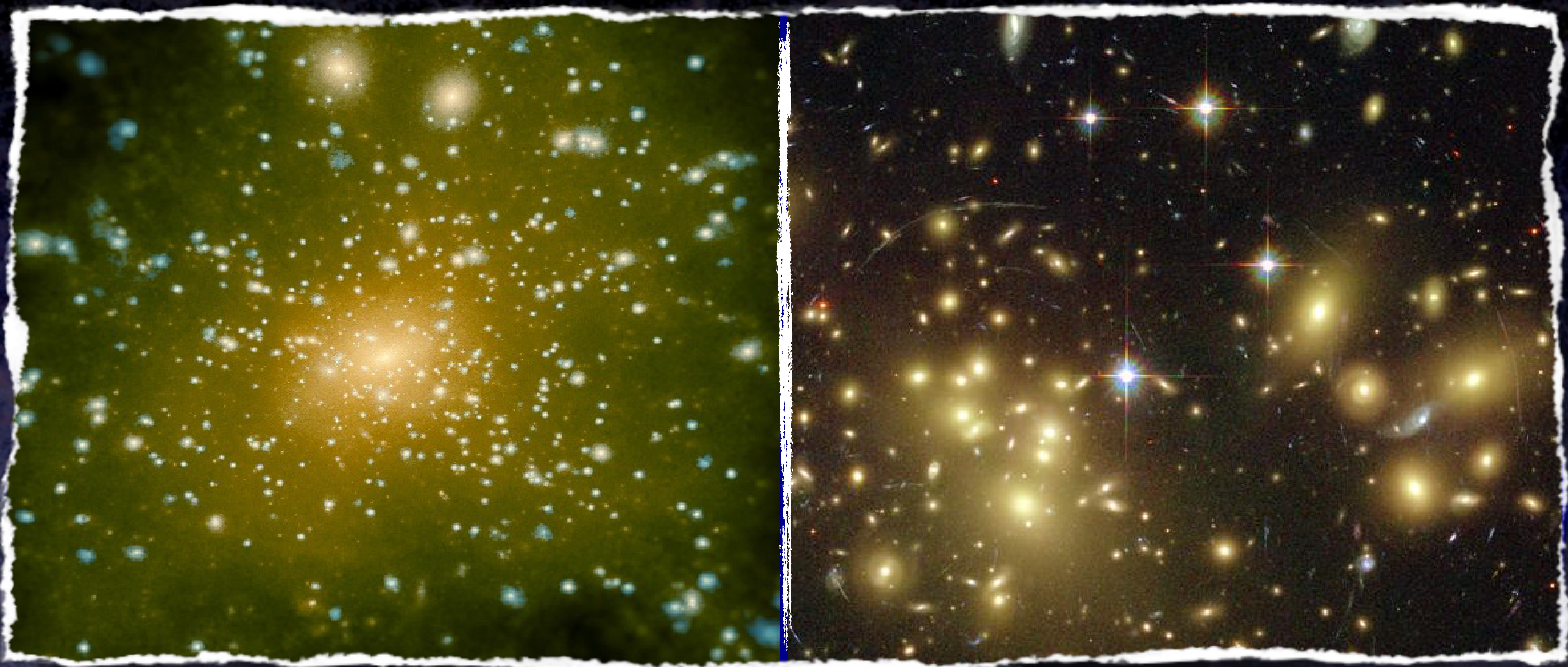
Structure Formation: from the linear to the non-linear regime



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In collaboration with:
Marcello Cacciato (HU), Surhud More (KICP),
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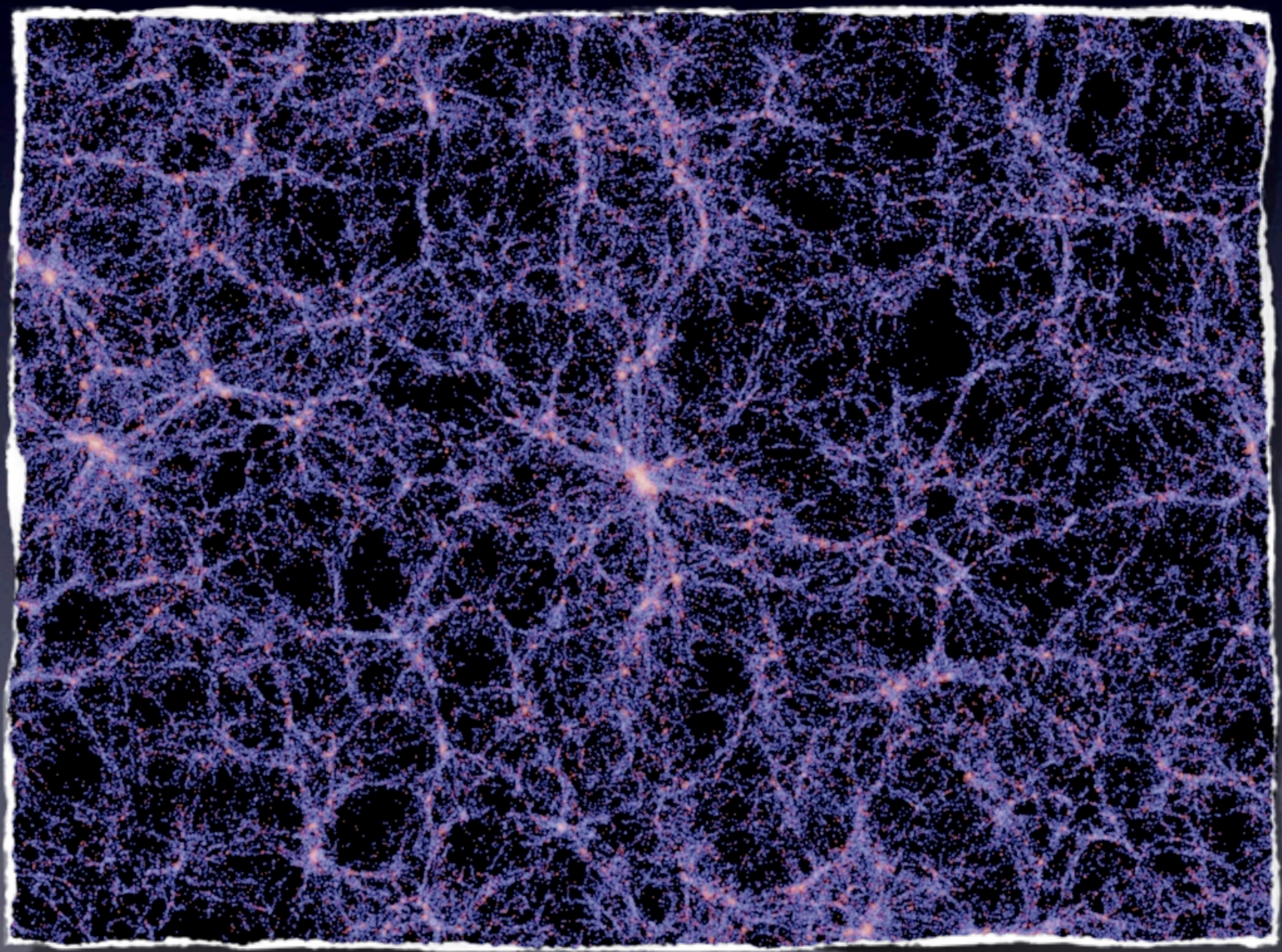
Outline

- How to describe the density field?
- The Evolving Density Field
- The Halo Model
- Halo Occupation Statistics
- Application 1: Galaxy Clustering
- Application 2: Galaxy-Galaxy Lensing
- Application 3: Constraining Cosmology

How to describe the matter distribution?

Let $\rho(\vec{x})$ be the density distribution of matter at location $\vec{x}$

It is useful to define the corresponding overdensity field $\delta(\vec{x}) = \frac{\rho(\vec{x}) - \overline{\rho}}{\overline{\rho}}$



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$$\mathcal{P}(\delta_1, \delta_2, ..., \delta_N)\, \mathrm{d}\delta_1\, \mathrm{d}\delta_2 \dots \mathrm{d}\delta_N$$

where $\delta_1 = \delta(\vec{x}_1)$, etc.

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This probability distribution is completely specified by the moments

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ergodic principle: ensemble average = spatial average

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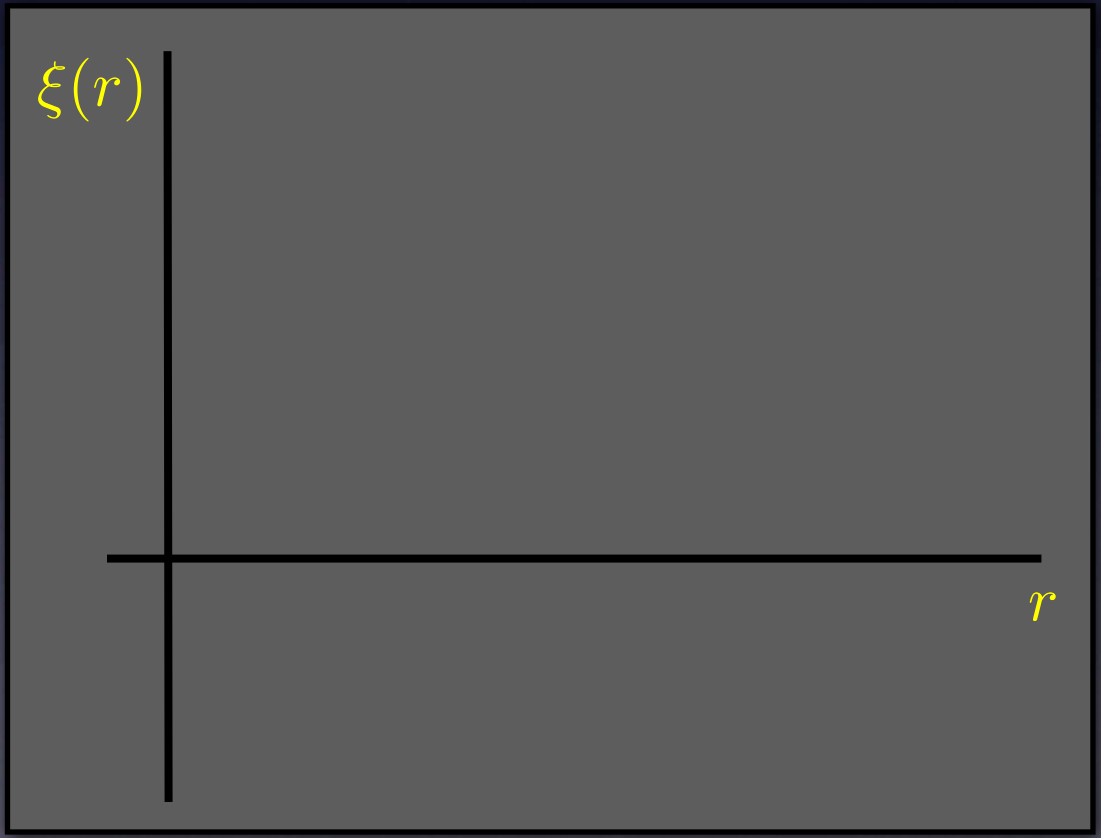

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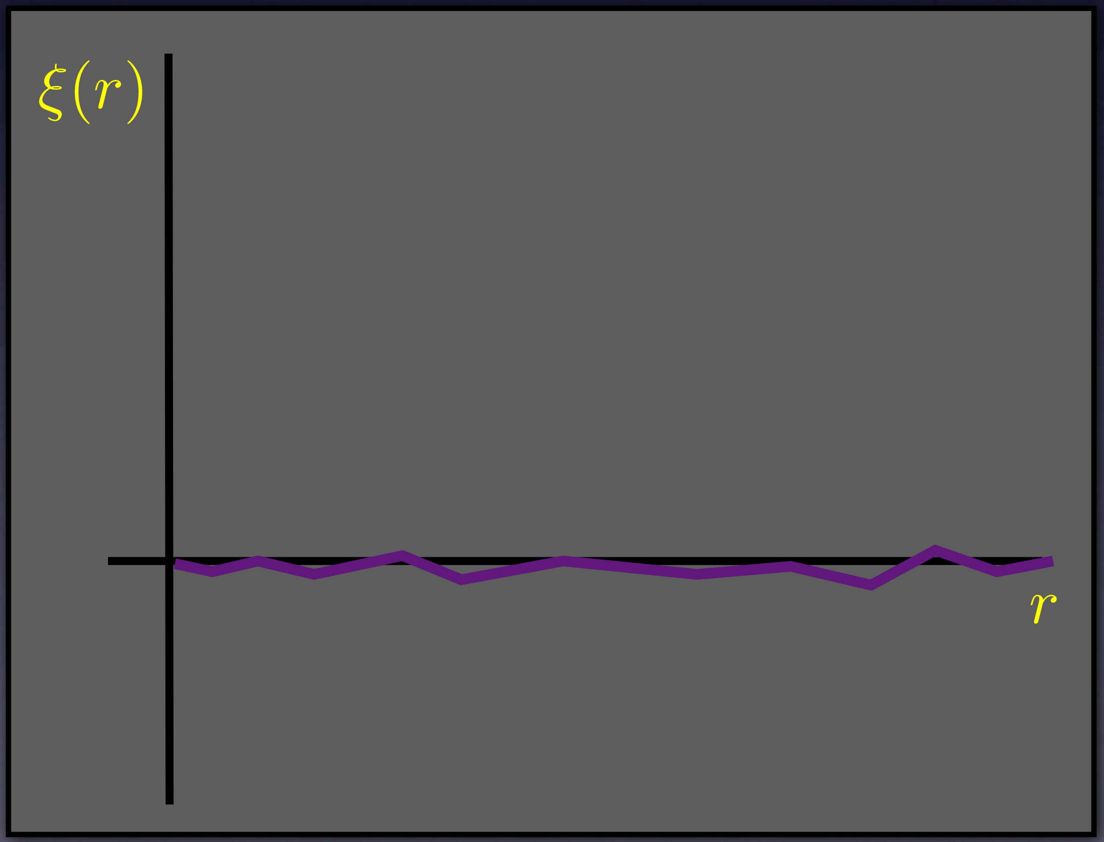

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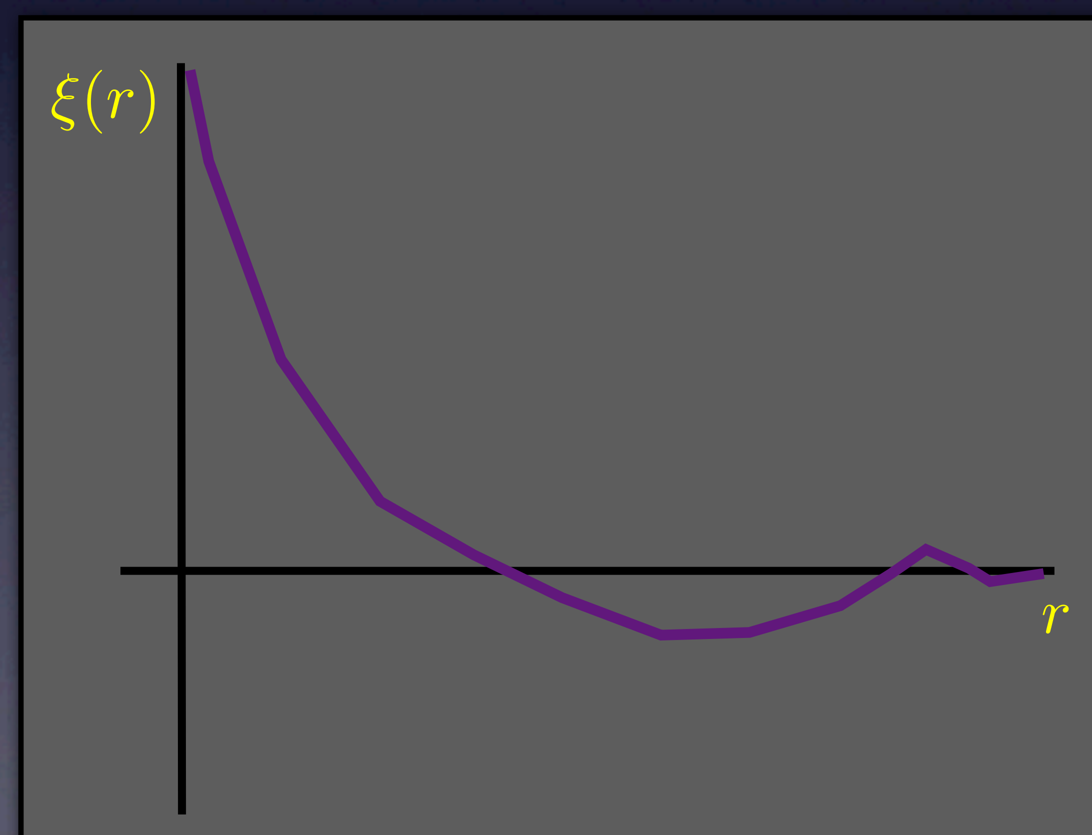
Clustered distribution



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A random field $\delta(\vec{x})$ is said to be Gaussian if the distribution of the field values at an arbitrary set of N points is an N-variate Gaussian:

$$\mathcal{P}(\delta_1, \delta_2, ..., \delta_N) = \frac{\exp(-Q)}{[(2\pi)^N \det(\mathcal{C})]^{1/2}}$$

$$Q \equiv \frac{1}{2} \sum_{i,j} \delta_i \, (\mathcal{C}^{-1})_{ij} \delta_j$$

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As you can see, such a Gaussian random field is completely specified by its second moment, the two-point correlation function $\xi(r)$!!!!

How to describe the matter distribution?

Often it is very useful to describe the matter field in Fourier space:

$$\delta(\vec{x}) = \sum_k \delta_{\vec{k}}\, e^{+i\vec{k}\cdot\vec{x}} \qquad \delta_{\vec{k}} = \frac{1}{V}\int \delta(\vec{x})\, e^{-i\vec{k}\cdot\vec{x}}$$

Here V is the volume over which the Universe is assumed to be periodic.

The Fourier transform of the two-point correlation function is called the power spectrum and is given by

$$\begin{aligned} P(\vec{k}) &\equiv V\langle|\delta_{\vec{k}}|^2\rangle \\ &= \int \xi(\vec{x})\, e^{-i\vec{k}\cdot\vec{x}} \\ &= 4\pi \int \xi(r)\, \frac{\sin kr}{kr}\, r^2\, \mathrm{d}r \end{aligned}$$

A Gaussian random field is completely specified by either the two-point correlation function $\xi(r)$, or, equivalently, the power spectrum $P(k)$

The Evolving Density Field

The Evolving Density Field

Inflation predicts that the power spectrum, immediately after inflation, is given by a simple power-law

$P(k) \propto k^n$ with $n \simeq 1$

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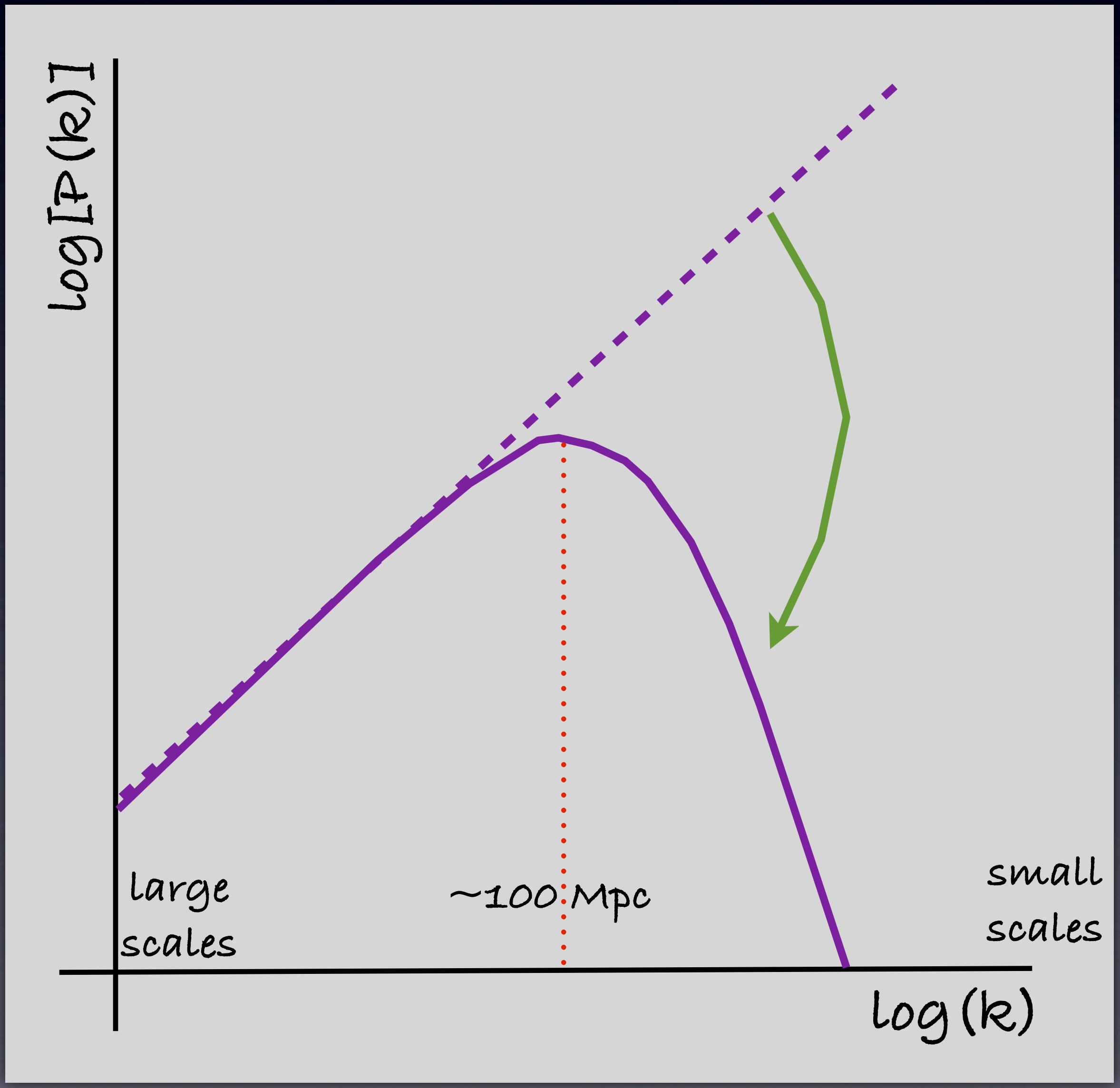


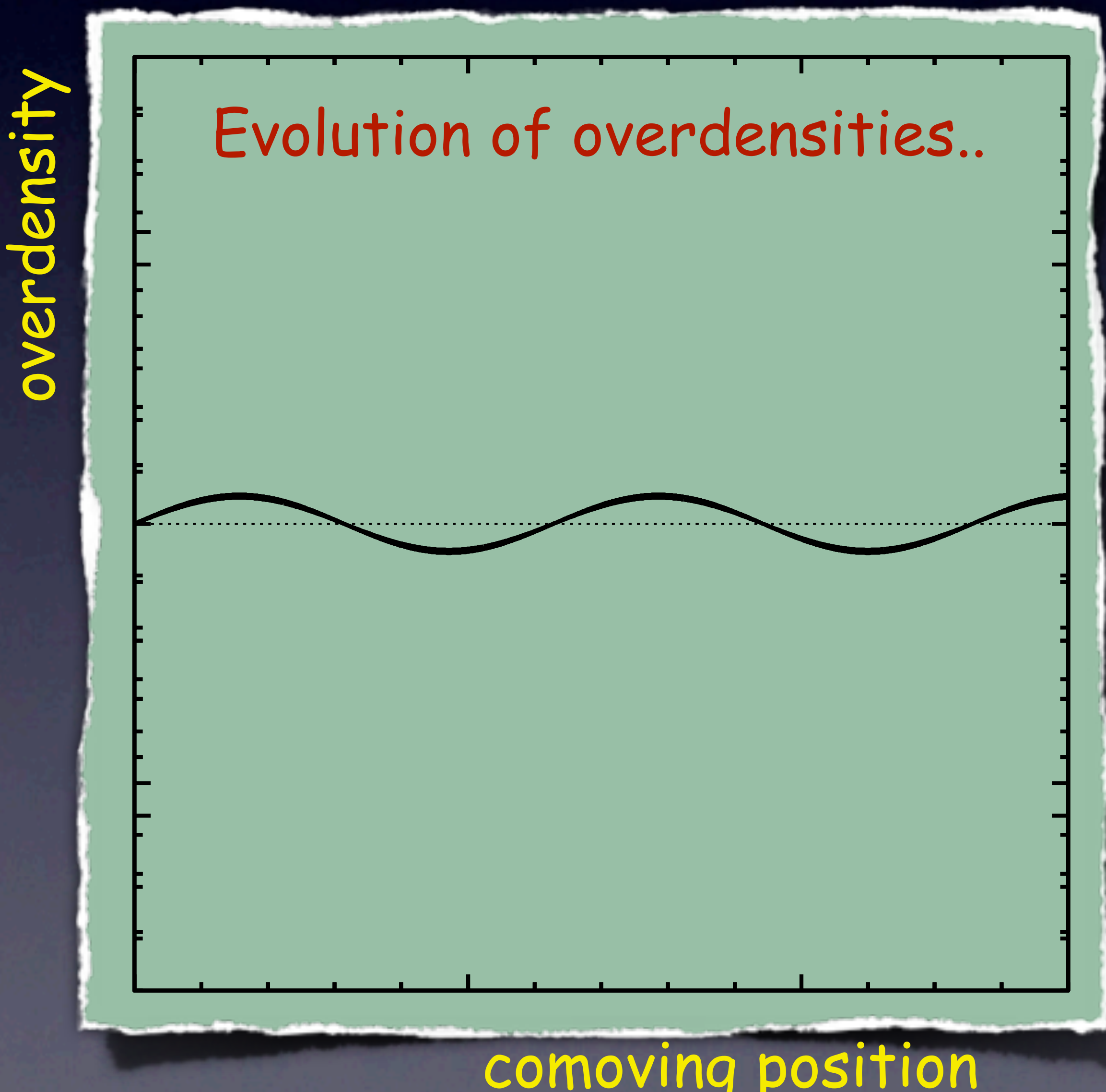
During radiation domination, matter perturbations that are inside the horizon cannot grow. This is called `stagnation' or the Meszaros effect.

Because of this, after recombination there is a characteristic scale in the matter power spectrum, which corresponds to the sound horizon at matter-radiation equality.

The Evolving Density Field

Gravitational Instability: slightly denser regions attract matter thus becoming even denser, etc.

