming.net/revision/force&motion/ncananim.htm
Orbital Speed of Planets

Although orbits of planets are elliptical, they are close to circular. We can use Kepler’s third law to derive \textit{circular velocity} = velocity of a perfectly circular orbit. Consider a test particle (i.e. of negligible mass) in orbit around object of mass $M$:

$$P^2 = \frac{4\pi^2}{GM}R^3$$

According to Kepler’s third law, 

For a circular orbit, we also have that:

$$P = \frac{2\pi R}{v_c}$$

where $v_c$ is the circular velocity.

Combining both expressions for $P$:

$$\frac{4\pi^2 R^2}{v_c^2} = \frac{4\pi^2 R^3}{GM}$$

$\Rightarrow$ \textit{circular velocity} \hspace{1cm} $v_c = \sqrt{\frac{GM}{R}}$

This \textit{Keplerian fall-off} is in perfect agreement with observations in Solar system.
Orbital Speed of Planets

Although orbits of planets are elliptical, they are close to circular. We can use Kepler’s third law to derive \textit{circular velocity} = velocity of a perfectly circular orbit. Consider a test particle (i.e. of negligible mass) in orbit around object of mass \( M \):

\[
\begin{align*}
\text{Alternative derivation based on the centripetal force:} \\
\text{Along circular orbit, the centripetal acceleration is due to the gravitational acceleration} \\
\end{align*}
\]

\[
\begin{align*}
a_c &= \frac{v_c^2}{r} = \frac{GM}{r^2} \\
v_c &= \sqrt{\frac{GM}{R}}
\end{align*}
\]
Using elegant geometry, Newton proved that the gravitational force at a point $P$ inside a spherical shell is always exactly zero. In the illustration, the gravitational force at $P$ due to area $A$ is exactly opposite and equal to that due to area $B$; the latter is more massive, but further away.

Consequently, point $P$ only experiences a non-zero gravitational force from shell $C$. Shells $A$ and $B$, which are located beyond the radius of $P$, add zero gravitational force! The gravitational force at $P$ is EXACTLY the same as if all the mass of $C$ is located at the origin $O$. 

Point $P$ only feels a net gravitational force from shell $C$. Shells $A$ & $B$ contribute zero.
Spherical Matter Distributions

The gravitational force at P is exactly the same in all three situations shown. In situation 3, the sphere C has the same mass as shell C in situation 2.

\[ F_P = \frac{G \, M(<r) \, m_P}{r^2} \]

\( r = \text{distance from center of mass distribution} \)

\( M(<r) = \text{mass enclosed within sphere of radius} \ r \)

\( m_P = \text{mass of } P \)

As long as the mass distribution is spherical, it does not matter how the mass within \( r \) is distributed; point mass, homogeneous sphere, shell, etc. etc.

**WARNING:** this only holds in spherical symmetry!!!
The gravitational force at P is exactly the same in all three situations shown. In situation 3, the sphere C has the same mass as shell C in situation 2.

\[ v_c = \sqrt{\frac{GM(<r)}{r}} \]

- \( r \) = distance from center of mass distribution
- \( M(<r) \) = mass enclosed within sphere of radius \( r \)
- \( v_c \) = circular velocity at \( r \)

As long as the mass distribution is spherical, it does not matter how the mass within \( r \) is distributed; point mass, homogeneous sphere, shell, etc. etc.

**WARNING:** this only holds in spherical symmetry!!!
In physics a **field** is a region in which a force is present.

- gravitational force  --> gravitational field.
- magnetic force  --> magnetic field, etc.

M₁ experiences a gravitational force from M₂; We say M₁ is located in gravitational field of M₂. Similarly, M₂ is located in gravitational field of M₁.

Physicists depict the gravitational field as a **vector field**: to each point in space you assign a vector whose direction and amplitude indicate the direction and strength of the gravitational force, respectively.

- The gravitational field of a point mass looks like a porcupine...
- There is no gravitational field inside a spherical mass shell
- Gravitational fields are **additive**; the gravitational field of two point masses is a field where the vector at each point in space is the vector sum of the vectors associated with the two separate fields.
Energy

Energy is a quantity that expresses the ability of a physical system to produce changes on itself or another physical system. This comes about when energy is transformed from one form to another or from one system to another system.

The unit of energy is the Joule: \(1 \text{ J} = 1 \text{ kg m}^2 \text{ s}^{-2}\) MKS units

Energy comes in many different forms: thermal energy (‘heat’), kinetic energy, potential energy, chemical energy, nuclear energy, electrical energy, mechanical energy, etc.

ENERGY CONSERVATION LAW
The total amount of energy of a closed system is always conserved.
The **kinetic energy** of an object is the energy which it possesses due to its motion.

- An object of mass $m$ moving at velocity $v$ has a kinetic energy $E_{\text{kin}} = \frac{1}{2}mv^2$.

The **potential energy** of an object is the energy which it possesses due to its position in a force field (e.g., gravitational field).

- An object of mass $m$ located at a distance $r$ from another mass $M$ has a potential energy $E_{\text{pot}} = -\frac{GmM}{r} = -m \frac{GM}{r^2} r = -ma_r r$.

The **total energy** of an object moving in a gravitational field is the sum of its kinetic & potential energy. This is often called the **mechanical energy**.

$$E_{\text{tot}} = E_{\text{kin}} + E_{\text{pot}} = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

In absence of friction, the mechanical energy is a conserved quantity. During motion in gravitational field, kinetic energy is converted into potential energy and vice versa.
**Kinetic & Potential Energy**

**NOTE:** potential energy is, by convenient definition, negative!!

Hence, the total (mechanical) energy can be positive, negative or zero.

- Bound orbits have negative total (mechanical) energy
- Unbound orbits have positive or zero total (mechanical) energy

**Escape velocity**

the velocity required to escape (i.e., reach infinity with zero velocity)

\[
v_{\text{esc}}(r) = \left[ \frac{2GM(r)}{r} \right]^{1/2}
\]

**Circular velocity**

the velocity along a circular orbit

\[
v_{\text{circ}}(r) = \left[ \frac{GM(r)}{r} \right]^{1/2}
\]
Potential energy is the energy stored in an object due to its position in a force field (e.g., gravitational field).

\[
v_{\text{esc}}(r) = \left[\frac{2GM(r)}{r}\right]^{1/2}
\]

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the velocity along a circular orbit

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- Unbound orbits have positive or zero total (mechanical) energy
Summary:
Newtonian Gravity
Uniform Circular Motion
**Newton’s law of gravity**

\[ F = \frac{GM_1 M_2}{r^2} \]

**Uniform circular motion**

\[ v = \frac{2\pi r}{P} = \omega r \]

\[ a_c = |\vec{a}_c| = \frac{v^2}{r} = \omega^2 r \]

**Frictionless motion in gravitational field conserves:**

\[ E_{\text{tot}} = E_{\text{kin}} + E_{\text{pot}} = \frac{1}{2}mv^2 - \frac{GMm}{r} \]

- **Circular speed**
  \[ v_{\text{circ}} = \left[ \frac{GM(<r)}{r} \right]^{1/2} \]

- **Escape speed**
  \[ v_{\text{esc}} = \left[ \frac{2GM(<r)}{r} \right]^{1/2} \]

Point P only feels a net gravitational force from shell C. Shells A & B contribute zero.
Consider the space shuttle on a circular orbit around Earth, with uniform velocity $v$.

Q: what happens to the orbit if at location $x$ the shuttle accelerates (burst of booster rockets)?
Changing Orbits

Consider the space shuttle on a circular orbit around Earth, with uniform velocity \(v\).

**Q:** what happens to the orbit if at location \(x\) the shuttle accelerates (burst of booster rockets)?

**A:** the orbit becomes an ellipse with \(x\) at the perigee.

**Q:** what happens to the orbit if at location \(x\) the shuttle decelerates (burst of booster rockets direction opposite of motion)?

**A:** the orbit becomes an ellipse with \(x\) at the apogee.

**apogee:** point along ellipse furthest from earth

**perigee:** point along ellipse closest to earth
You want to construct a roller-coaster with a looping as shown. Ignoring friction, what is the minimum height $h$ from which you have to start your cart so that it safely completes the looping?

In order to reach $B$, centripetal acceleration has to be larger than gravitational acceleration: $a_c = \frac{v_B^2}{R} \geq g$

For convenience, define potential energy to be zero at $h=0$

$E_A = m \cdot g \cdot h$

$E_B = \frac{1}{2} \cdot m \cdot v_B^2 + 2 \cdot m \cdot g \cdot R$

Energy conservation requires $E_A = E_B$:

$m \cdot g \cdot h = \frac{1}{2} \cdot m \cdot v_B^2 + 2 \cdot m \cdot g \cdot R \quad \Rightarrow \quad h = \frac{v_B^2}{2 \cdot g} + 2 \cdot R \geq \frac{R}{2} + 2 \cdot R = \frac{5}{2} \cdot R \quad \Rightarrow \quad h \geq \frac{5}{2} \cdot R$
Elementary Particles, and the Fundamental Forces of Nature
The electrical force is very similar to the gravitational force:

**Gravitational Force**

\[ F = G \frac{m_1 m_2}{r^2} \]

\( m \) = gravitational mass

**Electrical Force**

\[ F = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r^2} \]

\( q \) = electrical charge

Masses are always positive, so that force is always attractive.

\[ G = 6.7 \times 10^{-11} \text{N m}^2 \text{kg}^{-2} \]

Charges can be either positive or negative. Opposite charges attract, equal charges repel.

\[ \frac{1}{4\pi \epsilon_0} = 9.0 \times 10^9 \text{N m}^2 \text{C}^{-2} \]

Although electrical force is very strong, on large scales there are as many positive charges as negative charges, making large objects effectively neutral. On large scales electrical force is negligible.
The Particle Nature of Matter

We distinguish 3 phases (or states) of matter: solid, liquid & gas. In astronomy we are mainly concerned with gases, as stars (and hence galaxies) are all gaseous.

In a gas, the constituent particles have a mean random velocity $\langle v \rangle \propto T^{1/2}$
Here $T$ is the absolute temperature, measured in degrees Kelvin.

Collisions among the constituent particles give rise to pressure $P \propto \rho T$
Here $\rho$ is the density of the gas.

The absolute temperature is also a measure for the thermal energy content of the gas: $E_{th} \propto T$
The Particle Nature of Matter

Regular (baryonic) matter is made up of elementary particles. The particles that are most relevant for ASTR 170 are

- \( \text{P} \) proton; positive charge (q=+1)
- \( \text{n} \) neutron; no electrical charge (q=0)
- \( \text{e} \) electron; negative charge (q=-1)

\[
m_p \sim m_n \sim 2000m_e \\
q_p = -q_e = 1.6 \times 10^{-19} \text{C}
\]

when talking about elementary particles, we express electrical charge in units of electric charge of proton

Atoms are made up of a nucleus, consisting of protons and neutrons, surrounded by electrons. The number of electrons is equal to the number of protons, so that atoms carry no net electrical charge.

Molecules are electrically neutral groups of at least two atoms held together by chemical bonds (i.e., water, ethanol, carbon-monoxide, etc.)

Ions are atoms or molecules in which the number of electrons is either larger or smaller than the number of protons. If the number of electrons is zero, the atom is said to be fully ionized

Isotopes are atoms that contain the same number of protons, but a different number of neutrons (i.e. Carbon-12 and Carbon-14 are isotopes).
The Particle Nature of Matter

Hydrogen and Deuterium are isotopes

Neutral hydrogen is called \( \text{HI} \)
Ionized hydrogen is called \( \text{HII} \)
Molecular hydrogen is called \( \text{H}_2 \)

~74%

In astronomy, all elements heavier than helium are called \textbf{metals}

~24%

~2%
The Particle Nature of Matter

Hydrogen and Deuterium are isotopes

H (Hydrogen)

H (singly ionized Helium)

He (Helium)

He (double ionized Helium)

In astronomy, all elements heavier than helium are called **metals**
Question: why doesn’t the nucleus of an atom `blow’ apart. After all, the electric force between protons expells them from each other. Neutrons are neutral and therefore don’t feel the electric force.....

Answer: because protons and neutrons are held together by a force which is even stronger than the electric force, called the strong force.
The Four Fundamental Forces of Physics

**Gravity**: weakest of them all, but dominates on large scale

**Electromagnetism**: stronger than gravity, but can be small due to neutral charge

**Weak Force**: mainly responsible for radioactive decay

**Strong Force**: holds protons & neutrons together in atomic nuclei, and binds gluons together to form protons and neutrons
Protons and neutrons are made up of three quarks each, held together by **gluons**. There are a total of six different quarks (known as `flavors'') giving rise to many different particles (pions, kaons, hyperons, etc.). All of these are unstable (will decay) except for proton. Quarks can never be isolated; they always combine to make composite particles (called **hadrons**).

### Quarks & Co

<table>
<thead>
<tr>
<th>FLAVOR</th>
<th>CHARGE*</th>
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<tbody>
<tr>
<td>up</td>
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<tr>
<td>down</td>
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</tr>
<tr>
<td>charm</td>
<td>+2/3</td>
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<tr>
<td>strange</td>
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* electron has charge of -1

The existence of quarks was proposed by Murray Gell-Mann in order to put order to the zoo of elementary particles.
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\* electron has charge of -1

Strong force between quarks becomes constant at separations larger than ~hadron (at ~10,000 N). As a consequence, you can’t pull two quarks apart; the very energy applied to pull two quarks apart will turn into new quarks that pair up again with the original ones. This is called confinement.

Murray Gell-Mann
Entered Yale (JE) at age 15, 1969 Nobel Prize in Physics

The existence of quarks was proposed by Murray Gell-Mann in order to put order to the zoo of elementary particles.
Most hadrons are unstable and will decay into other hadrons. This occurs because weak force causes flavor change of quarks.

Important example of such decay is the beta-decay of a neutron into a proton plus an electron plus an anti-neutrino. Since (anti)-neutrino has no electric charge, the total charge is conserved (ALWAYS!!!)

\[
\begin{align*}
\bar{\nu} &\rightarrow e^- \\
W^- &\rightarrow \bar{\nu} + e^- \\
\end{align*}
\]

**Beta-decay** is a type of radioactive decay in which a beta particle (electron or positron) is emitted.

- A free neutron has a mean lifetime (before it decays to a proton) of ~15 minutes !!!
  However, when neutron is inside a nucleus, bound to protons and other neutrons, it is stable (i.e., does not undergo beta-decay)....

- As far as we can tell, protons are stable (i.e., do not decay into other particles).
The Standard Model for Particle Physics is a highly successful, quantum-field theory, which has been tested (and verified) in great detail: it describes all of physics except gravity.

It states that there are three generations of matter, and four `bosons' (the force carriers).

It has one `annoying' problem: it predicts that all particles have zero mass, and move at the speed of light (just like the photon)...

The `solution' to this problem is to postulate that the vacuum has energy-density, described by a quantum-field called the Higgs field, after its `inventor' Peter Higgs.

Actually, was invented by 6 scientists in 1964 (Englert, Higgs, Brout, Hagen, Guralnik & Kibble), but the name “EHBHGK” didn’t stick....
The Higgs Mechanism

This Higgs field is different than any other field, in that it has no sources. EM-field is sourced by charged particles (electrons, protons, etc). Strong-field is sourced by quarks. Higgs-field is property of vacuum; it exists even in absence of matter.

The Standard Model for particle physics claims that all particles have zero mass. However, they have interaction with Higgs field, which slows them down (gives them mass), similar to you being slowed down if you have to move through water (or syrup)

<table>
<thead>
<tr>
<th>Stronger interaction</th>
<th>Slower</th>
<th>More massive (more inert)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weaker interaction</td>
<td>Faster</td>
<td>Less massive (less inert)</td>
</tr>
<tr>
<td>No interaction</td>
<td>$v = c$</td>
<td>Zero mass (e.g., photon)</td>
</tr>
</tbody>
</table>

Energy density of Higgs field has to be constant in space and time, or Universe would be a mess; changing energy density of Higgs changes all particle masses...

Enough energy density can create `ripples` in Higgs field, which manifest themselves as Higgs particles: if Standard Model is correct, Higgs particle (boson) has to exist!
The Hunt for the Higgs Particle
Last March, scientists at the LHC announced they had discovered (at 4.9σ significance) a new Boson with a mass of 125 ± 0.6 Gev. This most likely is the awaited Higgs boson.
Problems with the Vacuum

The Cosmological Constant Problem: UNSOLVED

Recent measurements of supernovae have revealed that the cosmological constant, which represents the energy density of the vacuum is not zero. The measured energy density of this `dark energy' is

\[ u_\Lambda \sim 10^{-9} \text{ J/m}^3 \]

But, we have seen that the vacuum contains the Higgs field, whose energy density is

\[ u_H \sim 10^{+46} \text{ J/m}^3 \sim 10^{55} u_\Lambda \]

If the energy density of Higgs field indeed where that of the vacuum, the Universe would expand to a billion times its current size in less than a nanosecond....clearly, this is unacceptable. NEW PHYSICS REQUIRED

ALSO: according to some theorists, the Higgs field could be unstable....
Electromagnetic Radiation
Waves are structures that transport energy associated with (periodic) displacements. Displacement of water due to droplet, displacement of hand attached to rope, and displacement of air due to subwoofer. In all these cases, the wave is transported through a medium (water, rope, air).

Electromagnetic (EM) radiation, of which light is one form, also consists of waves. EM waves differ from the other waves above in that they travel through empty space!!

Q: so what is waving?
A: the strength of the electromagnetic field
Waves

Waves are characterized by:

- **Frequency**, \( f \): rate at which the cycles of the wave motion are repeated \([f] = \text{Hz} = \text{s}^{-1}\)
- **Wavelength**, \( \lambda \): distance between repeated features in the wave pattern \([\lambda] = \text{m}\)
- **Wave speed**, \( v \): speed with which a wave-phase propagates \([v] = \text{m s}^{-1}\)

These parameters are related according to

\[
\lambda = \frac{v}{f}
\]

Note: the wave speed is a property of the medium in which the wave propagates (tension of rope, temperature of air, etc.). In case of EM waves, the wave speed is the speed of light: \( v = c \)

One distinguishes two types of waves:

**Longitudinal waves:**
- displacement is in same direction as that in which the wave moves
- **Example**: sound waves, seismic P waves

**Transverse waves:**
- displacement is perpendicular to direction in which wave moves
- **Example**: water waves, mechanical waves, EM waves, mexican wave...

Movie: [http://www.youtube.com/watch?v=MoVz2ENjb8M&feature=related](http://www.youtube.com/watch?v=MoVz2ENjb8M&feature=related)
The ElectroMagnetic Spectrum

Penetrates Earth's Atmosphere?

Radiation Type | Wavelength (m) | Approximate Scale of Wavelength | Frequency (Hz) | Temperature of objects at which this radiation is the most intense wavelength emitted
--- | --- | --- | --- | ---
Radio | $10^3$ | Buildings | $10^4$ | $1\text{ K}$
Microwave | $10^{-2}$ | Humans | $10^8$ | $-272\text{ °C}$
Infrared | $10^{-5}$ | Butterflies | $10^{12}$ | $100\text{ K}$, $-173\text{ °C}$
Visible | $0.5\times10^{-6}$ | Needle Point | $10^{15}$ | $10,000\text{ K}$, $9,727\text{ °C}$
Ultraviolet | $10^{-8}$ | Protozoans | $10^{16}$ | $10,000,000\text{ K}$, $\sim10,000,000\text{ °C}$
X-ray | $10^{-10}$ | Molecules | $10^{18}$ | 
Gamma ray | $10^{-12}$ | Atoms | $10^{20}$ |
Optical light is just one of many types of EM radiation that only differs from `light' in its wavelength. Regular (white) light is a mixture of light of different wavelengths (different colors).
Not all EM radiation reaches the Earth’s surface. In fact, most radiation is absorbed by the Earth’s atmosphere. There are only two ‘windows’ (wavelength intervals) for which the radiation reaches the Earth’s surface:

- optical and near-IR
- radio

That is why you will only find optical and radio telescopes on Earth. For all other wavelength regimes, you need to use balloon’s or satellites.
The Andromeda Galaxy in a different light...
Galaxies from Optical to X-rays

Visible | Ultraviolet | X-ray
--- | --- | ---
![M81](image1) | ![M81](image2) | ![M81](image3)

Visible | Ultraviolet | X-ray
--- | --- | ---
![M63](image4) | ![M63](image5) | ![M63](image6)

Visible | Ultraviolet | X-ray
--- | --- | ---
![Arp 319](image7) | ![Arp 319](image8) | ![Arp 319](image9)

Visible | Ultraviolet | X-ray
--- | --- | ---
![Arp 319](image10) | ![Arp 319](image11) | ![Arp 319](image12)
The ElectroMagnetic Spectrum

Question: What happens if you shine the red light coming out of the prism through another prism???

Answer: It simply remains red light; it is not dispersed any further... This experiment, first conducted by Newton, demonstrates that a prism does not `make the colors', but that `white' light is already composed of light of different colors.
Observing EM Radiation

**Optical Telescopes:** one distinguishes two types of optical telescopes

- Refractor (Galileo)
- Reflector (Newton)

- Virtually all modern telescopes are reflectors.
- The newest technologies include active & adaptive optics with laser guidestars.....
The two Keck Telescopes (Mauna Kea, Hawaii)

These are currently the biggest telescopes on Earth, with a mirror of 10m diameter.
The European Very Large Telescope (VLT) array (Paranal, Chile)

Four telescopes of 8.2m diameter each, which can be combined to mimic a single telescope of ~16m diameter
Observing EM Radiation

Radio Telescopes:

- Arecibo (Puerto Rico)
- Parkes Radio Telescope (Australia)
- Very Large Array (VLA), New Mexico

Radio telescopes typically operate at GHz frequencies (corresponding to wavelengths of the order of cm - m).

Radio galaxy, observed with VLA at 6cm

A radio telescope reflects radio waves to a focus at the antenna.
Each wavelength range requires very different detector technology.
Quantum Physics & Emission Mechanisms
Continuum Radiation

Since vibrating electrons emit EM radiation, and since any object with an absolute temperature $T > 0^\circ$K has its electrons vibrating, any object with $T > 0^\circ$K emits EM radiation. If the object is a black body (i.e., a body that absorbs all incident radiation), then the radiation has a continuum spectrum that has a Planck curve.

**Wien's displacement law**

Here $\lambda_{\text{max}}$ is in meters, and $T$ in degrees Kelvin.

$$\lambda_{\text{max}} = \frac{2.9 \times 10^{-3}}{T}$$

**Conversion between Kelvin (K) and Fahrenheit (F)**

- $K = (F - 32) \times \frac{5}{9} + 273.15$
- $F = (K - 273.15) \times \frac{9}{5} + 32$
A system is called a **black body**, and hence emits continuum radiation with a **Planck Spectrum**, if it obeys the following two conditions:

1) the body is in **thermal equilibrium**
2) it absorbs every incident photon

Two systems in contact are in **thermal equilibrium** when there is no exchange of energy between them. This implies that they have the same temperature.

