Scaling Relations for Disk Galaxies and their Interpretation

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Sample of \( \sim 1300 \) disk galaxies with H\( \alpha \) rotation curves.

Rotation velocities measured at 2.2 disk scale lengths.

Uniform inclination & extinction corrections.  

\textit{Courteau+07}

\textbf{NOTE:} TF residuals are not correlated with surface brightness (size).
Galaxy Scaling Relations

**Tully-Fisher (TF) Relation**

\[ L \propto V_{\text{rot}}^\alpha \quad (\alpha \sim 3.5) \]

scatter NOT correlated with size

**Faber-Jackson (FJ) Relation**

\[ L \propto \sigma^\beta \quad (\beta \sim 4) \]

scatter correlated with size

**Fundamental Plane:** \[ L \propto \sigma^\beta R_e^\gamma \]

**Origin of TF and FJ relations** is believed to be that DM halos have same density:

\[ V_{\text{vir}} \propto R_{\text{vir}} \propto M_{\text{vir}}^{1/3} \]

Using that less massive halos are more concentrated, this becomes

\[ V_{\text{max},h} \propto M_{\text{vir}}^{0.29} \]

This scaling is similar to observed stellar mass TF & FJ relations

\[ V_{2.2} \propto M_\star^{0.28} \quad [\text{Dutton et al. 2010}] \]

\[ \sigma_e \propto M_\star^{0.29} \quad [\text{Gallazzi et al. 2006}] \]

\( V_{2.2} \) is disk rotation velocity at 2.2 disk scale lengths

\( \sigma_e \) is velocity dispersion inside effective radius
The Origin of Galaxy Scaling Relations

For the $V_{\text{max},h} - M_{\text{vir}}$ relation to be the direct origin of the TF & FJ relations requires that $V_{\text{obs}}/V_{\text{max},h}$ and $M_*/M_{\text{vir}}$ are both constants! ✤

These requirements are neither “natural” nor consistent with observations

Galaxy Formation Efficiency

$V_{\text{obs}}/V_{\text{max},h}$

self-gravity

$M_*/M_{\text{vir}}$

Response of Dark Matter Halo

Here $V_{\text{obs}} = V_{2.2}$ for late-types, and $V_{\text{obs}} = \sigma_e$ for early-types
**Dark Halo Response**

When baryons collect at center, the dark matter halo contracts...

In the limit where the process is slow, the response is adiabatic

spherical symmetry: \( r_i M_i(r_i) = r_f M_f(r_f) \)

no shell crossing: \( M_{h,i}(r_i) = M_{h,f}(r_f) \)

initially well mixed: \( M_{b,i}(r_i) = f_b M_{h,i}(r_i) \)

\[
\frac{r_f}{r_i} = \Gamma_{AC} = \frac{M_{h,i}(r_i)}{M_{b,f}(r_f) + (1 - f_b)M_{h,i}(r_i)}
\]

Blumenthal et al. (1986)

In general, system is not spherically symmetric and the process of galaxy formation may not be adiabatic. It is useful to adopt the more general form:

\[
\frac{r_f}{r_i} = \Gamma_{\nu} = \Gamma_{AC}^{-\nu}
\]

\[
\begin{align*}
\nu = 1 & \quad \text{standard AC} \\
\nu = 0 & \quad \text{no contraction} \\
\nu < 0 & \quad \text{expansion}
\end{align*}
\]

Here \( \nu \) is a free parameter, to be constrained by the data:

[Based on hydro-simulations, Gnedin+04 suggest \( \nu \approx 0.8 \), while Abadi+10 find \( \nu \approx 0.4 \)]
Simultaneously matching LF & TF has been a long-standing problem for CDM-based models (White & Frenk 1991; Kauffmann et al. 1993; Cole et al. 2000; and many more...)

**GALICS**
- Giovanelli+97
- Cored halos
- No AC

**GALFORM**
- Appears reasonably successful...

**MUNICH**
- Claim success, but assume that $V_{\text{rot}} = V_{\text{max}}$
- $V_{\text{rot}} = 1.5 V_{\text{max}}$

**GALACTICUS**
- Sophisticated model including disk self-gravity and AC: “fails to predict correct sizes and velocities of disk galaxies”

(Hatton et al. 2003)

(Benson et al. 2003)

(Croton et al. 2006; Guo et al. 2011)

(Benson & Bower 2010)
The Optical-to-Virial Velocity Ratio

NOTE: assuming $V_{2.2} = V_{\text{max}}$ is equivalent to assuming halo expansion

$M_*/M_h = 0.05$

$\lambda_{\text{gal}} = 0.048$

$\log M_{\text{vir}} = 13, 12, 11$

$M_*/M_h = 0.05$

$\lambda_{\text{gal}} = 0.048$
- TFR has min. scatter (0.036 ± 0.005 dex) when using $M^\star_{\text{Bell}}$ and $V_{80}$ (Reyes et al. 2011)

- The velocity-mass (TF) and size-mass residuals are uncorrelated; this constrains the contribution of the disk to the measured rotation velocity (Courteau & Rix 1999)

- When using $M_{\text{bar}}$ and $R_{\text{bar}}$, instead of $M^\star$ and $R^\star$, the slope of the residual correlation is -0.15. Hence, $R_{\text{bar}}$ is a third parameter in the baryonic TFR (Avila-Reese et al. 2008)
Model Predictions

Naive prediction:
If haloes of same mass yield disk galaxies of same $M^*$, then scatter in spin parameter can yield large scatter in $V_{\text{rot}}$. This scatter is anti-correlated with disk size....