During linear evolution, $(\delta \ll 1)$, all modes $\delta_{\vec{k}}$ evolve independently from each other according to $\delta_{\vec{k}} \propto D(t)$. Hence, $\delta(\vec{x},t) = \sum_i \delta_{\vec{k}} e^{+i\vec{k}\cdot\vec{x}} \propto D(t)$



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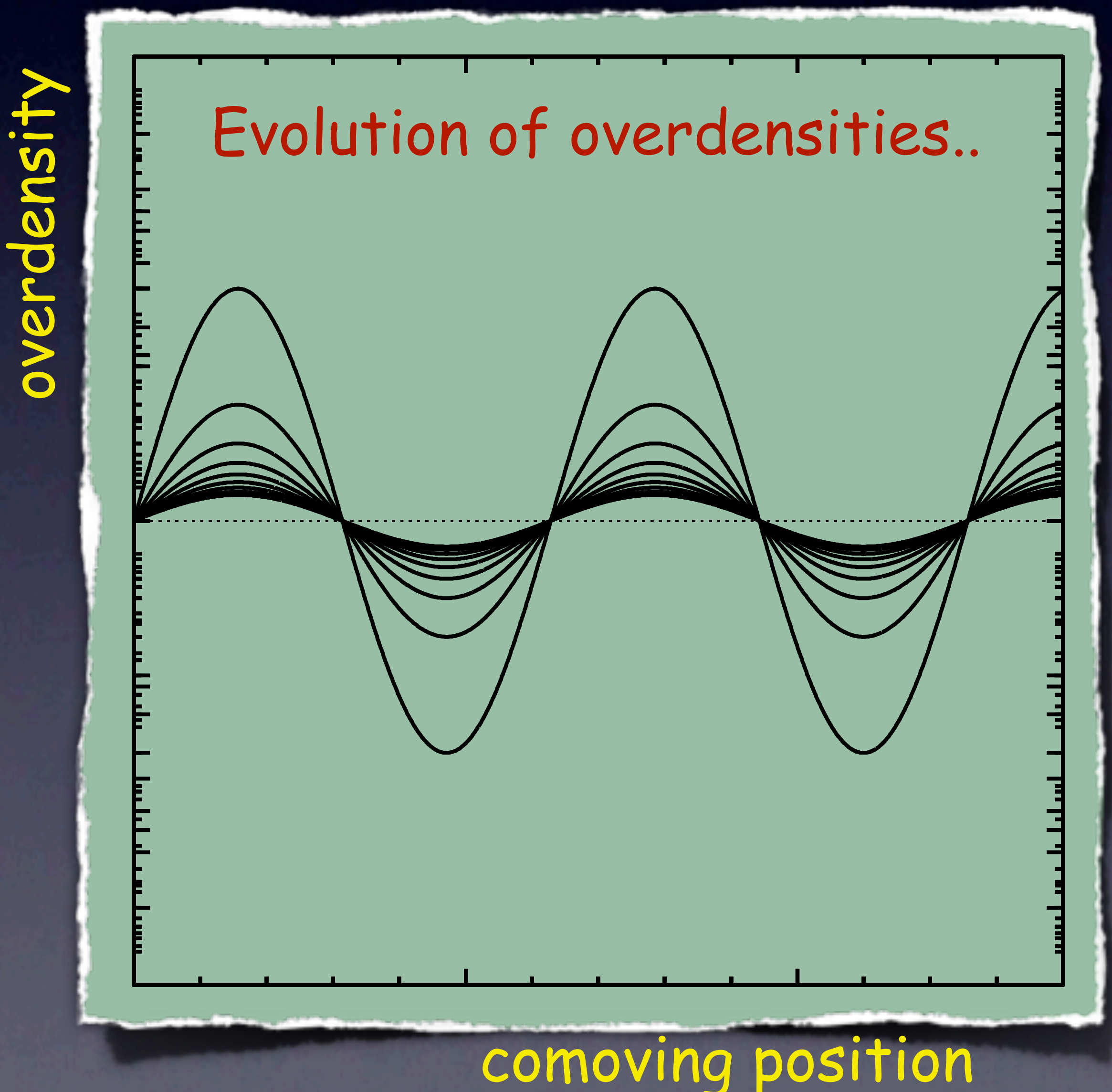

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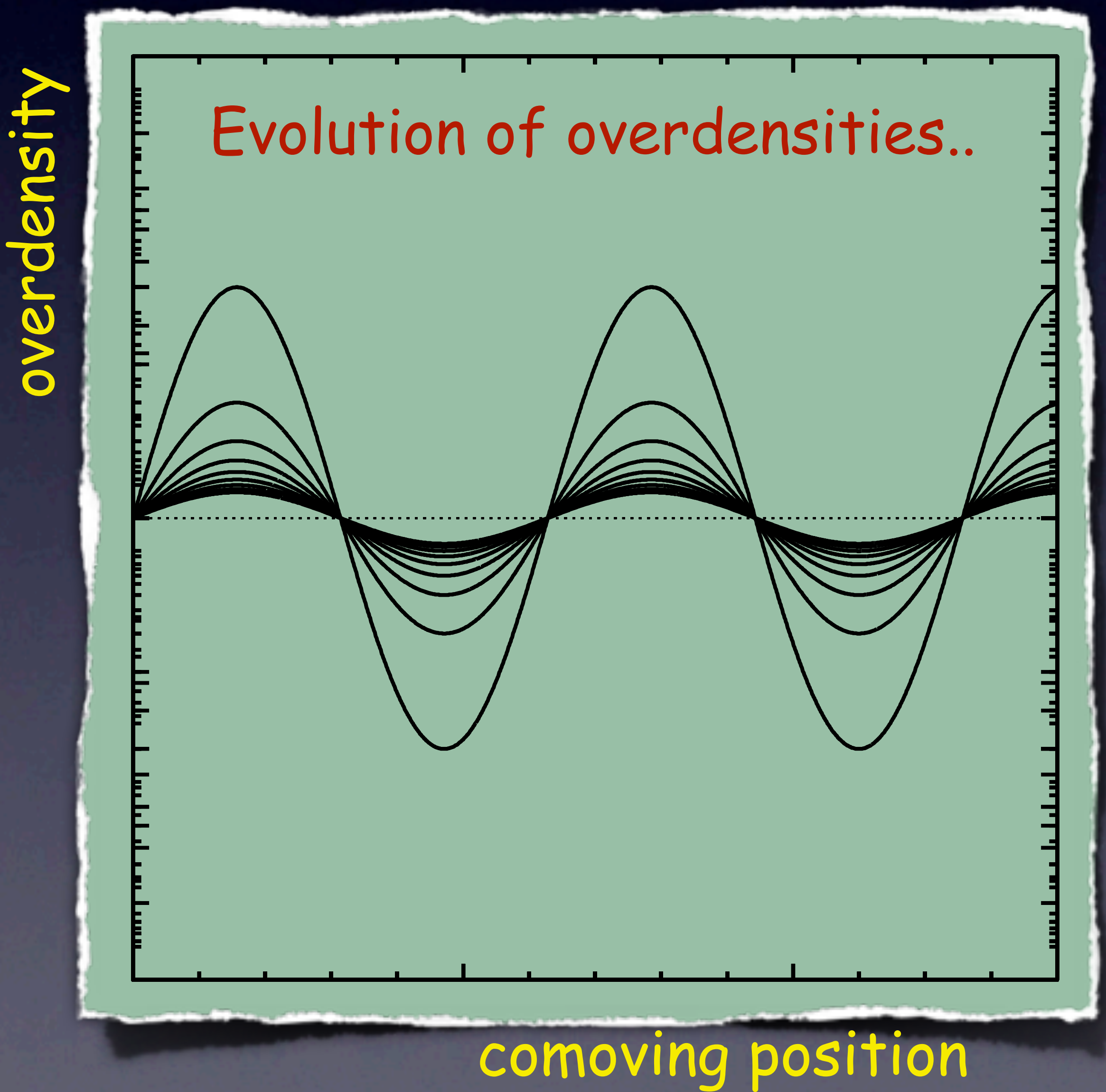

$D(t)$ is called the linear growth rate
It is cosmology dependent. Accurate fitting functions available in literature.

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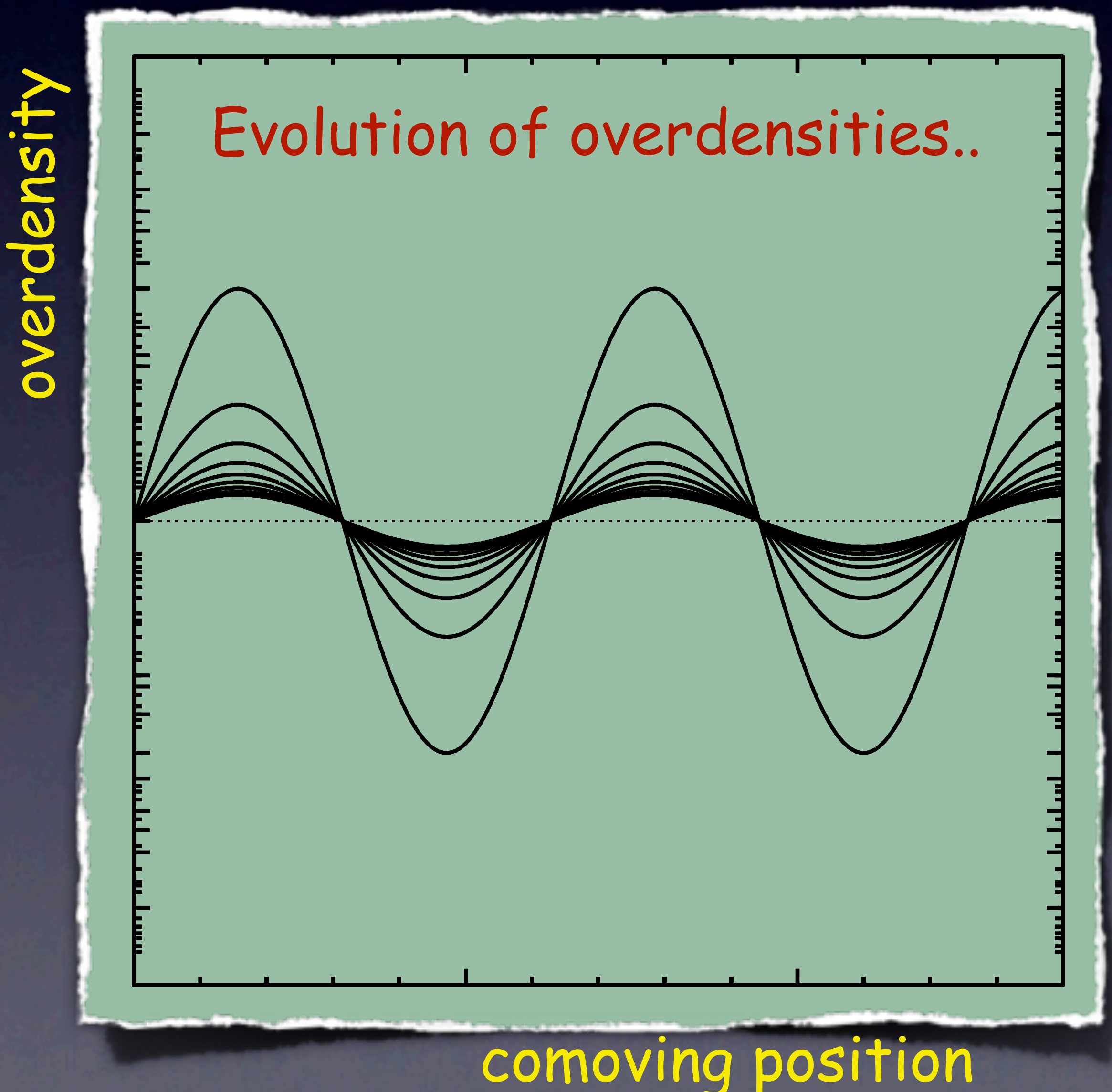

$D(t)$ is called the linear growth rate
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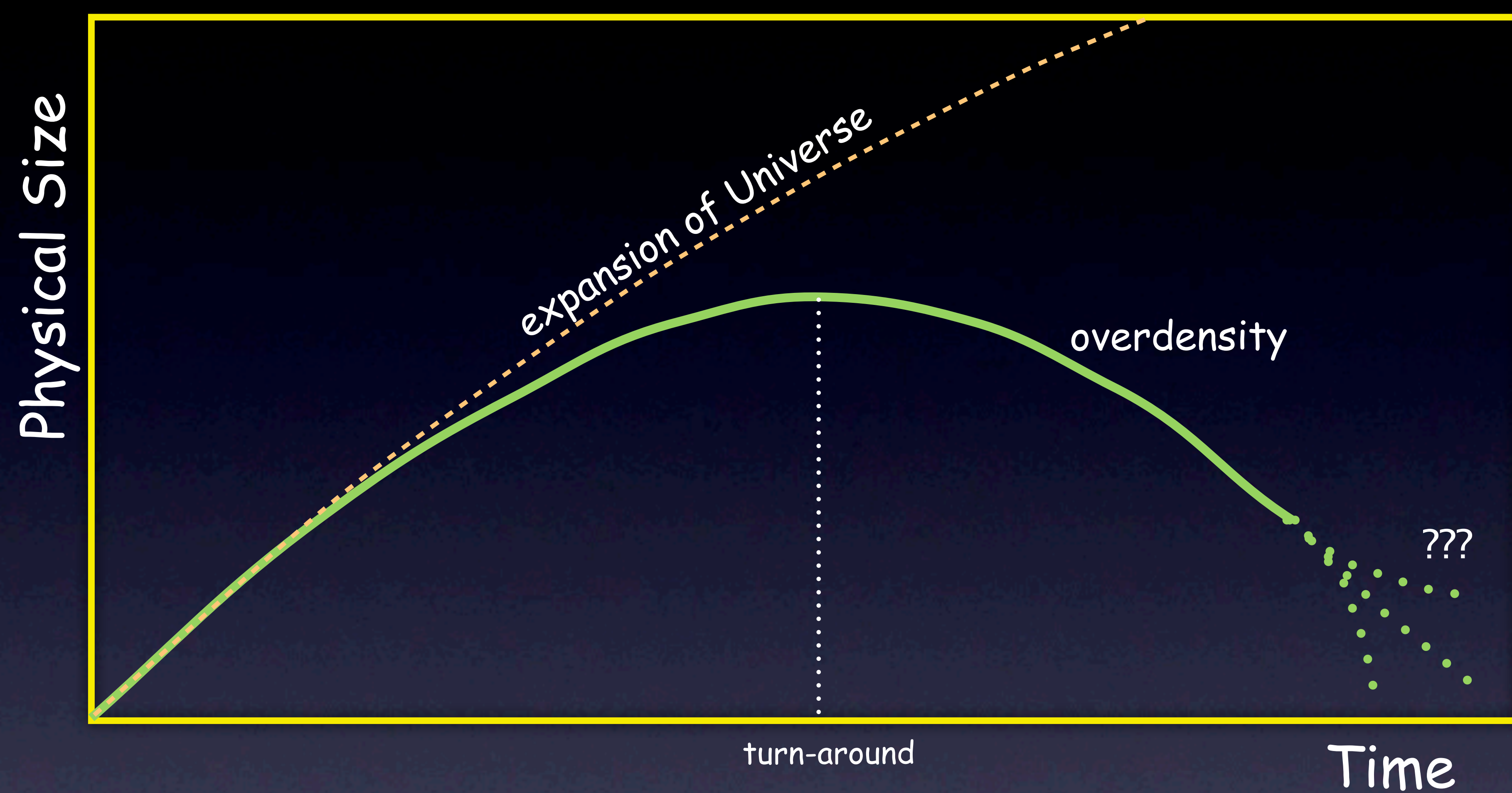
This process continues until overdensities are of order unity.
At that point, overdensities `turn around' (stop expanding) and start to collapse...
According to spherical collapse model, collapse happens when $\delta = \delta_c \simeq 1.686$

The Evolving Density Field



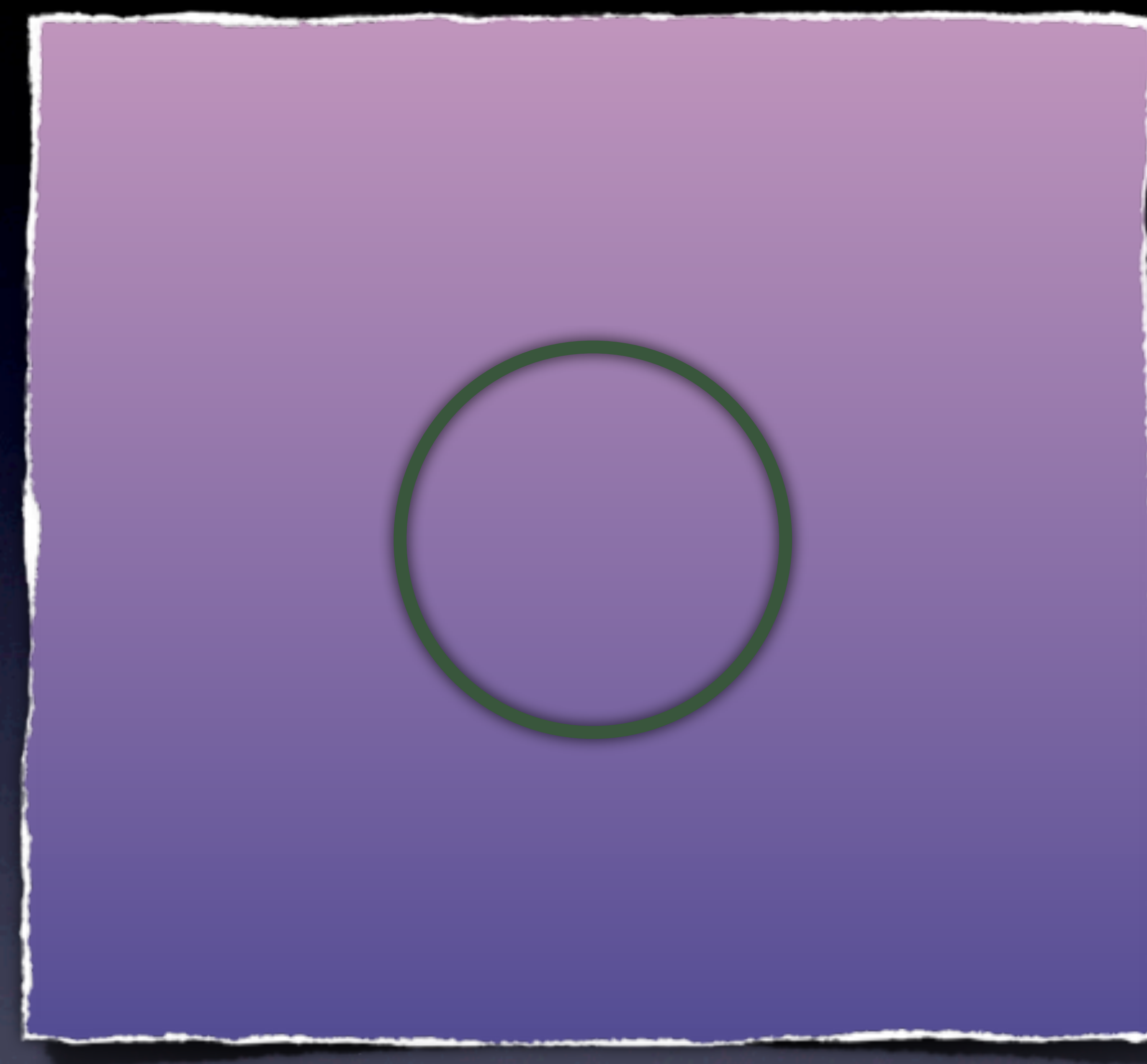
Evolution after turn-around depends on nature of matter

Dark Matter = collisionless ⟶ shell crossing

Baryonic Matter = collisional ⟶ shock heating

The Evolving Density Field

you can think of overdensity as consisting of many individual thin mass shells



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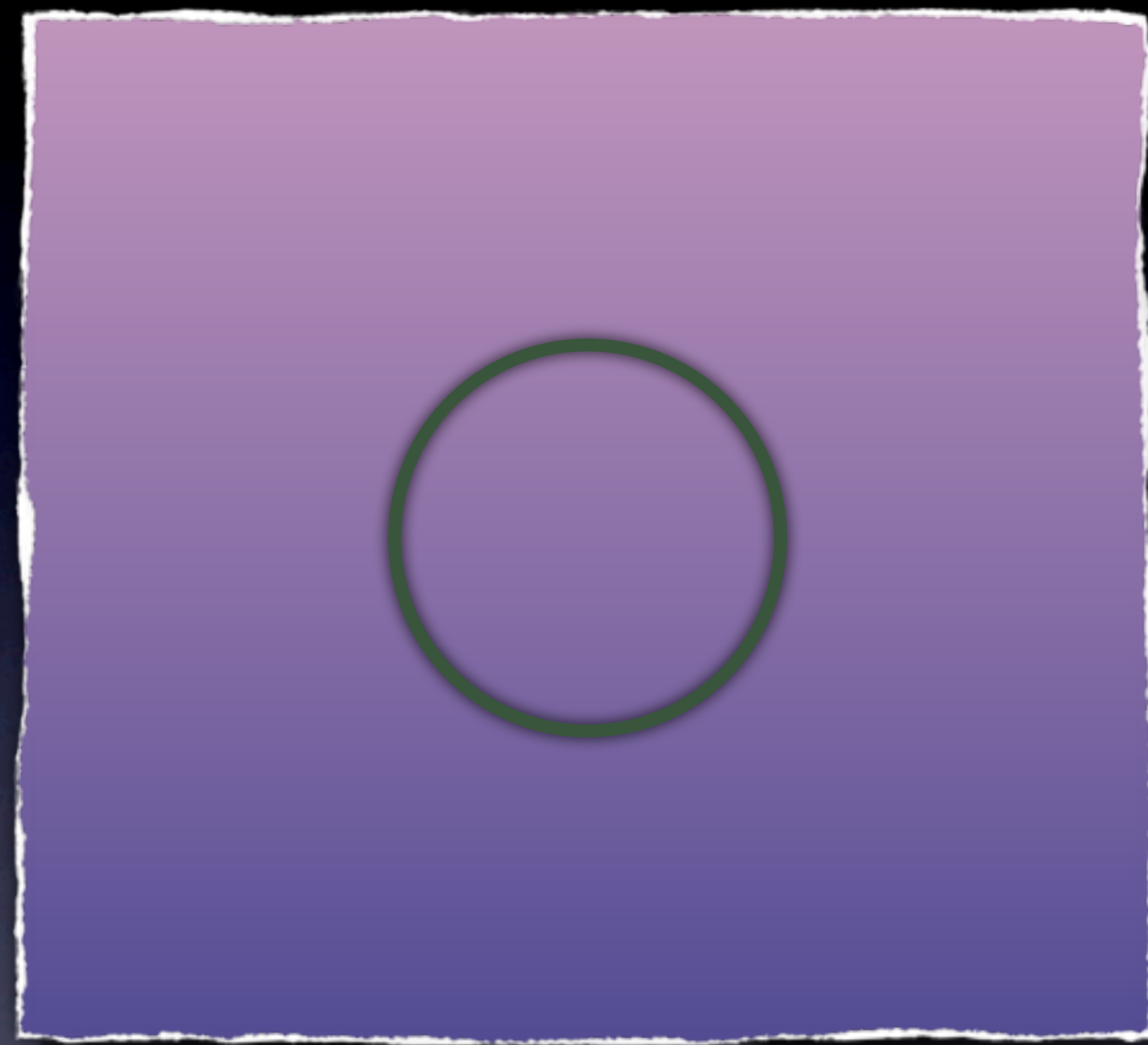

Because dark matter has no pressure,
shell crosses itself and starts to oscillate

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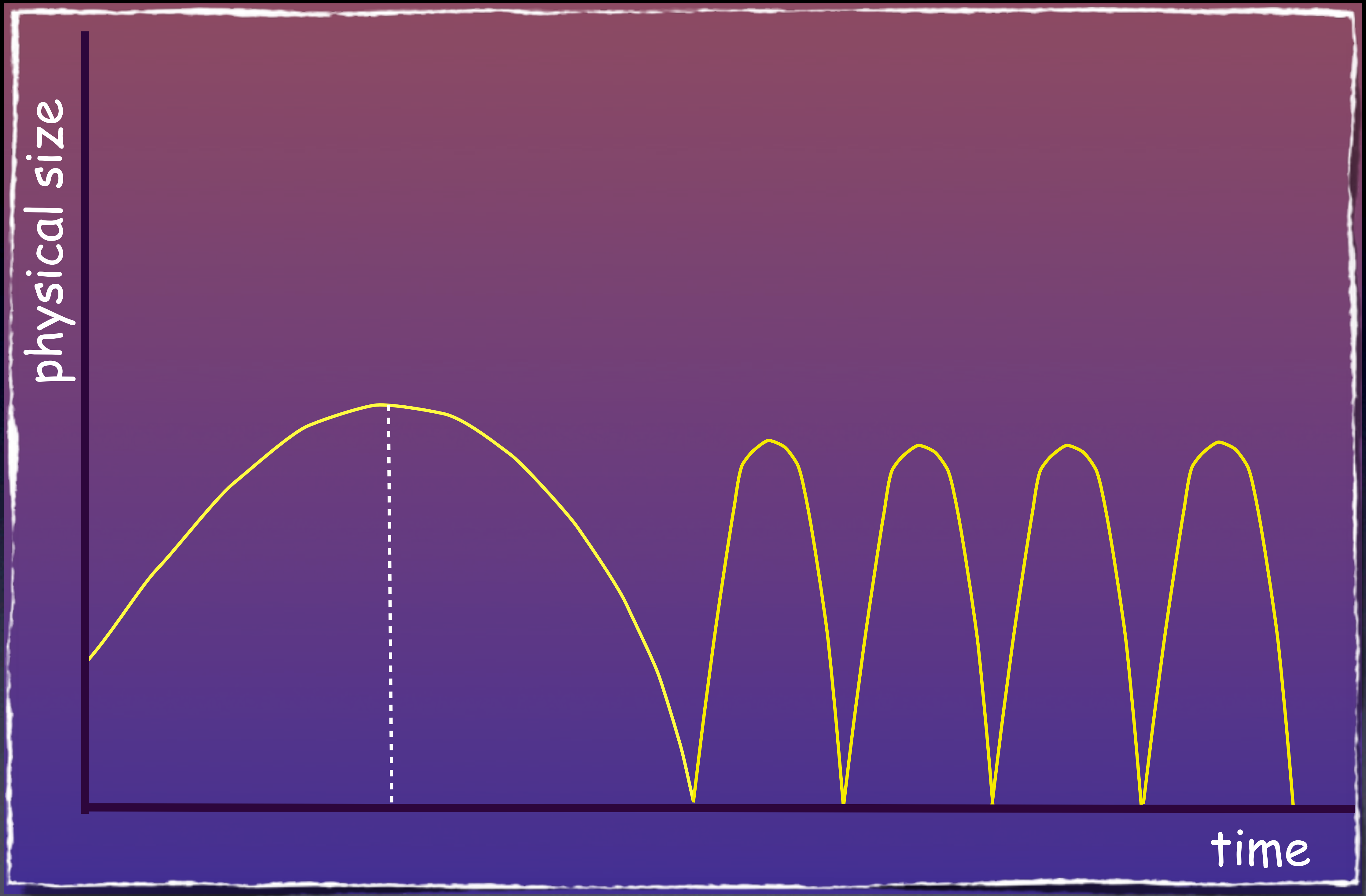