A single isolated system is said to be in **thermal equilibrium** if it has a uniform temperature.

The best known black body in nature, by far, is the **cosmic microwave background**.

Stars are reasonable black bodies, except that they are surrounded by a cooler atmosphere, and are only in **local** thermal equilibrium [i.e. \( T = T(r) \)]
Bremsstrahlung

When a charged particle (i.e., electron) is accelerated, it emits electromagnetic radiation. This radiation is called *bremsstrahlung*, which literally translates into 'braking radiation'.

Clusters of galaxies contain a large amount of very hot gas (*intra-cluster medium*), which is completely ionized (no atoms; only free electrons & atomic nuclei). These free electrons are constantly decelerated by the nuclei, causing *bremsstrahlung*, which is visible as a large halo of X-ray emission.

Image of X-ray emission due to bremsstrahlung (in purple) of a cluster of galaxies, overlaid on an optical image.
Bremsstrahlung

When a charged particle (i.e., electron) is accelerated, it emits electromagnetic radiation. This radiation is called bremsstrahlung, which literally translates into ‘braking radiation’.

Problem: If electrons in atom circle nucleus, they should also emit bremsstrahlung. Due to this energy loss, they should spiral to nucleus.....atoms can’t survive...

This was a big problem for physics in beginning of 20th century....
The Photoelectric Effect

Shining green or blue light on potassium, one notices that free electrons are kicked out.... If the light is red, nothing happens however, independent of the intensity of the red light!

The (maximum) kinetic energy of liberated electrons depends on the frequency (=color) of the incident light.

In 1905 Einstein concluded that light must be quantized (birth of quantum physics): Light is made of quanta, or particles, (called photons), rather than waves.

The photoelectric effect is easy to understand if (i) a threshold energy is required to liberate an electron, and (ii) the energy of light quanta increases with the frequency of the light. **Bluer** light has higher frequency --> more energy per quanta. The energy of red photons is apparently less than the threshold energy. Quanta of **bluer** light have more surplus energy, resulting in a higher kinetic energy of the liberated electrons...

Energy of photon of frequency \(f\) is: \(E = hf\)

Here \(f\) is the frequency of the photon and \(h\) is the Planck constant: \(h = 6.6 \times 10^{-34} \text{ m}^2 \text{ kg s}^{-1}\)
Quantum Physics
&
Emission Mechanisms
In 1913 the Danish physicist **Niels Bohr** invented a model for the structure of the hydrogen atom that can explain the quantized nature of radiation.

He postulated that the single electron can only orbit the nucleus in a number of discrete (quantized) circular orbits, each corresponding to a different discrete (quantized) energy.

This solved the *bremsstrahlung problem*: electron can’t spiral in, because it only is allowed to have quantized energy states...

In modern physics, we believe that ALL atoms have a shell like structure for their electrons, with each shell having a discrete energy.
The Particle-Wave Duality

So we have seen that light is an electromagnetic wave, but then we said that light consists of particles (photons). So what is it, wave or particle?

Answer: both

In fact, elementary particles (electrons, protons, neutrons), are ALSO both particle and wave. This is evident from famous double-slit experiment

_movie:_ http://www.liveleak.com/view?i=3e3_1283308378 (5m12s)

This particle-wave duality of matter & radiation is a strange aspect of physical reality in the quantum world. Think of particles (including photons) as small wave packages (localized waves)... 

Heisenberg’s Uncertainty Principle:

certain properties of elementary particles, such as position and velocity, cannot be known simultaneously to arbitrary precision

\[ \Delta x \Delta v \geq \frac{h}{4\pi} \]
Interaction of Light with Matter

Absorption of photon

$E = hf = E_2 - E_1$

Emission of photon

$E = hf = E_2 - E_1$

$E_1 < E_2 < E_3$
Absorption Lines:
arise when continuum photons from for example a star are absorbed by atoms and ions in intervening gas along the line of sight (or in atmosphere of star). Only the photons at specific frequencies that excite atoms are absorbed.

Emission Lines:
arise when continuum photons from for example a star excite atoms in a gas cloud. The electrons in an excited state spontaneously fall back to lower energy states (ultimately making it back to ground state), emitting photons at specific frequencies associated with the transitions.
Each element has a unique set of energy-levels. When that element emits or absorbs photons, it results in a set of spectral lines that are unique to that element; spectral lines are the fingerprint of an element.
Spectral Lines of Hydrogen

Since hydrogen is the most common element in the Universe, it is useful to investigate its spectral lines up close.

Rydberg Formula: \( \frac{1}{\lambda} = R_\infty \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \)

Rydberg constant: \( R_\infty = 1.1 \times 10^7 \text{m}^{-1} \)

\( n_2 > n_1 \) integers indicating energy levels
- \( n_1 = 1 \) Lyman series (Ly)
- \( n_1 = 2 \) Balmer series (H)
- \( n_1 = 3 \) Paschen series (Pa)

Hydrogen lines are indicated by one or two letters indicating series (Ly, H, Pa, etc), followed by a greek letter, indicating order.

The first 3 Balmer lines

<table>
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Summary: Emission Mechanisms

**Continuum Radiation**
thermal emission (black body radiation) from objects with non-zero temperature.
bremsstrahlung from free electrons in a fully or partially ionized (hot) gas.

**Absorption Lines**
arise when atoms/ions in relatively cold gas are excited due to incident radiation. This removes photons of specific frequencies.

**Emission Lines**
arise when electrons in excited atoms decay to lower excitation levels. This produces photons of specific frequencies. NOTE: this requires radiation/shocks to excite atoms....

**example:**
- cosmic microwave background
- stars (approximately)
- dust clouds (low T --> IR)

- hot intra-cluster medium
  hot gas is fully ionized; electrons are decelerated by protons

- cold gas clouds
  if seen along line of sight (los) to a continuum source

- stellar atmospheres
  relatively cold gas in atmosphere absorbs continuum radiation from star

- star forming regions
  young, hot stars excite gas in their surroundings
The Doppler Effect
The Doppler Effect explained:

When source of waves is moving toward observer, each successive wave crest is emitted from a position closer to the observer than the previous wave. Therefore each wave takes slightly less time to reach the observer than the previous wave. Therefore the time between the arrival of successive wave crests is reduced, causing an increase in the frequency. Conversely, if source of waves is moving away from observer, each wave is emitted from a position farther from the observer than the previous wave, causing an increase in arrival time between successive waves, reducing the frequency.
The Doppler Effect

For waves propagating in a medium (e.g., sound waves, water waves), the velocity of observer and source are relative to medium in which the waves are transmitted. The Doppler effect may therefore result from motion of the source, the observer, or the medium. For EM waves, which do not require a medium, only the relative difference in velocity between observer and source needs to be considered.

In ASTR 170 we only consider the Doppler effect for EM waves (e.g. light)

Galaxy not moving wrt Earth: no Doppler effect

Galaxy receding from Earth: redshift (spectral lines move to red site)

Galaxy approaching Earth: blueshift (spectral lines move to blue site)
The Doppler Effect

We can measure blue/red-shift by comparing observed wavelength, $\lambda_{\text{obs}}$, of emission/absorption lines to their rest-wavelength, $\lambda_0$, measured in laboratory on Earth.

\[
\frac{\Delta \lambda}{\lambda_0} = \frac{\lambda_{\text{obs}} - \lambda_0}{\lambda_0} = \frac{\Delta v}{c}
\]

- $\Delta v > 0$ Object is receding from Earth; redshift
- $\Delta v < 0$ Object is approaching Earth; blueshift

Hence, by measuring the observed wavelengths of spectral lines in the spectrum of a star, we can measure its speed wrt us along the line-of-sight (=line connecting us with object).

In case of circular motion, the wavelength of a spectral line is a function of time. By measuring $\Delta \lambda(t)$ one can determine the masses of the objects orbiting each other!!
Example: Measuring the mass of a Black Hole

Physical Situation

Observational Measurement

\[
\frac{\Delta \lambda}{\lambda} = \frac{v_c}{c}
\]

\[
P = \frac{2\pi r}{v_c}
\]

\[
v_c^2 = \frac{GM_{BH}}{r}
\]

\[
M_{BH}
\]
Stars
Luminosity and Flux

An important property of a star is its **luminosity**, $L$, which is the total amount of energy it radiates per second

$$[L] = \frac{J}{s} = W = \text{Watt}$$

What we observe from a star is what we call the **flux**, $f$. This is the amount of energy that reaches our eye/telescope per unit time per unit surface area of the detector (eye/telescope). Another word for flux that is sometimes used is **intensity**.

$$[f] = \frac{J}{s \cdot m^2} = W \cdot m^{-2}$$

The luminosity and flux of an object are related to each other via the **inverse square law**

$$f = \frac{L}{4\pi r^2}$$

Here $r$ is the distance to the source: moving an object twice as far away reduces the flux by a factor four.
Astronomers are interested in measuring the **luminosity** of a star (or galaxy). However, what they can directly measure is only a **flux**. In order to turn that into a luminosity, they first need to estimate the **distance** to the star (or galaxy).

\[ L = 4\pi r^2 f \]

For ~1000 nearest stars, we can use the **annual parallax** to determine their distance. For more distant stars we have to use other, indirect methods, some of which we will discuss at later stages....
The Stefan-Boltzmann Law

Stars are big, gaseous spheres. When we look at a star (e.g., the Sun), most of the radiation we see (in optical) comes from a thin surface layer, called the photosphere. We can’t look deeper inside the star, because the photons are scattered by the ionized atoms in the stellar interior.

To good approximation, photosphere of a star is a Black Body. As we have seen, Black Bodies of a higher temperature emit (i) more radiation and (ii) at lower wavelengths (Wien’s law).

The Stefan-Boltzmann Law

A black body of temperature, $T$, emits a flux, $f$, (energy per unit time per unit emitting area) equal to

$$f = \sigma_{SB} T^4$$

Here $\sigma_{SB} = 5.7 \times 10^{-8} \text{W m}^{-2} \text{K}^{-4}$ is the Stefan-Boltzmann constant.

Hence, a star with a (photospheric) radius $R$ has a luminosity $L$ given by

$$L = 4\pi R^2 \sigma_{SB} T^4$$

surface area of the star
Magnitudes

Unfortunately, astronomers often express fluxes and luminosities in a stupid, archaic unit-system called magnitudes.

Flux, \( f \), is expressed in terms of an apparent magnitude, \( m \)
Luminosity, \( L \), is expressed in terms of an absolute magnitude, \( M \)

\[
m = -2.5 \log f + C_1
\]

\[
M = -2.5 \log L + C_2
\]

Here \( C_1 \) and \( C_2 \) are two constants

Because of the minus-signs in the definitions of magnitudes, an object that appears brighter (larger flux) has a smaller (more negative) apparent magnitude, and a more luminous object has a smaller (more negative) absolute magnitude.

Example: Absolute magnitude of Sun is 4.75, while that of the Milky Way is -20.5
Apparent magnitude of Sun is -26.7, while that of Sirius is -1.4, and that of the Andromeda galaxy is 3.4....

yes, this is stupid and annoying, but so is a government shutdown
The Physics of Stars

At any given radius inside of a star, there is a competition between gravity, which pulls the shell inwards (makes it want to collapse), and pressure, which pushes outwards (makes it want to expand).

Most stars, especially those on the Main Sequence are in a state of equilibrium (neither noticeably contracting or expanding). This is called hydrostatic equilibrium.

In order for a star to be in hydrostatic equilibrium, the pressure needs to increase with decreasing radius. Recall that $P \propto \rho T$. We therefore expect that density and/or temperature are higher in core than at surface....In fact, both density and temperature increase strongly towards the center.

In case of the Sun:

$T_{\text{core}} = 1.5 \times 10^7 \text{K}$

$\rho_{\text{core}} = 1.5 \times 10^5 \text{kg m}^{-3}$

$= 150 \rho_{\text{water}}$

Deeper layers have more gas on top of them and therefore experience a stronger gravitational compression; they need greater outward pressure to compensate.

Under such extreme conditions, nuclear burning takes place....
Nuclear Burning: The Energy Source of Stars

At the temperatures and densities in the interiors of stars, Hydrogen (which is fully ionized) is converted into Helium via a three step process known as the proton-proton (pp) chain. During this nuclear burning, \(6H \rightarrow \text{He} + 2H\). Hence, effectively, one converts 4 Hydrogen nuclei into one Helium nucleus.

Note: this reaction requires high temperature and density for the protons to overcome their repulsive electromagnetic force, and to come close enough to each other for the strong force to take over. Fusion only takes place in the core of the Sun (inner 25%).

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Mass Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4\text{H})</td>
<td>(6.69 \times 10^{-27}) kg</td>
</tr>
<tr>
<td>(\text{He})</td>
<td>(6.64 \times 10^{-27}) kg</td>
</tr>
<tr>
<td>Difference</td>
<td>(0.05 \times 10^{-27}) kg</td>
</tr>
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</table>

This tiny amount of mass is converted into energy according to Einstein’s famous \(E = mc^2\) law.

Per pp-chain, this results in \(4.5 \times 10^{-12}\) J.

Sun has almost \(10^{38}\) pp-chain reactions per second resulting in an energy production rate of \(3.8 \times 10^{26}\) W.

This depletes \(6.3 \times 10^{11}\) kg of hydrogen per second.

For comparison, the mass of the Sun is \(2 \times 10^{30}\) kg.

This energy production heats the core of the Sun, providing the pressure needed to maintain hydrostatic equilibrium. All this energy ultimately makes its way to the surface of the star, and is radiated away: \(L_\odot = 3.8 \times 10^{26}\) W.
Energy Production reduces
Temperature decreases
Pressure decreases
Gravitational contraction
Increase in density and temperature
Nuclear Burning rate increases
Energy Production increases
Temperature decreases
Pressure decreases
Gravitational contraction
Increase in density and temperature
Nuclear Burning rate increases
Energy Production increases
How is energy produced in the central region of a star due to nuclear burning transported to its photosphere, from which it is subsequently radiated away?

In general, there are three mechanisms for transport of heat (thermal energy):

**conduction**
transfer of vibrational energy (heat) due to atoms touching each other, but without actual bulk-motion of the atoms. Requires atoms to be in close contact. Therefore only important in extremely dense regions of some stars...

**convection**
transfer of heat due to bulk motion of atoms or molecules in a fluid (gas or liquid). Hot fluid is less dense than cold fluid --> in gravitational field, hot fluid will rise wrt cold fluid.

**radiation**
transport of photons (which carry energy)
Energy Transport in the Sun

In inner 70% of Sun, main mode of heat transport is radiation.

In outer 30% of Sun, main mode of heat transport is convection.

Close-up of Sun’s photosphere reveals the convective bubbles, called granules, which have average size of ~1000 km.

More massive stars have less convection and more radiation transport....

Question: how long does it take for an average photon to travel from nucleus of Sun to the base of the convective zone?

Answer: roughly 200,000 years!!! Photons produced in core (due to nuclear burning), when traveling outward are continuously absorbed and re-emitted by atoms or scattered by free electrons (Thompson scattering). Once photon reaches photosphere, it only needs an additional 8min to travel to Earth....
Thomson scattering is the scattering of photons (electromagnetic radiation) by free charged particles (mainly electrons).

Photon (local vibration in EM field), when coming close to electron (or other charged particle), causes electron to vibrate. Effectively this means the photon has transferred its energy to the electron. Frequency with which electron now vibrates is exactly the same as the frequency of the photon. A vibrating charge emits EM radiation. Hence, our vibrating electron once again emits a photon, with exactly the same energy as the original photon, but emitted in a random direction........
Based on the spectra of stars, astronomers have categorized stars in different spectral types: \textit{O, B, A F, G, K & M}.

This is mainly a sequence in temperature (and hence color!): going from \textit{blue} \textit{O} stars, with T\textit{$\sim$}30,000K, to \textit{red} \textit{M} stars with T\textit{$\sim$}3000K.

Each of these classes is subdivided in 10 subclasses: \textit{O0 - O9, B0 - B9, A0 - A9}, etc.

Our Sun is a \textit{G2} star, and the temperature of its photosphere is \textit{$\sim$}5800K.

How to remember the Spectral Types?

\textbf{O Be A Fearsome Gorilla, Kill Me}

As we will see, along the sequence from \textit{O} to \textit{M}, stars also become fainter, less massive, and smaller.
The Hertzsprung-Russell Diagram

The Hertzsprung-Russell (HR) diagram is a diagram in which you plot the luminosity (or absolute magnitude) of a star vs. its temperature (or, equivalently, its spectral class). When plotting many stars in such a HR-diagram, they fall in separate regions:

- Most stars (~90%) fall on the Main Sequence: these are called main sequence (MS) stars.
- In the upper right corner, one finds a class of stars that are relatively cold for their luminosity (compared to MS stars). These are the Giants and SuperGiants.
- In the lower left corner, one finds a class of stars that are relatively hot for their luminosity (compared to MS stars). These are the White Dwarfs.

The yellow lines are lines of constant radius. Recall that \( L = 4\pi R^2 \sigma_{SB} T^4 \) so that for a given luminosity & temperature, you obtain a unique radius. Giants are bigger than main sequence stars, white dwarfs are smaller than main sequence stars.
Main-Sequence Stars

Main Sequence (MS) stars are stars that are burning hydrogen into helium in their central region (Sun is a MS star).

Observations have shown that MS stars follow a narrow relation between mass and luminosity:

\[ L \propto M^{3.5} \]

Hence, more luminous stars are more massive....

Using the above relation, we can estimate the lifetime of stars on the Main Sequence:

\[
\text{lifetime} = \frac{\text{amount of hydrogen}}{\text{rate of hydrogen consumption}}
\]

amount of hydrogen \( \propto \text{mass of star} = M \)

rate of hydrogen consumption \( \propto \text{luminosity of star} = L \)

\[
t_{\text{MS}} \propto \frac{M}{L} \propto \frac{M}{M^{3.5}} \propto M^{-2.5}
\]

Hence, more massive stars have shorter MS lifetimes!!
Brown Dwarfs

Stars come in a fairly restricted range of masses: \( 0.08 \leq M/M_\odot \leq 20 \)

- Most massive stars are about 20\( \times \) more massive than Sun. Stars more massive than this cannot exist because they are unstable (radiation pressure is too large).

- Least massive stars have mass about 8 percent of that of Sun. Stars less massive than this do not reach sufficient temperatures and densities in their cores to ignite hydrogen fusion \( \rightarrow \) brown dwarfs.
The Evolution of Low Mass Stars

**Stage (1)**

At some point, star will run out of \( H \) in core, causing nuclear fusion to come to a halt.

Loss of pressure support causes core to contract and heat up (converting grav. potential energy into thermal energy).

\( H \rightarrow He \) fusion starts in shell outside of core. Radiation pressure due to core + shell causes outer layers to expand, cool and become red: star becomes **Red Giant**

Main Sequence Star $\rightarrow$ Red Giant $\rightarrow$ Red SuperGiant

$M_{MS} < 8M_\odot$

Planetary Nebula $\rightarrow$ White Dwarf
The Evolution of Low Mass Stars

Stage (2)

Core continues to contract until density and temperature are high enough that Helium starts to fuse into Carbon & Oxygen.

Once Helium in core is exhausted, core starts to contract again, igniting Helium fusion in second shell (first shell is still fusing Hydrogen into Helium). Radiation pressure from both shells causes further expansion to Red SuperGiant.

\[ M_{\text{MS}} < 8M_\odot \]
The Evolution of Low Mass Stars

Stage (3)

Strong radiation pressure starts to expell outer layers of the SuperGiant, exposing the much hotter inner regions.

Radiation from central star excites atoms in expelled material, signaling formation of a Planetary Nebula.

Main Sequence Star $\rightarrow$ Red Giant $\rightarrow$ Red SuperGiant

$M_{MS} < 8M_\odot$

Planetary Nebula $\rightarrow$ White Dwarf
The Evolution of Low Mass Stars

Stage (4)

After H-burning and He-burning shells are blown away as well, luminosity drops rapidly.

Surviving core has no nuclear fusion, and hence no significant energy production; it slowly cools over time. At this stage the star is called a white dwarf.

Main Sequence Star → Red Giant → Red SuperGiant

\[ M_{\text{MS}} < 8M_\odot \]

Planetary Nebula → White Dwarf
Planetary Nebulae

Mysteriously Beautiful
The Evolution of Low Mass Stars

Main Sequence Star \( (M_{MS} < 8M_\odot) \)

1. Core runs out of Hydrogen \( \rightarrow \) Red Giant
2. Ignition of Helium fusion in core
3. Core runs out of Helium \( \rightarrow \) Planetary Nebula
4. \( \rightarrow \) White Dwarf
5. \( \rightarrow \) Red SuperGiant
The Evolution of Massive Stars

When massive stars run out of hydrogen in the nucleus, they initially behave the same as low mass stars: they start hydrogen burning in a shell, and expand, thus becoming red supergiants. Core contraction causes ignition of Helium fusion, until this runs out and Helium fusion continues in shell.

Core contracts again, reaching sufficient conditions to ignite next fusion reaction.... runs out --> shell burning --> core contraction --> next fusion. Continues until core is made out of iron. Subsequent contraction does not ignite new fusion, but results in Supernova Explosion.
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Core contracts again, reaching sufficient conditions to ignite next fusion reaction. runs out --> shell burning --> core contraction --> next fusion. Continues until core is made out of iron. Subsequent contraction does not ignite new fusion, but results in Supernova Explosion.
Iron is the **most stable element** in the Universe:
- elements less massive than iron can produce energy via nuclear **fusion** (H-bomb)
- elements more massive than iron can produce energy via **fission** (atomic bomb)
- trying to fuse elements more massive than iron requires energy
- nuclear fusion in stars can only proceed up to iron

A = number of protons + neutrons
The Crab Nebula
SN of 1054 A.D.
Tycho's Supernova (1572)

Observed by Tycho Brahe
Part of the Vela Supernova Remnant exploded ~12,000 years ago.
Supernova 1987A
Large Magellanic Cloud

artist's impression

Supernova 1987A
Large Magellanic Cloud
Under Pressure

A stellar core in which there is no fusion taking place contracts. This contraction continues until (i) new fusion reaction starts, or (ii) a new source of pressure prevents further collapse.

- \( M_{\text{core}} < 1.4M_\odot \)
- \( M_{\text{MS}} < 8M_\odot \)

Core collapse ultimately stopped by electron degeneracy pressure ("you can’t push electrons too close together"). White dwarfs consist of Carbon/Oxygen core supported by electron degeneracy pressure.