Ways out:
- $M^*$ is correlated with spin parameter. Natural outcome of SF threshold
  \[ \text{Firmani & Avila-Reese (2000)} \]
- Self-gravity of disk is reduced (add feedback)
- Adiabatic contraction does not happen or is counter-acted \[ \text{Dutton et al. (2007)} \]

NOTE: model assumes flat LCDM cosmology with $\sigma_8=1$ and no feedback (illustration only)
Towards a Working Model...

Reducing $V_{2.2}/V_{\text{vir}}$ requires halo expansion ($\nu = -1$) and low spin parameters ($\lambda_{\text{gal}} \approx \lambda_{\text{halo}}/2$). Note that this model predicts a significant correlation in the residual plot for the baryonic relations, which has since been confirmed by Avila-Reese+08.
The Stellar Mass - Halo Mass Relation

- Use galaxy-galaxy lensing or satellite kinematics to infer $M^* - M_h$ relation.
- Convert halo mass to $V_{\text{vir}}$.
- Use stellar mass TFR to convert stellar mass to $V_{\text{opt}}$.
- This yields $V_{\text{opt}} / V_{\text{vir}}$ as function of $M^*$

[Dutton+10, Reyes+12]
Different analyses agree with each other at 2σ-level: $1.0 < \frac{V_{\text{opt}}}{V_{200c}} < 1.5$

Error bars still too large to place firm constraints: dominated by errors on $M_*/M_{\text{vir}}$
The `standard' picture of disk formation can nicely explain the evolution in the size-mass relation of disk galaxies. See also Firmani & Avila-Reese (2009)
Evolution in Disk Galaxy Scaling Relations

Same model is also successful in explaining observed evolution in TF relation. (see also Tonini et al. 2011)
Scaling relations & angular momentum

Structure of disk galaxies is governed by their angular momentum distribution.

In ‘standard model’, this angular momentum arises from cosmological torques, and is conserved during cooling.

As shown by Mo, Mao & White (1998), in this case one has that

$$R_d = \frac{1}{\sqrt{2}} \lambda \left( \frac{j_d}{m_d} \right) R_{\text{vir}} F_{R}^{-1} F_{E}^{-1/2}$$

halo spin parameter  
halo profile  
adiabatic contraction

But what about $j_d/m_d$ (almost always assumed to be unity) ???

$j_d$: fraction of angular momentum that ends up in disk
$m_d$: fraction of baryonic matter that ends up in disk
Methodology

Observed Scaling Relations

- $M_h$ vs. $M_*$
- $M_g$ vs. $M_*$
- $R_d$ vs. $M_*$
- $R_b$ vs. $M_*$
- $B/D$ vs. $M_*$
- $R_g$ vs. $R_d$

Rotation Curve

$$V_c(r) = \sqrt{V_h^2 + V_d^2 + V_b^2 + V_g^2}$$

Model Parameters

- $M_h$, $M_d$, $M_b$, $M_g$
- $R_d$, $R_b$, $R_g$
- $\Delta_{IMF}$, $\nu$

Constrain

$\Delta_{IMF}$ & $\nu$

compare to data

Sampling of $M_h$

TF relation
With `standard' adiabatic contraction (B86; $\nu=1$), the stellar IMF needs to be significantly more top-heavy than a Chabrier IMF (unrealistic).

With Chabrier IMF, disk scaling relations suggest halo expansion...
Methodology

**Observed Scaling Relations**
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**Model Parameters**
- $M_h$, $M_d$, $M_b$, $M_g$
- $R_d$, $R_b$, $R_g$
- $\Delta_{\text{IMF}}$
- $\nu$

**Rotation Curve**

$$V_c(r) = \sqrt{V_h^2 + V_d^2 + V_b^2 + V_g^2}$$

**SDSS**

**Dutton+vdB 2012**

**Formulae**
- $m_d \equiv \frac{M_{\text{gal}}/M_{\text{vir}}}{\Omega_b/\Omega_m}$
- $j_d/m_d \equiv \frac{J_{\text{gal}}/J_{\text{vir}}}{M_{\text{gal}}/M_{\text{vir}}}$

$$\lambda'_{\text{vir}} = 0.031$$
The Assembly of Mass and Angular Momentum

The gray-shaded areas mark region in `galaxy-formation-space' that are required in order to yield disks with the observed scaling relations.

- $m_d$ has strong halo-mass dependence, $j_d/m_d$ does not.

- This is NOT a `natural' outcome of a scenario in which disks form `inside-out'
More sophisticated models with SN feedback and angular momentum transfer (disk→halo) fair only slightly better; no 'natural' explanation within standard 'framework' of disk formation.

Hydro-simulations of Sales et al. (2009) predict relation between \( j_d/m_d \) and \( m_d \) similar to that of naive `inside-out-cooling-model'; outflows in simulations preserve rank-order of \( E_{\text{binding}} \).
Are Disk Galaxy Scaling Relations a `success’ for the LCDM paradigm?

**NO**
TF zero-point and angular momentum catastrophe have caused too much of a problem.

Are Disk Galaxy Scaling Relations a `failure’ for the LCDM paradigm?

**NO**
We now know what is needed to make it work, although this is not `natural’

- The formation of disk galaxies causes the central regions of dark matter halos to expand (or at least, not to contract). **HOW?**
- The small fraction of baryons that end up in disk have a disproportionate fraction of the specific angular momentum. **WHY?**
- Do we need to modify models for disk formation to account for cold flow feeding of disks? **[e.g., Kimm+11; Power+11; Pichon+11]**
- secular redistribution of angular momentum? **[e.g., Tonini+11; Minchev+12; Roškar+12]**