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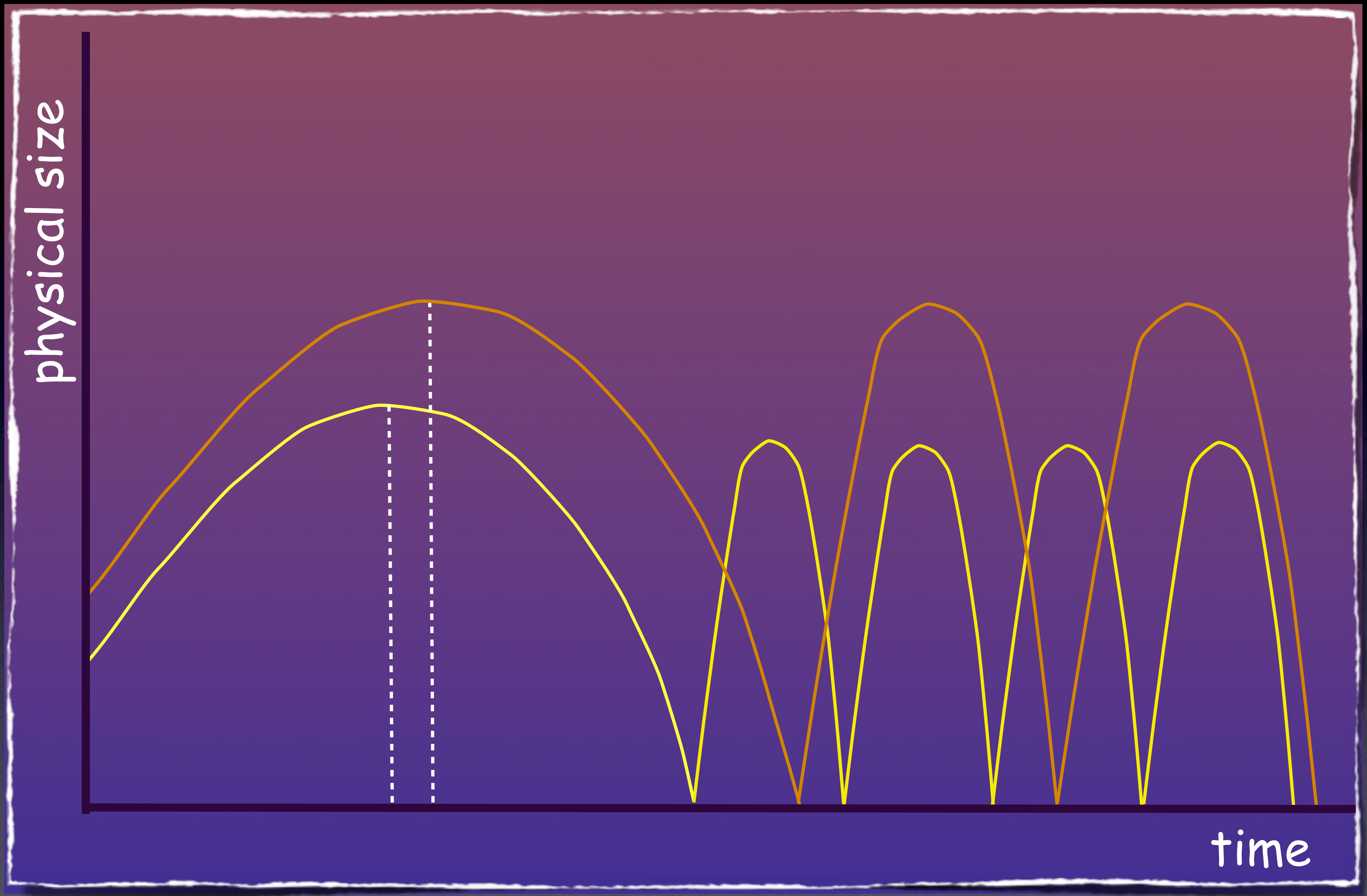

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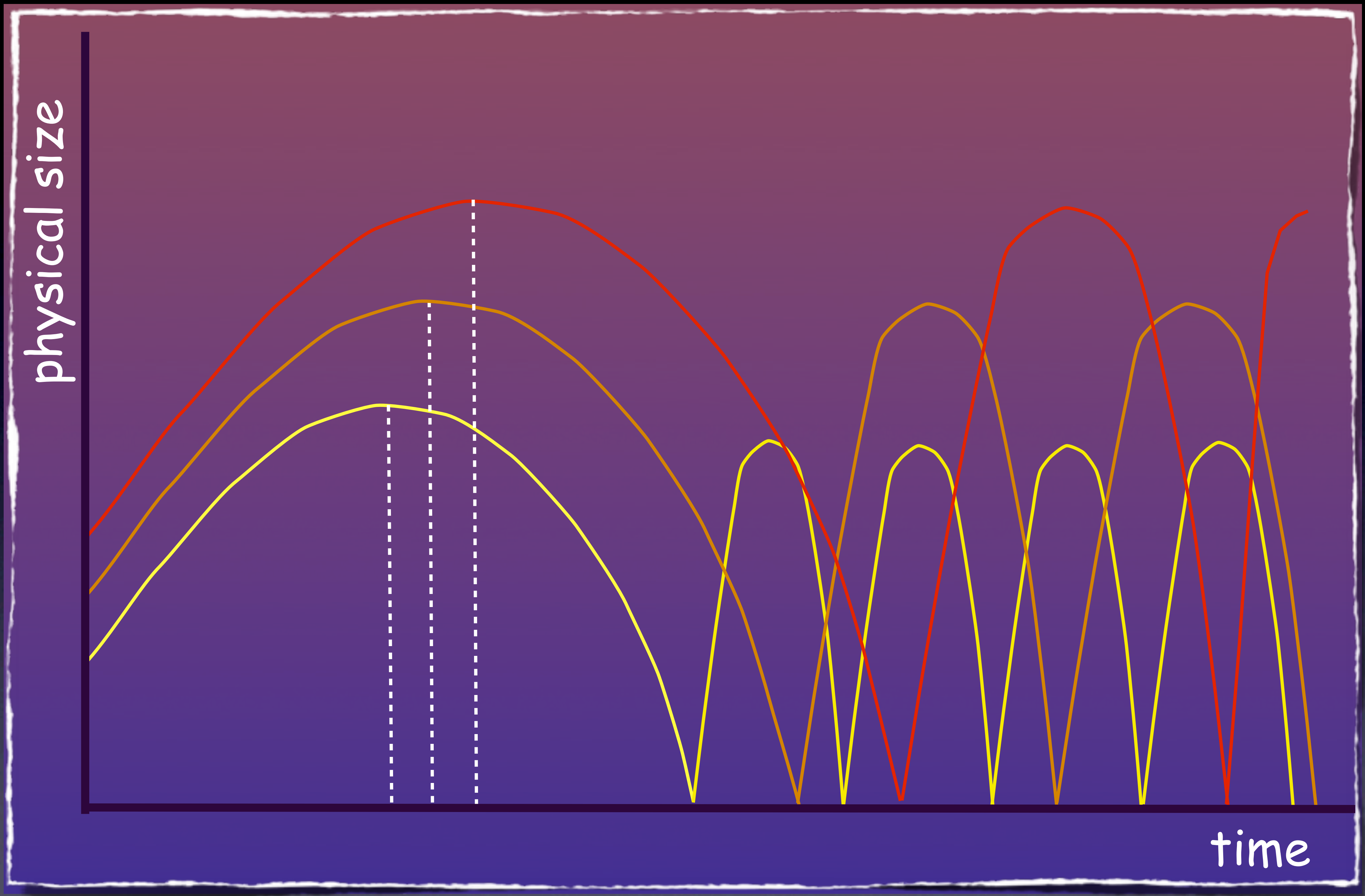

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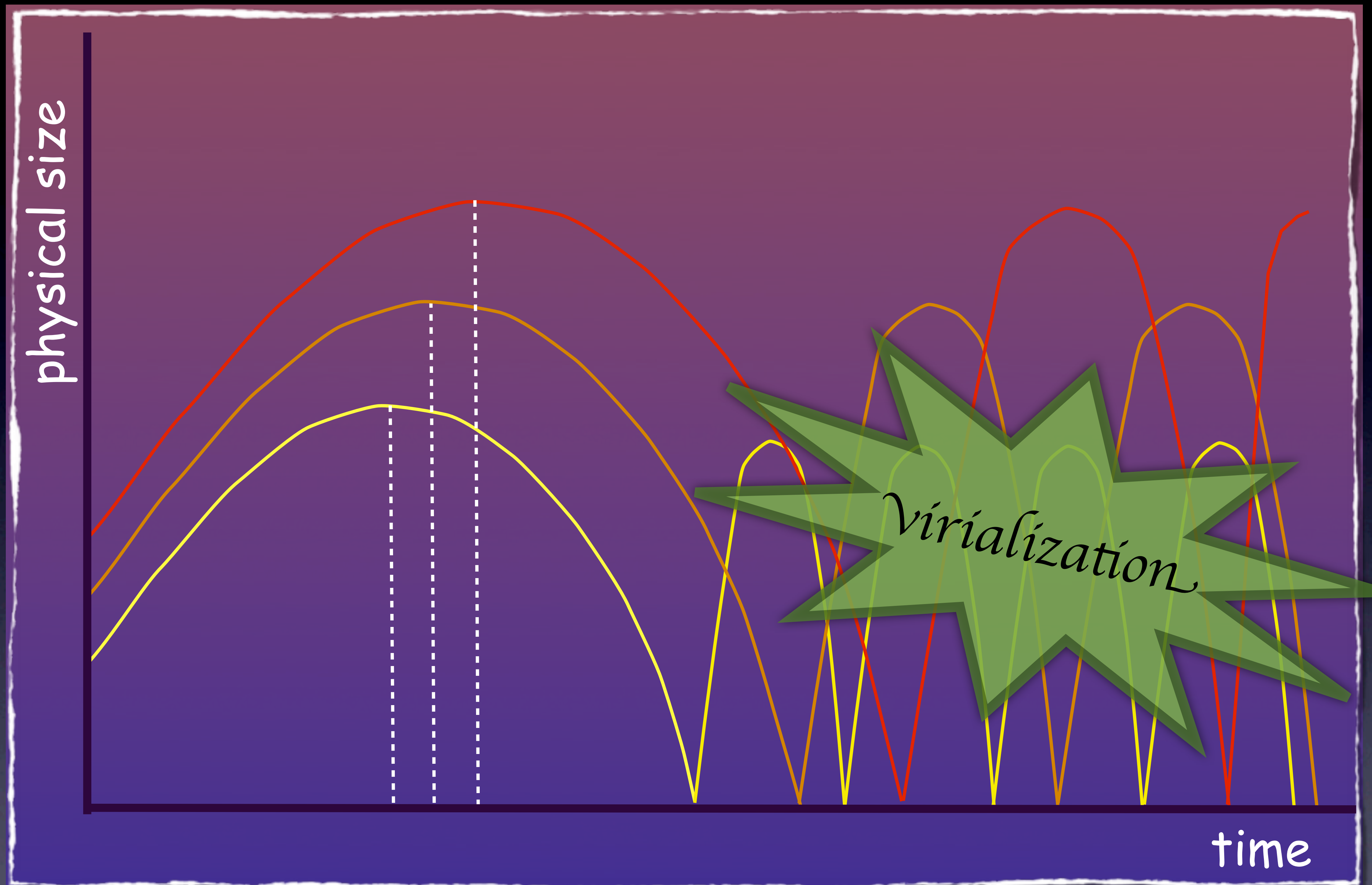

Individual oscillating shells interact gravitationally, exchanging energy (virializing), giving rise to a relaxed dark matter halo

The Evolving Density Field

Non-Linear Evolution:

During non-linear evolution modes start to couple to each other. One can no longer describe the evolution of the density field with a simple (linear) growth rate

Because of this mode-coupling, the density field looses its Gaussian properties, i.e., in the non-linear regime, we no longer have a Gaussian random field.

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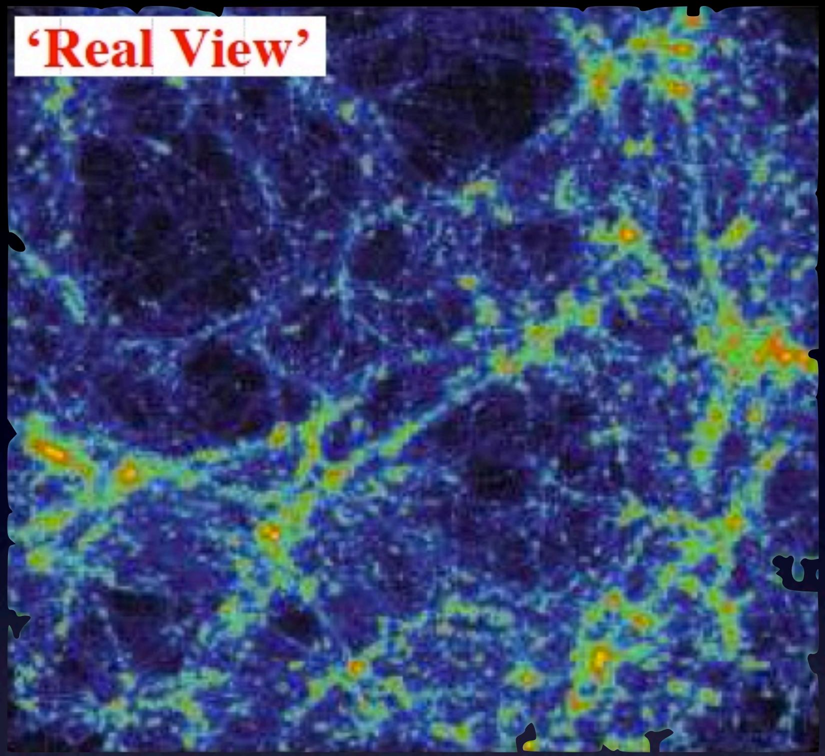
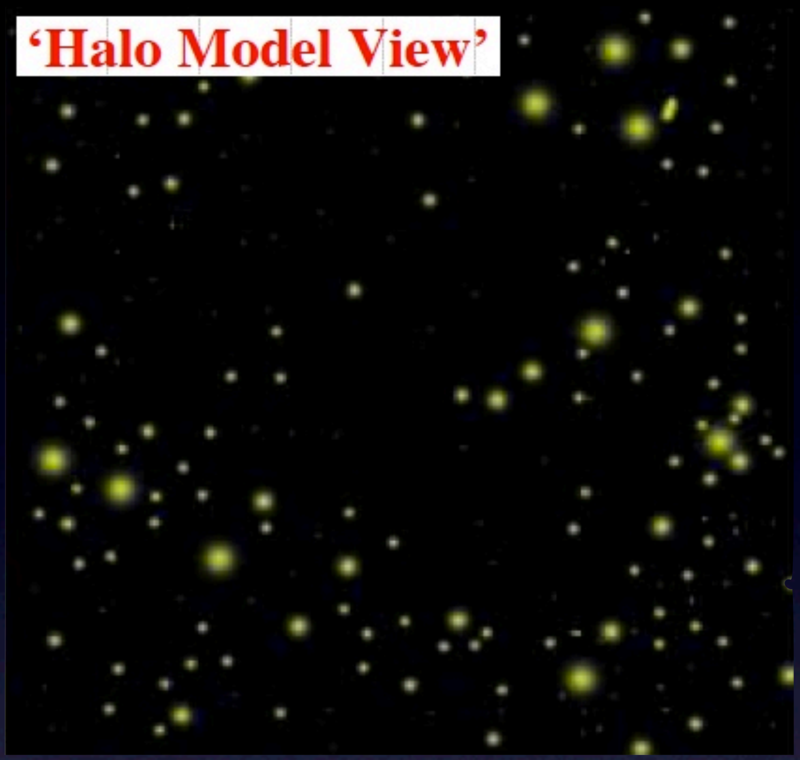




Halo model describes dark matter density distribution in terms of its halo building blocks, under ansatz that all dark matter is partitioned over haloes.

Throughout we assume that all dark matter haloes are spherical, and have a density distribution that only depends on halo mass:

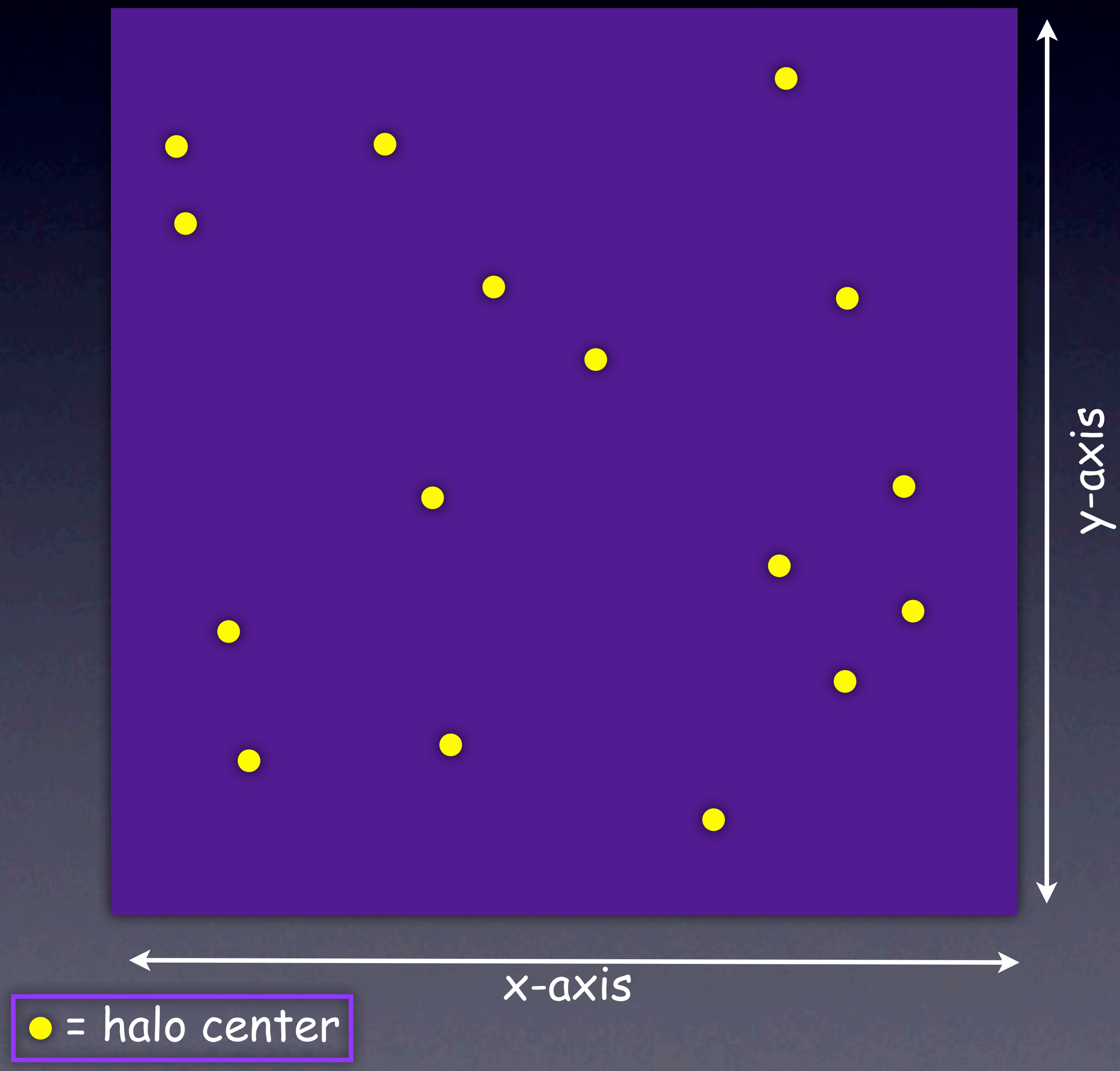
$$\rho(r|M) = M\, u(r|M)$$

Here $u(r|M)$ is the normalized density profile:

$$\int \mathrm{d}^3\vec{x}\, u(\vec{x}|M) = 1$$

The Halo Model: Density field = sum over halo building blocks

Imagine space divided into many small volumes, ΔV_i ,which are so small that none of them contain more than one halo center.



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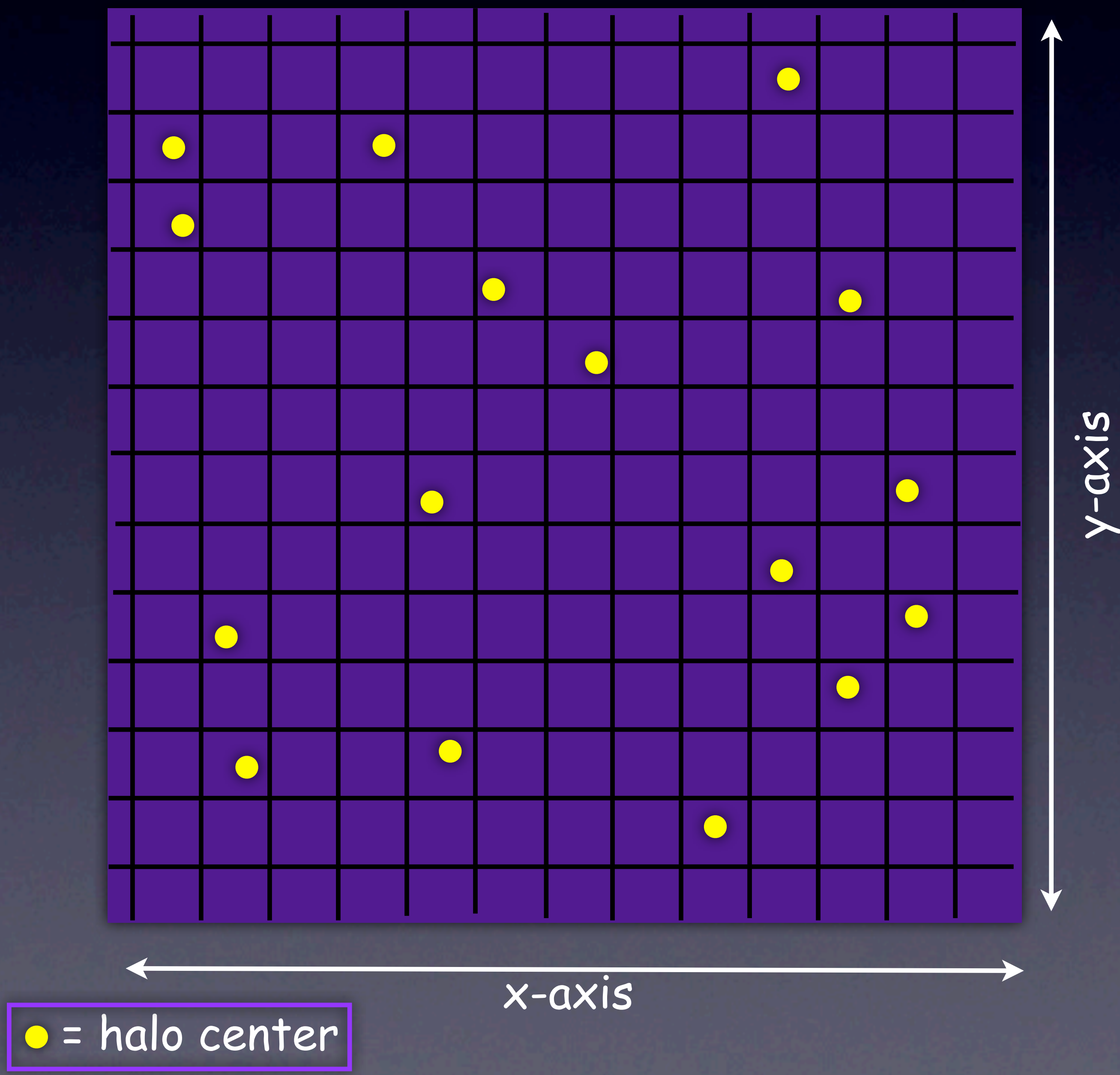

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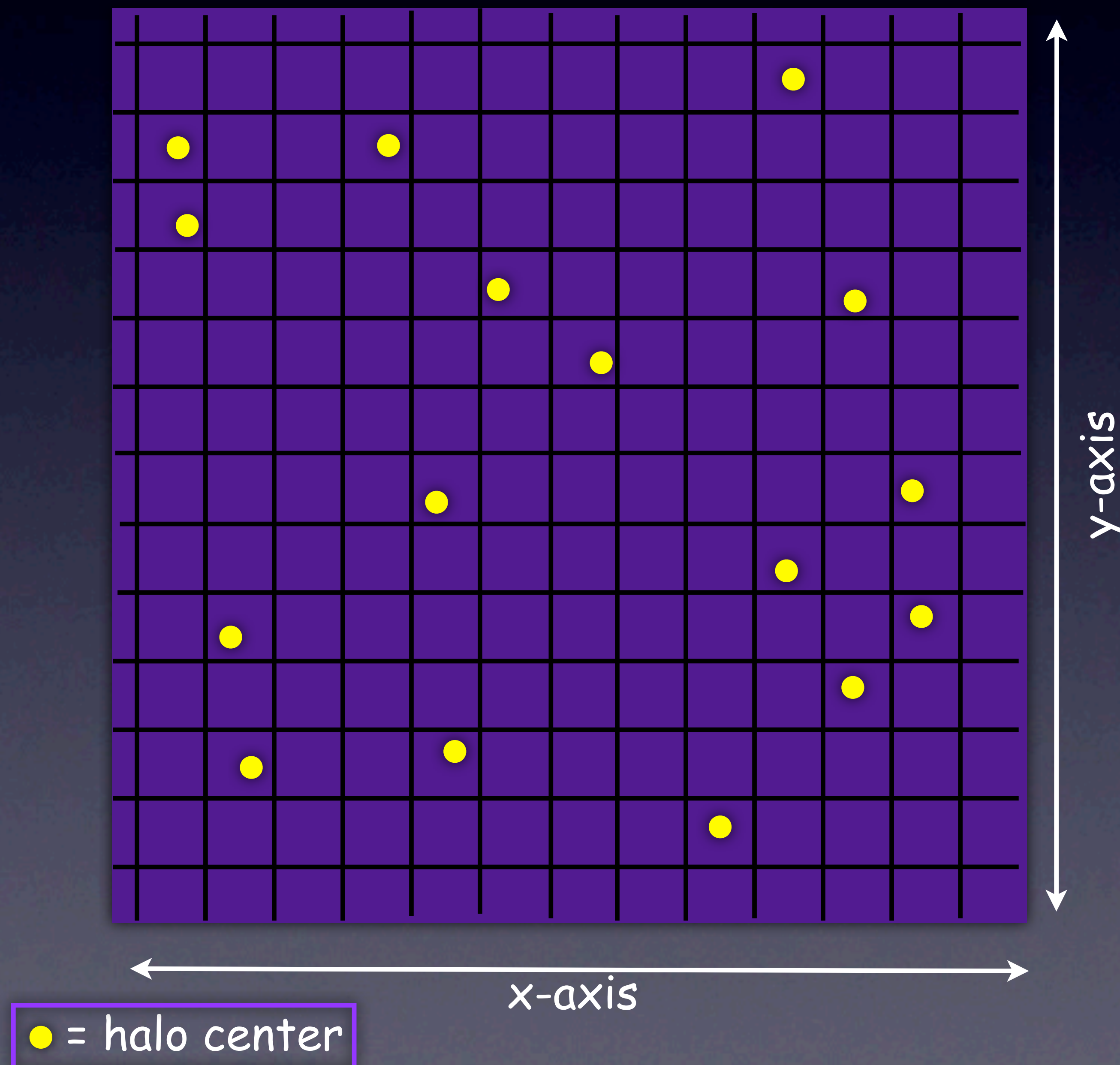

Let $\mathcal{N}_i$ be the occupation number of dark matter haloes in cell i

Then we have that $\mathcal{N}_i = 0, 1$
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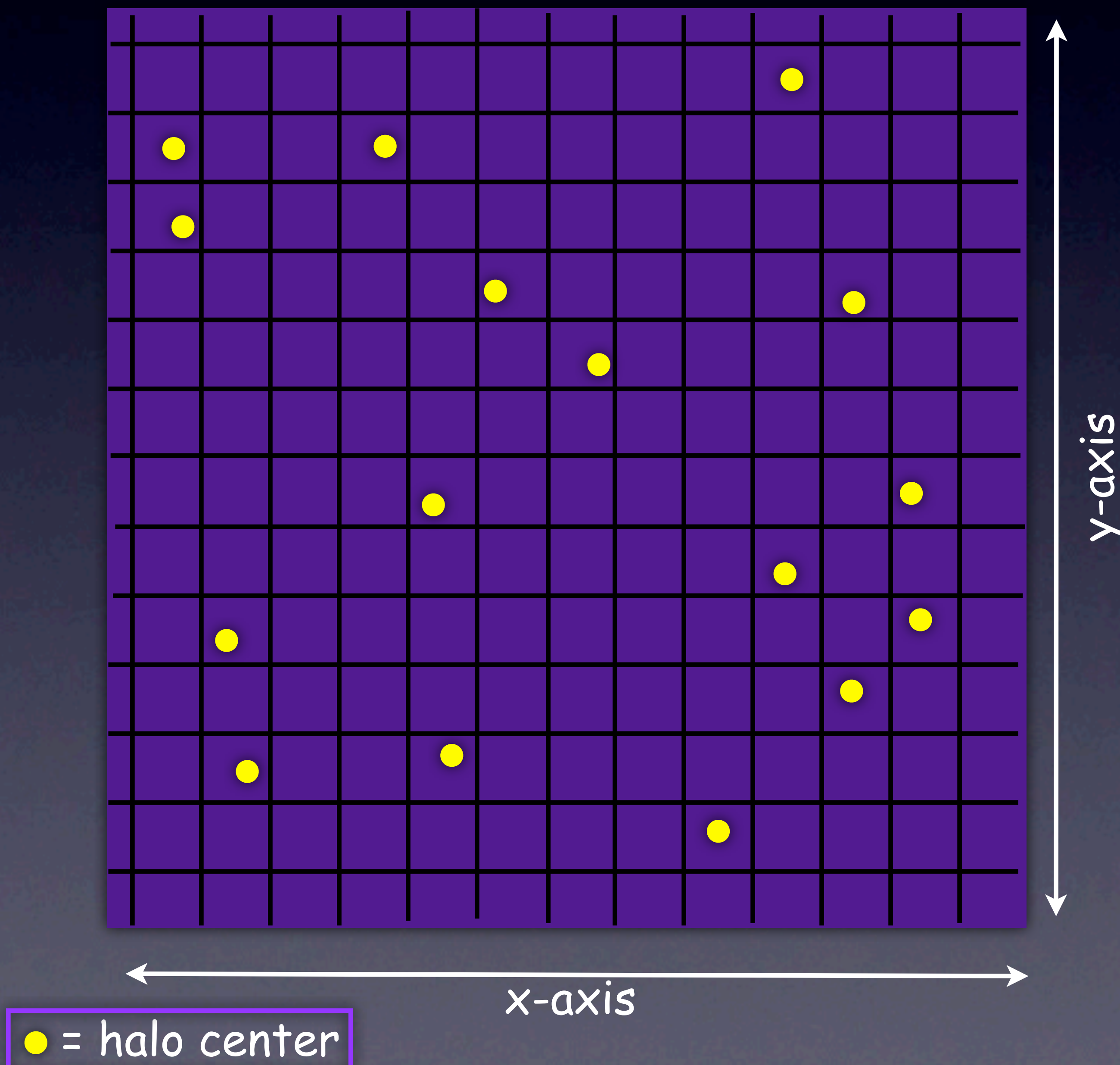

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This allows us to write the matter density field as a summation:

$$\rho(\vec{x}) = \sum_i \mathcal{N}_i \, M_i \, u(\vec{x} - \vec{x}_i | M_i)$$

The Halo Model

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$$= \sum_i \int \mathrm{d}M \, M \, n(M) \, \Delta V_i \, u(\vec{x} - \vec{x}_i | M)$$

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$$\overline{\rho} = \int \rho(\vec{x}) \, \mathrm{d}^3\vec{x} = \langle \rho(\vec{x}) \rangle = \langle \sum_i \mathcal{N}_i \, M_i \, u(\vec{x} - \vec{x}_i | M_i) \rangle$$

$$= \sum_i \langle \mathcal{N}_i \, M_i \, u(\vec{x} - \vec{x}_i | M_i) \rangle$$

$$= \sum_i \int \mathrm{d}M \, M \, n(M) \, \Delta V_i \, u(\vec{x} - \vec{x}_i | M)$$

halo mass function

The Halo Model

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$$= \int \mathrm{d}M \, M \, n(M)$$

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$$= \int \mathrm{d}M \, M \, n(M)$$

$$= \overline{\rho}$$

Q.E.D.