One sugar cube of white dwarf material weighs \( \sim 4000 \) kg

- \( 1.4M_\odot < M_{\text{core}} < 3.0M_\odot \)
- \( 8M_\odot < M_{\text{MS}} < 20M_\odot \)

If core mass is larger than 1.4 Msun (Chandrasekhar limit), electron degeneracy pressure is insufficient to prevent further collapse. Electrons & protons are squeezed together and `fuse' to form neutrons. If core mass is less than \( \sim 3 \) Msun neutron degeneracy pressure ("you can’t push neutrons too close together") prevents further collapse: neutronstar

One sugar cube of neutronstar material weighs \( \sim 10^{12} \) kg

- \( M_{\text{core}} > 3.0M_\odot \)
- \( M_{\text{MS}} > 20M_\odot \)

If core mass is larger than \( \sim 3 \) Msun, even neutron degeneracy pressure can not prevent gravitational collapse. In fact, nothing can prevent collapse, and core collapses to form a black hole.

One sugar cube of black hole material weighs \( > 10^{25} \) kg
Black Holes

Recall that the escape velocity from an object of mass $M$ and radius $R$ is given by

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

Hence, if such an object collapses (has its radius decrease), the escape velocity increases.

The radius for which the escape speed is equal to the speed of light is called the **Schwarzschild radius**, and is given by

$$R_s = \frac{2GM}{c^2}$$

$R_s,\odot \approx 3\text{ km}$

$R_s,\oplus \approx 1\text{ cm}$

An object whose radius is smaller than Schwarzschild radius is called a **Black Hole**.

The surface $R = R_s$ is called the **event horizon**: no information from inside the event horizon can ever get out.

**Question:** What is inside the event horizon?

**Answer:** We don’t know, and never will. If object collapses to black hole, no known force can halt collapse.. collapse proceeds to **singularity**: ($R = 0, \rho = \infty$)?
Black Holes for Movie Buffs

Prof. Charles Bailyn from Yale chasing black holes:
http://www.youtube.com/watch?v=rw5trKz_kdc&feature=related

Journey into a Black Hole
http://www.youtube.com/watch?v=eI9CvipHl_c&feature=related

The Ultimate Guide to Black Holes (1hr BBC documentary)
http://www.youtube.com/watch?v=VF1QI0a-Lxs&feature=player_embedded

National Geographic: Monster Black Holes (45min documentary)
http://www.youtube.com/watch?v=MlsljXCFcRc

Black Hole in the Office (funny)
http://www.youtube.com/watch?v=P5_Msrdr3Hk&feature=related
Stellar Populations & Globular Clusters
Consider a stellar population of a single age (i.e., all stars of the population formed at the same time). Shortly after the formation, all stars are on the Main Sequence.
Evolution of Stellar Population

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- After a few million years, the massive O-stars run out of hydrogen in their core and start to move to the right in HR-diagram.

HR diagram of a single-age stellar population
Evolution of Stellar Population

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- After O and B stars have gone supernova, the A and F stars start to leave the MS...
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- After \textit{O} and \textit{B} stars have gone supernova, the \textit{A} and \textit{F} stars start to leave the MS...
- The \textit{G} stars are next, slowly emptying out the MS from top-left to bottom-right. The population as a whole becomes both fainter and redder as time goes on...
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- The $G$ stars are next, slowly emptying out the MS from top-left to bottom-right. The population as a whole becomes both fainter and redder as time goes on...
- Since MS-lifetime of $K$ and $M$ stars are longer than age of Universe, they still haven’t turned off from main sequence.
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- Since MS-lifetime of K and M stars are longer than age of Universe, they still haven’t turned off from main sequence.

Stellar Populations that are blue are young (ongoing star formation), while those that are red are old (star formation has been quenched).
Globular Clusters

Globular clusters are dense stellar systems, consisting of 10,000 - 100,000 stars that all formed at roughly the same time (large coeval stellar population).

Illustration of real HR diagrams of various globular and open star clusters. Clearly, they span a wide range in ages....
The Milky Way

- Galactic halo
- Galactic disk
- Galactic bulge
- Galactic center
- Gas and dust
- Open cluster
- Globular clusters
- O, B stars
- Sun
- Emission nebula

The Milky Way is a spiral galaxy located in the local group of galaxies. It contains over 100 billion stars, including our solar system. The galaxy is approximately 100,000 light-years in diameter and 30,000 light-years thick at its center.
Mass-to-Light Ratios

If all stars in a stellar population (i.e. galaxy) are the same as the Sun, then if the luminosity of the system is $L = 10^{10} L_\odot$, then its total stellar mass is $M_* = 10^{10} M_\odot$

We say that the stellar mass-to-light ratio of the population is $\frac{M_*}{L} = 1 \frac{M_\odot}{L_\odot}$

If one knows the stellar mass-to-light ratio, one can trivially convert the luminosity of a stellar population (i.e., galaxy) into a stellar mass. In general, however, stellar populations do not consist solely of Suns....

In general, for a single age stellar population, $\frac{M_*}{L}$ increases with population age.

For realistic stellar populations, stellar mass-to-light ratios cover the range $0.1 \frac{M_\odot}{L_\odot} < \frac{M_*}{L} < 10 \frac{M_\odot}{L_\odot}$

Stellar Populations that are blue have low stellar mass-to-light ratios while those that are red are have high stellar mass-to-light ratios.
Galaxies
In 1610, Galileo Galilei argued, based on his observations with his telescope, that the Milky Way band consists of many thousands of stars...

In 1750, the English Astronomer Thomas Wright, speculated in his “An original theory or new hypothesis of the Universe” that the Milky Way might be a rotating body of many, many stars held together by gravity akin to the Solar system, but on much larger scale. He correctly interpreted the Milky Way band as a geometric projection effect of being inside.... Note that he did not place the Sun at the center!

In 1755, the German Philosopher Immanuel Kant elaborated on Wright’s ideas and he postulated (without any proof) that the many nebulae visible through larger telescopes are separate worlds similar to ours (i.e. the Milky Way). He coined the concept of Island Universes.
From 1758 to 1782 the French Astronomer Charles Messier was trying to find comets, and to prevent wasting his time, he decided to make a catalog of all 'fluffy-looking objects' that could be mistaken for comets in small telescopes.

His list contains over 100 diffuse objects (emission nebula, planetary nebula, supernova remnants, globular clusters & galaxies), and is still in use today for naming objects...The `M' stands for Messier.
Sir William Herschel

German born, English astronomer (1738 - 1822 AD)

Discovered Uranus in 1781 (also discovered 2 moons of Saturn and 2 moons of Uranus)

Composed 24 symphonies, and constructed more than 400 telescopes

Together with his sister Caroline, he made a map of the Milky Way based on counting stars in every direction. Assuming a uniform density of stars, this translates into probing the relative size of the Universe (= Milky Way) in each direction.

Herschel's map of the Milky Way

Towards galactic center: reduced counts due to absorption by dust

Herschel's 40-foot telescope
Jacobus Kapteyn

Dutch Astronomer (1851-1922)

Used extensive observational campaign, and variety of techniques to probe the spatial distribution of stars in the Milky Way.

Similar to Herschel, Kapteyn underestimated the amount of interstellar extinction (absorption by dust). This caused him to underestimate the diameter of the Milky Way disk by factor ~2.

He placed Sun at ~2000pc from center of Milky Way (real distance is ~8000pc)
The `Great' Debate

On April 26, 1920 the National Academy of Sciences hosted a debate between astronomers Harlow Shapley & Heber Curtis at the Smithsonian Museum of Natural History in Washington DC. The issue was the Size of the Universe.

Both astronomers gave a brief talk, and then entered a debate. They also each published a corresponding paper describing their opposing views on the size of the Milky Way and the nature of the nebulae (extra-galactic or not)...

<table>
<thead>
<tr>
<th>The view of Shapley</th>
<th>The view of Curtis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of Milky Way is ~100 kpc</td>
<td>Diameter of Milky Way is ~10 kpc</td>
</tr>
<tr>
<td>Distance to galactic center is ~20 kpc</td>
<td>Sun is close to galactic center</td>
</tr>
<tr>
<td>This large Milky Way is entire Universe; spiral nebulae are population of gaseous objects within it.</td>
<td>Spiral nebulae are extra-galactic. They are island universes as envisioned by Kant.</td>
</tr>
</tbody>
</table>

Based his arguments on Cepheid distances to globular clusters in MW, and on (erroneous) observations of proper motion in spiral nebulae by Van Maanen.

Based his arguments on observations of novae in spiral nebulae, assuming they are similar to those in MW. Also argued that spiral nebulae have correct angular size for extra-galactic objects similar in size to MW.
Cepheid Variable Stars

Cepheid variables are a special class of variable stars (stars whose luminosity varies with time). The variability is periodic (with typical periods of a couple of days to months). The variability is due to radial pulsations of the star (becoming bigger and smaller).

Cepheids are very important because they have a tight relation between their pulsational period and their average luminosity. Hence, they can be used as distance indicators.

- Measure apparent brightness (flux) of Cepheid star as a function of time, and determine period.
- Determine luminosity using empirical period-luminosity relation.
- Using inverse-square law between luminosity & flux you can now determine the distance.
Arguably most influential astronomer of 20th century

- Settled Shapley-Curtis debate in favor of Curtis
- Discovered expansion of the Universe
- Introduced classification scheme of galaxies

In 1922-1923 Hubble used the 2.5m Hooker Telescope on Mount Wilson (the largest telescope in the world at that time) to observe the Andromeda galaxy (M31). He discovered Cepheids, and using their Period-Luminosity relation was able to conclusively demonstrate that M31 was extra-galactic. This once-and-for-all settled the Shapley-Curtis debate, and signaled the birth of extra-galactic astronomy.
Edwin Hubble's Classification Scheme

Ellipticals

E0 E3 E5 E7 S0

Spirals

Sa Sb Sc

SBa SBb SBC

“Early-Types”

“Late-Types”

S' zero', not SO! Disks without spiral structure
**Galaxy Bimodality**

**Late-Type Galaxies**
- Disk-like morphology, often with central bulge component, spiral arms and/or bar
- **Blue** colors, indicative of young stellar populations --> ongoing star formation and small stellar mass-to-light ratios
- Presence of dust and gas (mainly atomic and molecular hydrogen)
- Prevalent in relatively isolated environments (“the field”)

**Early-Type Galaxies**
- Spheroidal morphology. Blend, smooth appearance with little substructure
- **Red** colors, indicative of old stellar populations --> quenched star formation and large stellar mass-to-light ratios
- Virtually free of both dust and gas
- Prevalent in dense environments (clusters)
**Definition:** A galaxy is a gravitationally bound system that consists of stars, brown dwarfs, stellar remnants, and often an interstellar medium consisting of gas & dust.

**Note:** Although globular clusters obey the definition given above, they are NOT considered to be galaxies. Later we will therefore revise our definition of a galaxy.
The Sloan Digital Sky Survey (SDSS)

In 2000, a large worldwide collaboration of hundreds of astronomers started an ambitious project, to obtain deep, multi-waveband images (to measure colors) and spectra of 1 million galaxies.

The project used a dedicated 2.5 meter telescope at the Apache Point Observatory in New Mexico, and took 8 years to complete.

Currently, a follow-up project is underway using the same telescope but with new and improved detectors, in order to probe the galaxy distribution at even larger distances. Yale is a proud member of this collaboration.

The SDSS has revolutionized extra-galactic astronomy, and provided the astronomical community with many years of work to analyze all the data....
Typical Spectrum of a Red Galaxy
The Bewildering Variety of Galaxies

Galaxies display a bewildering variety of shapes, colors, sizes and luminosities.

\[ 10^3 L_{\odot} < L < 10^{12} L_{\odot} \quad (L_{MW} \approx 5 \times 10^{10} L_{\odot}) \]

Galaxies with \( L < 10^9 L_{\odot} \) are typically called dwarf galaxies.

They typically appear as smudges of star-light on photographic plates. Since they are so faint, and have such low surface brightness, they are very difficult to detect. Even at the present day, we continue to detect dwarf galaxies in our nearby environment...
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**Galaxy sizes:** galaxies don’t have sharp edges. Therefore, sizes are often defined as radii that encompass a certain fraction of the entire flux. The most often used radius is the effective radius, \( R_e \), defined as the radius enclosing half the total flux (also called half-light radius).

\[ 0.1 \text{kpc} < R_e < 10 \text{kpc} \quad (R_{e, MW} \approx 5 \text{kpc}) \]
Disk stars are mainly on close to circular orbits. Net sense of rotation.

Stars in spheroidal components (bulge, halo) have no (significant) net sense of rotation.

**NOTE:** Disk orbits are not perfect ellipses, as around point mass. This is due to fact that mass distribution is extended, and that disk is not spherically symmetric.

Gas cannot be on self-intersecting orbits, since this would cause shocks (gas would lose angular momentum and fall to center of galaxy). Hence, gas can only move on circular orbits.

Rotation velocity of gas reflects circular velocity.
Protons, electrons and neutrons have spin. Hydrogen atoms in which proton & electron have aligned spins has slightly more energy than one in which spins of proton & electron are anti-aligned. When spin flip occurs, photon is emitted with a wavelength of 21cm (in radio).

Radio observations at 21cm are ideally suited to probe neutral gas distribution in galaxies. Using Doppler effect, we can also use 21cm observations to probe the rotation of a galaxy. Since gas has to be on circular orbits the motion of neutral gas in a galaxy reflects circular velocity $V_{\text{circ}}(r)$.
21 cm emission from Neutral Hydrogen

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Consider a disk galaxy. We can measure its rotation curve by probing the line-of-sight velocities using emission and/or absorption lines and the Doppler effect.

\[ \frac{\Delta \lambda}{\lambda_0} = \frac{\lambda_{\text{obs}} - \lambda_0}{\lambda_0} = \frac{\Delta v}{c} \]
Correcting for inclination angle

The angle \( i \) is called the inclination angle.

\[
V_{\text{obs}} = V_{\text{circ}} \sin i
\]

Hence, one can obtain circular velocity from observed velocity along line-of-sight, as long as one knows inclination angle. One can estimate \( i \) from the observed flattening:

\[
\cos i = \frac{b}{a}
\]

Here \( a \) and \( b \) are the semi-major and semi-minor axes, respectively.
The Discovery of Dark Matter

In 1978, Vera Rubin took spectra of some nearby disk galaxies, and measured their rotation curves (using the Hα line to probe the dynamics of the gas).

To her surprise, and that of everyone else, there was no expected Keplerian decline at large radii. Rather, the rotation curves remained flat out to last measured point.

\[ M(< R) = \frac{R V_c^2(R)}{G} \]

Hence, the fact that the rotation curve is flat (i.e., \( V_c(R) = \text{constant} \)), implies that \( M(< R) \propto R \)

Thus, when going to larger and larger radii from the center of a galaxy, the enclosed mass continues to increase, even though there is no light (no stars).

This alleged matter that we can’t see directly is called Dark Matter.
Evidence for Dark Matter

Contours show distribution of neutral hydrogen as probed by 21cm emission

$V_c^2(R) = \frac{GM(< R)}{R} = \frac{G [M_d(< R) + M_h(< R)]}{R}$

Disk contribution: $M_d(< R) = \left(\frac{M_*}{L}\right) L(< R)$

The enclosed mass of the dark matter halo, $M_h(< R)$, follows from rotation curve...
The Milky Way

- Galactic halo
- Galactic disk
- Galactic bulge
- Galactic center
- Gas and dust
- Open cluster
- Globular clusters
- O, B stars
- Sun
- Emission nebula

The Milky Way is a spiral galaxy located in the local group of galaxies. The galaxy is approximately 100,000 light-years in diameter and contains over 100 billion stars. The Sun is located in the spiral arm known as the Orion Arm, approximately 28,000 light-years from the galactic center.
Dark Matter in the Milky Way

The distance to the Galactic Center is $R_{GC} = 8 \text{kpc}$

Velocity with which Sun is circling Galactic Center is $v_{rot} = 220 \text{ km/s}$

Period in which the Sun goes around Galactic Center: $P = \frac{2\pi R_{GC}}{v_{rot}} = 2.4 \times 10^8 \text{ yr}$

Mass enclosed within distance to GC: $M(< R_{GC}) = \frac{R_{GC} v_{rot}^2}{G} = 9 \times 10^{10} M_\odot$

The enclosed mass within this radius is: $M(< R) \simeq 5 \times 10^{11} M_\odot$

The enclosed luminosity within this radius is: $L(< R) \simeq 3 \times 10^{10} M_\odot$

$$\frac{M}{L}(< R) \simeq 17 \frac{M_\odot}{L_\odot}$$

Too high for a stellar population!

**Dark Matter**
All galaxies that astronomers have looked at in detail reveal flat (or rising) rotation curves out to the largest radii probed.

This suggests that all galaxies are embedded in large haloes of dark matter...
The Ubiquity of Dark Matter

All galaxies that astronomers have looked at in detail reveal flat (or rising) rotation curves out to the largest radii probed.

This suggests that all galaxies are embedded in large haloes of dark matter...

Since each dark matter halo is more massive than the galaxy which it hosts, this suggests that most matter in the Universe is dark!

The One Million Dollar Question: what is “dark matter”??
**Definition:** A galaxy is a gravitationally bound system that consists of stars, brown dwarfs, stellar remnants, an interstellar medium (gas & dust) and an important but poorly understood component tentatively dubbed dark matter.

**Note:** Globular clusters are NOT galaxies, since they do not seem to have any dark matter.
Consider the (roughly) circular motion in a disk:

$$\omega(R) = \frac{V_c(R)}{R}$$

$V_c(R)$ is constant (flat rotation curves)

$$\omega(R) \propto R^{-1} \implies P = \frac{2\pi}{\omega} \propto R$$

Hence, stars and gas on smaller radii take less time to complete an orbit. This aspect is called **differential rotation**.

Now consider a cloud of gas in which new stars are forming (i.e., blue blob in figure). Because of differential rotation, this blob will be sheared into a spiral arm, consisting of young stars:

**differential rotation --> spiral arms**

But there is a problem.....
Spiral Structure

Differential Rotation can explain some spiral structure we see, but only if spiral arms are continuously being created...

We believe that this is the case in spiral galaxies in which the spiral structure is messy, with multiple arms that cannot be traced over large parts of the disk. Such spirals are called *flocculent spirals*.

*Movie: flocculent spiral* [http://www.youtube.com/watch?v=wPbX1_C_I6o](http://www.youtube.com/watch?v=wPbX1_C_I6o)

However, some spirals have very pronounced spiral structure, with spiral arms that can be traced over large parts of the disks. These are called *Grand-Design spirals*, and they cannot be formed in the same way as flocculent spirals...

In 1964, Lin & Shu proposed an alternative theory for creating Grand-Design spiral structure: *Spiral Density Wave Theory*
In **spiral density wave theory** it is assumed that due to some perturbation the closed orbits (the ones followed by the gas) become **elliptical**, and that their orientations are correlated.

If the ellipses are rotated with respect to each other, a spiral pattern emerges due to **orbit crowding**.

In this picture, stars and gas clouds overtake (or are overtaken) by the density wave, but are not continuously part of it. A good analogy is the knot of traffic around a slow moving truck: the enhancement of traffic moves forward, but at each time consists of different cars...

Note that this is different from **flocculent spiral arms** (also called **material arms**), in which the spiral is always made-up of same material.

Movie of spiral density wave [http://www.youtube.com/watch?v=9B9i4vjj5D4&feature=related](http://www.youtube.com/watch?v=9B9i4vjj5D4&feature=related)
Spiral Density Wave Theory

**Question:** but how do spiral density waves form???

**Answer:** we are not exactly sure, but it may be that they are triggered by gravitational interactions with nearby galaxies...(e.g. M51)

**Question:** If grand-design spirals are indeed spiral density waves, in which stars and gas are continuously moving in and out of the spiral arms, then why do young (blue) stars preferentially reside in spiral arms?

**Answer:** Because density is enhanced in the spiral arms (due to orbit crowding). Hence, when a gas cloud moves into a spiral arm, it gets compressed, which promotes star formation.

Simulation: [http://www.youtube.com/watch?v=AD9OV1Zrs4I](http://www.youtube.com/watch?v=AD9OV1Zrs4I)
In the 1970s computers became sufficiently powerful that astronomers could run numerical N-body simulations of disk galaxies. Starting from an initial distribution of N point masses (where N is large) that resembles a disk galaxy, one computes each time-step the gravitational force on each point mass from all the other point masses (i.e. one solves the equations of motion for all N point masses due to gravity).

To their surprise, astronomers found that self-gravitating disks are violently unstable. In one or two rotations they `self-destruct'. In 1973, Jeremiah Ostriker & Jim Peebles (Princeton) suggested that disk stability requires them to be embedded in a massive halo (of faint stars...). Today we consider the instability of disks another argument in favor of dark matter haloes around (disk) galaxies.
At the present day, astronomers use large and powerful supercomputers to simulate disk galaxies, consisting of stars and gas, embedded in massive dark matter haloes.

These simulations, using millions of particles, show that spiral structure and bars develop “automatically”, due to small instabilities.

Hence, spiral structure and bars are natural features of disk galaxies.

Movie: Spiral Galaxy simulation with gas:
http://www.youtube.com/watch?v=v39UtTfOHAU&feature=related

Movie: Bar formation in disk with gas
http://www.youtube.com/watch?v=L16m5Vg5LbA
Active Galaxies & SuperMassive Black Holes
Radio Galaxies

In **1940s** astronomers discovered class of galaxies that are surprisingly luminous at radio wavelengths; these are called **radio galaxies**, and typically emit millions of times more energy in radio waves than does a normal galaxy.

Radio galaxies often consist of a bright **nuclear source** at the center of the galaxy, two blobs (the **radio lobes**) that are offset from the central galaxy, and **jets** that connect the nucleus with the lobes.

This clearly suggest that these structures are powered by something in the central region of these galaxies...

---

**Synchroton Radiation**

Radio emission arises when charged particles are accelerated by a (strong) magnetic field. Such radiation is called synchrotron radiation.
In 1960s astronomers discovered point-like radio sources. Although they look like stars, it was clear they are not: they were called Quasi-Stellar Radio Sources (quasars for short). Their spectra revealed emission lines that no one had ever seen before; at least, so it seemed...

....until Dutch astronomer Maarten Schmidt from CalTech discovered that the spectrum is familiar but is simply redshifted by a huge amount.... This implies that quasars are very, very far away (further than most galaxies), and therefore also that they are intrinsically extremely bright (luminosities that $1000\times$ that of MW).
Quasars

Today we have discovered thousands and thousands of quasars; because of their extreme luminosities, they can be seen out to very large distances. Quasars emit at all frequencies, from the radio to X-rays, and their spectra look nothing like that of a galaxy (=stars). Another property of quasars is that their brightness varies: In X-rays, it can go up and down by a factor of a few on time-scales of days. This implies that the quasar engine (the source that produces the energy) is smaller than a few light-days across!!!

![Comparison of spectra of star and quasar](image)

It takes at least a timescale

$$\Delta t = cR$$

for a source of size $R$ to increase its brightness by a significant amount

Deep imaging with the Hubble Space Telescope has shown that quasars are the nuclei of galaxies, but that these galaxies, in many cases, have very distorted morphologies...
Active Galaxies

Radio galaxies and Quasars are subclasses of what are called Active Galaxies. In general, an active galaxy is a galaxy that emits excessive radiation that cannot be due to stars. Since most of this excessive emission originates from the nuclei of these galaxies, they are also sometimes called AGN (for Active Galactic Nucleus).

We believe that AGN are powered by matter accreting onto supermassive black holes with masses in the range

$$10^6 M_{\odot} < M_{\text{BH}} < 10^9 M_{\odot}$$

This is based on the following facts:

- emission comes from very small region
  - immediately evident from radio images
  - variability on time-scale $\Delta t$ implies that source has size $R < c \Delta t$

- motion of stars and/or gas in nuclei imply presence of very massive object.
  In a few cases we have been able to convincingly prove that it has to be a massive black hole: Occam’s razor then suggests that it must ALWAYS be a black hole

- we simply don’t know an alternative source that can produce such amounts of energy

OCCAM’S RAZOR
To quote Isaac Newton, "We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances. Therefore, to the same natural effects we must, so far as possible, assign the same causes."
Astronomers have found that the **number density** of quasars is a strong function of time. In the past (~8-10 Gyrs ago) the number density was few hundred times larger than today (after correction for expansion of Universe). Since we don’t believe that Black Holes can disappear, their must be many **dormant black holes** in present day Universe. Since AGN are in nuclei of galaxies, we expect these dormant black holes to be at centers of `normal' galaxies....

This has been confirmed; in every galaxy we look at we find evidence for the presence of a supermassive black hole.

**The Black Hole Paradigm**

- All galaxies have a central supermassive black hole
- Mass of BH is proportional to the mass of the bulge/spheroid
- Whether a galaxy is active or not, simply depends on whether it happens to be accreting (significant) amounts of matter

Movie on SMBHs: [http://www.youtube.com/watch?v=KCADH3x56eE](http://www.youtube.com/watch?v=KCADH3x56eE)
At the very center of the Milky Way, astronomers have discovered a supermassive black hole with a mass of 3 million Solar masses.
Albert Einstein
(1879-1955)
Einstein’s Legacy

- Reformulated concepts of space and time (Special Relativity)
  - Space and time are not absolute quantities (time dilation & Lorentz contraction)
  - \( E = mc^2 \); paved the way to atomic energy (and atomic bomb)

- Opened the road to Quantum Mechanics
  - Light consists of particles (photons)
  - Photons have quantized, discrete energies depending on their wavelengths

- Presented new theory of gravity (General Relativity)
  - Matter tells space how to curve; Space tells matter how to move
  - Provided the foundations for modern cosmology
Einstein was born on March 15, 1879 in Ulm, Germany as the son of Hermann and Pauline Einstein, two “entirely irreligious Jewish parents”.

Hermann Einstein

Pauline Einstein

Albert Einstein in a class photo of his high-school in Munich, in 1889. He hated high-school because of the “memorization and obedience to arbitrary authority”. He studied his own interests at home (math, philosophy, geometry...).

According to Einstein himself, his first scientific insight occurred at age 5, when his father showed him a compass. “I realized that something in empty space acted upon the needle”. He later called this one the revealing events of his life....

One of his uncles was an engineer who stimulated Einstein’s curiosity.
A teacher at his high-school in Munich suggested that Einstein leave school since “his very presence destroyed the other student’s respect for the teachers”. Einstein indeed quit school at age 15, and joined his parents for a trip to Italy for “a glorious half year of freedom”.

In 1895 he took entrance examination at the Swiss Federal Institute of Technology (ETH) in Zurich, but he failed. He was advised to study at a school in Aarau (Switzerland), which he did. At this school the teachers were humane and his ideas were set free: He studied Maxwell’s theory of Electromagnetism...

After successfully finishing the school in Aarau, Einstein enrolled at ETH. There he worked hard, but mainly on his own (he didn’t like lectures). Fortunately, his good friend Marcel Grossman, took excellent notes and shared them with Einstein, who later wrote: “I would rather not speculate on what would have become of me without these notes”
The Patent Office

After graduating from ETH (with no exceptional grades), he failed to get a job in academia: he irritated all professors, who therefore didn't help him in getting a job...

In 1902 he finally got a job in the patent office in Bern. It was "a kind of salvation," he said. The regular salary and the stimulating work evaluating patent claims freed Einstein. He now had time to devote his thought to the most basic problems of physics of his time, and began to publish scientific papers. In the evenings he would meet with some friends (they called themselves `Olympia Academy`) in the bars of Bern to discuss physics; he used these opportunities to try out his outlandish ideas...

1905 Annus Mirabilis

In 1905, while still working at the patent office in Bern, Einstein published 4 papers that changed the world:

- March: Light consists of quanta (Nobelprize 1921)
- May: Brownian motion (Einstein's thesis topic)
- June: Special Relativity (new concepts of space & time)
- Sept: E=mc² (atomic energy & atom bomb)
Special Relativity

Common sense is the collection of prejudices acquired by age eighteen......(A. Einstein)

"You think you're pretty smart, don't you?"
Inertial Frames, Invariance & Covariance

A frame of reference is a standard relative to which motion and rest may be measured. Any set of points or objects that are at rest with respect to each other can serve as a frame of reference (i.e., coordinate system, WLH 208, Earth).

An inertial frame is a frame of reference that has a constant velocity with respect to the distant stars, i.e., it is moving in a straight line at a constant speed, or it is standing still. It is a non-accelerating frame, in which the laws of physics take on their simplest forms, because there are no fictitious forces.

A non-inertial frame is a frame of reference that is accelerating. In a non-inertial frame the motion of objects is affected by fictitious forces, such as centrifugal force.

An invariant is a property or quantity that remains unchanged under some transformation of the frame of reference (i.e., charge of an electron, Planck’s constant).

Covariance is the invariance of the physical laws or equations under some transformation of the frame of reference.
Galileo, and later Newton, realized that in an inertial frame there is no physical experiment that can reveal the velocity of that inertial frame.

The outcome of every experiment done by stickman is completely independent of the velocity of his inertial frame. When he throws his ball up in the air, it looks exactly the same as if he was at rest wrt distant stars...

You may be familiar with this concept; while waiting in the train at a station, the train next to you starts to move....a second later you suddenly realize that it is your train that is moving, not the one on the track next to yours.

This argues against the notion of absolute velocity; only relative motion is measurable in physics. This concept that there is no such thing as absolute velocity is called Newtonian relativity.
Newtonian Relativity

Two inertial frames of references, having a speed of $v$ wrt each other, are related by the Galilean transformation rules:

$$
\begin{align*}
  x' &= x - vt \\
  y' &= y \\
  z' &= z \\
  t' &= t
\end{align*}
$$

Newtonian relativity says that the physical laws are covariant under Galilean transformation; if you are not covariant under Galilean transformation, you cannot be a physical law...

This means that if a physical law describes the motion of an object in reference frame $A$, and it predicts an orbit $x(t)$, then the orbit as seen by an observed in reference frame $B$ is given by $x'(t') = x'(t) = x(t) - v t$

It also means that if an observer in $A$ sees an object move in the $x$-direction with velocity $u$, then an observer in reference frame $B$ sees it move with a velocity $u' = u - v$ (addition rule of velocities).
James Clerk Maxwell (1831-1879)

Scottish Physicist who formulated the (classical) theory of Electromagnetism, which had a huge influence on Einstein’s thinking.....

Prior to Maxwell the observed behavior of magnets (i.e., compasses) and electricity were thought to have nothing to do with each other. Maxwell (~1870) proposed that magnetism, electricity and light were all aspects of the same underlying set of physical laws: ElectroMagnetism

Maxwell came up with a set of 4 beautiful equations that completely describe all electrical and magnetic phenomena, and which describe electromagnetic waves (i.e. light).

But, Maxwell’s laws are NOT covariant under Galilean transformations...In fact, according to these laws the speed of light, c, is an invariant, whereas according to Newtonian relativity velocities are relative, and transform according to the addition rule....
The Luminiferous Aether

Maxwell’s law not being covariant under Galilean transformation constitutes a problem since it violates Newtonian relativity.....

Three alternative conclusions can be drawn:

- Maxwell’s equations are simply wrong
- Maxwell’s equations are valid in only ONE inertial frame. Hence, the Newtonian relativity principle is inapplicable.
- Maxwell’s equations do obey principle of relativity, but relationship between inertial frames is not given by Galilean transformation.

At end of 19th century, almost all physicists accepted the second of these alternatives: it was thought that like all mechanical waves, electromagnetic waves needed some medium for propagation, the luminiferous aether (not to be confused with Aristotle’s aether). Maxwell’s equations were only deemed valid in the inertial frame that is at rest with respect to this aether, and electromagnetic waves propagate with a speed $c$ with respect to this aether.

But, if an aether exists and is at rest wrt absolute space, and if light moves with constant speed wrt aether, then one should be able to detect absolute motion of an inertial frame; it should reveal itself as directional dependence of speed of light wrt the inertial frame.
In 1887, **Albert Michelson** and **Edward Morley** conducted an ingenious experiment to try an measure the speed of Earth wrt the aether; i.e., the tried to measure differences in the speed of light in different directions....
The Michelson-Morley Experiment

To everyone’s surprise, they could not detect any variations in the speed of light, whereas their experiment was sensitive enough to at least detect motion of Earth around Sun (30 km/s). Implication: speed of light is constant.
The Michelson-Morley Experiment

To everyone’s surprise, they could not detect any variations in the speed of light, whereas their experiment was sensitive enough to at least detect motion of Earth around Sun (30 km/s). Implication: speed of light is constant.
Special Relativity

Unlike most physicists at his time, Einstein accepted the results from the Michelson-Morley experiment, which led to his development of SR:

**The two postulates of Special Relativity:**
- All laws of physics are the same for all inertial observers
- The speed of light is the same for all inertial observers

**Implications**
- There is no luminiferous aether.
- No material object can travel at, or faster than, the speed of light: \( c = 300,000 \text{ km/s} \)
- Inertial frames do not transform according to Galilean transformation, but according to the Lorentz transformation, which leave the Maxwell equations invariant

\[
x' = \gamma (x - vt)
\]
\[
y' = y
\]
\[
z' = z
\]
\[
t' = \gamma (t - [v/c^2]x)
\]

\[
\gamma = \frac{1}{\sqrt{1 - (\frac{v}{c})^2}}
\]

\[
v \ll c \rightarrow \gamma \approx 1
\]

\[
x' = x - vt
\]
\[
y' = y
\]
\[
z' = z
\]
\[
t' = t
\]

Lorentz transformations

Galilean transformations
Special Relativity; more implications

- Space and Time are interwoven into **space-time**.
  Space-time intervals are **invariant**; space intervals or time intervals are not.

  \[
  \text{space-time invariant: } \Delta s^2 = c^2 \Delta t^2 - \Delta x^2 + \Delta y^2 + \Delta z^2
  \]

- **Time dilation**: the observed passage of time becomes slower for a moving object.
  This is not just an illusion, time really passes more slowly traveling at high speed: if you
  travel to a star 10 ly away at 99% of speed of light, it only takes you 1.4 years...

- **Lorentz Contraction**: observed length along line of motion of moving object becomes less
  than its length when measured at rest. In the above example, you, in your spaceship see
  the distance to the star to be only 1.4 ly, so you are not surprised reaching it in 1.4 years

- **E = mc^2**: (mass is a form of energy, and vice versa). This principle powers stars...

- Velocities do **not** add linearly....
Curved Space
Geometry (Ancient Greek: γεωμετρία; geo- "earth", -metri "measurement") "Earth-measuring" is a branch of mathematics concerned with questions of shape, size, relative position of figures, and the properties of space.

In ancient civilizations it was the art of land measurements, and it was used in the construction of mammoth works such as the Great Pyramid of Giza.

The Babylonians of 2000 BC and the Chinese of 300 BC used the rule that the circumference of a circle is three times its diameter (i.e., \( \pi = 3 \)). The Egyptians of 1800 BC used \( \pi = (16/9)^2 = 3.1605... \)

It was the Greeks who developed geometry into a science that climaxed in the axiomatic and definite treatise presented by Euclid.
Euclidean Geometry

Euclid (ca 300 BC)

Lived in Alexandria (Egypt)

Greek mathematician; considered “father of Geometry”

Wrote Elements, of which more copies have been sold than any other textbook in history

In the Elements, Euclid starts with a set of five axioms (a proposition that is not proved but considered self-evident), from which he derives and proves a large number of theorems about geometry.

One of Euclid’s five axioms is the parallel axiom (or postulate), which was considered the least self-evident of them all.

Some of Euclid’s theorems

- The bridge of asses theorem states that A=B and C=D.
- The sum of angles A, B, and C is equal to 180 degrees.
- Pythagoras’ theorem: The sum of the areas of the two squares on the legs (a and b) of a right triangle equals the area of the square on the hypotenuse (c).
- Thales’ theorem: if AC is a diameter, then the angle at B is a right angle.
Euclidean Geometry

The following “rules” you probably (hopefully) learned in highschool:

1) The shortest distance between two points is a straight line.
2) Parallel lines never intersect.
3) The circumference of a circle is $2\pi$ times its radius.
4) The (interior) angles of a triangle add up to 180 degrees.
5) Theorem of Pythagoras: $c^2 = a^2 + b^2$.

A space whose geometry is such that the above rules apply, is called a Euclidean space, or a flat space. Note that the meaning of the word ‘flat’ here is different from everyday usage of that word: a flat space is not necessarily two-dimensional, like a sheet of paper; a flat space is any space that obeys the above rules of Euclidean geometry.
For over two thousand years, the adjective "Euclidean" was unnecessary because no other sort of geometry had been conceived. Euclid's axioms seemed so intuitively obvious that any theorem proved from them was deemed true in an absolute, often metaphysical, sense.

The reason for this is Euclid's fifth postulate; the parallel postulate, which implicitly restricts the geometry to flat (i.e. non-curved) spaces. It wasn't until the beginning of the 19th century before mathematicians started to construct non-Euclidean geometries for curved spaces, in which Euclid's fifth postulate does not hold.

A defining breakthrough in non-Euclidean geometry was a lecture by the German mathematician Bernhard Riemann in 1854, which signaled the birth of Riemannian geometry; the geometry of `smooth' manifolds (i.e. spaces) of arbitrary curvature (which is allowed to vary from one point to the other in a smooth fashion).
Curvature

For a path, the curvature, $K$, reflects how fast the path changes direction: A circle of radius $r$ has a curvature $K = 1/r$. So a small circle has large curvature and a large circle has small curvature:

Small radius, large curvature

Large radius, small curvature

In general, curvature is a local parameter, which can vary from point to point: The local curvature derives from the radius of the circle that best matches the curve locally;

At this point, the local curvature is $K = 1/r$ with $r$ the radius of the circle that best matches the curve locally.

At this point the curvature $K$ is very large, as reflected by the small circle.
The dimensionality of a space is the number of coordinates needed to describe the location of a point in that space:

<table>
<thead>
<tr>
<th>Object</th>
<th>Dimensionality</th>
</tr>
</thead>
<tbody>
<tr>
<td>a hair</td>
<td>n=1 (n=3)</td>
</tr>
<tr>
<td>a piece of paper</td>
<td>n=2 (n=3)</td>
</tr>
<tr>
<td>a solid sphere</td>
<td>n=3</td>
</tr>
<tr>
<td>surface of a sphere</td>
<td>n=2</td>
</tr>
<tr>
<td>Einstein's space-time</td>
<td>n=4</td>
</tr>
</tbody>
</table>

Try yourself: what is the dimensionality of

n-Dimensional Space:

An n-dimensional space (or space-time) can be finite or infinite, and flat or curved. If it is flat, we say it has n-dimensional Euclidean geometry, if it is curved, we say it has n-dimensional Riemannian (or non-Euclidean) geometry.

Examples:

- line: infinite, 1-dimensional flat (Euclidean) space
- line segment: finite, 1-dimensional flat (Euclidean) space
- circle: finite, 1-dimensional curved (non-Euclidean) space
- sheet: (in)finite, 2-dimensional flat (Euclidean) space
- surface of a sphere: finite, 2-dimensional curved (non-Euclidean) space
- surface of a Pringle chip: finite, 2-dimensional curved (non-Euclidean) space
- absolute space according to Newton: infinite, 3-dimensional flat (Euclidean) space
- Einstein's space-time in SR: infinite, 4-dimensional flat (Euclidean) space
- Universe: ????

Movie: Dr. Quantum visits flatland: http://www.youtube.com/watch?v=BWyTxCsIXE4
**Curved Space**

In the realm of Riemannian manifolds, we distinguish manifolds that are positively curved, flat (=Euclidean) and negatively curved.

<table>
<thead>
<tr>
<th>Flat Space</th>
<th>Positive Curvature</th>
<th>Negative Curvature</th>
</tr>
</thead>
<tbody>
<tr>
<td>distance between parallel lines is everywhere the same</td>
<td>parallel lines cross each other</td>
<td>distance between parallel lines diverges</td>
</tr>
<tr>
<td>$\alpha + \beta + \gamma = 180^o$</td>
<td>$\alpha + \beta + \gamma &gt; 180^o$</td>
<td>$\alpha + \beta + \gamma &lt; 180^o$</td>
</tr>
<tr>
<td>circumference = $2 \pi r$</td>
<td>circumference &lt; $2 \pi r$</td>
<td>circumference &gt; $2 \pi r$</td>
</tr>
<tr>
<td>parallel transport of vector independent of path taken</td>
<td>parallel transport of vector depends on path taken</td>
<td>parallel transport of vector depends on path taken</td>
</tr>
</tbody>
</table>
### Drawing Dimensions

In **Euclidean** geometry, we can easily draw 1D and 2D objects on a sheet of paper. If we have to draw 3D objects, we try to use shadowing, or perspective to give the impression of a third dimension...Drawing 4D objects is virtually impossible....

In **Riemannian** geometry, we can easily draw 1D curved objects on a sheet of paper: one dimension is used to reflect the extent along the object, the second dimension is used to draw the curvature. Drawing 2D curved spaces/objects on paper is equivalent to drawing 3D flat spaces/objects....drawing curved spaces/objects for $n>3$ is basically impossible.

<table>
<thead>
<tr>
<th></th>
<th>$n = 1$</th>
<th>$n = 2$</th>
<th>$n = 3$</th>
<th>$n = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Euclidean</strong></td>
<td><img src="image1" alt="1D Euclidean" /></td>
<td><img src="image2" alt="2D Euclidean" /></td>
<td><img src="image3" alt="3D Euclidean" /></td>
<td>Impossible to draw...</td>
</tr>
<tr>
<td><strong>non-Euclidean (curved)</strong></td>
<td><img src="image4" alt="1D curved" /></td>
<td><img src="image5" alt="2D curved" /></td>
<td><img src="image6" alt="3D curved" /></td>
<td>Impossible to draw...</td>
</tr>
</tbody>
</table>
A uniform space is a space in which the curvature is the same at each point. Uniform spaces are **homogeneous** (all points are equivalent) and **isotropic** (all directions are equivalent).

We believe the Universe to be a uniform space.

The belief that the Universe is homogeneous and isotropic (on large scales) is called the **cosmological principle**: it has strong observational support! It also means the geometry of the Universe is described by one number: \( K \).

- All Euclidean spaces (independent of their dimensionality) are uniform.
- For \( n=1 \), the only uniform spaces are a straight line (Euclidean, \( K=0 \)) and a circle (\( K>0 \)). For \( n=1 \), no negatively curved space exists...
- For \( n=2 \), there exist three uniform spaces: a flat, infinite sheet (Euclidean space, \( K=0 \)), the surface of a sphere (‘spherical’ space, \( K>0 \)) and an infinite Pringle-like surface (‘hyperbolic’ space, \( K<0 \)).
- For \( n>2 \), there always exist three uniform spaces: a Euclidean space (\( K=0 \)), a spherical space (\( K>0 \)) and a hyperbolic space (\( K<0 \)).
Uniform Spaces & The Cosmological Principle

A uniform space is a space in which the curvature is the same at each point. Uniform spaces are homogeneous (all points are equivalent) and isotropic (all directions are equivalent).

We believe the Universe to be a uniform space.

The believe that the Universe is homogeneous and isotropic (on large scales) is called the cosmological principle; it has strong observational support!
It also means the geometry of the Universe is described by one number: $K$

What is $K$ of the Universe?
Thinking in Higher Dimensions

According to SR, our space-time is a four-dimensional space (mathematician often say “manifold”). If we take into account that it may be curved as well, this means we need 5 dimensions to “draw” such a manifold .... Impossible.

However, we can get insight by considering 2D manifolds:

<table>
<thead>
<tr>
<th>Flat Space</th>
<th>Sheet</th>
<th>K = 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spherical Space</td>
<td>Surface of sphere</td>
<td>K &gt; 0</td>
</tr>
<tr>
<td>Hyperbolic Space</td>
<td>Pringle chip</td>
<td>K &lt; 0</td>
</tr>
</tbody>
</table>

These 2D manifolds represent our 3D space; the third spatial dimension is used to indicate the curvature.... and we simply use time to represent time; if space expands, we imagine stretching this 2D space with time...
General Relativity
Problems with Gravity around 1905

- **Newton’s law of gravity** appears to give an accurate description of what happens, but gives no explanation of gravity.

- **Newton’s law of gravity** only holds in inertial systems and is covariant under Galilean transformations. However, according to SR inertial systems transform according to Lorentz transformations, which leave Maxwell equations invariant.

- Since there is matter in the Universe, and you can not shield yourself from it, true inertial frames do not exist...

- According to **Newton’s law of gravity**, moving a distant object has an immediate effect all throughout space; violation of Special Relativity.
The Vanishing Sun; Newton’s version

When Sun instantaneously disappears, Earth will immediately continue in straight-line orbit, according to Newton’s first law of motion.
The Vanishing Sun; Einstein's version

When Sun instantaneously disappears, Earth will continue on circular orbit for at least 8 more minutes which is the minimum time required for the information about the Sun's disappearance to reach Earth.
Problems with Gravity around 1905

- **Newton's law of gravity** appears to give an accurate description of what happens, but gives no explanation of gravity.

- **Newton's law of gravity** only holds in inertial systems and is covariant under Galilean transformations. However, according to SR inertial systems transform according to Lorentz transformations, which leave Maxwell equations invariant.

- Since there is matter in the Universe, and you can not shield yourself from it, true inertial frames do not exist...

- According to **Newton's law of gravity**, moving a distant object has an immediate effect all throughout space; violation of Special Relativity.

These issues deeply disturbed Einstein. In 1907, beginning with a simple thought experiment involving an observer in free fall, he embarked on what would be an eight-year search for a relativistic theory of gravity. This culminated in November 1915 when he presented what are now known as the **Einstein Field Equations** to the Prussian Academy of Science. These equations specify how the geometry of space and time is influenced by whatever matter is present, and form the core of Einstein's theory of **General Relativity**.
Einstein’s Thought Experiments

Consider *stick-man* in a windowless lab, moving with constant speed (i.e., his lab is an *inertial frame*)

According to *Special Relativity*, *stick-man* can perform no experiment from which he can determine his velocity!