The Halo Model

$$\rho(\vec{x}) = \sum_i \mathcal{N}_i \, M_i \, u(\vec{x} - \vec{x}_i | M_i)$$

Now that we can write the density field in terms of the halo building blocks, let's focus on two-point statistics $\xi_{\rm mm}(r) \equiv \langle \delta(\vec{x})\, \delta(\vec{x}+\vec{r}) \rangle = \frac{1}{\bar{\rho}^2} \langle \rho(\vec{x})\, \rho(\vec{x}+\vec{r}) \rangle - 1$

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$$\vec{x}_2 = \vec{x}_1 + \vec{r}$$

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$\vec{x}_2 = \vec{x}_1 + \vec{r}$

$$= \sum_i \sum_j \langle \mathcal{N}_i \, \mathcal{N}_j \, M_i M_j \, u(\vec{x}_1 - \vec{x}_i | M_i) u(\vec{x}_2 - \vec{x}_j | M_j) \rangle$$

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We split this in two parts: the 1-halo term $(i = j)$, and the 2-halo term $(i \neq j)$

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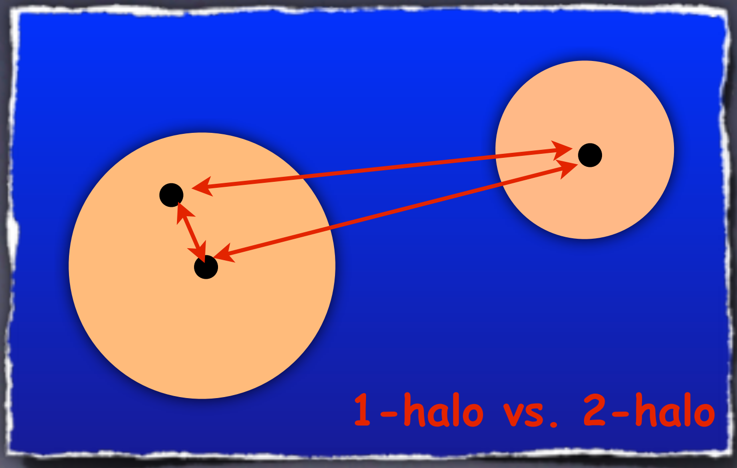

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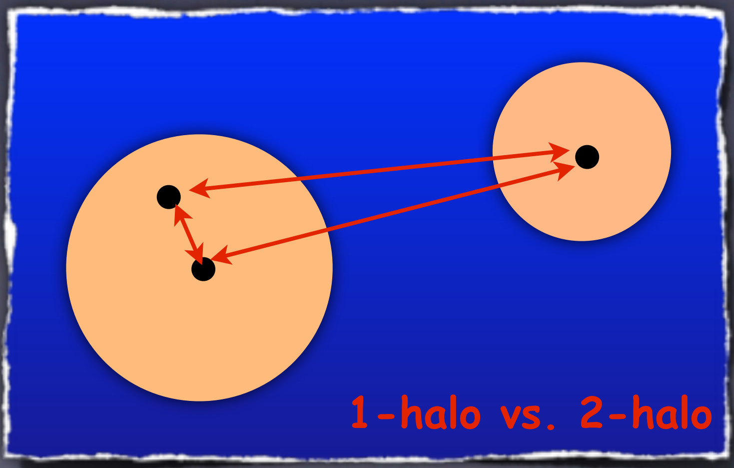

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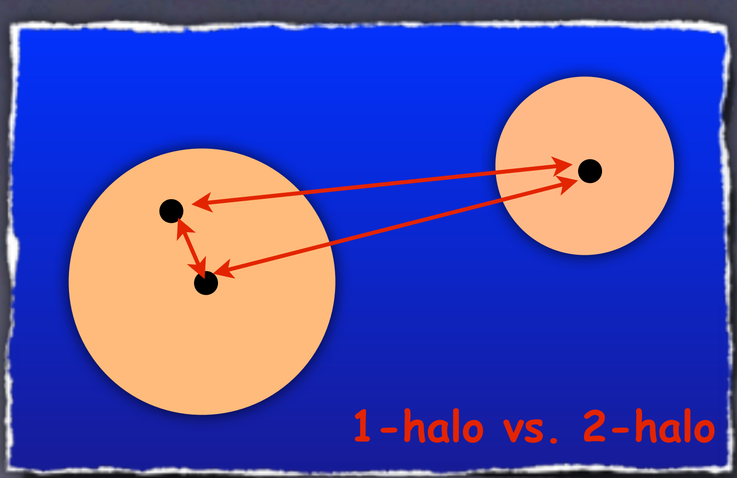

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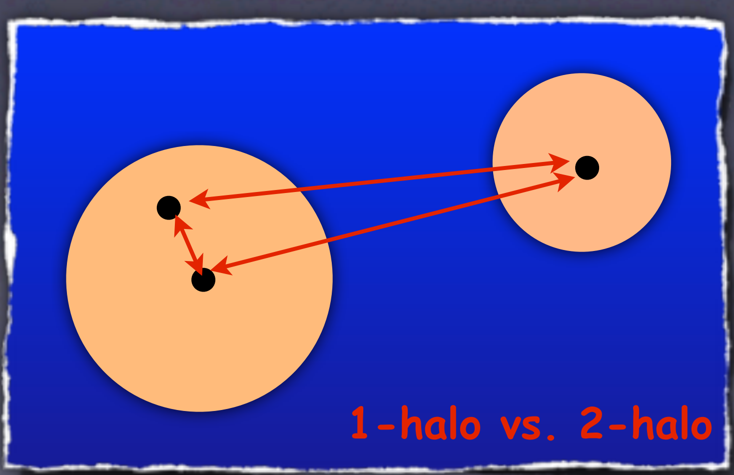

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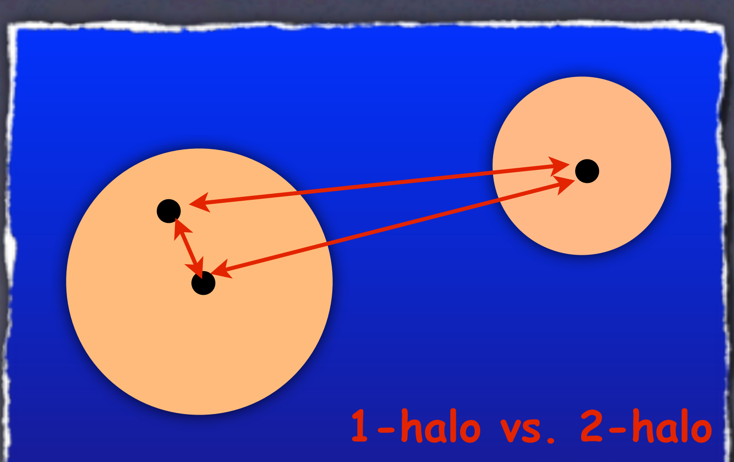

$$= \int {\rm d}M \, M^2 \, n(M) \int {\rm d}^3\vec{y} \, u(\vec{x}_1 - \vec{y} | M) u(\vec{x}_2 - \vec{y} | M)$$

convolution integral

The Halo Model

$$\rho(\vec{x}) = \sum_i \mathcal{N}_{\mathrm{h},i} M_i \, u(\vec{x} - \vec{x}_i | M_i)$$

For the 2-halo term we obtain:

$$\langle \rho(\vec{x}) \, \rho(\vec{x} + \vec{r}) \rangle_{2\mathrm{h}} = \sum_i \sum_{j \neq i} \langle \mathcal{N}_i \, \mathcal{N}_j \, M_i \, M_j \, u(\vec{x}_1 - \vec{x}_i | M_i) \, u(\vec{x}_2 - \vec{x}_j | M_j) \rangle$$

The Halo Model

$$\rho(\vec{x}) = \sum_i \mathcal{N}_{\mathrm{h},i} M_i \, u(\vec{x} - \vec{x}_i | M_i)$$

For the 2-halo term we obtain:

$$\langle \rho(\vec{x})\, \rho(\vec{x}+\vec{r}) \rangle_{2\mathrm{h}} = \sum_i \sum_{j \neq i} \langle \mathcal{N}_i \mathcal{N}_j \, M_i \, M_j \, u(\vec{x}_1 - \vec{x}_i | M_i)\, u(\vec{x}_2 - \vec{x}_j | M_j) \rangle$$

$$= \sum_i \sum_{j \neq i} \int \mathrm{d}M_1 \, M_1 \, n(M_1) \int \mathrm{d}M_2 \, M_2 \, n(M_2) \, \Delta V_i \, \Delta V_j \times$$

$$u(\vec{x}_1 - \vec{x}_i | M_1)\, u(\vec{x}_2 - \vec{x}_j | M_2)$$

The Halo Model

$$\rho(\vec{x}) = \sum_i \mathcal{N}_{\mathrm{h},i} M_i \, u(\vec{x} - \vec{x}_i | M_i)$$

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$$u(\vec{x}_1 - \vec{x}_i | M_1) \, u(\vec{x}_2 - \vec{x}_j | M_2) \;\; = \overline{\rho}^2$$

The Halo Model

$$\rho(\vec{x}) = \sum_i \mathcal{N}_{\mathrm{h},i} M_i \, u(\vec{x} - \vec{x}_i | M_i)$$

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$$\langle \rho(\vec{x}) \, \rho(\vec{x} + \vec{r}) \rangle_{2\mathrm{h}} = \sum_i \sum_{j \neq i} \langle \mathcal{N}_i \, \mathcal{N}_j \, M_i \, M_j \, u(\vec{x}_1 - \vec{x}_i | M_i) \, u(\vec{x}_2 - \vec{x}_j | M_j) \rangle$$

$$\stackrel{?}{=} \sum_i \sum_{j \neq i} \int \mathrm{d}M_1 \, M_1 \, n(M_1) \int \mathrm{d}M_2 \, M_2 \, n(M_2) \, \Delta V_i \, \Delta V_j \times$$

$$u(\vec{x}_1 - \vec{x}_i | M_1) \, u(\vec{x}_2 - \vec{x}_j | M_2) \;\; = \overline{\rho}^2$$

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$$\neq \sum_i \sum_{j \neq i} \int \mathrm{d}M_1 \, M_1 \, n(M_1) \int \mathrm{d}M_2 \, M_2 \, n(M_2) \, \Delta V_i \, \Delta V_j \, \times$$

$$u(\vec{x}_1 - \vec{x}_i | M_1) \, u(\vec{x}_2 - \vec{x}_j | M_2)$$

NO: dark matter haloes themselves are clustered, i.e., have a non-zero two point correlation function. This needs to be taken into account.

The Halo Model

$$\rho(\vec{x}) = \sum_i \mathcal{N}_{\mathrm{h},i} M_i \, u(\vec{x} - \vec{x}_i | M_i)$$

For the 2-halo term we obtain:

$$\langle \rho(\vec{x})\, \rho(\vec{x}+\vec{r}) \rangle_{2\mathrm{h}} = \sum_i \sum_{j \neq i} \langle \mathcal{N}_i \mathcal{N}_j \, M_i \, M_j \, u(\vec{x}_1 - \vec{x}_i | M_i) \, u(\vec{x}_2 - \vec{x}_j | M_j) \rangle$$

$$= \sum_i \sum_{j \neq i} \int \mathrm{d}M_1 \, M_1 \, n(M_1) \int \mathrm{d}M_2 \, M_2 \, n(M_2) \, \Delta V_i \, \Delta V_j \, \times$$

$$[1 + \xi_{\mathrm{hh}}(\vec{x}_i - \vec{x}_j | M_1, M_2)] \, u(\vec{x}_1 - \vec{x}_i | M_1) \, u(\vec{x}_2 - \vec{x}_j | M_2)$$

The halo-halo correlation function: dark matter haloes are biased tracers of the dark matter mass distribution!

$$\xi_{\mathrm{hh}}(r | M_1, M_2) = b(M_1) \, b(M_2) \, \xi_{\mathrm{mm}}^{\mathrm{lin}}(r)$$

Here $b(M)$ is called the halo bias function

Note: only valid on large (linear) scales!!!!

The Halo Model

$$\rho(\vec{x}) = \sum_i \mathcal{N}_{\mathrm{h},i} M_i \, u(\vec{x} - \vec{x}_i | M_i)$$

For the 2-halo term we obtain:

$$\langle \rho(\vec{x}) \, \rho(\vec{x} + \vec{r}) \rangle_{2\mathrm{h}} = \sum_i \sum_{j \neq i} \langle \mathcal{N}_i \mathcal{N}_j \, M_i \, M_j \, u(\vec{x}_1 - \vec{x}_i | M_i) \, u(\vec{x}_2 - \vec{x}_j | M_j) \rangle$$

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$$[1 + \xi_{\mathrm{hh}}(\vec{x}_i - \vec{x}_j | M_1, M_2)] \, u(\vec{x}_1 - \vec{x}_i | M_1) \, u(\vec{x}_2 - \vec{x}_j | M_2)$$

$$= \overline{\rho}^2 + \int \mathrm{d}M_1 \, M_1 \, n(M_1) \int \mathrm{d}M_2 \, M_2 \, n(M_2) \times$$
$$\int \mathrm{d}^3\vec{y}_1 \int \mathrm{d}^3\vec{y}_2 \, u(\vec{x}_1 - \vec{y}_1 | M_1) \, u(\vec{x}_2 - \vec{y}_2 | M_2) \, \xi_{\mathrm{hh}}(\vec{y}_1 - \vec{y}_2 | M_1, M_2)$$

The Halo Model

$$\rho(\vec{x}) = \sum_i \mathcal{N}_{\mathrm{h},i} M_i \, u(\vec{x} - \vec{x}_i | M_i)$$

For the 2-halo term we obtain:

$$\langle \rho(\vec{x})\, \rho(\vec{x}+\vec{r}) \rangle_{2\mathrm{h}} = \sum_i \sum_{j\neq i} \langle \mathcal{N}_i \mathcal{N}_j \, M_i \, M_j \, u(\vec{x}_1 - \vec{x}_i | M_i)\, u(\vec{x}_2 - \vec{x}_j | M_j) \rangle$$

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$$[1 + \xi_{\mathrm{hh}}(\vec{x}_i - \vec{x}_j | M_1, M_2)]\, u(\vec{x}_1 - \vec{x}_i | M_1)\, u(\vec{x}_2 - \vec{x}_j | M_2)$$

$$= \overline{\rho}^2 + \int \mathrm{d}M_1 \, M_1 \, n(M_1) \int \mathrm{d}M_2 \, M_2 \, n(M_2) \; \times$$

$$\int \mathrm{d}^3\vec{y}_1 \int \mathrm{d}^3\vec{y}_2 \, u(\vec{x}_1 - \vec{y}_1 | M_1)\, u(\vec{x}_2 - \vec{y}_2 | M_2)\, \xi_{\mathrm{hh}}(\vec{y}_1 - \vec{y}_2 | M_1, M_2)$$

$$= \overline{\rho}^2 + \int \mathrm{d}M_1 \, M_1 \, b(M_1)\, n(M_1) \int \mathrm{d}M_2 \, M_2 \, b(M_2)\, n(M_2) \times$$

$$\int \mathrm{d}^3\vec{y}_1 \int \mathrm{d}^3\vec{y}_2 u(\vec{x}_1 - \vec{y}_1 | M_1)\, u(\vec{x}_2 - \vec{y}_2 | M_2)\, \xi^{\mathrm{lin}}_{\mathrm{mm}}(\vec{y}_1 - \vec{y}_2)$$

The Halo Model

$$\rho(\vec{x}) = \sum_i \mathcal{N}_{\mathrm{h},i} M_i \, u(\vec{x} - \vec{x}_i | M_i)$$

For the 2-halo term we obtain:

$$\langle \rho(\vec{x})\,\rho(\vec{x}+\vec{r})\rangle_{\mathrm{2h}} = \sum_i \sum_{j\neq i} \langle \mathcal{N}_i \mathcal{N}_j \, M_i \, M_j \, u(\vec{x}_1 - \vec{x}_i|M_i)\, u(\vec{x}_2 - \vec{x}_j|M_j)\rangle$$

$$= \sum_i \sum_{j\neq i} \int \mathrm{d}M_1\, M_1\, n(M_1) \int \mathrm{d}M_2\, M_2\, n(M_2)\, \Delta V_i\, \Delta V_j \; \times$$
$$[1 + \xi_{\mathrm{hh}}(\vec{x}_i - \vec{x}_j|M_1, M_2)]\, u(\vec{x}_1 - \vec{x}_i|M_1)\, u(\vec{x}_2 - \vec{x}_j|M_2)$$

$$= \overline{\rho}^2 + \int \mathrm{d}M_1\, M_1\, n(M_1) \int \mathrm{d}M_2\, M_2\, n(M_2) \; \times$$
$$\int \mathrm{d}^3\vec{y}_1 \int \mathrm{d}^3\vec{y}_2\, u(\vec{x}_1 - \vec{y}_1|M_1)\, u(\vec{x}_2 - \vec{y}_2|M_2)\, \xi_{\mathrm{hh}}(\vec{y}_1 - \vec{y}_2|M_1, M_2)$$

$$= \overline{\rho}^2 + \int \mathrm{d}M_1\, M_1\, b(M_1)\, n(M_1) \int \mathrm{d}M_2\, M_2\, b(M_2)\, n(M_2) \times$$
$$\int \mathrm{d}^3\vec{y}_1 \int \mathrm{d}^3\vec{y}_2 u(\vec{x}_1 - \vec{y}_1|M_1)\, u(\vec{x}_2 - \vec{y}_2|M_2)\, \xi^{\mathrm{lin}}_{\mathrm{mm}}(\vec{y}_1 - \vec{y}_2)$$

convolution integral

The Halo Model: Summary (part I)

$$\xi(r) = \xi^{1h}(r) + \xi^{2h}(r)$$

$$\xi^{1h}(r) = \frac{1}{\bar{\rho}^2} \int \mathrm{d}M\, M^2\, n(M) \int \mathrm{d}^3\vec{y}\, u(\vec{x}-\vec{y}|M)\, u(\vec{x}+\vec{r}-\vec{y}|M)$$

$$\xi^{2h}(r) = \frac{1}{\bar{\rho}^2} \int \mathrm{d}M_1\, M_1\, b(M_1)\, n(M_1) \int \mathrm{d}M_2\, M_2\, b(M_2)\, n(M_2) \times \int \mathrm{d}^3\vec{y}_1 \int \mathrm{d}^3\vec{y}_2\, u(\vec{x}-\vec{y}_1|M_1)\, u(\vec{x}+\vec{r}-\vec{y}_2|M_2)\, \xi^{\rm lin}_{\rm mm}(\vec{y}_1-\vec{y}_2)$$

Halo Model Ingredients:

- the halo density profiles $\rho(r|M) = M\,u(r|M)$
- the halo mass function $n(M)$
- the halo bias function $b(M)$
- the linear correlation function of matter $\xi^{\rm lin}_{\rm mm}(r)$

The Halo Model: Summary (part I)

$$\xi(r) = \xi^{1\mathrm{h}}(r) + \xi^{2h}(r)$$

$$\xi^{1\mathrm{h}}(r) = \frac{1}{\bar{\rho}^2} \int \mathrm{d}M\, M^2\, n(M) \int \mathrm{d}^3\vec{y}\, u(\vec{x}-\vec{y}|M) u(\vec{x}+\vec{r}-\vec{y}|M)$$

$$\xi^{2\mathrm{h}}(r) = \frac{1}{\bar{\rho}^2} \int \mathrm{d}M_1\, M_1\, b(M_1)\, n(M_1) \int \mathrm{d}M_2\, M_2\, b(M_2)\, n(M_2) \times \int \mathrm{d}^3\vec{y}_1 \int \mathrm{d}^3\vec{y}_2 u(\vec{x}-\vec{y}_1|M_1)\, u(\vec{x}+\vec{r}-\vec{y}_2|M_2)\, \xi^{\mathrm{lin}}_{\mathrm{mm}}(\vec{y}_1-\vec{y}_2)$$

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- the halo bias function $b(M)$
- the linear correlation function of matter $\xi^{\mathrm{lin}}_{\mathrm{mm}}(r)$

All of these are (reasonably) well calibrated against numerical simulations.