Now imagine *stick-man’s* lab being *accelerated* due to the gravitational field of the Earth (i.e., *stick-man’s* lab is a *non-inertial* frame in *free-fall*)

What experiment(s) can *stick-man* do from which he can determine his acceleration?

**Answer:** NONE

*Stick-man* does not notice the acceleration since the *gravitational force* is exactly balanced (and hence cancelled) by the *centrifugal inertial force*.
Einstein’s Thought Experiments

**acceleration**

Stick-man’s lab is accelerated. He experiences an inertial force, which gives him a non-zero weight.

**gravity**

Stick-man’s lab is inhibited in its free-fall due to the normal force of the Earth. Consequently, stick-man experiences the gravitational force, giving him a non-zero weight.

Einstein realized that there is no experiment that Stick-man can do that tells him the difference between gravity and acceleration.

Principle of Relativity is really a principle of impotence: you are unable to tell the difference between being at rest, moving at constant speed or being in free-fall, and you’re unable to tell the difference between being in a gravitational field or being accelerated.

Einstein, who had this revelation in 1907, describes it as ‘the happiest thought of my life’.
Inertial Mass vs Gravitational Mass

Newton’s 2nd law of motion \( \vec{F} = m_i \cdot \vec{a} \)\( m_i \) = inertial mass

Newton’s law of gravity \( \vec{F}_g = \frac{G M\oplus m_g}{r^2} = m_g \cdot \vec{g} \)\( m_g \) = gravitational mass

Hence, for motion in a gravitational field:

\[ \vec{a} = \frac{m_g}{m_i} \cdot \vec{g} \]

Galileo and Newton have shown that all objects experience same acceleration (all objects fall at same rate). This implies that the ratio of inertial mass and gravitational mass must be a constant:

\[ \frac{m_g}{m_i} = \text{constant} \]

NOTE: this is not at all an obvious result...
Inertial Mass vs Gravitational Mass

Consider a circular orbit centered on the Earth:

\[
\begin{align*}
F_c &= m_i \cdot a_c = m_i \frac{v_c^2}{r} \\
\Rightarrow \quad F_c &= \frac{G M_{\oplus} m_i}{r^2} \\
v_c^2 &= \frac{GM_{\oplus}}{r} \\
F_g &= \frac{G M_{\oplus} m_g}{r^2}
\end{align*}
\]

The fact that gravitational force and centrifugal force exactly cancel, implies exact equality of inertial and gravitational masses:

\[m_i = m_g\]

If this would not be the case, then different objects would be on different free-fall orbits; in space-shuttle, objects would fly against the walls...
The equality of inertial mass and gravitational mass has been tested and confirmed to exquisite precision.

The first to test the equality of inertial & gravitational mass was the Hungarian physicist Loránd Eötvös, who used a nifty apparatus, called the torsion balance.

In 1889 Eötvös was able to show that there is no difference between inertial and gravitational masses to an accuracy of 1 part in 20 million.

Modern versions of the Eötvös torsion balance experiment show that

\[
\frac{m_i}{m_g} = 1 + \epsilon \quad |\epsilon| < 10^{-12}
\]
Einstein’s General Relativity Theory

Based on the thought experiments described above, Einstein postulated the following:

**Strong Equivalence Principle**: inertial and free-falling systems (reference frames) are entirely equivalent. SR applies to both

there exists no conceivable experiment that allow you to distinguish between inertial motion and free-fall. Special Relativity is valid in free-falling reference frames as well as in inertial frames. You can effectively ‘transform gravity away’ by going to a free fall frame....

**Gravity is property of space-time**: mass causes space-time to curve, and the curvature of space-time causes orbits to be deflected.

A useful, and powerful, analogy is the curvature of a rubber sheet if you put a bowling ball (a mass) on it. A marble at rest on the rubber sheet will now start to roll (=fall) towards the ball. A moving marble’s path will be deflected (bend) due to the curvature in the sheet.
Einstein’s General Relativity Theory

Based on the thought experiments described above, Einstein postulated the following:

**Strong Equivalence Principle**: inertial and free-falling systems (reference frames) are entirely equivalent. SR applies to both.

there exists no conceivable experiment that allow you to distinguish between inertial motion and free-fall. Special Relativity is valid in free-falling reference frames as well as in inertial frames.

**Gravity is property of space-time**: mass causes space-time to curve, and the curvature of space-time causes orbits to be deflected.

BEWARE: in GR, 4D space-time is curved (in a fifth dimension), whereas in our analogy a 2D-sheet is curved (in 3rd dim.) This is impossible to draw.....to the left is an attempt to depict curvature in 3D space...
Einstein’s General Relativity Theory

Einstein’s Field Equations

Riemann tensor; describes curvature of space-time

Energy-Momentum tensor; describes distribution of matter and energy (which also has mass)

For a weak gravitational field, Einstein’s Field Equations reduce to the standard equations of Newtonian Gravity. The strength of gravity is expressed via the parameter

\[ G = \left( \frac{v_{\text{esc}}}{c} \right)^2 \]

At surface of Earth: \( G \approx 2 \times 10^{-8} \)
At photosphere of Sun: \( G \approx 4 \times 10^{-6} \)

Only when \( G \approx 1 \) does Newton’s law start to fail

Gravity in Solar System is weak and well described by Newton’s law of gravity.
Brad Pitt is racing to his beloved Angelina, who is ready to deliver their 43rd child. Since he is a nice guy, he has bought his pregnant wife a helium balloon, which is floating against the ceiling in the back of the car...

Suddenly, a cat crosses the street, and Brad, being a nice guy, hits the brakes.

**Question:** what happens to the balloon?

**Hint:** use the strong equivalence principle
Gravitational Lensing
Gravitational Lensing

In stick-man’s accelerated lab, the laser-beam appears to follow a curved trajectory, which is simply a reflection of upwards acceleration. Based on the strong equivalence principle, the laser-beam must follow the same trajectory in gravitational field.

The fact that light should be deflected in a gravitational field is in accord with SR; photons have energy, which is equivalent to mass ($E = mc^2$).

This prediction of GR, called gravitational lensing, was confirmed in 1919 during a Solar eclipse when the British astronomer Sir Arthur Eddington observed the light from stars passing close to the sun to be slightly bent, so that they appeared slightly out of position. Einstein became a hero....
LIGHTS ALL ASKEW
IN THE HEAVENS

Men of Science More or Less
Agog Over Results of Eclipse
Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed
or Were Calculated to be,
but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

No More in All the World Could
Comprehend It, Said Einstein When
His Daring Publishers Accepted It.
Gravitational Lensing can cause multiple images of same object
Gravitational Lensing

Gravitational lensing is very common. Astronomers observe all kinds of lensing phenomena. Since the angle by which the light is bend is proportional to the mass of the lensing object, accurate observations of lensing systems provide accurate measurements of their masses!

Example: from the relative locations of the various images of the background galaxy (the blue `stuff`) astronomers can infer the mass of cluster of galaxies (the foreground lens). A comparison with the total amount of stellar light in the cluster shows that its mass is dominated by dark matter.

All the blue distorted looking galaxies are (distorted) images of one and the same background galaxy that is being lensed due to the massive cluster in the foreground.
Hubble's Expansion Law
Recall from Lecture 9; we can measure blue/red-shift by comparing observed wavelength, $\lambda_{\text{obs}}$, of emission/absorption lines to their rest-wavelength, $\lambda_0$, measured in laboratory on Earth.

\[
\frac{\Delta \lambda}{\lambda_0} = \frac{\lambda_{\text{obs}} - \lambda_0}{\lambda_0} = \frac{v_{\text{los}}}{c}
\]

$v_{\text{los}} > 0$ Object is receding from Earth; redshift

$v_{\text{los}} < 0$ Object is approaching from Earth; blueshift

Here $v_{\text{los}}$ is the component of the velocity along the line-of-sight.

**Definition:** the redshift of an object is defined as

\[
z = \frac{\Delta \lambda}{\lambda_0}
\]

Note that an object with a negative redshift $(z < 0)$ is blueshifted.

If the redshift of an object is interpreted as due to the Doppler effect, the velocity of the object along the line-of-sight is equal to

\[
v_{\text{los}} = c \cdot z
\]
In 1912, Vesto Slipher, an astronomer working at Lowell’s Observatory in Flagstaff, Arizona took spectra of spiral nebulae (which Lowell believed to be planetary systems in formation). He noticed that almost all spiral nebulae have very large redshifts. If interpreted as a Doppler shift, the inferred recession velocities were of the order of 1000-2000 km/s.

In 1920, Heber Curtis, used Slipher's observations in The Great Debate to argue that spiral nebulae are extra-galactic island Universes.

In 1923, Edwin Hubble, used Cepheid variables to demonstrate that spiral nebulae are galaxies.

In 1929, Edwin Hubble, noticed a linear relation between the recession velocity of a galaxy (as inferred from its redshift) and its distance (obtained using Cepheids or similar techniques).

In 1931, Edwin Hubble and his assistant, Milton Humason (an uneducated mule-driver at Mt. Wilson), confirm the Hubble relation, extending the distance scales out to which it is measured by an order of magnitude.
The Hubble Expansion Law

The figure shows the graph from Hubble’s original 1929 paper in which he announces the relation between distance and recession velocity of spiral nebulae, currently called the Hubble Expansion Law.

**Hubble Expansion Law**

\[ v = H_0 d \]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v )</td>
<td>recession velocity</td>
<td>( \text{km s}^{-1} )</td>
</tr>
<tr>
<td>( H_0 )</td>
<td>Hubble constant</td>
<td>( \text{km s}^{-1} \text{ Mpc}^{-1} )</td>
</tr>
<tr>
<td>( d )</td>
<td>distance</td>
<td>( \text{Mpc} )</td>
</tr>
</tbody>
</table>

Due to usage of an incorrect Period-Luminosity relation for Cepheid’s, Hubble miscalculated the distances to these galaxies, and ended up with a Hubble constant of

\[ H_0 \sim 500 \text{ km s}^{-1} \text{ Mpc}^{-1} \]

Today, after much detailed work, we know that

\[ H_0 \sim (72 \pm 2) \text{ km s}^{-1} \text{ Mpc}^{-1} \]

**Hubble constant is the slope of this relation**
The Hubble Expansion Law

We can use the Hubble expansion law to determine the distances to galaxies and quasars:

\[ v = H_0 d \]
\[ v = z \cdot c \]

\[ d = \frac{c}{H_0} z \]

(Only valid for \( z \ll 1 \))

This is the main method by which astronomers determine the distances to galaxies, quasars and other extra-galactic objects; take a spectrum of the object, from which you can determine the redshift. If you know the value of the Hubble constant, you then use the above equation to determine the distance.

**Example 1:** A galaxy is observed to have a redshift of \( z = 0.1 \); what is its distance in Mpc?

\[ d = \frac{c}{H_0} z = \frac{300,000 \text{ km s}^{-1}}{72 \text{ km s}^{-1} \text{ Mpc}^{-1}} \cdot 0.1 = 417 \text{ Mpc} \]

**Example 2:** A quasar is observed to have a redshift of \( z = 2 \); what is its recession velocity?

\[ v = z \cdot c = 2c \]

But according to Einstein’s SR, nothing can move faster than speed of light.... so how can \( z > 1 \)? To understand this we need to look at expansion of space.
The Expanding Universe
So the Universe is expanding, but....

(1) how can it be that objects are expanding away from us at velocities larger than speed of light?

(2) does expansion of space mean that the distance between Earth and Sun increases with time?

(3) where is the center of the expansion?

(4) what was there before the Big Bang?

(5) what is at the edge of the Universe, or, in other words, what is the Universe expanding into?
**What is expanding?**

**Question:** Does the distance between Earth and Sun increase with time?

**Answer:** No. The expansion of space does not affect objects that are bound together by gravity or some other force.

**Analogy:** Think of two bowling balls on a rubber sheet that you stretch; the balls, due to the curvature they induce in the rubber sheet, stay next to each other...

Therefore, the following objects do not expand:

- Solar system
- Milky Way and other galaxies
- Clusters of galaxies
- Sun and other stars
- Earth and other planets
- you and I
- atoms
- protons

What **does** expand is the space between galaxies and clusters that are not gravitationally bound to each other.
Consider the following patch of space: the `stars` symbolize galaxies that are NOT bound to each other. The dotted lines indicate a coordinate system.

**NOTE:** the stars (=galaxies) getting bigger is **not** realistic! In reality galaxies do not expand. This is an artifact of how I made this slide!!!!!
NOTE: the stars (=galaxies) getting bigger is not realistic! In reality galaxies do not expand. This is an artifact of how I made this slide!!!!!

NOTE: none of the stars/galaxies are moving with respect to space. They maintain their position wrt the (comoving) coordinate system!!
distance between two
galaxies is 2.4 grid-cells
distance between two galaxies is still 2.4 grid-cells

This distance, expressed in units of a coordinate system that is expanding with space itself, is called **comoving distance**, and corresponding coordinates are called **comoving coordinates**.

The **physical distance** between two objects, i.e. the one measured with a yard stick, is related to the **comoving distance** according to:

\[
d_{\text{phys}}(t) = a(t) \, d_{\text{com}}(t)
\]

\(a\) is called the **scale factor**, and its evolution with redshift completely describes the **expansion history** of the Universe. By definition we set the present-day value to \(a_0 = 1\).
If all galaxies are moving away from each other, i.e., the scale factor is increasing with time, then if we go back in time, all galaxies must have been closer together; the scale factor was smaller at earlier times.

This implies that there must have been a point in time at which \( a(t) = 0 \). This instant in time is called the Big Bang.

The Big Bang is often depicted as some kind of explosion (often, in movies, involving a loud `bang'-sound). This is a very distorting view of reality, as it suggests a specific point in space from where the Universe emerged (and the sound part is utterly wrong, as sound does not propagate through empty space).

The correct way to think about the Big Bang is as the event that created space-time; prior to the Big Bang space and time did not exist, and it is therefore meaningless to ponder about a time before the Big Bang.
NOTE: because of the expansion of space, the wavelength of the photons is also stretched, resulting in all objects being redshifted wrt each other. Yet, no object is really moving; cosmological redshift is NOT related to Doppler effect, but simply to expansion of space. \( z > 1 \) is not a violation of special relativity!!!
Cosmological Redshift

Here is another look at it: Note how the expansion of space causes the photons to become redder (their wavelength is stretched)
The relation between redshift and scale factor

The relation between physical and comoving distance is specified by the scale factor:

\[ d_{\text{phys}}(t) = a(t) d_{\text{com}} \quad \quad d_{\text{phys},0} = a_0 d_{\text{com}} = d_{\text{com}} \]

Thus the comoving distance is the same as the physical distance at the present day.

Since the wavelength of a photon is a physical distance, we also have that

\[ \lambda(t) = a(t) \lambda_{\text{com}} \]

Hence, for a photon observed today

\[ \lambda_{\text{obs}} = a_0 \lambda_{\text{com}} = \lambda_{\text{com}} \]

Since wavelength at time of emission is equal to rest-wavelength, we have that

\[ \lambda(t_{\text{em}}) = \lambda_0 = a(t_{\text{em}}) \lambda_{\text{com}} \]

Hence, for the redshift we can write that

\[ z = \frac{\Delta \lambda}{\lambda_0} = \frac{\lambda_{\text{obs}} - \lambda_0}{\lambda_0} = \frac{\lambda_{\text{obs}}}{\lambda_0} - 1 = \frac{\lambda_{\text{com}}}{a(t_{\text{em}})\lambda_{\text{com}}} - 1 = \frac{1}{a(t_{\text{em}})} - 1 \]

Thus, the light of object at redshift \( z \) was emitted when scale factor of Universe was

\[ a(z) = 1/(1 + z) \]

e.g., the light from a \( z=2 \) quasar was emitted when Universe was 1/3 of its present size.
The Hubble Expansion Law

**Question:** But if nothing is really moving, then what is the meaning of the velocity $v$ in Hubble’s expansion law ($v = H_0 d$)?

**Answer:** Simply think of it as the rate (in m/s) at which the *physical distance* between two points that are at rest wrt *comoving* coordinate system increases with time. Since this rate has the units of velocity, we refer to it as the *expansion velocity*.

**NOTE:** the distance $d$ in Hubble’s expansion law is the *physical* distance, not the *comoving* distance!!!
Thus far, we have assumed that galaxies are at rest wrt the comoving coordinate system. This is not realistic; in reality galaxies have non-zero velocities, due to the fact that they are accelerated by nearby galaxies (i.e., the Milky Way and Andromeda galaxies are moving towards each other due to their mutual gravitational force).

Such a velocity is called “peculiar velocity” in order to distinguish it from the “velocity” associated with the Hubble expansion law (hereafter called “expansion velocity”).

Note how all galaxies have moved wrt comoving coordinates due to their peculiar velocities.
The redshift of a galaxy has two components: one due to the expansion of the Universe (cosmological redshift), and one due to its peculiar velocity (Doppler effect).

\[ v = c \cdot z = v_{\text{exp}} + v_{\text{pec}} = H_0 d_{\text{phys}}(t) + v_{\text{pec}} \]

(Only valid for \( z \ll 1 \))

The peculiar velocities of galaxies are responsible for the scatter in the Hubble diagram. If you blindly use the Hubble expansion law to infer the distance to a galaxy, you make an error due to the non-zero peculiar velocity:

\[ d = \frac{c}{H_0} z = \frac{c}{H_0} \frac{v_{\text{exp}} + v_{\text{pec}}}{c} \]

Hubble expansion law using above equation
Peculiar Velocity

The redshift of a galaxy has two components: one due to the expansion of the Universe (cosmological redshift), and one due to its peculiar velocity (Doppler effect)

\[
v = c \cdot z = v_{\text{exp}} + v_{\text{pec}} = H_0 d_{\text{phys}}(t) + v_{\text{pec}}
\]

(Only valid for \(z \ll 1\))

The peculiar velocities of galaxies are responsible for the scatter in the Hubble diagram. If you blindly use the Hubble expansion law to infer the distance to a galaxy, you make an error due to the non-zero peculiar velocity:

\[
d = \frac{c}{H_0} \cdot z = \frac{c}{H_0} \cdot \frac{v_{\text{exp}} + v_{\text{pec}}}{c}
\]

\[= \frac{v_{\text{exp}}}{H_0} + \frac{v_{\text{pec}}}{H_0} = d_{\text{phys}}(t) + \frac{v_{\text{pec}}}{H_0}
\]

Astronomers have determined that the peculiar velocities of galaxies rarely exceed ~1000 km/s: Hence, the distance error due to peculiar velocities becomes negligible for galaxies with

\[
d_{\text{phys}} \gg \frac{1000}{72} = 14 \text{Mpc}
\]
Expansion of Universe; where is the center?

NOTE: the stars getting bigger is *not* realistic! In reality the stars do not expand. This is an artifact of how I made this slide!!!!!
Expansion of Universe; where is the center?

NOTE: you can clearly see the Hubble Expansion Law at work: more distant galaxies seem to move away with higher "velocity"
Expansion of Universe; where is the center?
**Expansion of Universe; where is the center?**

**ANSWER:** nowhere and everywhere. From every point in space, all objects are "moving" away from that point, yet, that point is not special in any way. There is no "center of the expansion"; it is expansion, not explosion!!!!

This notion that no point in space is special, is called the **cosmological principle**.
The Cosmological Principle

**Cosmological Principle:** The Universe is **homogeneous** & **isotropic**.

- **homogeneous:** the universe looks the same from all locations
- **isotropic:** the universe looks the same in all directions

**NOTE:** Clearly, the Universe on small scales is neither **homogeneous** (i.e., location of Sun is very different from that at center of Milky Way) nor **isotropic** (i.e., Universe looks different towards center of Milky Way than towards the Virgo cluster). The cosmological principle only holds on very large scales (larger than few hundred Mpc).

The **Cosmological Principle** can be considered a generalized **Copernican Principle:** our location in the Universe should be typical and should not be distinguished in any fundamental way from any other location.

Large surveys of galaxies provide observational evidence for the cosmological principle: For example, the image to the right shows the distribution of galaxies in the Sloan Digital Sky Survey (SDSS) along two different directions; color indicates the local density of galaxies. On small scales there are clearly differences, but on large enough scales, things start to look similar.
The Geometry of the Universe

**Cosmological Principle:** The Universe is **homogeneous** & **isotropic**.

- **homogeneous**: the universe looks the same from all locations
- **isotropic**: the universe looks the same in all directions

Recall (lecture 14): A space that is **homogeneous** and **isotropic** is called a **uniform** space: A uniform space is a space in which the curvature is the same at each point; it is characterized by a single parameter, namely the global curvature, $K$.

The cosmological principle therefore implies that the Universe is a uniform 3D Riemannian manifold, in which the curvature, $K$, is everywhere the same. This leaves three possibilities: space is flat (Euclidean space, $K=0$), positively curved (spherical space, $K>0$), or negatively curved (hyperbolic space, $K<0$).

NOTE: To be able to facilitate illustrations, we have reduced the dimensionality of space from 3D to 2D. This allows us to draw curved spaces. We will adopt this simplification throughout.
Expansion of Universe; where is the edge?

Where is the center of expansion?

as we have seen, there is not center of expansion.

Where is the edge of the Universe?

astronomers assume that a flat space is infinite (i.e., has no edge). Although we don’t have proof that this is true, it is the only assumption that is consistent with the cosmological principle; if there is an edge, not every point in space is equivalent...

What is the Universe expanding into?

itself....space is infinite, but keeps on expanding (getting bigger).

Case I: flat (Euclidean) space

Assume that the Universe has a Euclidean geometry (i.e., is flat). A useful 2D analogy is a flat rubber sheet that is being stretched....
Expansion of Universe; where is the edge?

Case II: curved space

Assume that the Universe is positively curved. An analogy of an expanding, positively curved space is the inflation of a balloon; note that the 2D surface of the balloon reflects the Universe.

Where is the center of expansion?
in the center of the balloon, but that is NOT part of the Universe itself. On the surface of the balloon (=Universe) no point is special...So in this case there is a “center of expansion” but it lies outside of the Universe itself (in an extra dimension).

Where is the edge of the Universe?
there is no edge; you can travel over the surface of the balloon (=travel through the Universe) indefinitely, and you will never encounter an edge. In the above image the balloon has an edge in the same dimension as the one in which it is curved, but this dimension is not part of the space that makes up the Universe itself.

What is the Universe expanding into?
the extra dimension, which is not part of the Universe itself.
So the Universe is expanding, but....

(1) how can it be that objects are expanding away from us at velocities larger than speed of light?

(2) does expansion of space mean that the distance between Earth and Sun increases with time?

(3) where is the center of the expansion?

(4) what was there before the Big Bang?