The Halo Model in Fourier Space

$$P(k) = P^{1\mathrm{h}}(k) + P^{2\mathrm{h}}(k)$$

$$P^{1\mathrm{h}}(k) = \frac{1}{\bar{\rho}^2} \int \mathrm{d}M\, M^2\, n(M)\, |\tilde{u}(k|M)|^2$$

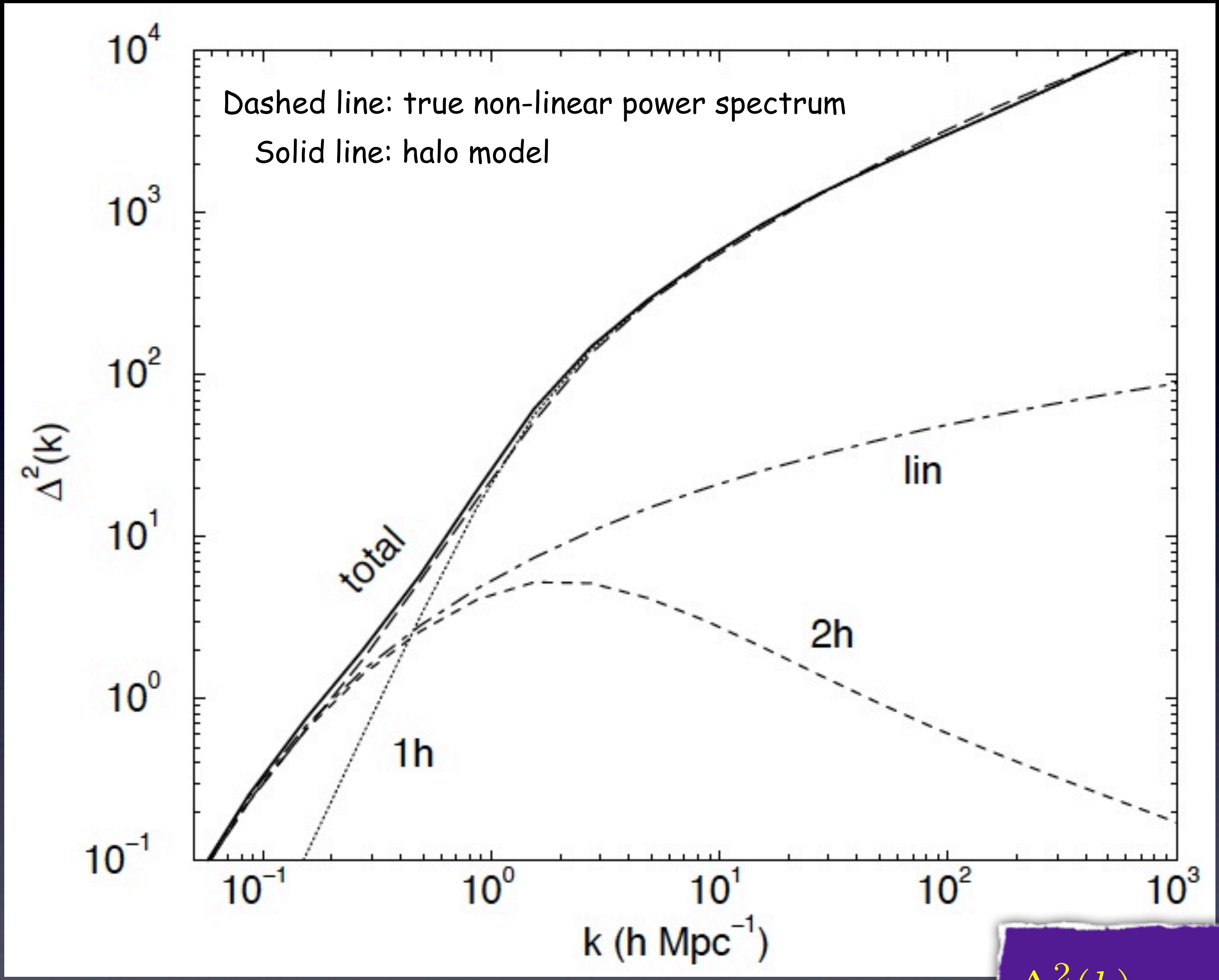
$$P^{2\mathrm{h}}(k) = P^{\mathrm{lin}}(k) \left[\frac{1}{\bar{\rho}} \int \mathrm{d}M\, M\, b(M)\, n(M)\, \tilde{u}(k|M)\right]^2$$

$$P^{\mathrm{lin}}(k) = \int \xi_{\mathrm{mm}}^{\mathrm{lin}}(\vec{x}) e^{-i\vec{k}\cdot\vec{x}} \mathrm{d}^3\vec{x} = 4\pi \int_0^\infty \xi_{\mathrm{mm}}^{\mathrm{lin}}(r) \frac{\sin kr}{kr}\, r^2\, \mathrm{d}r$$

$$\tilde{u}(\vec{k}|M) = \int u(\vec{x}|M) e^{-i\vec{k}\cdot\vec{x}}\, \mathrm{d}^3\vec{x} = 4\pi \int_0^\infty u(r|M) \frac{\sin kr}{kr}\, r^2\, \mathrm{d}r$$

Since convolutions in real-space become multiplications in Fourier space, the halo model expression for the power spectrum is much easier. Therefore, in practice, one computes $P(k)$ and then uses Fourier transformation to obtain two-point correlation function $\xi(r)$

The Halo Model in Fourier Space



from: Cooray & Sheth (2002)

$$\Delta^2(k) = \frac{1}{2\pi^2} k^3 P(k)$$

Dimensionless power spectrum

The Halo Model: complications

$$P^{1h}(k) = \frac{1}{\bar{\rho}^2} \int \mathrm{d}M \, M^2 \, n(M) \, |\tilde{u}(k|M)|^2$$

$$P^{2h}(k) = P^{\mathrm{lin}}(k) \left[\frac{1}{\bar{\rho}} \int \mathrm{d}M \, M \, b(M) \, n(M) \, \tilde{u}(k|M) \right]^2$$

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However, this is ONLY true under the simplifying assumption that

$$\xi_{\rm hh}(r|M_1, M_2) = b(M_1) \, b(M_2) \, \xi^{\rm lin}_{\rm mm}(r)$$

In reality, on small scales, in the (quasi)-linear regime, this description of the halo-halo correlation function becomes inadequate for two reasons:

The Halo Model: complications

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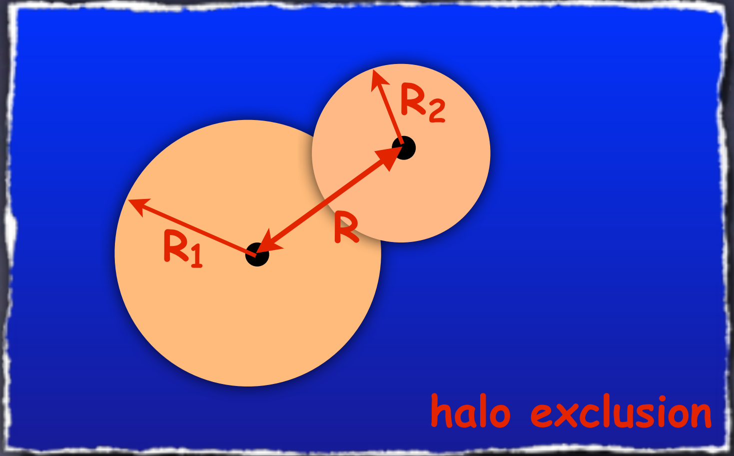
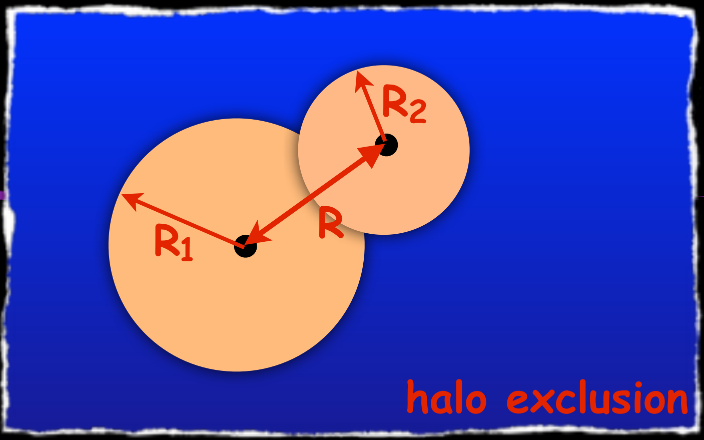
In reality, on small scales, in the (quasi)-linear regime, this description of the halo-halo correlation function becomes inadequate for two reasons:

- $\xi^{\rm lin}_{\rm mm}(r)$ is no longer adequate
- halo exclusion



The Halo Model: complications



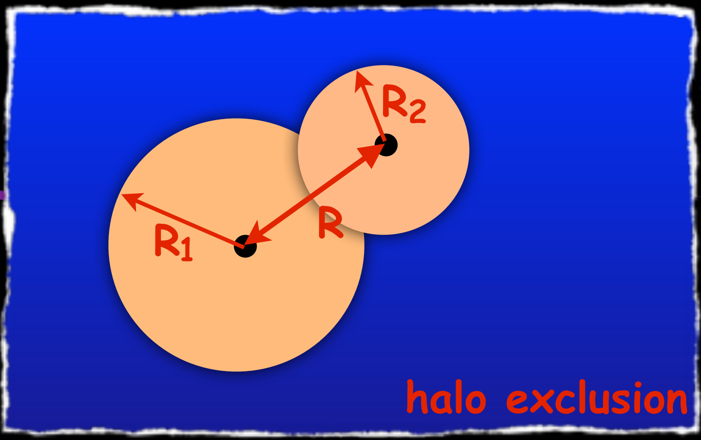
Because of these complications, the 2-halo term needs to be modified to the following, much more complicated form

$$P^{2h}(k) = \frac{1}{\bar{\rho}^2} \int \mathrm{d}M_1\, M_1\, n(M_1)\, \tilde{u}(k|M_1) \int \mathrm{d}M_2\, M_2\, n(M_2) \tilde{u}(k|M_2)\, Q(k|M_1, M_2)$$

Here $Q(k|M_1, M_2) = 4\pi \int_{r_{\rm min}}^{\infty} [1 + \xi_{\rm hh}(r|M_1, M_2)]\, \frac{\sin kr}{kr} r^2 \mathrm{d}r$

describes the fact that dark matter haloes are clustered, as described by the halo-halo correlation function, $\xi_{\rm hh}(r|M_1, M_2)$, and takes halo exclusion into account by having $r_{\rm min} = R_1 + R_2$

The Halo Model: complications



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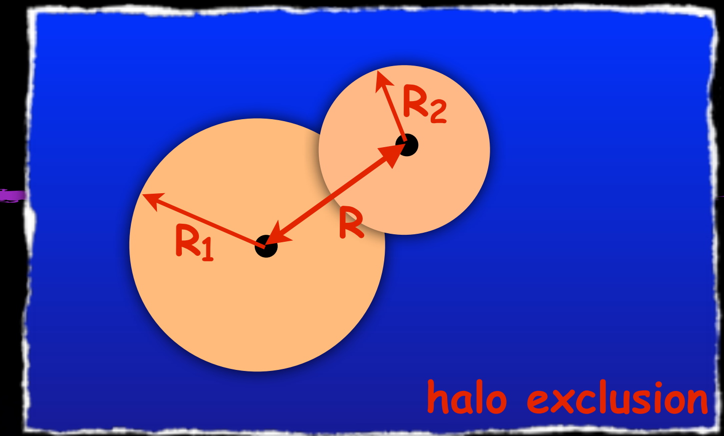
Also, the halo-halo correlation function itself has to be modified to:

$$\xi_{\rm hh}(r|M_1, M_2) = b(M_1)\, b(M_2)\, \zeta(r)\, \xi_{\rm mm}(r)$$

Here $\zeta(r)$ is called the radial bias function, and which is a higher-order bias correction. No analytical model for $\zeta(r)$ exists; use empirical calibration...

The Halo Model: complications



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Worst of all; $\xi_{\rm mm}(r)$ is the non-linear matter-matter correlation function

The Halo Model: Summary (part II)

The simple, `linear', halo model

$$P^{1\mathrm{h}}(k) = \frac{1}{\bar{\rho}^2} \int \mathrm{d}M\, M^2\, n(M)\, |\tilde{u}(k|M)|^2$$

$$P^{2\mathrm{h}}(k) = P^{\mathrm{lin}}(k) \left[\frac{1}{\bar{\rho}} \int \mathrm{d}M\, M\, b(M)\, n(M)\, \tilde{u}(k|M)\right]^2$$

Only accurate to ~40-50% in the 1-halo to 2-halo transition region (~1 Mpc/h)
Can still be adequate for certain applications....

The more accurate halo model

$$P^{1\mathrm{h}}(k) = \frac{1}{\bar{\rho}^2} \int \mathrm{d}M\, M^2\, n(M)\, |\tilde{u}(k|M)|^2$$

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Accurate to ~5% level, but requires $\xi_{\mathrm{mm}}(r)$ as input....
Still, this halo model is very useful, as it can also be used to model the correlation function (or power-spectrum) of galaxies !!!!

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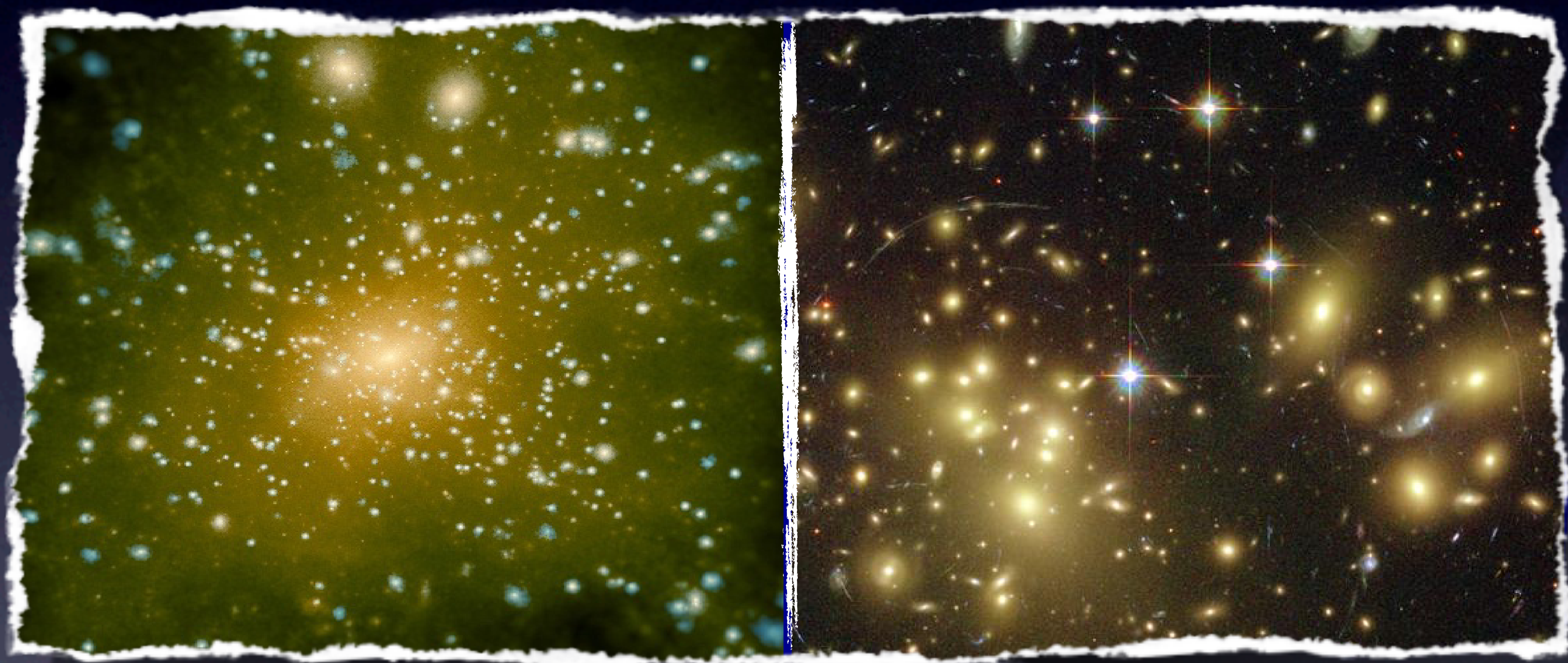
Halo Occupation Modeling

Halo Occupation Modelling: Motivation & Goal

Our main goal is to study the Galaxy-Dark Matter connection;
i.e., what galaxy lives in what halo?

- To constrain the physics of Galaxy Formation
- To constrain cosmological parameters



Four Methods to Constrain Galaxy-Dark Matter Connection:

- Large Scale Structure
- Galaxy-Galaxy Lensing
- Satellite Kinematics
- Abundance Matching

The Galaxy-Galaxy Correlation Function

$$P^{1h}(k) = \frac{1}{\bar{\rho}^2} \int \mathrm{d}M \, M^2 \, n(M) \, |\tilde{u}(k|M)|^2$$

$$P^{2h}(k) = \frac{1}{\bar{\rho}^2} \int \mathrm{d}M_1 \, M_1 \, n(M_1) \, \tilde{u}(k|M_1) \int \mathrm{d}M_2 \, M_2 \, n(M_2) \tilde{u}(k|M_2) \, Q(k|M_1, M_2)$$

The above equations describe the non-linear matter power-spectrum.

It is straightforward to use same formalism to compute power spectrum of galaxies:

Simply replace

$$\frac{M}{\bar{\rho}_{\rm m}} \rightarrow \frac{\langle N \rangle_M}{\bar{n}_{\rm g}}$$

$$\tilde{u}(k|M) \rightarrow \tilde{u}_{\rm g}(k|M)$$

where $\langle N \rangle_M$ describes the average number of galaxies (with certain properties) in a halo of mass M. Thus, the halo model combined with a model for the halo occupation statistics, allows a computation of $\xi_{\rm gg}(r)$

The Conditional Luminosity Function

The CLF $\Phi(L|M)$ describes the average number of galaxies of luminosity L that reside in a halo of mass M.

$$\Phi(L) = \int \Phi(L|M)\, n(M)\, \mathrm{d}M$$

$$\langle L \rangle_M = \int \Phi(L|M)\, L\, \mathrm{d}L$$

$$\langle N \rangle_M = \int \Phi(L|M)\, \mathrm{d}L$$

- Describes occupation statistics of dark matter haloes
- Links galaxy luminosity function to halo mass function
- Holds information on average relation between light and mass

see Yang, Mo & vdBosch 2003

The CLF Model

We split the CLF in a central and a satellite term:

$$\Phi(L|M) = \Phi_{\rm c}(L|M) + \Phi_{\rm s}(L|M)$$

For centrals we adopt a log-normal distribution:

$$\Phi_{\rm c}(L|M){\rm d}L = \frac{1}{\sqrt{2\pi}\sigma_{\rm c}} \exp\left[-\left(\frac{\ln(L/L_{\rm c})}{\sqrt{2}\sigma_{\rm c}}\right)^2\right]\frac{{\rm d}L}{L}$$

For satellites we adopt a modified Schechter function:

$$\Phi_{\rm s}(L|M){\rm d}L = \frac{\phi_{\rm s}}{L_{\rm s}}\left(\frac{L}{L_{\rm s}}\right)^{\alpha_{\rm s}} \exp\left[-(L/L_{\rm s})^2\right]{\rm d}L$$

Note: $\{L_{\rm c}, L_{\rm s}, \sigma_{\rm c}, \phi_{\rm s}, \alpha_{\rm s}\}$ all depend on halo mass

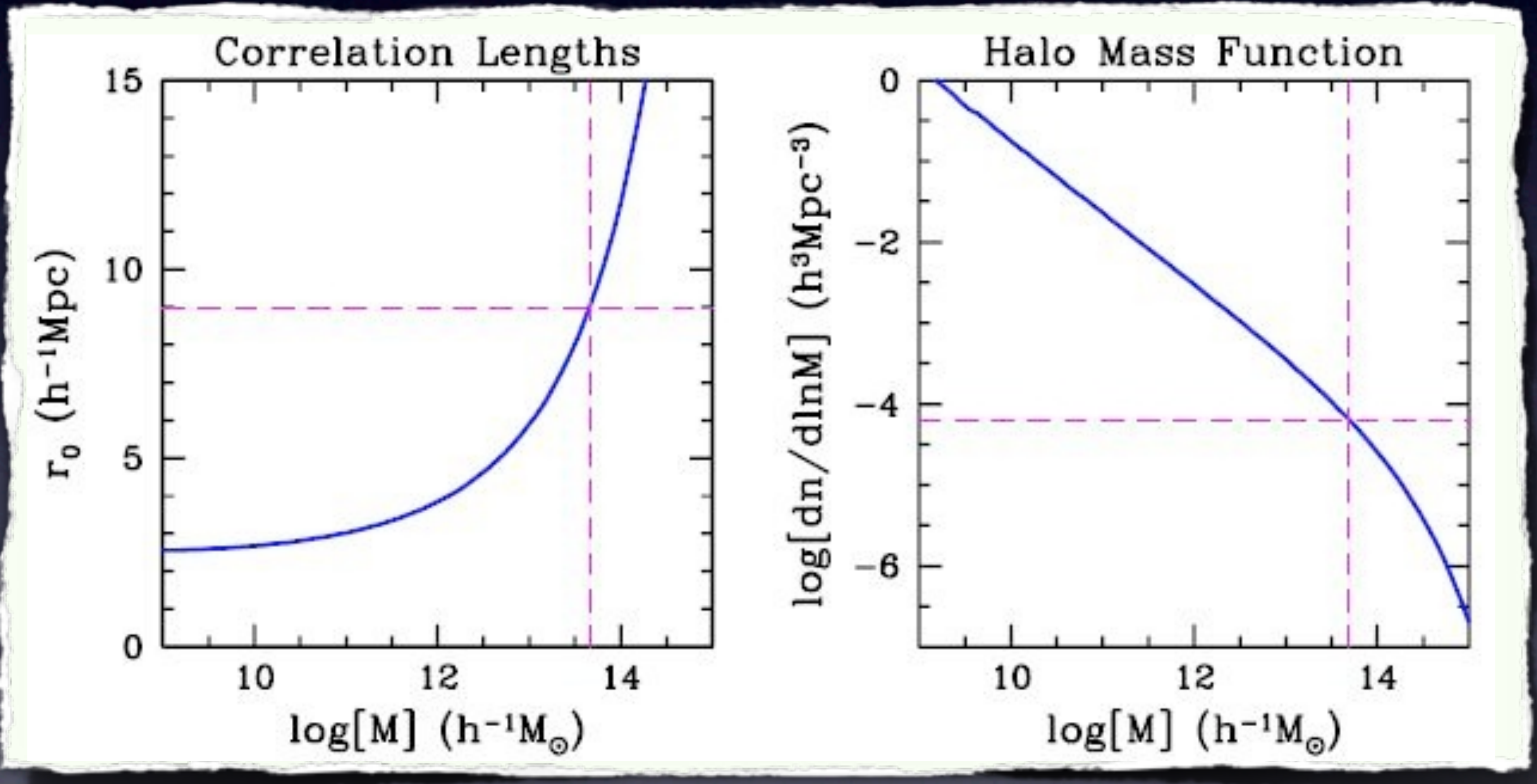
Free parameters are constrained by fitting data.

Galaxy Clustering

Occupation Statistics from Clustering

- Galaxies occupy dark matter halos
- CDM: more massive halos are more strongly clustered
- Clustering strength of given population of galaxies indicates the characteristic halo mass

Clustering strength measured by correlation length r_0



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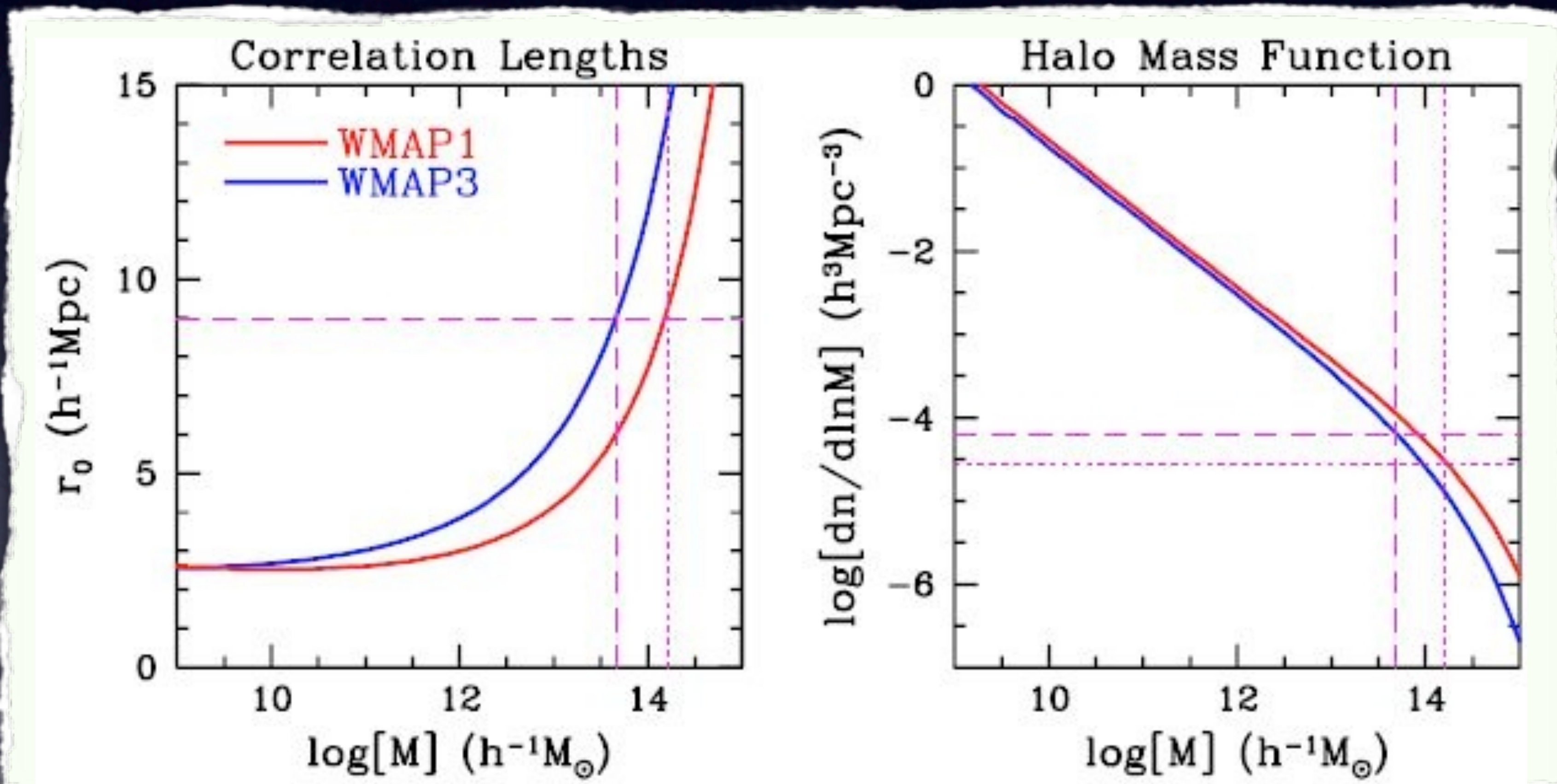


WMAP1

$\Omega_m = 0.30$

$\Omega_\Lambda = 0.70$

$\sigma_8 = 0.90$

WMAP3

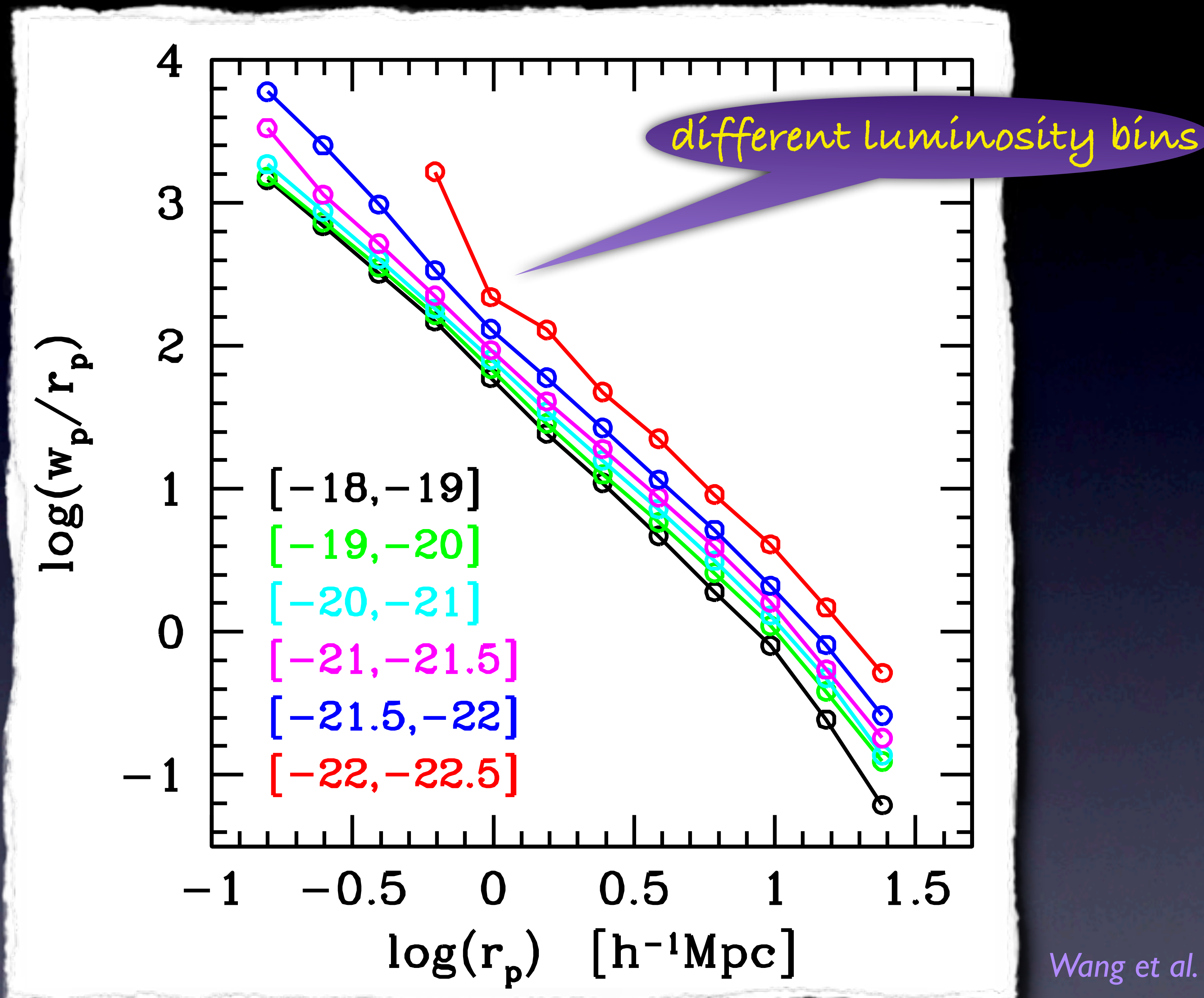
$\Omega_m = 0.24$

$\Omega_\Lambda = 0.76$

$\sigma_8 = 0.74$

CAUTION: results depend on cosmology

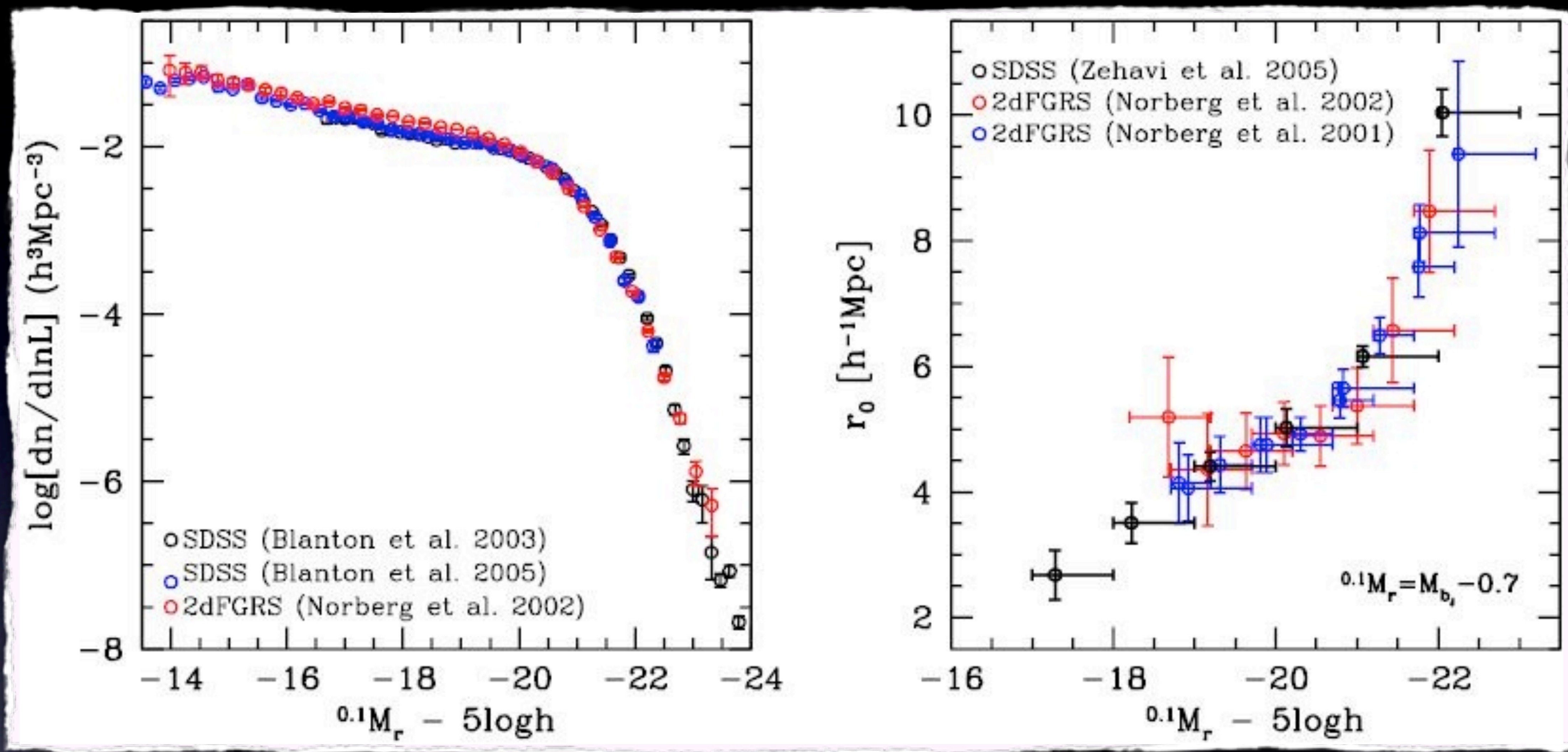
Galaxy Clustering: The Data



Wang et al. (2007)

More luminous galaxies are more strongly clustered

Luminosity and Correlation Functions

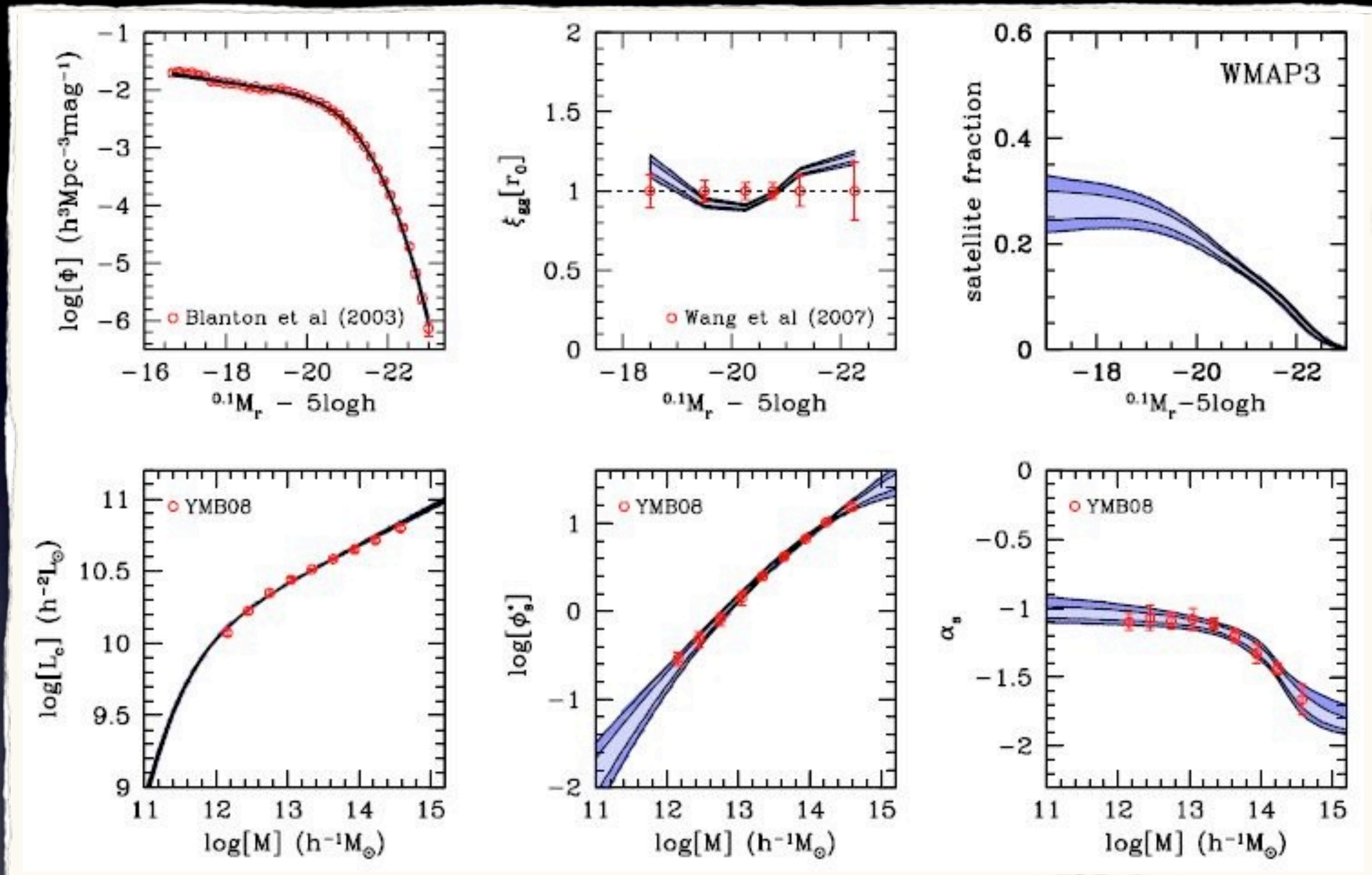


DATA: more luminous galaxies are more strongly clustered

LCDM: more massive halos are more strongly clustered

CONCLUSION: more luminous galaxies reside in more massive halos

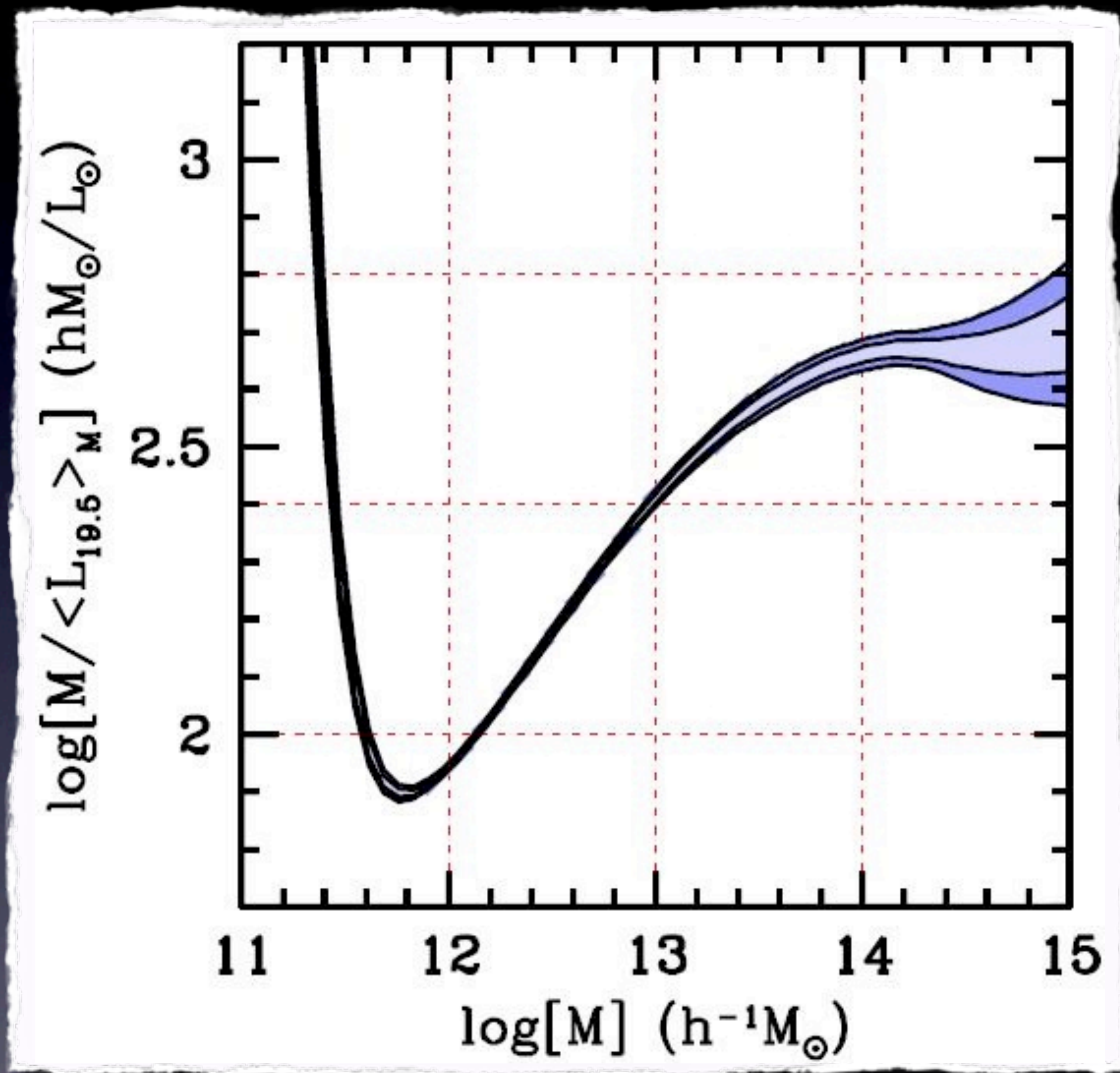
Results from MCMC Analysis



Cacciato, vdB et al. (2009)

- Model fits data extremely well with
- Same model in excellent agreement with results from SDSS galaxy group catalogue

Cosmology Dependence



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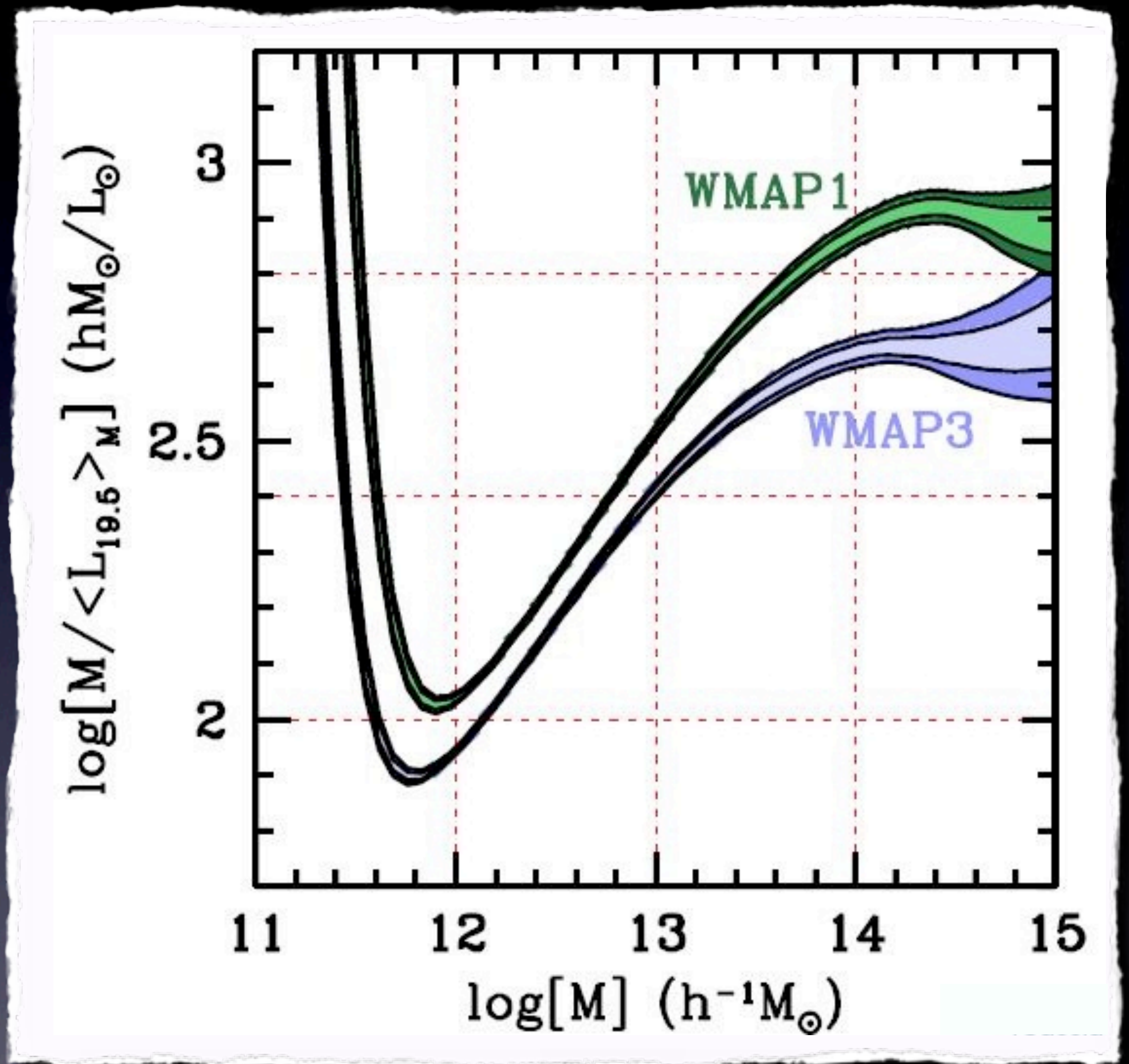
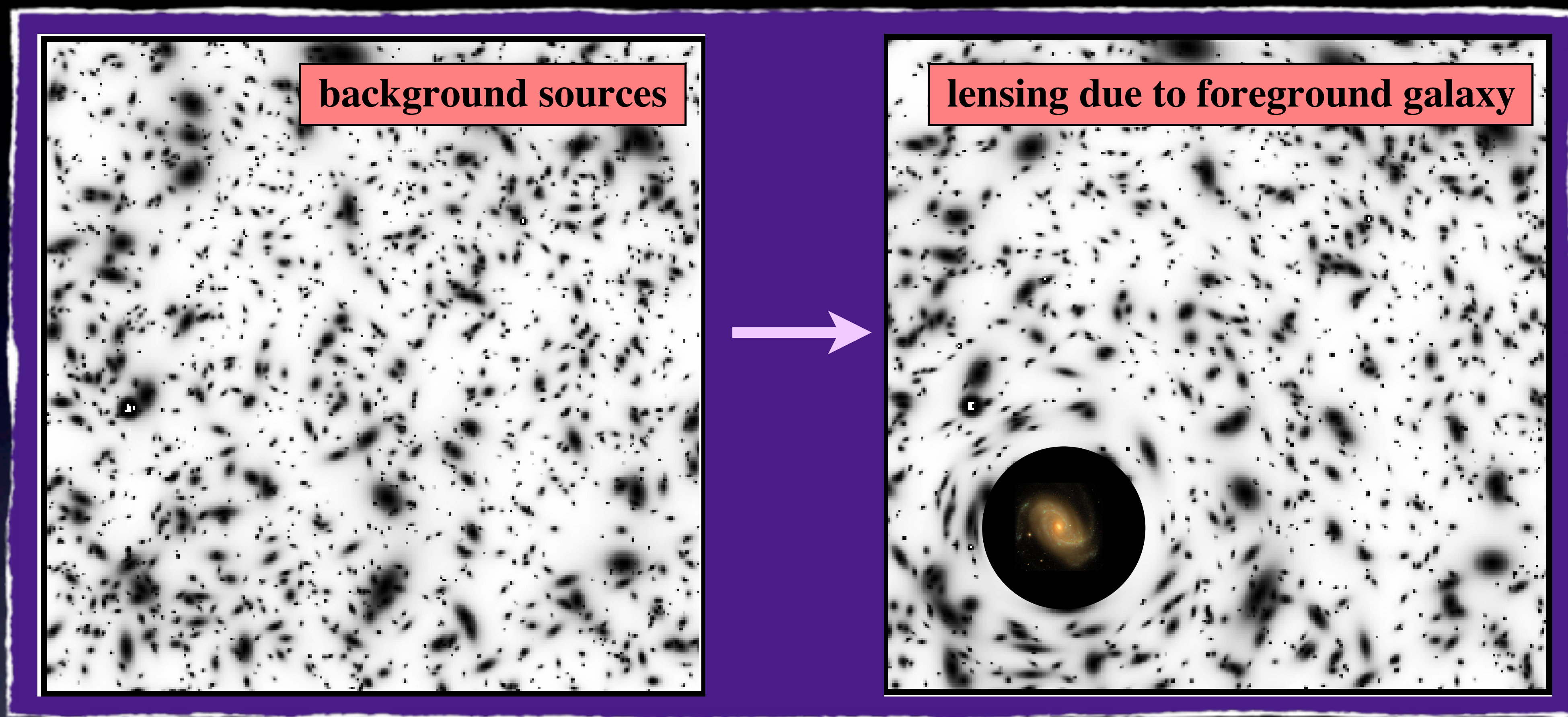


Galaxy-Galaxy Lensing

Galaxy-Galaxy Lensing

The mass associated with galaxies lenses background galaxies



Lensing causes correlated ellipticities, the tangential shear, $\gamma_{\rm t}$, which is related to the excess surface density, $\Delta\Sigma$, according to

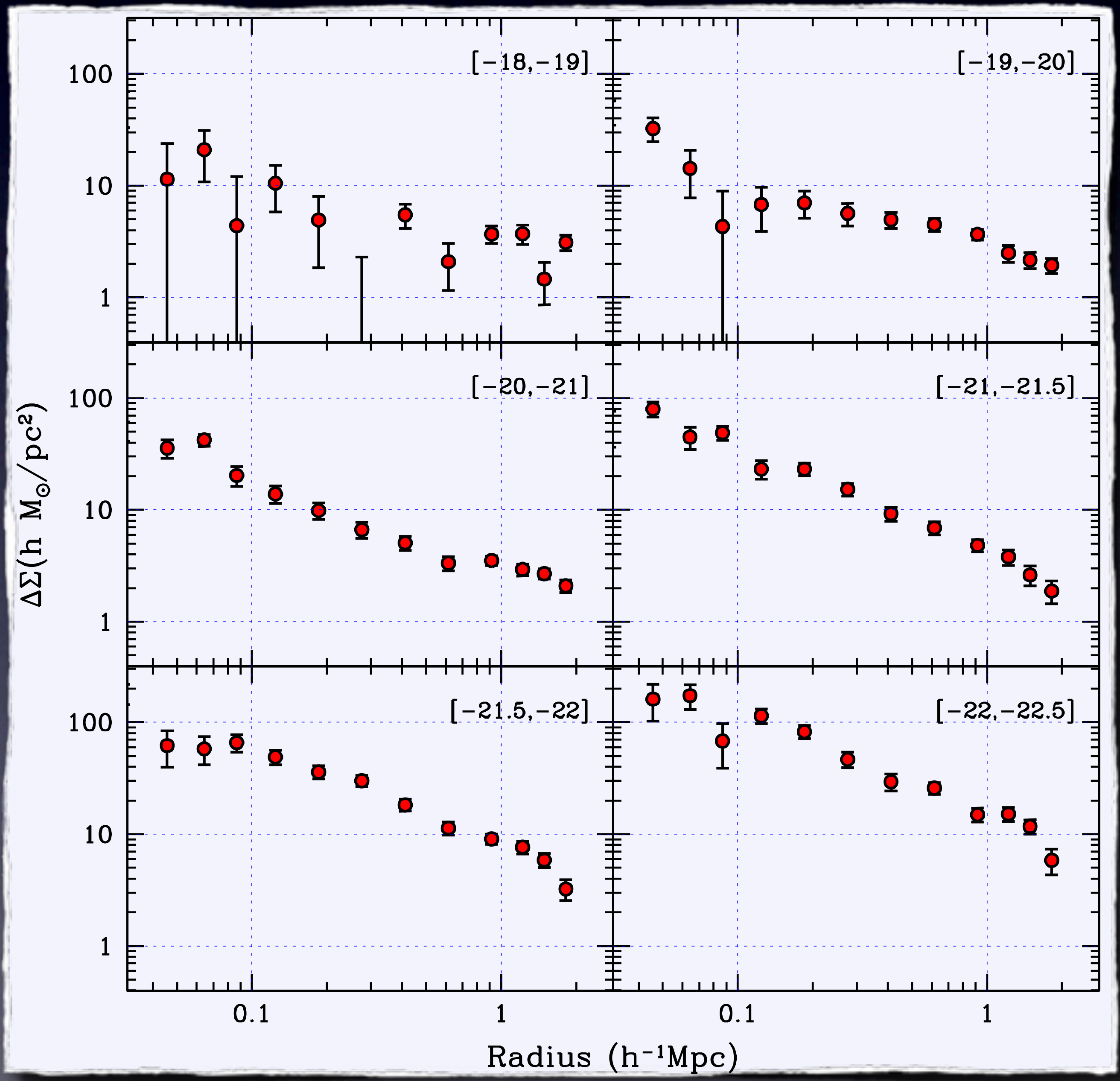
$$\gamma_{\rm t}(R)\Sigma_{\rm crit} = \Delta\Sigma(R) = \bar{\Sigma}(< R) - \Sigma(R)$$

$\Delta\Sigma$ is line-of-sight projection of galaxy-matter cross correlation

$$\Sigma(R) = \bar{\rho}\int_0^{D_{\rm s}} [1 + \xi_{\rm g,dm}(r)]\, {\rm d}\chi$$

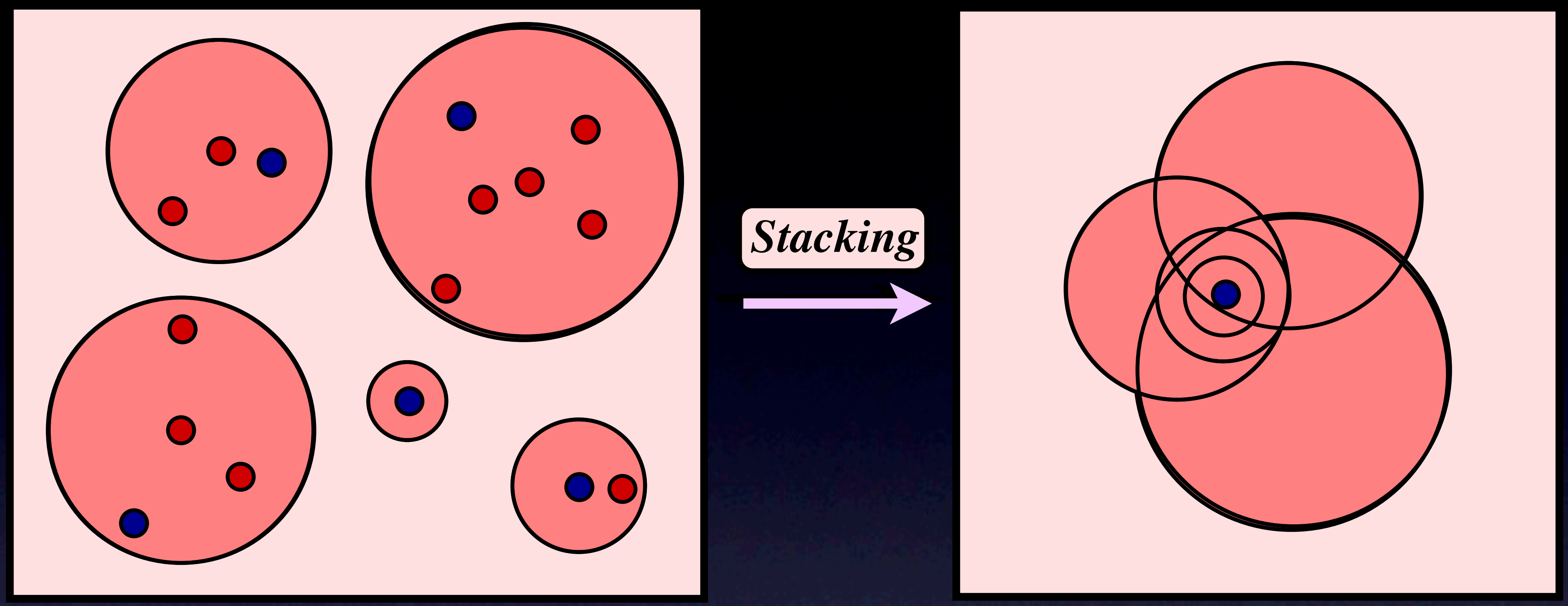
Galaxy-Galaxy Lensing: The Data

- Number of background sources per lens is limited
- Measuring shear with sufficient S/N requires stacking of many lenses
- $\Delta\Sigma(R|L_1, L_2)$ has been measured using the SDSS by Mandelbaum et al. (2006), using different bins in lens-luminosity



Mandelbaum et al. (2006)

How to interpret the signal?