(5) what is at the edge of the Universe, or, in other words, what is the Universe expanding into?
The Big Bang

Because of the expansion, the scale factor $a(t)$ increases with time: i.e., the physical distance between two objects at rest wrt comoving coordinates becomes larger and larger as time goes on...

$$d_{\text{phys}}(t) = a(t) d_{\text{com}}$$

Recall that, by definition, $a_0 = a(\text{today}) = 1$. Hence, the comoving distance between two objects is identical to the physical distance at the present day.

Now imagine going back in time. The physical distance between any two objects becomes smaller and smaller, and there comes a point in time when $d_{\text{phys}} = 0$ between all objects.

This moment, defined by $a(t) = 0$, is called the Big Bang. It signals the creation of space-time. Since there is no space or time prior to Big Bang, the often posed question “what was there before the Big Bang?” is meaningless. And it certainly is not part of science, since it can never be tested with any experiment!! Note that we do not `understand' the Big Bang, since our physics brakes down at $t < 10^{-43}$ sec

This image shows up when you Google `Big Bang'.....this is the cause of much confusion, since it implies that the Big Bang was an explosion localized in space. It is neither an explosion, nor is it localized; it created all of space & time.
The Age of the Universe

What is the age of the Universe, i.e., how many years ago did the Big Bang occur?

We can answer this question using the Hubble expansion law:

\[ v_{\text{exp}} = H_0 \, d_{\text{phys},0} \]

where the index zero refers to the present.

At the time of the Big Bang we have that \( d_{\text{phys}} = 0 \)

If we assume that the expansion velocity is constant, then we have that

\[ d_{\text{phys}}(t) = v_{\text{exp}} \, t \]

Hence, the physical distance at the present day obeys

\[ d_{\text{phys},0} = v_{\text{exp}} \, t_0 \]

where \( t_0 \) is the present-day age of the Universe

Combining this with the Hubble expansion law above, we see that

\[ t_0 = \frac{1}{H_0} \]

In words; the age of the Universe is simply the reciprocal of the Hubble constant.

Using that \( H_0 = 72 \text{ km s}^{-1} \text{ Mpc}^{-1} \) we obtain an age of 14.3 Gyr. However, this is only true if the expansion velocity is constant. This is not true in general, but this method nevertheless yields an approximate age. More sophisticated treatments yield an age of \((13.73 +/\ 0.12)\) Gyr.
Expanding your Horizon

The fact that the Universe has a finite age, and that the speed of light is finite, means that we can only see a limited extent of the entire Universe.

For simplicity, let us start by considering a non-expanding Universe:

If that Universe is \( t_0 \) years old, light can only have travelled a distance \( d_H = c t_0 \).

We call this distance the particle horizon (or simply `horizon'), as no information from any object at a distance \( d > d_H \) could have reached the observer by the present day. Such objects are said to lie outside the observer's horizon.

As time goes on (and \( t_0 \) increases), more and more objects fall inside the observer's horizon (i.e., \( d_H \) increases); we say that as time goes on objects `enter our horizon'.

Note that an object outside of our horizon cannot affect us in any way; we don’t even notice its gravitational field; this is similar to the Earth only `realizing' the disappearance of the Sun 8 minutes after it has gone....

The existence of a horizon also means that we can never test whether the Universe is really infinite, since we can never probe past the horizon...

In case of expansion, the horizon is larger than without, since space expands while light travels from horizon to observer....
Expansion of Space; the horizon

The horizon grows faster than the expansion rate of the Universe. Consequently, larger and larger parts of the Universe fall within the horizon.
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Expansion of Space; the horizon

The horizon grows faster than the expansion rate of the Universe. Consequently, larger and larger parts of the Universe fall within the horizon.
We have seen that Hubble's expansion law implies that the Universe is expanding. This implies a 'creation event', which we have termed the Big Bang. Since the Universe is filled with matter (clusters, galaxies, stars, and apparently dark matter), and all matter pulls on all other matter via gravity, we expect that this gravitational pull should slow down the expansion....

Consider a spherical region of radius $R$, centered on an observer, called $O$. And let the mass inside this sphere be distributed uniformly (cosmological principle). If the average mass density is $\bar{\rho}$ then the total mass of all the attracting matter (galaxies etc.) inside this sphere is

$$M = \frac{4}{3} \pi R^3 \bar{\rho}$$

While space expands, and the radius $R$ of our sphere increases, the total mass within this expanding sphere remains the same; the galaxies furthest away (at distance $R$) are moving the fastest (ignoring peculiar velocities), so no galaxies at smaller $R$ can overtake them; hence $M$ is conserved...
Now consider galaxy $A$, exactly at the edge of our sphere. From the point of view of observer $O$, this galaxy, which has a mass $m$, has an energy

$$E = E_{\text{kin}} + E_{\text{pot}} = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

Since galaxy $A$ is moving freely (no friction), this energy is a conserved quantity. Similar to the situation with a ball thrown in the air, galaxy $A$ will be bound (to the observer) if $E < 0$. If so, it means that at some point in time the distance between $A$ and $O$ will reach a maximum, after which $A$ will start moving towards $O$; in other words, if $E < 0$ the expansion of the Universe will come to a halt, and is followed by a collapse. Similarly, if $E > 0$, the Universe will continue to expand indefinitely (i.e., is unbound). In the limiting case, $E = 0$, expansion will come to halt when $A$ and $O$ are infinitely far apart.
Newtonian Cosmology

Let us focus on this limiting case, \( E = 0 \):

\[
E = E_{\text{kin}} + E_{\text{pot}} = \frac{1}{2}mv^2 - \frac{GMm}{R}
\]

\[
E = 0 \quad \Rightarrow \quad v = H_0 R
\]

\[
\frac{1}{2}mH_0^2 R^2 = \frac{GMm}{R}
\]

\[
M = \frac{4}{3} \pi R^3 \bar{\rho}
\]

\[
H_0^2 R^2 = \frac{8\pi G \rho^2}{3 \bar{\rho}}
\]

\[
\bar{\rho} = \frac{3H_0^2}{8\pi G}
\]

Thus, the limiting case, \( E = 0 \), corresponds to a Universe in which the density is given by this particular value. We call this density the **critical density**, which we denote by \( \rho_{\text{crit}} \)

In fact, in cosmology we express the average density of the Universe in units of this critical density via the unitless quantity “omega”:

\[
\Omega = \frac{\bar{\rho}}{\rho_{\text{crit}}}
\]

\[
E = 0 \quad \Leftrightarrow \quad \bar{\rho} = \rho_{\text{crit}} \quad \Leftrightarrow \quad \Omega = 1
\]

**GR**: matter distribution sets space-time geometry

\[
K = 0
\]
# The Fate of the Universe

<table>
<thead>
<tr>
<th>“Closed” Universe</th>
<th>“Flat” Universe</th>
<th>“Open” Universe</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E &lt; 0$ $\bar{\rho} &gt; \rho_{\text{crit}}$ $\Omega &gt; 1$</td>
<td>$E = 0$ $\bar{\rho} = \rho_{\text{crit}}$ $\Omega = 1$</td>
<td>$E &gt; 0$ $\bar{\rho} &lt; \rho_{\text{crit}}$ $\Omega &lt; 1$</td>
</tr>
</tbody>
</table>

Expansion history, $a(t)$

Universe reaches maximum size, followed by collapse & Big Crunch. Gravity prevails over expansion.

Universe reaches maximum size, followed by halt after infinite amount of time. Gravity exactly balances expansion.

Expansion of Universe continues indefinitely with finite rate. Expansion prevails over gravity.

$K > 0$

Space-Time has positive curvature

$K = 0$

Space-Time has Euclidean geometry

$K < 0$

Space-Time has negative curvature
The Expansion History

What would my `Hubble diagram’ (redshift vs distance) look like in each of these cases?

Looking at larger distances is looking back further in time. From the above plot we see that in all three cases, the expansion rate of the Universe was larger in the past. Hence, in all three cases the Universe is decelerating over time.

From plot above, it is clear that a closed Universe is younger than an open Universe. Both are younger than the age we would infer assuming a constant expansion rate (which results in $t_0 = 1/H_0$). Since the scale factor is directly related to redshift, i.e. $a = 1/(1 + z)$, we see that light from a certain redshift was emitted a longer time ago in a Universe with a lower density; hence, in a Universe with lower density, a given redshift corresponds to a larger distance.
Thus, we can determine the density of the Universe, and therefore also the curvature of the Universe, by simply measuring redshifts and distances of (many) galaxies. Measuring redshifts is easy (just take a spectrum); however, measuring distances is hard. We can only measure Cepheids out to small distances.

However, there is another method to measure distances, which works out to much larger distances; Supernova Ia

Supernova Ia are a special class of supernovae, which all happen to have the same luminosity at their maximum brightness (during the explosion). Hence, if we measure the flux during peak brightness, and we know the associated luminosity, we can infer the distance!

To everyone’s surprise when SNIa data became available in late 1990’s, it revealed that the Universe is accelerating, rather than decelerating. How could this be? Gravity is an attractive force, and should cause acceleration.
Our analysis thus far used simple Newtonian considerations. Proper cosmology, however, has to rely on Einstein’s General Relativity (after all, we are talking about curved space-times).

\[ G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \]

The Universe is homogeneous & isotropic

\[ H^2(t) = \frac{8\pi G}{3} \rho(t) - \frac{Kc^2}{a^2(t)} + \frac{\Lambda c^2}{3} \]

Hubble parameter: \( H(t) \)
describes the rate at which the Universe is expanding at time \( t \)
Hubble ‘constant’ is \( H_0 = H(t_0) \)

Curvature Constant: \( K \)
describes the curvature of the uniform and isotropic space-time; normalized to be either -1, 0 or +1.

Cosmological Constant: \( \Lambda \)
accounts for the fact that the vacuum may also contribute energy. introduced by Einstein to allow for static Universe (without expansion).
Relativistic Cosmology

The density appearing in the **Friedmann equation** describes the density of both matter and energy. Since both photons and matter have energy, they both contribute to this energy density:

\[ \bar{\rho} = \bar{\rho}_m + \bar{\rho}_r \]

Einstein’s **cosmological constant** may be interpreted as a property of the vacuum. In particular, the vacuum has an energy-density given by:

\[ \bar{\rho}_\Lambda = \Lambda \frac{c^2}{8\pi G} \]

This means we can write the **Friedmann equation** at the present time, \( t = t_0 \), as:

\[ H_0^2 = \frac{8\pi G}{3} \left[ \bar{\rho}_{m,0} + \bar{\rho}_{r,0} + \bar{\rho}_{\Lambda,0} \right] - \frac{K c^2}{a_0^2} \]

where a subscript \( 0 \) refers to the present time. Using the definition of the **critical density**, and the fact that \( a_0 = 1 \), this can be rewritten as

\[ 1 - \Omega_0 = -K \left( \frac{c}{H_0} \right)^2 \quad \text{where} \quad \Omega_0 = \Omega_{m,0} + \Omega_{r,0} + \Omega_{\Lambda,0} \]

This makes it explicit that the curvature of space-time is directly related to its energy density, which has contributions from matter, radiation and (maybe) the vacuum!!!
The Cosmological Constant

Let's take another look at our expanding sphere...

\[ E = E_{\text{kin}} + E_{\text{pot}} = \frac{1}{2}mv^2 - \frac{GMm}{R} \]

While the sphere expands (R increases), its binding energy decreases. But now let's have a look at what happens if we add vacuum energy:

\[ E = E_{\text{kin}} + E_{\text{pot}} + E_{\text{vac}} \]
\[ = \frac{1}{2}mv^2 - \frac{GMm}{R} + \frac{4}{3}\pi R^3 \rho_\Lambda \]

Expansion of the sphere now corresponds to an increase in vacuum energy. When R becomes sufficiently large, \( \rho_\Lambda \) becomes the dominant source of energy. At that point, expansion actually causes an increase in the total energy E. This corresponds to an acceleration of the expansion (like adding energy to space shuttle at take off by burning fuel...).

The SNIa data clearly indicates that our Universe is accelerating its expansion. This therefore indicates that the energy density in our Universe is dominated by vacuum energy.
But what is vacuum energy?

It is a property of space itself.

Hence, the energy density of the vacuum is a constant, i.e.:

\[
\rho_\Lambda(x) = \bar{\rho}_\Lambda = \bar{\rho}_\Lambda,0 = \frac{\Lambda c^2}{8\pi G}
\]

Quantum-field theory actually `predicts' that the vacuum has energy. However, it predicts that \(\bar{\rho}_\Lambda \simeq 10^{120} \rho_{\text{crit}}\), whereas according to our cosmological data \(\bar{\rho}_\Lambda < \rho_{\text{crit}}\).

In other words, the quantum-physical prediction is off by \(~120\) orders of magnitude!!! This is an embarrassing inconsistency (aka a “problem”).

Because of this, astronomers and physicists sometimes assume that there simply is no vacuum energy, but that there is something else that “behaves like vacuum energy”. This something else is called dark energy.

But what is dark energy?

It is a scalar field

A scalar field assigns to each point in space a value (which reflects its local energy-density). It is different from a vector field, which assigns to each point in space a vector, which has both amplitude and direction (i.e., electromagnetic field, gravitational field). A scalar field has no direction, just amplitude. A scalar field is different from cosmological constant in that its energy density may vary in space and time....
Consider once again our spherical volume, containing a mass $M$, expanding with time. The average density of matter inside this volume evolves with time according to:

$$\bar{\rho}_m(t) = \frac{3M}{4\pi R^3(t)}$$

Here $R(t)$ is the physical distance, which can be written as $R(t) = R_{\text{phys}}(t) = a(t)R_{\text{com}}$

Using that the comoving distance $R_{\text{com}}$ is the same as the physical distance at the present day (because $a_0 = 1$), we find that the matter density evolves as

$$\bar{\rho}_m = \bar{\rho}_{m,0} a^{-3} = \bar{\rho}_{m,0} (1 + z)^3$$

For brevity, I have not explicitly written down the time dependence of $\bar{\rho}_m$ and $a$

Thus, at redshift $z = 1$, when the Universe was half the present-day size, the average matter density was eight times higher than it is today. At $z = 9$ it was a thousand times higher than it is today, etc. NOTE: this does not mean that galaxies, or stars, were denser; only that galaxies that were unbound to each other were closer to each other!!!!
Now consider the **radiation density** in that volume. This is proportional to the number of photos in the volume, $N_{\text{ph}}$, (which is constant if the Universe is homogeneous and isotropic) and the energy per photon, $E_{\text{ph}}$

$$\bar{\rho}_r(t) = \frac{3N_{\text{ph}}E_{\text{ph}}}{4\pi R^3(t)}$$

As before, we have that $R(t) = R_{\text{phys}}(t) = a(t)R_{\text{com}}$, but we also have that the wavelength of the photons change with time according $\lambda_{\text{ph}}(t) \propto a(t)$. In particular, we have that

$$E_{\text{ph}} = hf_{\text{ph}} = hc/\lambda_{\text{ph}} = hc/\lambda_{\text{ph},0}a = E_{\text{ph},0}a^{-1}$$

Hence, we finally obtain that the radiation density of the Universe evolves according to:

$$\bar{\rho}_r = \bar{\rho}_{r,0}a^{-4} = \bar{\rho}_{r,0}(1 + z)^4$$

Thus, at redshift $z = 1$, when the Universe was half the present-day size, the average radiation density was sixteen times higher than it is today. At $z = 9$ it was ten-thousand times higher than it is today, etc. Note that the radiation density decreases faster than the matter density. As we shall see, this has important implications for the early Universe!!!
The energy density of the vacuum doesn’t change with time; it is simply a property of the vacuum, and is not affected by the expansion of space.

$$\rho_\Lambda = \rho_\Lambda,0$$

Consequently, the various densities evolve as indicated in the figure to the left. At early times radiation dominates the energy density of the Universe, followed by matter, and ultimately the vacuum (cosmological constant).

As we will see, the radiation content of the Universe is dominated by the cosmic microwave background (CMB), which has a perfect black body curve. As the Universe evolves, the wavelengths of the photons redshift, but the distribution remains that of a black body.

A black body obeys Wien’s displacement law: $$\lambda_{\text{max}} = 2.9 \times 10^{-3}/T.$$ Here $T$ reflects the temperature of the photons. Since $$\lambda_{\text{max}} \propto a$$ we find that $$T \propto a^{-1}.$$ Hence, the Universe was hotter at earlier times; going back towards the Big Bang, the temperature becomes infinitely high!!!

It may seem strange to talk about “the temperature of the Universe”. Here is how to think of it: suppose you place a black body of temperature $T=0$ in this universe. It will absorb all photons that hit it (definition of black body), and will establish thermal equilibrium with these photons; hence it temperature becomes that of the Universe.
Cosmic Microwave Background

I. Discovery & Data
1965: Penzias & Wilson serendipitously discovered CMB while testing Bell Lab’s horn-antenna on Crawford Hill, New Jersey. They were awarded the 1978 Nobel Prize in Physics!
Observing the CMB: do it yourself

Roughly 1 percent of the static on your TV is CMB!!!
The Cosmic Microwave Background (CMB) radiation reveals a perfect black body curve corresponding to a temperature of $T=2.726$ K. This is the most accurate Planck Curve ever measured; physicists in their laboratories cannot make better Planck Curves!
Increasing temperature sensitivity

\[ \frac{\Delta T}{T} = 1.5 \]

\[ \frac{\Delta T}{T} = 3 \times 10^{-3} \]

\[ \frac{\Delta T}{T} = 7 \times 10^{-5} \]
The CMB all sky map, after removal of the radiation coming from the Milky Way disk.
The CMB all sky map, after removal of the radiation coming from the Milky Way disk.
The CMB all sky map, after removal of the radiation coming from the Milky Way disk.

“cold” spot; 
$T = 2.7262 \, \text{K}$

“hot” spot; 
$T = 2.7266 \, \text{K}$

$\frac{\Delta T}{T} = 7 \times 10^{-5}$
For comparison....
The Anisotropy Power Spectrum

How to measure the Power Spectrum

1. Measure the average temperature inside a circular region with a diameter of $X$ degrees centered on random spot.

2. Repeat this for many random spots.

3. Measure the variance ('scatter') in these average temperatures. This is the power on a scale of $X$ degrees.

4. Repeat steps 1-3 for different values of $X$. The power as function of $X$ is called the power spectrum...

$X=20$ degrees
The Anisotropy Power Spectrum

How to measure the Power Spectrum

1. Measure the average temperature inside a circular region with a diameter of X degrees centered on random spot.
2. Repeat this for many random spots.
3. Measure the variance ('scatter') in these average temperatures. This is the power on a scale of X degrees.
4. Repeat steps 1-3 for different values of X. The power as function of X is called the power spectrum...

X=1 degree
Cosmic Microwave Background

II. Physics of the CMB
Going Back in Time...

In present day Universe, the mean free path of photons (the average distance a photon can travel before it is scattered or absorbed) exceeds the horizon; The average photon therefore traverses the visible Universe without a single interaction (which is why we can see galaxies etc.)

Going back in time, the Universe becomes denser & hotter (and its photons become more energetic). There comes a time when the photons are so energetic that they ionize all the available hydrogen atoms: \( H \rightarrow p + e \)

At earlier times, photons interact strongly with free electrons (Thomson scattering). The negatively charged free electrons, in turn, have strong electro-magnetic interactions with positively charged protons. Hence, the protons, electrons and photons are all tightly coupled together. This is called the photon-baryon fluid.

In the tightly coupled photon-baryon fluid, the mean free path of photons is tiny; the photons are trapped, and can’t get ‘out of the box’. At this point in time the Universe is completely opaque, like a very, very thick fog...
Shortly after Big Bang, the Universe consists of dark matter and photon-baryon fluid; the photons are prevented from traveling freely for a significant distance because of Thomson scattering; they are trapped. About 380,000 years after Big Bang, the Universe has cooled to the point where electrons and protons combine to form hydrogen atoms ($p + e \rightarrow H$). This is called recombination and happens very suddenly. It removes the free electrons, bringing an end to Thomson scattering. Hence, after recombination the mean free path of the photons suddenly increases to being larger than the size of the Universe; they travel freely throughout the Universe. These photons make up the Cosmic Microwave Background (CMB).
You can think of the CMB as the photons that are suddenly released from a small jack-in-the-box; at some point the box is opened and all its photons stream out in all directions.....

Now imagine one such jack-in-the-box at every point in expanding space (at every point in space there are photons that are trapped)

The opening of the jack-in-the-box coincides with recombination. Since recombination occurs when the Universe has cooled down to a low enough temperature, it occurs everywhere at the same time.

Hence, all jack-in-the-box’s will open exactly at the same time.... From every point in space, suddenly photons start to fly off in all directions...
Observations in an expanding Universe

shortly after the Big Bang...
Observations in an expanding Universe

Universe expands, horizon grows....
Observations in an expanding Universe

An event happens (a photon-box is opened) at comoving coordinates (+1.5,+1.5)
Observations in an expanding Universe

light from the event is on its way to us....
Observations in an expanding Universe

light from the event is on its way to us....
Observations in an expanding Universe

light from the event is on its way to us....
location of event has entered our horizon, but light from event has not reached us yet...
we “see” the event
All simultaneous events that are equidistant from us are observed at the same time. If all four events shown happened simultaneously, we observe them at the same time...
At present we observe all photons released at recombination from all jack-in-the-box’s located on the blue, dashed circle...On their way to us, these photons have been redshifted due to expansion of the Universe...Since recombination occurred 380,000 yrs after the Big Bang, when scale factor was $a = 1/(1 + z) \simeq 0.0009$, the CMB corresponds to a redshift of $z \simeq 1100$. 
Observing the CMB

At each point in time, I observe CMB photons coming from jack-in-the-box’s at different locations:

- The origin of the CMB photons that I observed 2 Gyrs ago
- The origin of the CMB photons that I observe today
- The origin of the CMB photons that I will observe 100,000 yrs from today
The Last Scattering Surface

We can't see any photons from an event that happened before recombination... Those photons were locked up in their jack-in-the-boxes, and could not get out.

Hence, using photons we can only probe back to the epoch of recombination, about 380,000 years after the Big Bang. When we look at CMB at given point in time, we see photons that came from thin, spherical shell centered on us. We call this shell the last scattering surface, since on that shell the photons experienced their last scattering before the jack-in-the-box opened (at recombination).

Last Scattering Surface is like photosphere of Sun; no information from before recombination can reach us (except for neutrinos & gravitational waves)

At recombination $T \sim 3000K$. Because of the expansion of the Universe, the CMB photons have been redshifted (and cooled down) to $T \sim 2.7 K$, which is the temperature of the CMB spectrum.
Cosmic Microwave Background

III. Anisotropies
The Origin of the Dipole

Our peculiar motion is made up of:
- Motion of Earth around Sun (~30 km/s)
- Motion of Sun around MW center (~220 km/s)
- Motion of MW towards Virgo cluster (~300 km/s)

Total vector sum of 369 km/s

Photons coming from the direction in which we are moving are blueshifted (as if that direction is moving towards us). Photons of a shorter wavelength correspond to photons of a higher temperature (recall Wien's law).
Origin of the Acoustic Peaks

Postulate:
shortly after the Big Bang some mechanism created small perturbations in the density distribution of the dark matter....