Because of stacking the lensing signal is difficult to interpret

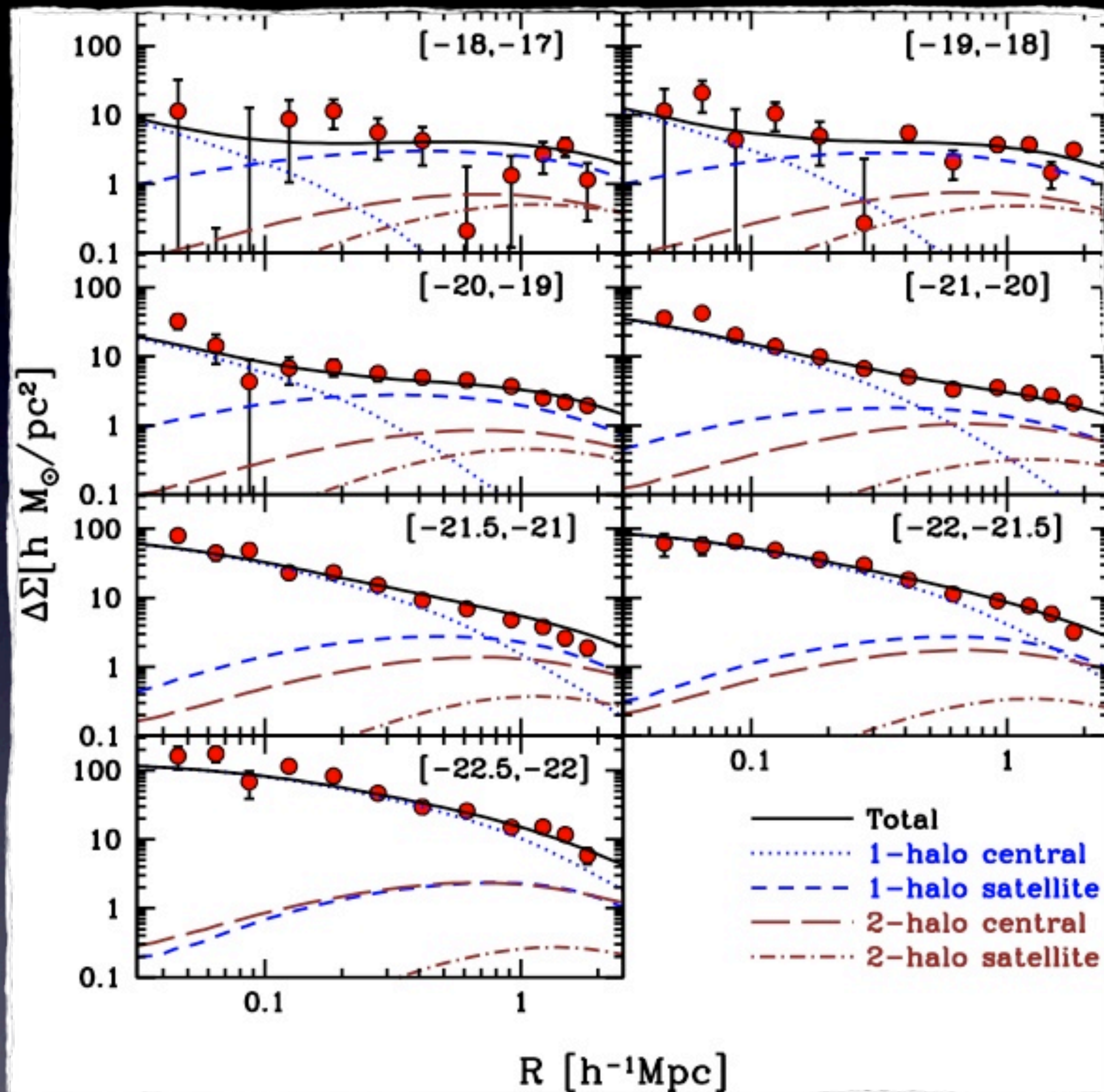
In order to model the data, what is required is:

$P_{\rm cen}(M|L)$ $P_{\rm sat}(M|L)$ $f_{\rm sat}(L)$

These can all be computed from the CLF...

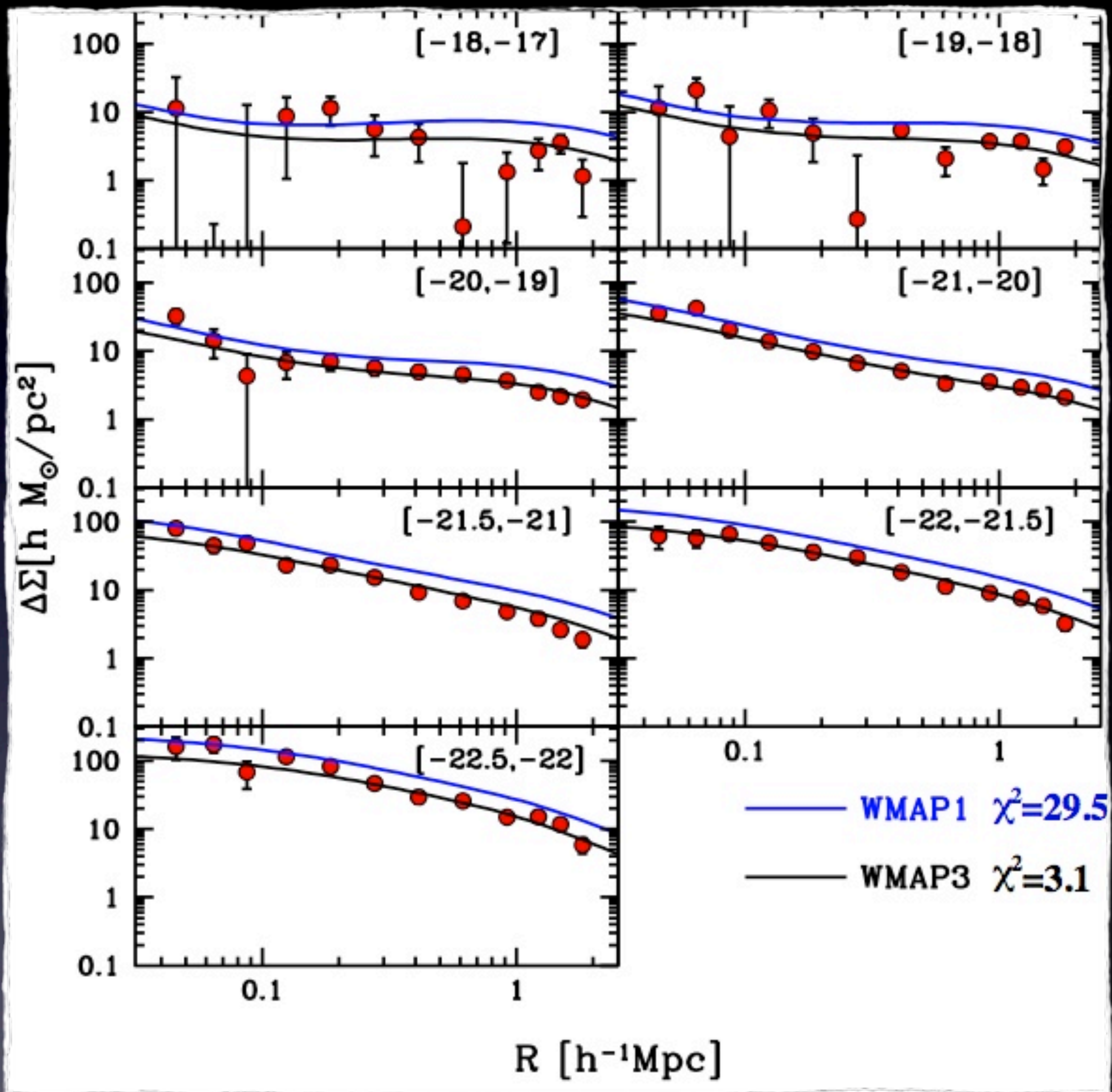
For a given $\Phi(L|M)$ we can predict the lensing signal $\Delta\Sigma(R|L_1, L_2)$

Galaxy-Galaxy Lensing: Results



NOTE: this is not a fit, but a prediction based on CLF

Galaxy-Galaxy Lensing: Results



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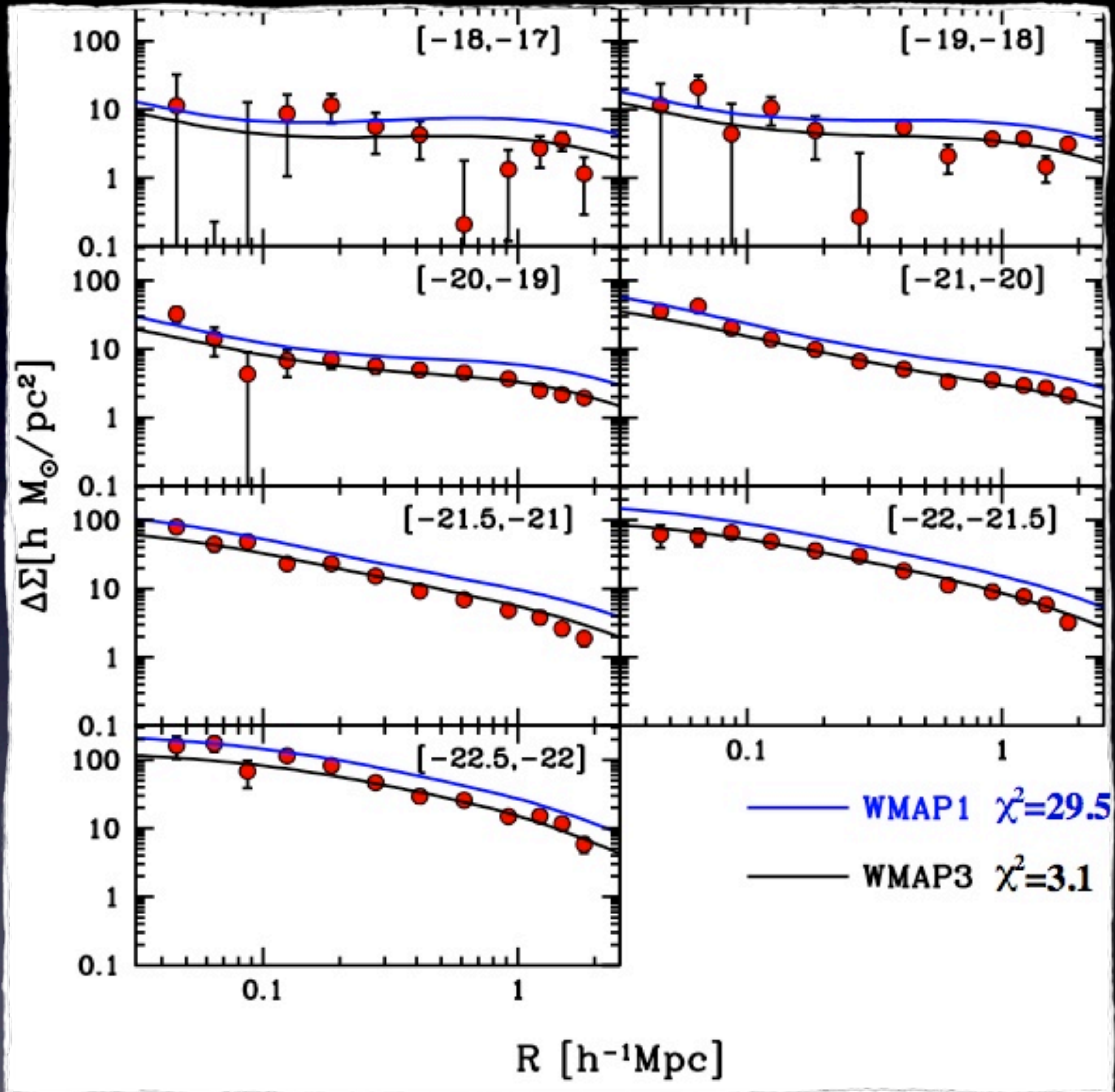
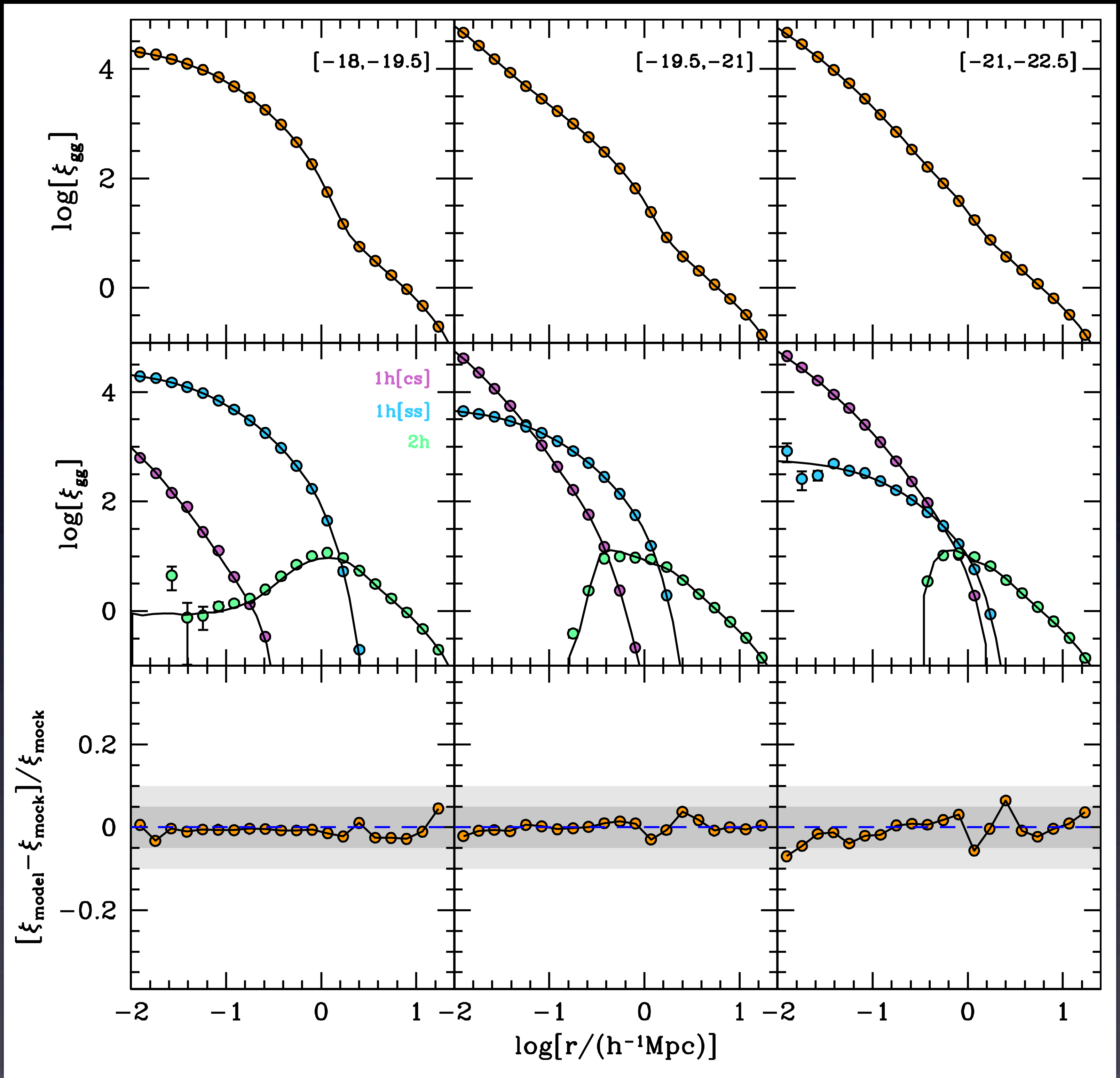


Combination of clustering & lensing can constrain cosmology!!!

Constraining Cosmology

Comparison with Mock Catalogues



- Run numerical simulation of structure formation (DM only)
- Identify DM haloes, and populate them with galaxies using a model for the CLF.
- Compute galaxy-galaxy correlation functions for various luminosity bins.
- Use analytical model to compute the same, using the same model for the CLF.

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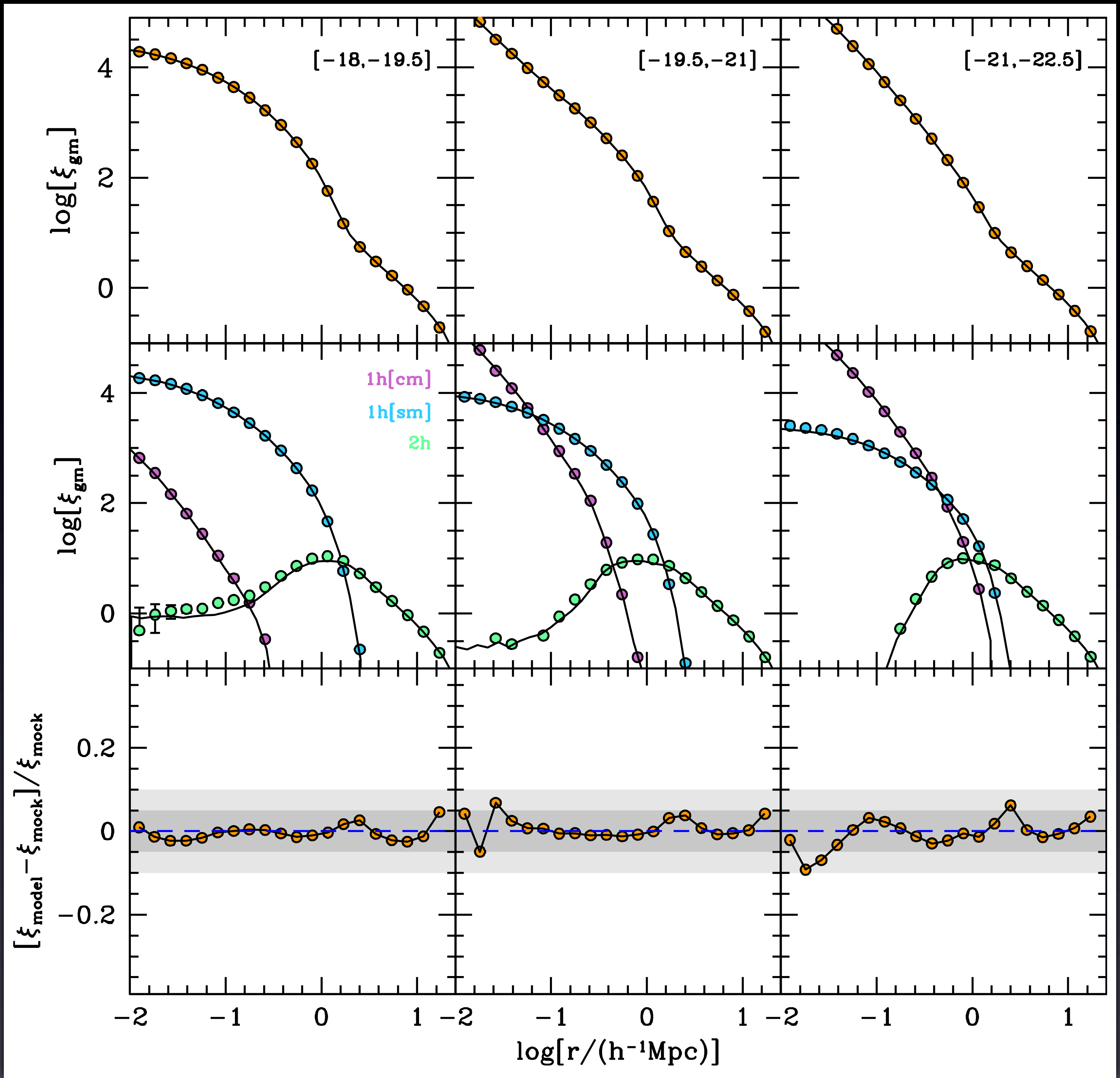

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Our model is accurate to better than ~5%

Residual Redshift Space Distortions

To avoid redshift space distortions, one typically uses projected correlation function

$$w_{\rm p} = 2\int_0^{\infty} \xi_{\rm gg}(r_{\rm p}, r_{\pi})\, {\rm d}r_{\pi} = 2\int_{r_{\rm p}}^{\infty} \xi_{\rm gg}(r)\, \frac{r\,{\rm d}r}{\sqrt{r^2 - r_{\rm p}^2}}$$

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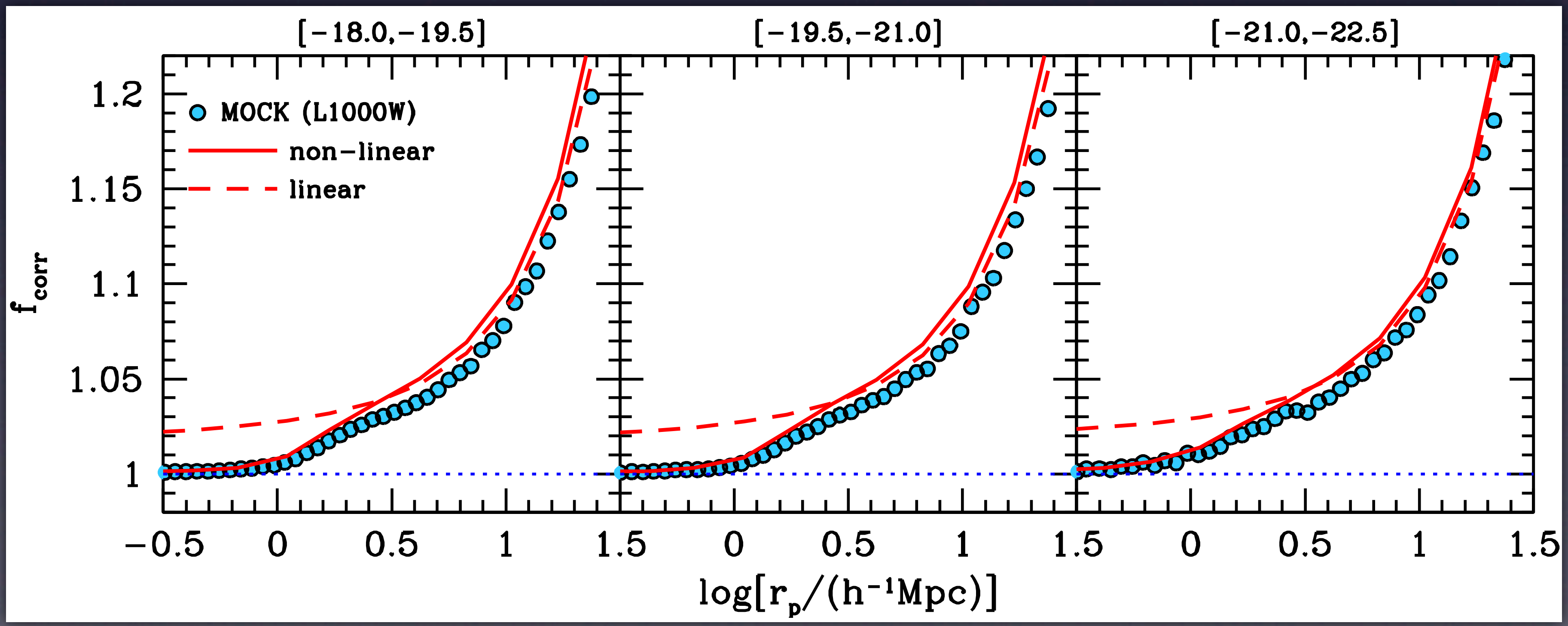
The resulting, residual z-space distortions easily exceed 20% at r_p ~20 Mpc/h (Norberg et al. 2009).



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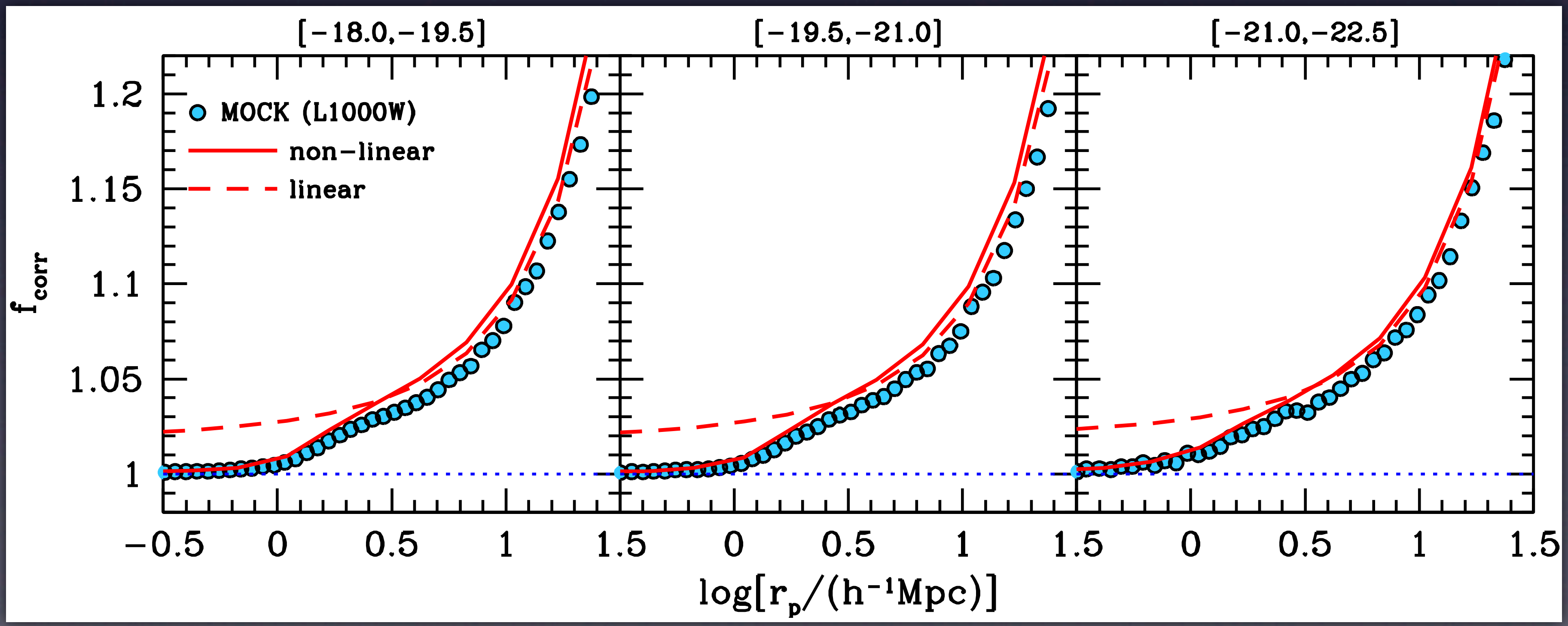
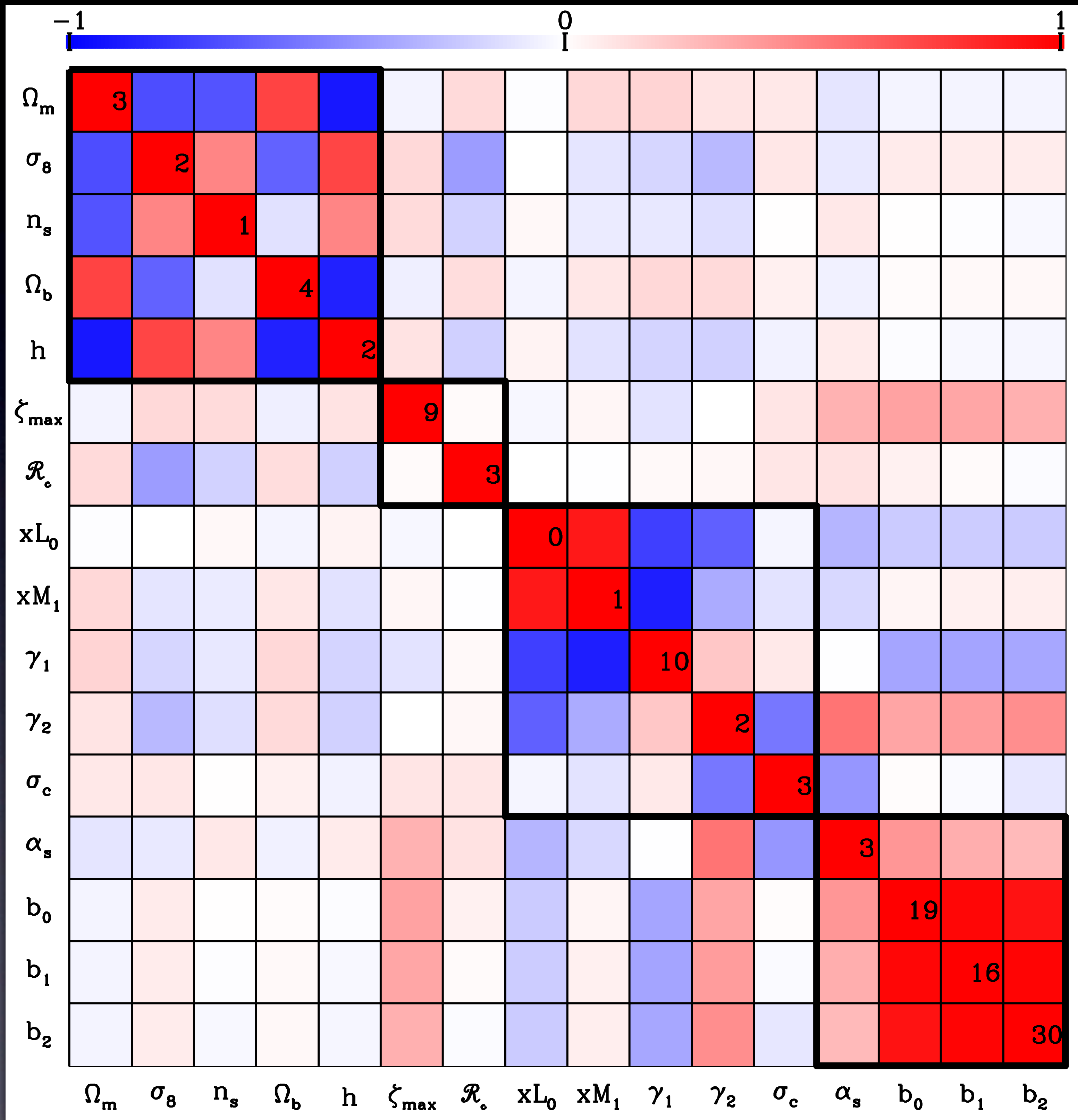


We correct for these residual redshift space distortions using the linear Kaiser formalism. Mocks show that this is accurate to few percent.