Potential well of an overdensity in dark matter

Overdensities pull matter towards them. We say that overdensities are associated with a potential well (regions of more negative potential energy). The baryons (which are part of the tightly coupled photon-baryon fluid) feel the gravitational pull from these overdensities, and start to fall into their potential wells....

However, photon-baryon fluid has enormous pressure (is very incompressible due to the photons bouncing off electrons). This pressure resists gravitational compression, giving rise to oscillations (compression --> rarefaction --> compression --> rarefaction). This sets off sound wave in photon-baryon fluid and creates temperature fluctuations.

Compressing a gas heats it up, expanding a gas cools it down

Temperature fluctuations
Origin of the Acoustic Peaks

Previously we only looked at a single overdensity. In reality, Universe is filled with both over- and under-densities. We can think of this 'perturbed' Universe as being built up from a large sum of sinusoidal perturbations of different wavelengths (we call these the different perturbation *modes*). Below is an illustration of one such mode...

![Diagram of acoustic peaks](image)

**Compression results in higher temperature**

**Rarefaction results in lower temperature**

**Oscillations:** Compression in valley *(hot)* & rarefaction at hill *(cold)* is followed by rarefaction in valley *(cold)* & compression at hill *(hot)* is followed by compression in valley *(hot)* & rarefaction at hill *(cold)*, etc
Origin of the Acoustic Peaks

Density perturbation field is a combination of perturbations with different wavelengths (different ‘modes’).

Modes with smaller wavelengths oscillate faster (consequence of constant sound speed).

At recombination, photons are released, and the pressure of the photon-baryon fluid drops to (almost) zero. This stops the oscillations; they are frozen in the configuration in which they happen to be at the moment of recombination.
Origin of the Acoustic Peaks

Shown is the time-evolution of a single perturbation mode, and the locations of six 'jack-in-the-boxes'.

Oscillations

Space

Space
Origin of the Acoustic Peaks

At recombination, jack-in-the-boxes open (photons `decouple') and the photons start to free-stream through space. The observer sees this mode as angular temperature fluctuation on the sky, with a characteristic angular scale set by the wavelength of the mode.
The first acoustic peak is due to the mode that just reaches maximal compression in valley/rarefaction on hill top for first time at recombination.
Temperature fluctuations at troughs are not zero! Although photon-baryon fluid has constant temperature, motions in the fluid cause Doppler shifts.
The second acoustic peak is due to mode that just reaches maximal rarefaction in valley/compression on hill top for first time at recombination.
Cosmic Microwave Background

IV. Lessons Learned
The Curvature of the Universe

One such triangle comes from angular scale of first acoustic peak: this angular scale corresponds to the wavelength of the mode that just managed to collapse (reach maximal compression) at recombination (i.e., at $t = t_{\text{rec}}$)

\[ \lambda/2 = v_{\text{sound}} t_{\text{rec}} \]
\[ v_{\text{sound}} = c/\sqrt{3} \]
\[ \lambda = (2/\sqrt{3}) c t_{\text{rec}} = \lambda_{\text{fp}} \]

This wavelength is a constant, the value of which is known. We also know the distance to last scattering surface (lss)

RESULT: Our Universe is flat (K=0), i.e., has Euclidean Geometry
The Baryon-to-Dark Matter Ratio

Increasing the density of the baryons relative to that of the dark matter causes stronger compression in valleys (due to the self-gravity of baryons), and less compression on hill tops. Since odd peaks (first, third, etc) correspond to compression in valleys, whereas even peaks (second, fourth, etc) correspond to compression on hill tops, the baryon-to-dark matter ratio controls the ratio of odd-to-even peak heights.

RESULT: dark matter density ~6x higher than baryon density
CMB data supports Dark Energy

Detailed modelling of the CMB anisotropies shows that the dark and baryonic matter have a combined density

\[ \bar{\rho}_m \simeq 0.25 \rho_{\text{crit}} \quad \Leftrightarrow \quad \Omega_m \simeq 0.25 \]

However, the fact that the geometry of the Universe is Euclidean (i.e., \( K = 0 \)), indicates that

\[ \Omega_0 = 1.0 \]

This suggests the presence of another component of energy density, other than baryonic or dark matter. The most obvious candidate is dark energy, whose presence is also inferred from the observed accelerated expansion. The total energy density of dark energy has to be

\[ \bar{\rho}_\Lambda \simeq 0.75 \rho_{\text{crit}} \quad \Leftrightarrow \quad \Omega_\Lambda \simeq 0.75 \]
Summary

- The CMB is a prediction of Big-Bang theory. Its presence is considered one of the strongest pieces of evidence supporting Big-Bang cosmology.

- The CMB reveals tiny temperature perturbations. The acoustic peaks in the temperature power spectrum were predicted to exist well before they were observed.

- The location and heights of the acoustic peaks indicate that Universe is flat, and that its matter content is dominated by dark matter.

- The amounts of dark matter and baryonic matter inferred from the CMB anisotropies are not enough to make the Universe flat. Hence, the CMB data supports the notion that the total energy density is dominated by dark energy.
The Early Universe
The History of the Universe

$ t = 10^{-43} \text{s} $ $ T = 10^{32} \text{K} $  
$ t = 10^{-35} \text{s} $ $ T = 10^{28} \text{K} $  
$ t = 10^{-33} \text{s} $ $ T = 10^{27} \text{K} $  
$ t = 10^{-6} \text{s} $ $ T = 10^{13} \text{K} $  
$ t = 10^{-4} \text{s} $ $ T = 10^{12} \text{K} $  
$ t = 10 \text{s} $ $ T = 10^{9} \text{K} $  
$ t = 300 \text{s} $ $ T = 10^{9} \text{K} $  
$ t = 10^{5} \text{yr} $ $ T = 10^{4} \text{K} $  
$ t = 10^{6} \text{yr} $ $ T = 3000 \text{K} $  
$ t = 9 \text{Gyr} $ $ T = 5 \text{K} $  
$ t = 13.73 \text{Gyr} $ $ T = 2.726 \text{K} $  

Big Bang

Quark Soup

Hadron era

Lepton era

radiation era

matter era

- quark-hadron phase transition
- matter-radiation equality
- recombination --> CMB
- Solar system forms
- structure formation

$ BBN = \text{Big-Bang Nucleosynthesis} $
Particle Physics
The Particle Nature of Matter

Regular (baryonic) matter is made up of elementary particles. The particles that are most relevant for ASTR 170 are:

- **Proton** (p); positive charge \((q=+1)\)
- **Neutron** (n); no electrical charge \((q=0)\)
- **Electron** (e); negative charge \((q=-1)\)

When talking about elementary particles, we express electrical charge in units of electric charge of proton:

\[
m_p \sim m_n \sim 2000m_e
\]
\[
q_p = -q_e = 1.6 \times 10^{-19}\text{C}
\]

**Atoms** are made up of a nucleus, consisting of protons and neutrons, surrounded by electrons. The number of electrons is equal to the number of protons, so that atoms carry no net electrical charge.

**Molecules** are electrically neutral groups of at least two atoms held together by chemical bonds (i.e., water, ethanol, carbon-monoxide, etc.)

**Ions** are atoms or molecules in which the number of electrons is either larger or smaller than the number of protons. If the number of electrons is zero, the atom is said to be fully ionized.

**Isotopes** are atoms that contain the same number of protons, but a different number of neutrons (i.e. Carbon-12 and Carbon-14 are isotopes).
### The Particle Nature of Matter

<table>
<thead>
<tr>
<th>Proton</th>
<th>Neutron</th>
<th>Electron</th>
<th>H (Hydrogen)</th>
<th>D (Deuterium)</th>
<th>Hydrogen and Deuterium are isotopes</th>
</tr>
</thead>
</table>
|        |         |          | ![H](Hydrogen) | ![D](Deuterium) | Neutral hydrogen is called HI  
Ionized hydrogen is called HII  
Molecular hydrogen is called H₂ |
|        |         |          | ~74%         |               |                                    |

<table>
<thead>
<tr>
<th>He (Helium)</th>
<th>He⁺ (singly ionized Helium)</th>
<th>He⁺⁺ (double ionized Helium)</th>
<th>In astronomy, all elements heavier than helium are called metals</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="Helium" alt="He" /></td>
<td>![He⁺](singly ionized Helium)</td>
<td>![He⁺⁺](double ionized Helium)</td>
<td>~2%</td>
</tr>
<tr>
<td>~24%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Quarks & Co

Protons and neutrons are made up of three quarks each, held together by gluons. There are a total of six different quarks (known as `flavors') giving rise to many different particles (pions, kaons, hyperons, etc.). All of these are unstable (will decay) except for proton. Quarks can never be isolated; they always combine to make composite particles (called hadrons).

<table>
<thead>
<tr>
<th>FLAVOR</th>
<th>CHARGE*</th>
</tr>
</thead>
<tbody>
<tr>
<td>up</td>
<td>+2/3</td>
</tr>
<tr>
<td>down</td>
<td>-1/3</td>
</tr>
<tr>
<td>charm</td>
<td>+2/3</td>
</tr>
<tr>
<td>strange</td>
<td>-1/3</td>
</tr>
<tr>
<td>top</td>
<td>+2/3</td>
</tr>
<tr>
<td>bottom</td>
<td>-1/3</td>
</tr>
</tbody>
</table>

* electron has charge of -1

Total charge:
proton: $+2/3 + 2/3 - 1/3 = +1$
neutron: $+2/3 - 1/3 - 1/3 = 0$
**Anti-Matter**

All charged elementary particles (i.e., mainly electrons and protons) have an **anti-particle**, which has the same mass but **opposite electrical charge**. The anti-particle of the **electron** is the positively charged anti-electron (or **positron**), and the anti-particle of the **proton** is the anti-proton. The neutral **neutron** also has an anti-particle (the anti-neutron), which has the same mass and the same electrical charge as the neutron; it only differs from the neutron in that it is made up of anti-quarks, instead of quarks.....

In what follows, we will indicate **anti-particles** by a little bar over the particle’s letter:

We use $e$ to indicate an electron, and $\bar{e}$ to indicate a positron (some books use $e^+$ instead)
We use $p$ to indicate a proton, and $\bar{p}$ to indicate anti-proton (some books use $p^-$ instead)
We use $n$ to indicate a neutron, and $\bar{n}$ to indicate an anti-neutron, etc..

The existence of anti-particles was predicted by **Paul Dirac** in 1928, based on some symmetry arguments in quantum theory. Only a few years later, in 1932, the first positrons were observed in particle accelerator experiments. Anti-protons and anti-neutrons were discovered soon afterwards.
Annihilation & Pair Production

When a particle `bumps' into its anti-particle, they annihilate (are transformed into two photons). During annihilation, the entire rest-mass energy of the particle and anti-particle is transformed into pure energy (in the form of photons).

\[
e + \bar{e} \rightarrow \gamma + \gamma
\]

**NOTE:** from here onwards, we will use the greek symbol \( \gamma \) to indicate a photon.

Interestingly, when a photon has sufficient energy, it can create (from its energy) a particle plus its anti-particle...

\[
\gamma \rightarrow e + \bar{e} \quad \text{creation of electron-positron pair}
\]
\[
\gamma \rightarrow n + \bar{n} \quad \text{creation of pair of neutron and anti-neutron}
\]

In the first example, the photon energy needs to be at least twice the rest-mass energy of an electron \( E_\gamma = hf \geq 2m_e c^2 \). In the second example, the photon energy needs to be at least twice the rest mass energy of a neutron....

Shortly after the **Big Bang**, the Universe is full of very energetic photons.... They have enough energy to create matter and anti-matter (in equal amounts).
Consider a Universe with only photons, electrons and positrons. Shortly after Big Bang, there are only photons. These produce electron-positron pairs. Once temperature of Universe becomes so low that individual photons can no longer create electron-positron pairs, they quickly annihilate away, yielding once again a Universe consisting only of photons.

**Puzzle:** The same should apply to protons/anti-protons and to neutrons/anti-neutrons. So why doesn't the current Universe consist only of photons? Where did all the stars and galaxies come from???
Baryo-Genesis

**Puzzle 1:** The same should apply to protons/anti-protons and to neutrons/anti-neutrons.... So why doesn’t the current Universe consist only of photons? Where did all the stars and galaxies come from????

**Solution:** the Universe is expanding...by the time pair-creation is no longer possible, the Universe has become so large and dilute, that the probability that a particle and an anti-particle meet each other has become very small.....

**Puzzle 2:** if particles and anti-particles survived, then where are the anti-particles? Once I create stars and galaxies, densities increase, and if I try to make objects from combination of matter and anti-matter, they should annihilate...... Did matter and anti-matter somehow segregate and create stars and anti-stars? Where are the anti-galaxies?

**Solution:** the anti-matter has disappeared! There are no anti-stars and/or anti-galaxies. We believe that in early Universe, for some (unknown) reason, there was a slight excess of matter over anti-matter. When Universe cooled down, all anti-matter annihilated with matter, leaving a little bit of excess matter. All galaxies, stars, planets, etc are made out of that little bit of excess matter.

The process of producing an *asymmetry* between baryons and anti-baryons is called *baryo-genesis*. At the present we have no clear understanding of how this happened......
In 1930, the Austrian physicist Wolfgang Pauli postulated the existence of a new particle, called the neutrino (meaning “small neutral one” in Italian), in order to satisfy energy conservation in beta decay: it was found that the energy of the observed decay products (electron plus proton) did not add up to that of the original particle, the neutron. Pauli postulated that the excess energy was carried away by a neutrino; an electrically neutral particle with very, very small, but non-zero mass. Neutrinos are denoted by the Greek letter \( \nu \) (nu).

Because of its small mass, and because it has no electrical charge (it does not feel the electromagnetic force), it is able to pass through ordinary matter almost unaffected. This makes them extremely difficult to detect (experimental confirmation for the existence of neutrinos had to wait until 1956).

Neutrinos are created in various nuclear reactions (for example, beta-decay). They are also produced, in copious quantities in the Sun as a by-product of hydrogen-fusion.

Most neutrinos passing through the Earth emanate from the Sun. Every second, about 65 billion Solar neutrinos pass through every square centimeter of your body...
**Neutrino Experiments**

SuperKamiokande, a neutrino observatory under Mount Kamioka near the city of Hida in Japan. It is located in a mine, 3300 ft under the surface of the Earth. It consists of a steel tank that holds 50,000 tons of ultra-pure water, and its walls are mounted with 11,146 photo-multipliers. Neutrinos that interact with electrons in the water produce a certain type of radiation (Cherenkov radiation) that can be detected with these photo-multipliers.
On Nov 12, 2001, about 6,600 of the photo-multiplier tubes (costing $3000 each) imploded, in a chain reaction, as the shock wave from the concussion of each imploding tube cracked its neighbours...
Going back in time, the Universe becomes hotter and denser. At sufficiently early times, there are no hadrons (particles made up of quarks), simply because the energy of the photons and the quarks is too large to bind the quarks together into compound particles...

At this point in time \( t < 10^{-6} \text{ s}, T > 10^{13} \text{ K} \), the Universe is made up of a primordial “soup” consisting of photons, neutrinos & anti-neutrinos, electrons & positrons, quarks and anti-quarks.

(we can ignore dark matter & dark energy for now)

Baryo-genesis has already done its job, so that there are more quarks and electrons than anti-quarks and anti-electrons. The difference is tiny though; for every \( 1,000,000,001 \) quarks there are \( 1,000,000,000 \) anti-quarks. This tiny asymmetry is sufficient to result in a Universe that we observe today...

**Main reactions occurring in quark soup**

\[
\gamma \rightarrow q + \bar{q} \\
q + \bar{q} \rightarrow \gamma + \gamma \\
\gamma \rightarrow e + \bar{e} \\
e + \bar{e} \rightarrow \gamma + \gamma
\]
The Hadron Era

When the Universe is about 1 millisecond old, and its temperature has cooled down to \( \sim 10^{13} \text{ K} \), the Universe undergoes the quark-hadron phase transition: quarks bind together to form hadrons (mainly protons and neutrons)

In fact, quarks make protons & neutrons, while anti-quarks make anti-protons & anti-neutrons.

Universe now consists of photons, neutrinos & anti-neutrinos, electrons & positrons, protons & anti-protons, and neutrons & anti-neutrons (ignoring dark matter & dark energy).

Main reactions during Hadron era

- \( \gamma \rightarrow e + \bar{e} \)
- \( \gamma \rightarrow p + \bar{p} \)
- \( \gamma \rightarrow n + \bar{n} \)
- \( n + \bar{n} \rightarrow \bar{p} + e \)
- \( n + \nu \leftrightarrow p + e \)
- \( e + \bar{e} \rightarrow \gamma + \gamma \)
- \( p + \bar{p} \rightarrow \gamma + \gamma \)
- \( n + n \rightarrow \gamma + \gamma \)
- \( n + \bar{e} \leftrightarrow p + \nu \)
- \( n + e \leftrightarrow p + \nu \)

After about \( 10^{-4} \text{ s} \), when Universe has cooled to a temperature of about \( 10^{12} \text{ K} \), the photons no longer have enough energy to produce protons/anti-protons pairs or neutrons/anti-neutron pairs. The anti-protons and anti-neutrons annihilate away, leaving a very small number of protons and neutrons (compared to number of photons). This signals the end of the hadron era...
The Lepton Era

At the end of the Hadron era, the Universe is left with photons, neutrinos & anti-neutrinos, electrons & positrons, and a `hand-full' of protons & neutrons. Since there are about 1 billion times as many electrons as protons (they are continuously being created out of photons), this period is called the Lepton era (leptons are elementary particles that are not made out of quarks; electrons & neutrinos).

Main reactions during Lepton era

\[ \gamma \rightarrow e + e \]
\[ n + \nu \leftrightarrow p + e \]
\[ e + \bar{e} \rightarrow \gamma + \gamma \]
\[ n + e \leftrightarrow p + \bar{\nu} \]

After about 10 s, when Universe has cooled to a temperature of about $10^9$ K, the photons no longer have enough energy to produce electron-positron pairs. The positrons annihilate away, leaving a very small number of electrons (the same number as protons). This signals the end of the lepton era...

At the end of the Lepton era, the Universe is left with photons, neutrinos & anti-neutrinos, and a `hand-full' of electrons, protons and neutrons that take part in weak-reactions that convert protons into neutrons and vice-versa. For every proton there is roughly one neutron, one electron and ~2,000,000,000 (2 billion) photons.
At the end of the Lepton era, the main reaction in the Universe is $n + \nu \leftrightarrow p + e$

Since the neutron is slightly more massive than the proton, it is energetically easier to convert a neutron in a proton than the other way around.

At early times, when the Universe is still very hot (and all particles have lots of energy), this difference is negligible, and both reactions occur equally frequent. However, by the end of the Lepton era, it becomes basically impossible to create the heavier neutrons out of the lighter protons.

If this reaction would continue, all neutrons would disappear. However, because of expansion of Universe, reaction rate become negligibly small (probability that neutron encounters neutrino becomes very, very small). Detailed calculations show that when this reaction stops, $n_n/n_p \approx 0.2$

But neutrons continue to decay via the above beta-decay reaction. Unless something happens, all neutrons will have disappeared before Universe is 1 year old....
Big-Bang Nucleosynthesis

Fortunately for mankind, something does happen: Big-Bang Nucleosynthesis (BBN): neutrons and protons start to combine to produce Helium nuclei. Once a neutron is locked up in a nucleus, it can no longer decay (it is stable against beta-decay).

The reactions involved in BBN are very similar to the pp-chain reactions that cause Hydrogen fusion in the interior of stars, with one small difference. In stellar interiors: $4p \rightarrow \text{He}$, whereas in BBN: $2p + 2n \rightarrow \text{He}$.

**Question:** what is the reason for this difference?

The reason for this difference is that there are no free neutrons in stellar interiors (they are all locked up in He nuclei ever since BBN).

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Event</th>
<th>Temperature (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-43}$</td>
<td>inflation</td>
<td>$10^{28}$</td>
</tr>
<tr>
<td>$10^{-35}$</td>
<td>BBN</td>
<td>$10^{27}$</td>
</tr>
<tr>
<td>$10^{-33}$</td>
<td>Quark Soup</td>
<td>$10^{28}$</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>Hadron era</td>
<td>$10^{13}$</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>Lepton era</td>
<td>$10^{12}$</td>
</tr>
<tr>
<td>$300$</td>
<td>BBN</td>
<td>$10^{9}$</td>
</tr>
</tbody>
</table>

**NOTE:** in principle, a direct reaction such as $2n + 2p \rightarrow \text{He}$, would be much more effective. However, this is a four-body interaction, which is extremely rare (four particles need to `bump' into each other simultaneously.)
Big-Bang Nucleosynthesis

BBN proceeds very rapidly. Once the conditions are such that Deuterium can be produced via the reaction $p + n \rightarrow D + \gamma$, which happens when Universe is about 100 seconds old, the entire network of reactions proceeds rapidly. About 300 seconds after the Big-Bang all neutrons have been locked-up in Helium nuclei, and BBN is over.

Since all neutrons end up in helium nuclei, we can calculate the mass fraction of Helium at end of BBN. Let $n_n$ and $n_p$ indicate number densities of neutrons and protons at onset of BBN, respectively. If we ignore the tiny difference in mass between protons and neutrons, we can write the mass fraction of Helium at the end of BBN simply as:

$$Y = \frac{4(n_n/2)}{n_p + n_n} = \frac{2(n_n/n_p)}{1 + (n_n/n_p)}$$

Here we have used that (i) you require 2 neutrons to make one Helium nucleus, and (ii) a Helium nucleus weights 4 times as much as a proton or neutron.

Detailed calculations show that, at the onset of BBN, the neutron-to-proton ratio is $n_n/n_p \simeq 0.14$. This implies a Helium mass abundance of $Y \simeq 0.25$.

Observations show that, throughout Universe, baryonic matter is 75% H, and 25% He. This is another confirmation of Big Bang cosmology (in addition to the CMB).
**Question**: why doesn’t BBN continue to convert Hydrogen and Helium into Carbon, Oxygen, Nitrogen, etc, etc, as in the interiors of (massive) stars?

**Answer**: because those reactions require higher densities and temperatures than Hydrogen fusion. Once neutrons have fused into Helium, Universe is cooler and less dense than when it started; conditions do not allow formation of any elements heavier than Lithium (which is a minute byproduct of Helium formation).

**Emergence of a concordance cosmology**: the observed element abundances are in excellent agreement with predictions based on baryon density inferred from CMB data...
We’re all stardust....

About 3 minutes after the Big Bang, the only elements available in the Universe are **Hydrogen** (~75%), **Helium** (~25%) and a tiny amount of **Lithium**.