Covariance Matrix



- Covariance matrix has block diagonal form.
- Little correlation between cosmological parameters, and other parameters.
- Nuisance parameters are mainly correlated with the satellite CLF parameters
- Our results are robust to our particular parameterization of the CLF.

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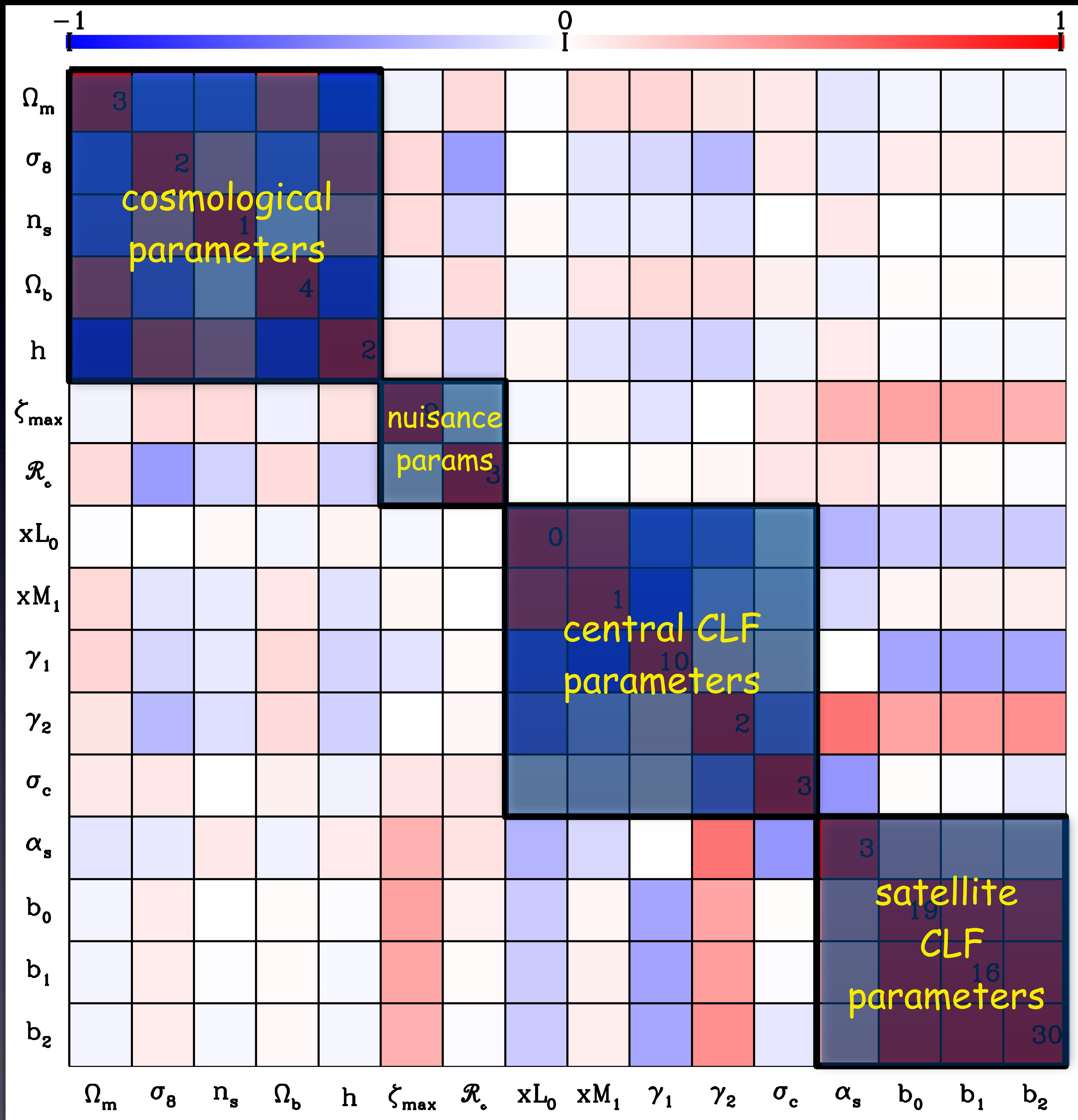

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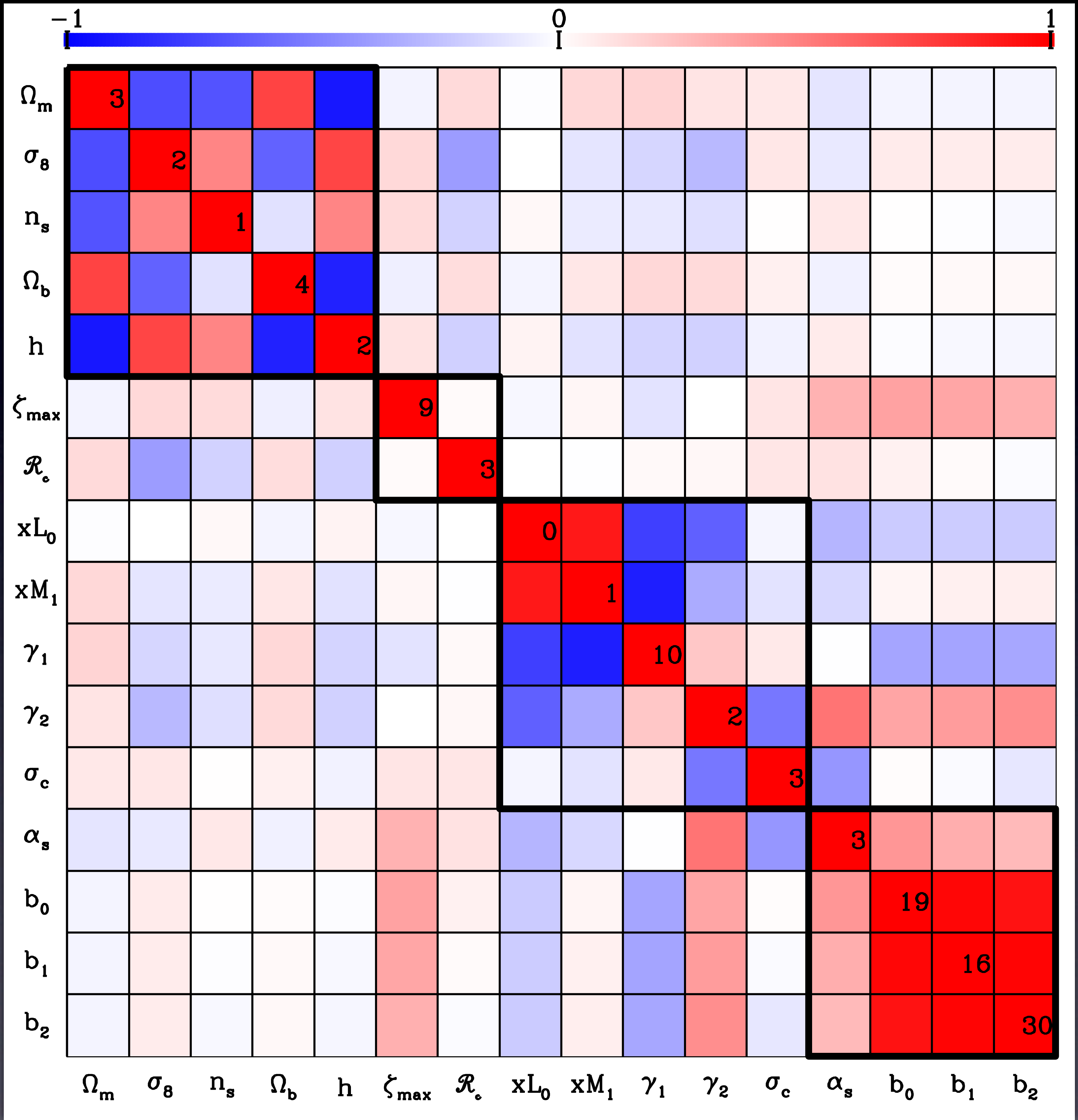

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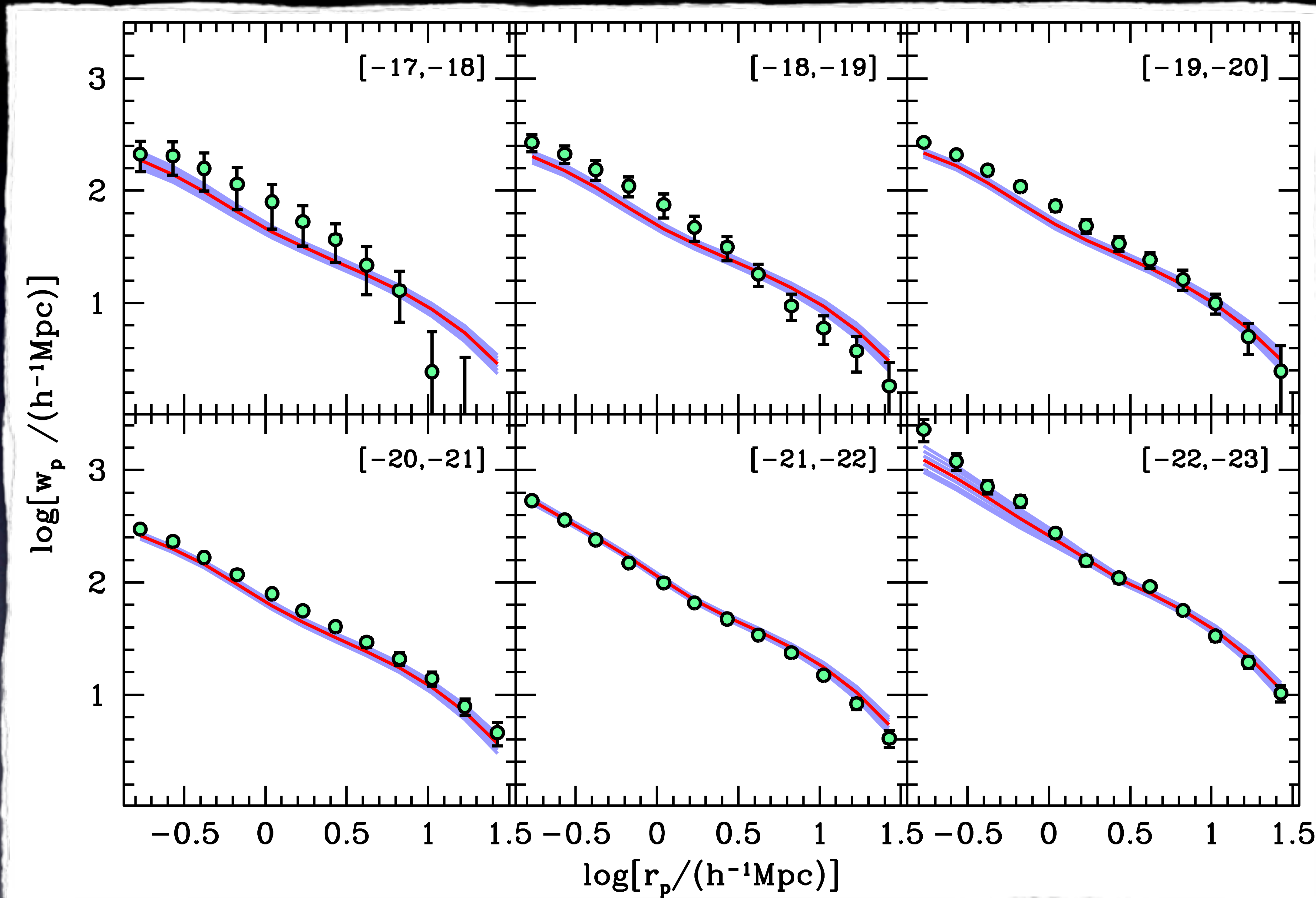
Fiducial Model

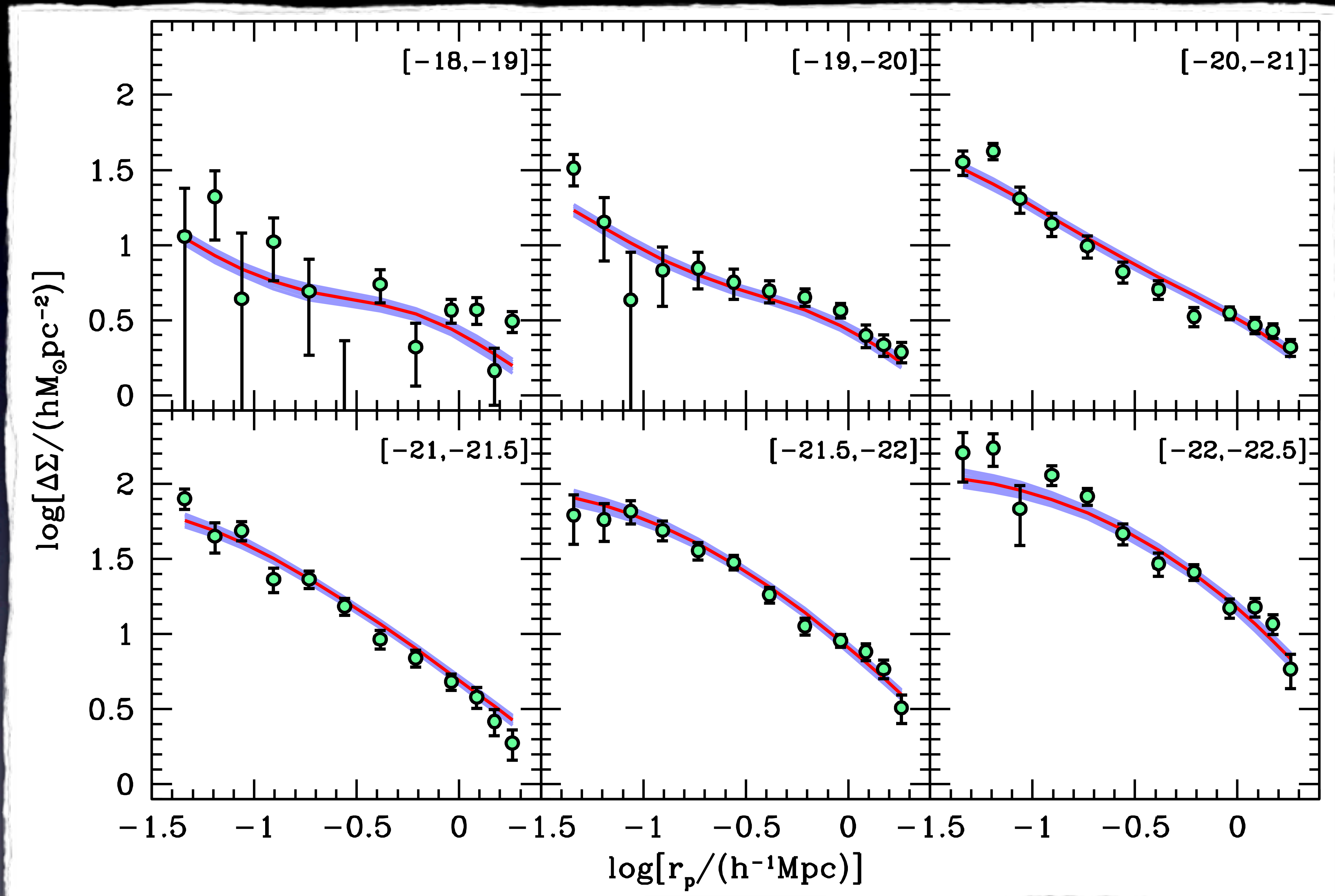
- Total of 16 free parameters:
  - 9 parameters to describe CLF
  - 5 cosmological parameters; $\Omega_{\rm m}, \Omega_{\rm b}, \sigma_8, n_{\rm s}, h$
  - 2 nuisance parameters; $\zeta_{\rm max}, \mathcal{R}_{\rm c}$

  Total of 176 data points.
- WMAP7 priors on $\Omega_{\rm b}, n_{\rm s}, h$
- Correction for residual redshift space distortions
- Dark matter haloes follow NFW profile + marginalize over 10% uncertainty in c(M) relation
- Radial number density distribution of satellites follows that of dark matter particles.
- Halo mass function and halo bias function of Tinker et al. (2009,2010).

Results: Clustering Data

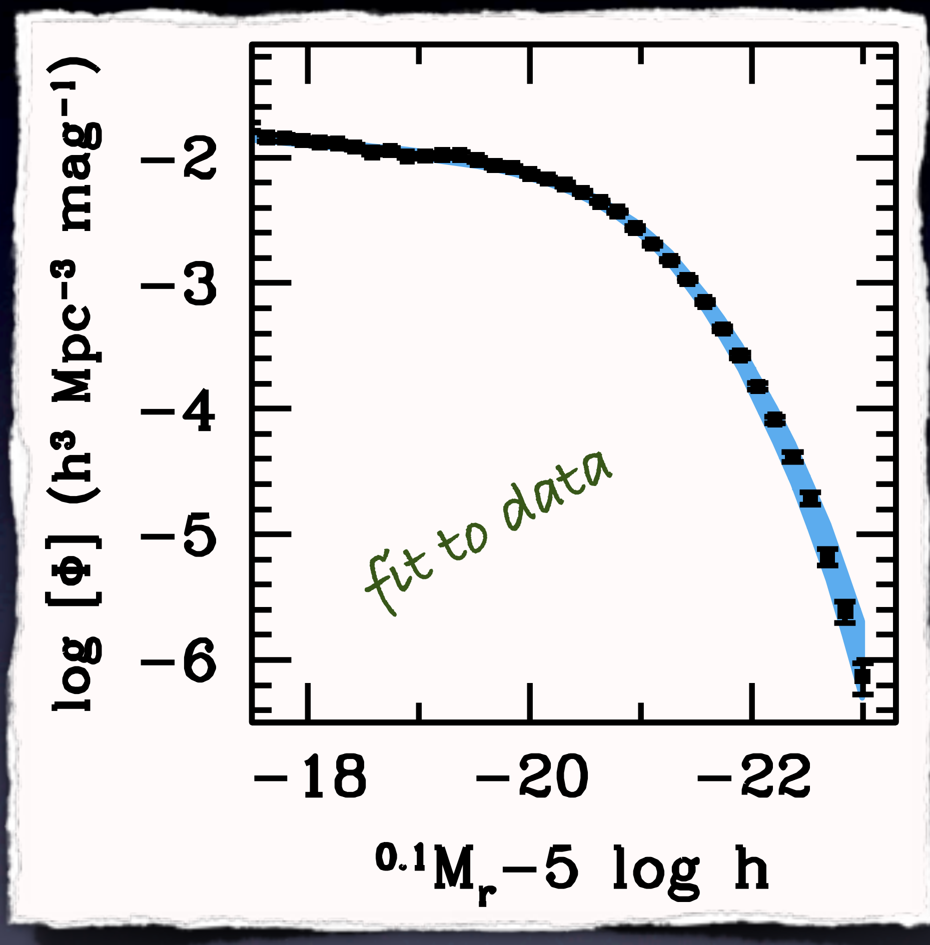


Results: Lensing Data

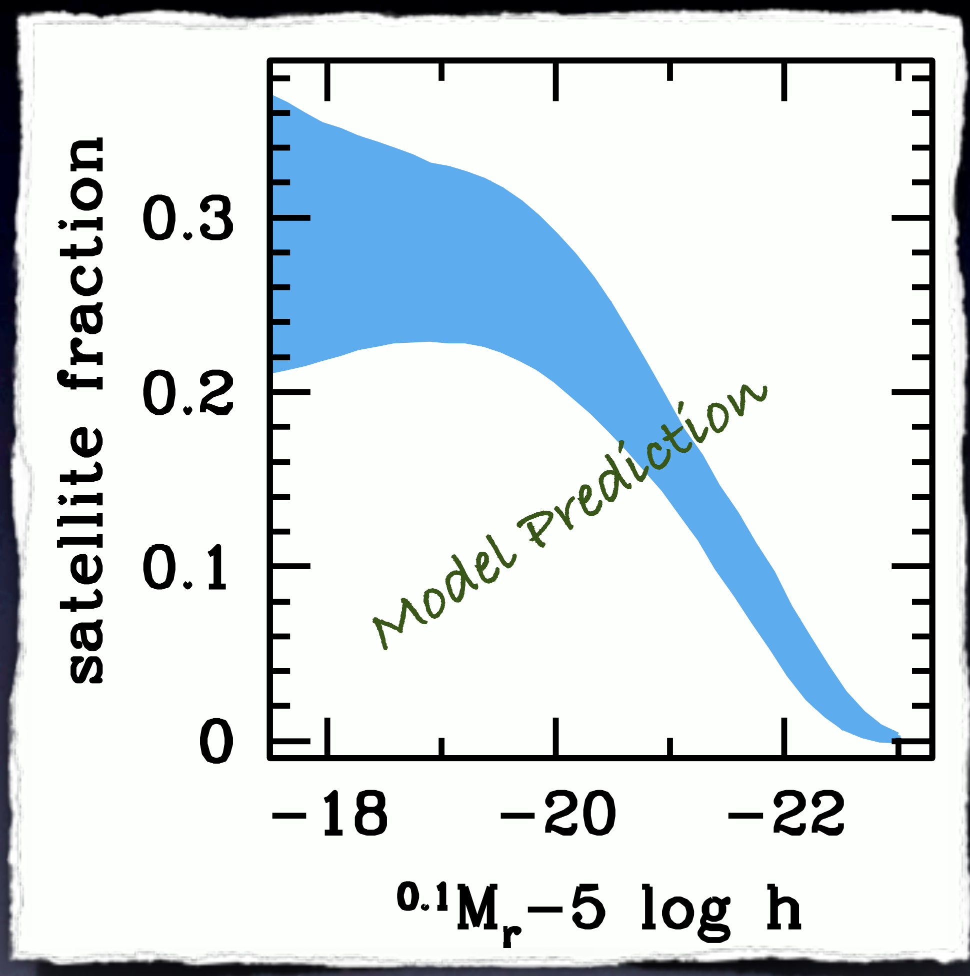


Luminosity Function & Satellite Fractions

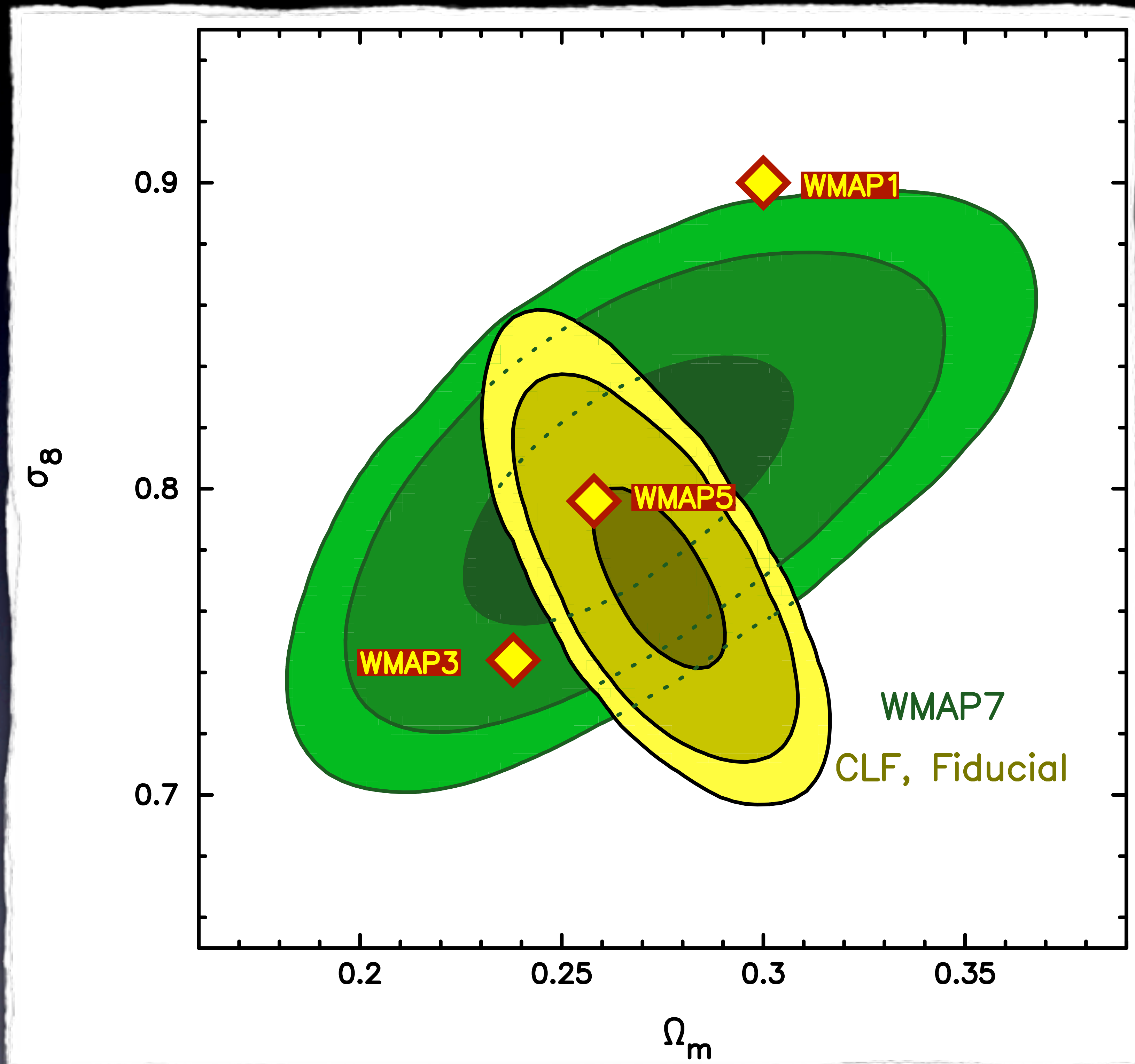
Luminosity Function



Satellite Fractions



Cosmological Constraints



Conclusions

- Recent years have seen enormous progress in establishing the galaxy-dark matter connection, including its scatter!
- Different methods (group catalogues, satellite kinematics, galaxy-galaxy lensing, clustering & abundance matching) now all yield results in good mutual agreement.
- Combination of galaxy clustering and galaxy-galaxy lensing can constrain cosmological parameters.
    - This method is complementary to and competitive with BAO, cosmic shear, SNIa & cluster abundances.
    - Preliminary results are in excellent agreement with CMB constraints from WMAP7
    - Forecasting for constraints on neutrino mass, WDM and modified gravity very promising.

The End