**Supernovae, the cradles of life....**

All elements heavier than Lithium have since been synthesized inside stars, and distributed over the Universe via stellar winds and supernovae.
The Composition of the Universe

Today At Recombination

\[ \log(a) \]

Note: neutrinos behave as photons, and their energy density therefore falls off as radiation (i.e., faster than matter)

Our concordance cosmology is called $\Lambda$CDM (\textit{`Lambda-CDM'}, to indicate that it postulates a Universe comprised of both (Cold) dark matter and Dark Energy (represented by the cosmological constant $\Lambda$). At present, only $\sim 4$ percent of the total energy density in the Universe is in the form of regular (baryonic) matter.
The Radiation Era

At the end of BBN, the Universe consists of photons, neutrinos & anti-neutrinos, and a `hand-full' of free electrons, hydrogen nuclei and helium nuclei (in addition to dark matter; at this point in time dark energy is still completely negligible)

The energy density is dominated by radiation, hence the name `radiation era'. Not much happens during this period; the Universe continues to expand and cool down....

After ~100,000 years, when temperature has dropped to ~10,000K, matter (baryonic + dark) becomes dominant component of the Universe (we enter `matter era'). A little bit later, when Universe is 380,000 years old and T = 3000K, free electrons combine with H and He nuclei to produce neutral atoms (recombination). This is the epoch of the CMB

For the next ~13Gyr, the Universe continues to expand and cool down; slowly the tiny density perturbations visible in the CMB grow to produce galaxies and clusters, until dark energy takes over and quenches structure growth....
### Summary

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 10^{-43}$ s</td>
<td>Big Bang</td>
<td>$T = 10^{32}$ K</td>
</tr>
<tr>
<td>$t = 10^{-35}$ s</td>
<td>inflation</td>
<td>$T = 10^{28}$ K</td>
</tr>
<tr>
<td>$t = 10^{-33}$ s</td>
<td>Quark Soup</td>
<td>$T = 10^{27}$ K</td>
</tr>
<tr>
<td>$t = 10^{-6}$ s</td>
<td>Hadron era</td>
<td>$T = 10^{13}$ K</td>
</tr>
<tr>
<td>$t = 10^{-4}$ s</td>
<td>Lepton era</td>
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<tr>
<td>$t = 10^5$ yr</td>
<td>radiation era</td>
<td>$T = 10^4$ K</td>
</tr>
<tr>
<td>$t = 9$ Gyr</td>
<td>matter era</td>
<td>$T = 5$ K</td>
</tr>
<tr>
<td>$t = 13.73$ Gyr</td>
<td>present</td>
<td>$T = 2.726$ K</td>
</tr>
</tbody>
</table>

- Solar system forms
- structure formation
- matter era
- radiation era
- BBN
- recombination $\rightarrow$ CMB
- matter-radiation equality
- BBN = Big-Bang Nucleosynthesis
- quark-hadron phase transition

**Temporally Relevant Events**

- $t = 300$ s: BBN
- $t = 10^{-4}$ s: Hadron era
- $t = 10^{-6}$ s: Lepton era
- $t = 10^5$ yr: radiation era
- $t = 9$ Gyr: matter era
- $t = 13.73$ Gyr: present
Inflation
After the discovery of the CMB, the Big Bang model was the main cosmological model. It was strongly supported by (i) observed expansion of Universe, (ii) presence of Cosmic Microwave Background (CMB) and (iii) the observed abundances of Hydrogen and Helium (BBN).
However, there were also four potentially important problems with Big Bang Theory:

- The Horizon Problem
- The Flatness Problem
- The Magnetic Monopole Problem
- The Perturbation Problem

As we shall see, none of these were really problems in the sense that they falsified Big-Bang cosmology; one could `solve' them by simply postulating certain initial conditions of the Universe. Nevertheless, cosmologists were unhappy with such a `solution', and were hoping that a more satisfactory solution would arise from quantum-gravity (a `unification' of Einstein’s General Relativity and Quantum Theory)... To this date, we do not understand quantum-gravity!!
The blue circles indicate the horizon of each jack-in-the-box, the moment they are opened. Observations show us that the photons from each of these jack-in-the-boxes are (almost) the same ($\Delta T/T < 10^{-5} K$).....but how could they have agreed on their temperature?
Consider two jack-in-the-boxes, located 180 degrees away from each other on the last-scattering surface. The material in each jack-in-the-box can only have been in causal contact (i.e., “communicated with”) material within its own horizon. It therefore could only have established thermal equilibrium within its own horizon. But then how did all jack-in-the-boxes get to be at the same temperature (how did they establish thermal equilibrium)?
Here is another illustration of the Horizon Problem. It shows a space-time diagram. At the top is the observer at the present. The light-gray triangle indicates the observer's horizon; he can only have received photons from events (things that happened at a location at a given time) within this triangle.

It also shows the horizons of two jack-in-the-boxes opening at recombination (=two events). Although both events lie within the observer's horizon, the two jack-in-the-boxes cannot have communicated with each other prior to recombination; hence, they cannot have established thermal equilibrium.

Prior to 1980, the “solution” to this Horizon Problem was simply to postulate that after the Big Bang the initial conditions were such that the temperature in space was everywhere the same (except for tiny fluctuations, the origin of which was unclear)...
The Horizon Problem

What would we have observed if the temperature of the Universe was NOT homogeneous?

Everywhere in space, *jack-in-the-boxes* open when their *local* temperature reaches 3000K.

In direction of box A, I see photons that have been *redshifted* more than in direction of B. Hence, in direction of A my *CMB* appears *colder* than in direction of B.
The Flatness Problem

If present-day Universe is flat ($\Omega_0 = 1$), then $\Omega$ has been unity since the Big Bang.

If present-day Universe is positively curved ($\Omega_0 > 1$), then $\Omega$ was only slightly larger than unity shortly after the Big Bang: At $t=10^{-43}$ s we need to have that $\Omega = 1 + 10^{-59}$

If present-day Universe is negatively curved ($\Omega_0 < 1$), then $\Omega$ was only slightly less than unity shortly after the Big Bang: At $t=10^{-43}$ s we need to have that $\Omega = 1 - 10^{-59}$

Since densities evolve with time (due to expansion of Universe), and since the critical density evolves with time, due to evolution of Hubble parameter, the density parameter $\Omega$ evolves with time. The diagram to the left shows how....
The Flatness Problem

This constitutes the so-called **Flatness Problem**: unless Universe was perfectly flat from the start, it must have been extremely close to `flat' shortly after the Big Bang. There were two schools of thought for how to `solve' this problem:

1. If Universe must have been arbitrarily close to flat, it is more `likely' that it always has been exactly flat. However, the total observed matter density (dark & baryonic) is insufficient to make the Universe flat (dark energy was not considered a realistic option)

2. Initial conditions of Universe were simply such that $\Omega$ was initially extremely close to, but not exactly equal to, unity. This results in fine-tuning problem: to end up with present-day value for $\Omega$ that is close to unity, you need to fine-tune $\Omega$ shortly after the Big-Bang to extreme precision
The Magnetic Monopole Problem

**Question**: what happens to this magnet when you cut it in half?

- **A**: one South magnet + one North magnet
- **B**: two new North-South magnets

Experience tells us that there are no **magnetic monopoles**; a hypothetical particle that is a magnet with only one magnetic pole. This is an important difference between electricity and magnetism: whereas you have electrical monopoles (i.e., electron, proton), there seem to be no magnetic monopoles...
The Magnetic Monopole Problem

Maxwell Equations

\( \nabla \cdot \mathbf{E} = 4\pi \rho_e \)
\( \nabla \cdot \mathbf{B} = 0 \)
\( \nabla \times \mathbf{B} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} - \frac{4\pi}{c} \mathbf{j}_e \)

...with magnetic monopoles

\( \nabla \cdot \mathbf{E} = 4\pi \rho_e \)
\( \nabla \cdot \mathbf{B} = 4\pi \rho_m \)
\( \nabla \times \mathbf{B} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} - \frac{4\pi}{c} \mathbf{j}_m - \frac{4\pi}{c} \mathbf{j}_e \)

Maxwell’s equations basically state that there are no magnetic monopoles. Maxwell obtained his equations using empirical fact that no magnetic monopoles had ever been observed.

If magnetic monopoles do exist, his equations need to be modified slightly; they would become even more symmetric (more beautiful!!)

Quantum Field Theory (QFT) predicts the existence of magnetic monopoles. According to QFT the magnetic monopoles are very massive and abundant. In fact, according to QFT there should be so many magnetic monopoles that they dominate the matter density in the Universe. However, to this date, no-one has ever seen a single magnetic monopole. This is called the Magnetic Monopole Problem.
The Cosmic Microwave Background reveals tiny perturbations in the temperature of the Universe at the epoch of recombination ($\Delta T/T < 10^{-5}K$). As we have seen, these temperature perturbations are caused by perturbations in the density of the dark matter.

But where did these density perturbations come from? This constitutes the perturbation problem.

Prior to 1980, it was simply assumed that these density perturbations were part of the initial conditions....for some reason the Universe simply started out with tiny fluctuations in its density...
In 1980, the theoretical physicist Alan Guth (then at Cornell) came up with a concept, called inflation, to solve the magnetic monopole problem. He soon realized inflation also solves the horizon problem, the flatness problem, and even the perturbation problem... Ever since, inflation has been a key-ingredient of Big-Bang cosmology.

So what is inflation? It is a period in which Universe experiences exponential expansion.

Before we investigate what might cause this to happen, let’s investigate why this solves the various problems mentioned above....

Exponential expansion means that the scale factor evolves according to

\[ a(t) \propto e^{t/\tau} \]

Here \( \tau \) is called the e-folding time; it is the time in which the scale factor (and therefore the size of the Universe) increases by a factor \( e = 2.71828 \)....

After 2 e-folding times, the Universe has increased by a factor \( e^2 = 7.8 \)
After 10 e-folding times, the Universe has increased by a factor \( e^{10} = 2.2 \times 10^4 \)
After 60 e-folding times, the Universe has increased by a factor \( e^{60} = 1.1 \times 10^{26} \)
Inflation

Let's assume that very shortly after the Big Bang (at $t \sim 10^{-35}$ s) the Universe enters a period of exponential expansion. If the e-folding time is sufficiently short, say $\tau \sim 10^{-35}$ s then after a very short period, say at $t \sim 10^{-33}$ s, the Universe would have experienced exponential expansion for $\sim 100$ e-folding times, which would have increased the size of the Universe by a factor $e^{100} = 2.7 \times 10^{43}$ !!! Two particles that at $t \sim 10^{-35}$ s are 1 mm away from each other, at $t \sim 10^{-33}$ s will be separated by a bewildering $8.7 \times 10^{17}$ Mpc.

For comparison:

- Our present-day horizon is only about 3000 Mpc.
- Since recombination, (about 13 Gyrs ago) the Universe has only expanded by a factor $1000$:
  \[
a_{\text{rec}} = \frac{1}{1 + z_{\text{rec}}} = \frac{1}{1101} \approx 0.001
\]
The Horizon Problem Solved

If inflation lasted for at least 60 e-folding times, the horizon problem is solved.

If inflation lasted long enough, then each point on the last-scattering surface could have been inside each other's horizon the moment inflation started. Detailed calculations show that the number of e-folding times required is at least ~60.

In fact, the entire volume inside our present-day horizon was at the moment inflation started located inside a tiny, minuscule volume, that was much smaller than the size of the horizon at that time; i.e., all regions within our present-day horizon have been in causal contact with each other prior to inflation.
Inflation solves the flatness problem, simply because it `predicts' that the Universe is (very, very, very, very) close to flat: it was stretched into a flat geometry... This is independent of the geometry it had before the onset of inflation.

Inflation predicts a Universe that is very, very close to flat; there is no flatness problem.
If magnetic monopoles already existed before inflation started, then inflation will have reduced their number density by a factor $e^{3N}$, where $N$ is number of e-folding times that inflation lasted. Starting from the number density predicted before inflation, we expect to find roughly one magnetic monopole within our entire visible Universe: it is not a surprise that we haven't seen that one yet......

Inflation dilutes the Universe by a huge factor. This solves the magnetic monopole problem.
The Physics of Inflation

This material is not required for the exam
What causes Inflation?

Having established that inflation, an early period of exponential expansion, can solve many problems, we now examine what physical mechanism can cause such exponential expansion.

So what causes inflation? The answer is simple; a scalar field, called the inflaton.

Recall: dark energy is also a scalar field. As we have seen, if it dominates the energy density, then it causes the Universe to accelerate its expansion. It is straightforward to show that in fact, the expansion is exponential. The only difference between inflation and dark energy is that the inflaton dominates the energy density shortly after the Big Bang, and has an extremely small e-folding time ($\tau \sim 10^{-35} \text{s}$), while dark energy dominates the energy density in the present-day Universe and has a much longer e-folding time of $\tau \sim 10 \text{Gyr} \sim 3 \times 10^{17} \text{s}$

Inflation is caused by a scalar field, called the inflaton, which dominates the energy density of the Universe for a short period of $\sim 10^{-33} \text{s}$, starting $\sim 10^{-35} \text{s}$ after Big Bang.
Okay, so we have postulated the existence of (yet another) scalar field, the inflaton. But how do we establish that it dominates the energy density of the Universe only for a certain amount of time?

To understand this, we need to look at the physics of scalar fields:

A scalar field assigns to each point in space a value, $\varphi$. Associated with that value is a certain potential energy, $E_{pot} = V(\varphi)$. Here $V$ is some function; the exact form of this function is simply a property of the scalar field under consideration. Scalar fields differ from each other in the functional form of the potential energy...

In addition to potential energy, a scalar field also has kinetic energy. This is associated with the rate at which $\varphi$ changes over time. Let this rate be written as $\dot{\varphi}$. Then, the total energy density of the scalar field is

$$E_{tot}(\varphi) = E_{kin} + E_{pot} = \frac{1}{2} \dot{\varphi}^2 + V(\varphi)$$
Consider a scalar field with potential $V(\varphi)$, as illustrated. Consider point in space where scalar field has a value $\varphi$. It will then have a potential energy indicated by green ball. Just like a ball wants to roll down to center of valley, the scalar field will evolve towards state in which it has minimum potential energy; the scalar field starts to `roll down' its potential well, converting potential energy into kinetic energy. However, since its total energy is conserved, it will start an oscillation back-and-forth.

The amplitude of these oscillations depends on the energy of the scalar field at the location in question: if the scalar field has a high energy density, it will have large oscillations; if it has low energy density, the oscillations will be small....
Physics of Inflation

To have the inflaton behave the way we want, we need to couple it to the energy density of radiation (which dominates the Universe at early times). This effectively means that the inflaton is in thermal equilibrium with the radiation field. As Universe cools down, the energy density in radiation decreases, and because of coupling, energy density of inflaton decreases at exactly the same rate.

If this was the entire story, then while the energy density in radiation decreases, so does the energy density of the inflaton; it never becomes dominant, and we never enter a period of inflation....

We need one more trick.....

The shape of $V(\phi)$ changes with the temperature of the radiation field.
Below a critical temperature, the shape of the inflaton potential suddenly changes dramatically. From having one minimum, it suddenly takes on two global minima and one local minimum.

Initially, when $T>T_{\text{crit}}$, inflaton is coupled to radiation field, and its energy density decreases over time as seen before.

However, once the temperature drops below the critical temperature, the scalar field gets stuck in the local minimum around $\phi=0$; this is called the false vacuum state. From this point on, the energy density of the inflaton is equal to $E = V(\phi=0)$, and is larger than that of radiation: **INFLATION**
During inflation, inflaton is stuck in false-vacuum state. During this period energy density of inflaton dominates energy density of the Universe, and Universe experiences inflation.

**Question:** But how do we stop inflation?

**Answer:** Quantum Tunneling

Inflaton "tunnels" through potential barrier, after which it starts to fall towards true vacuum state, converting its energy into radiation and matter (this is the matter we see in our Universe; the matter that existed before inflation has been inflated away.....). Once inflaton has reached true vacuum state, its energy density is zero, and inflaton is no longer of any relevance for evolution of Universe.
But what about Perturbation Problem?

Quantum physics dictates that on very small scales, energy density associated with inflaton fluctuates...

During inflation, these quantum fluctuations get stretched out to fluctuations in energy density on HUGE scales; when inflation ends, and the latent heat of inflaton is converted into matter & radiation, these fluctuations become perturbations in matter density field.
Inflation Summary

Inflation is a short period (~10^{-33}s) of exponential expansion shortly after the Big Bang (starting at t\approx 10^{-35}s). It lasts for at least 60 e-folding times, which stretches space by a factor of at least 10^{26}. Inflation is driven by a scalar field, the inflaton, which initially is coupled to the radiation field, until it gets stuck in a false vacuum state. It then dominates the energy density of the Universe, causing exponential expansion, until it tunnels to the true vacuum state, converting its latent heat into radiation and matter.

**Inflation rocks because:**

- It solves the Horizon Problem; it assures that all matter inside our present-day horizon has been in causal contact.
- It solves the Flatness Problem; it predicts that Universe is very, very close to flat (i.e., that \Omega=1 to high precision).
- It solves Magnetic Monopole Problem; magnetic monopoles do exist, but they have been diluted to ~1 per horizon.
- It solves Perturbation Problem; it predicts existence of tiny perturbations on all scales. These arise from quantum fluctuations in inflaton that are stretched to large scales.
Structure Formation
We live in an expanding Universe.

Universe was hotter & denser at earlier times.

Presently, energy density of Universe is dominated by some unknown stuff called dark energy, which causes expansion rate to accelerate.

Most of the matter in the Universe is some unknown stuff called Cold Dark Matter...

Only ~4 percent of energy density in Universe is in form of `normal' (we say baryonic) matter.

Shortly after Big Bang, Universe experienced inflation, during which tiny density perturbations have been created. We believe these are the seeds of all structure formation...
The Universe in a Nutshell

Astronomers have mapped the distribution of millions of galaxies. Galaxies are found to be distributed in a filamentary web, the cosmic web.

Note the foamy, sponge-like distribution of galaxies.
The temperature perturbations observed in the CMB tell us that at the time of recombination, the density perturbations were tiny (less than 1 part in 100,000). At the present, however, we see huge density perturbations; for example, the average density of our own Milky Way is about 100,000 times larger than that of the Universe (we say that its overdensity is $\delta=100,000$).

How did these perturbations grow by more than 10 orders of magnitude?
Gravitational Instability

A region that is slightly denser than its environment will pull matter towards itself, thus becoming more over-dense. An under-dense region becomes more under-dense because its matter is pulled away towards an over-dense region...

Hence, density perturbations automatically grow with time, simply due to the working of gravity. This is called gravitational instability, and it is the mechanism that causes tiny perturbations to become big perturbation...

This process of growth continues until the perturbations are of order unity ($\delta \sim 1$). At that point in time, the perturbations `turn around' and start to collapse...
Initially, the physical size of an over-dense region increases due to the expansion of the Universe (i.e., the over-density is not gravitationally bound to itself and therefore experiences expansion). Consequently, its density goes down, while its over-density increases (in other words, its density decreases less rapidly than the average density of the Universe). Once the overdensity becomes of order unity ($\delta \sim 1$), the overdensity stops growing in size, and starts to collapse under its own gravity (i.e., we say that the overdensity `turns around').
The Evolution of a Shell of Dark Matter

It is useful to think of an over-density as a spherical region that consists of many individual shells, much like an onion...

The movie shows the evolution one such shell: initially it expands with the Universe; when the overdensity inside the shell reaches $\delta \sim 1$, the shell turns around and collapses towards the center. Dark matter (whatever it is) is assumed to be collisionless; this means that the shell simply moves through itself, and starts to expand again; it starts an oscillation, during which it converts kinetic energy into potential energy and vice versa....
The Formation of a Dark Matter Halo

Individual oscillating shells interact gravitationally, exchanging energy (they `virialize'). This ends the oscillations and results in a virialized dark matter halo. The word virialized simply expresses that the system finds a balance between kinetic and potential energy...
Evolution of a Shell of Baryonic Matter

Let's reconsider our onion model. Now imagine that each shell consists of baryonic rather than dark matter; unlike the dark matter, the baryonic matter is collisional!!!

This means that it has a net pressure; a shell of regular (baryonic) matter cannot just move through itself; rather it bumps into itself, creating a shock. This shock heats up the baryonic gas to a high temperature (depending on the mass of the halo $T \sim 10^4 - 10^7$ K). Hence, the end-product of the collapse of an overdensity consisting of both dark matter and baryonic matter is a virialized dark matter halo filled with hot, shock-heated gas.
The Collapse of Perturbations

Physical Size

Time

expansion of Universe

overdensity

turn-around

???

Evolution after turn-around depends on nature of matter

Dark Matter = collisionless  ➔ shell crossing

Baryonic Matter = collisional  ➔ shock heating
The Hierarchical Growth of Dark Matter Halos
A region in space in which 5 dark matter haloes have formed
Dark matter haloes attract each other gravitationally...
consequently, they move towards each other....
and merge together, to form bigger haloes....
with substructure
Numerical N-body Simulations

The detailed collapse and hierarchical formation/evolution of dark matter haloes, although only governed by gravity, is a very complex process; simply analytical methods are insufficient to make detailed predictions. Instead, astronomers use numerical N-body simulations on supercomputers to study these processes.

How to run a N-body simulation?

1) Distribute many (millions) of particles over a cubic box; make sure their distribution is not random, but reveals tiny perturbations, similar to those seen in the CMB...

2) Let the box ‘expand’ (mimic Universal expansion), and compute the gravitational force on each particle due to all other particles in the box.

3) Propagate each particle according to the gravitational force (acceleration) it feels...

4) Go to 2, and repeat this process many, many times (many ‘time-steps’), until the total time covered represents the age of the Universe.
Distribution of dark matter with tiny fluctuations in initial density
Note the foamy, sponge-like distribution of the dark matter; this is very reminiscent of the observed distribution of galaxies ($z = 0$).
A Close-Up View of a Dark Matter Halo
The Formation of Galaxies
Cooling & Disk Formation

Note that the dark matter does NOT cool.

Inside the disk the density gets very high, causing fragmentation and star formation: a disk galaxy is born...

Hot gas radiates, emits photons which carry away energy: the gas cools.

Due to pressure loss, gas starts to contract.

Because of angular momentum conservation, the cooling baryons spin up and form a thin disk.

Movie of Simulation of Disk Formation: http://www.youtube.com/watch?v=h9za1CP9ImA
If dark matter haloes host galaxies, clusters are a natural outcome of hierarchical formation in CDM Universe.

But what happens when two galaxies collide??
When two disk galaxies collide...

...an elliptical galaxy is formed.
Small perturbations, due to quantum fluctuations, grow and collapse to form dark matter haloes.

- Baryonic gas is shock heated to high temperatures.
- Baryonic gas cools and settles in center of halo; angular momentum conservation --> disk galaxy.
- Disks merge giving rise to population of ellipticals especially in denser environments (clusters).

Galaxy Formation in a nutshell...