ASTRO 530: Problem Set 3

Problem 1: Consider a disk galaxy with a perfectly flat rotation curve, $V_{\rm rot}(R) = V_{\rm rot}$. If this galaxy has a bar with pattern speed $\Omega_{\rm p}$, what is the ratio of the corotation radius and the radius at which one finds the Outer Lindblad Resonance (OLR).

Problem 2: Consider a self-gravitating, virialized, infinitessimally thin, exponential disk of mass $M_{\rm d} = 10^{10} h^{-1} M_{\odot}$ and with scale length $R_{\rm d} = 3.0 h^{-1}$kpc, living in an EdS universe without dark matter.

a) Show that the total angular momentum and energy of the disk can be written as

$$J_{\rm d} = \alpha\, G^{1/2} M_{\rm d}^{3/2} R_{\rm d}^{1/2}$$

and

$$E_{\rm d} = \beta\, G M_{\rm d}^{2} R_{\rm d}^{-1}$$

where α and β are two constants. You can assume that all stars are on circular orbits.

b) The 'spin parameter', λ, of a system, which is a unitless expression of the system's specific angular momentum, is defined by

$$\lambda \equiv \frac{J_{\rm d}\, |E|^{1/2}}{G M_{\rm d}^{5/2}}$$

Tidal torques, operating during the linear regime of structure growth, impart protogalaxies typically with spin parameters $0.01 < \lambda < 0.1$ with a mean value of $\bar{\lambda} \simeq 0.04$. For our self-gravitating, infinitessimally thin exponential disk, one has that $\alpha \simeq 1.11$ and $\beta \simeq -0.147$. What is the corresponding value for the disk's spin parameter? Discuss what is needed in galaxy formation, in addition to tidal torques, to establish such a value.

c) Assume that our disk galaxy formed out of a spherical density perturbation of uniform density which experienced turn-around at redshift $z_{\rm ta}$. What was the proper size of this sphere at the epoch of turn-around?

(Hint: at turn-around the overdensity is $\delta_{\rm ta} \simeq 1$)

d) Under the assumption that disk formation conserves angular momentum, compute the spin parameter of the overdensity at turn-around, as function of $z_{\rm ta}$. Discuss the implications for disk formation.

Problem 3: Consider a population of galaxies with a comoving number density $\bar{n} = 0.1h^3\text{Mpc}^{-3}$.
a) Suppose these galaxies are randomly distributed. In that case, what is the probability that a random galaxy has no neighbours closer than $1h^{-1}$Mpc.

b) What is the corresponding probability if the galaxies are clustered with a two-point correlation function

$$\xi(r) = \left(\frac{r}{r_0}\right)^{-\gamma}$$

with $r_0 = 2h^{-1}$Mpc and $\gamma = 1.8$.
(Hint: assume Poisson sampling; see section 6.1.2)

Problem 4: According to the spherical collapse model, all virialized haloes at redshift z have an average overdensity of $\Delta_{\rm vir}(z)$. Bryan & Norman (1998) have shown that the overdensity of haloes is well approximated by

$$\Delta_{\rm vir}(z) = (18\pi^2 + 82x - 39x^2)/\Omega_{\rm m}(z)$$

Here $\Omega_{\rm m}(z) = \bar{\rho}(z)/\rho_{\rm crit}(z)$ is the cosmological density parameter and we have used the short-hand notation $x = \Omega_{\rm m}(z) - 1$.
a) What are the virial radius and dynamical time of a spherical dark matter halo of mass $M_{\rm vir} = 10^{12}h^{-1}M_\odot$ in a ΛCDM cosmology with $\Omega_{\rm m,0} = 0.3$?

b) Numerical simulations have shown that the density distribution of virialized dark matter haloes is well fitted by the spherical NFW profile

$$\rho(r) = \frac{\rho_{\rm s}}{(r/r_{\rm s})(1 + r/r_{\rm s})^2}$$

where $r_{\rm s}$ is the scale radius, and $\rho_{\rm s}$ is some normalization density. The circular velocity of a spherical NFW profile can be written as

$$V_{\rm circ}(r) = V_{\rm vir}\sqrt{\frac{f(cy)}{yf(c)}}$$

Here $c = r_{\rm vir}/r_{\rm s}$ is the halo concentration parameter, $V_{\rm vir}$ is the circular velocity at the virial radius, and $y = r/r_{\rm vir}$. Derive the functional form of $f(x)$.

c) It is straightforward to show that $V_{\rm circ}(r)$ of a spherical NFW halo reaches a maximum, $V_{\rm max}$, at a radius $r \simeq 2.163r_{\rm s}$. Derive the relation between the ratio

$V_{\rm max}/V_{\rm vir}$ and the halo concentration c, and evaluate the result for $c = 5$and $c = 20$ (the typical range of halo concentrations for galaxy-sized dark matter haloes).

Problem 5: In the central region of the seyfert galaxy NGC 4258 astronomers have discovered a disk of masers, which can be observed using radio telescopes. High resolution observations with VLBI (Very-Long Baseline Interferometry) has revealed that the inner edge of the masing gas disk is about 0.004 arcsec from the center. By measuring the motion of the masing spots across the sky, Herrnstein et al. (1999) have obtained a distance to NGC 4258 of 7.2 Mpc, and have derived that the gas at the inner edge of the masing disk moves with circular velocities of ~ 1100km/s.

a) Estimate the mass enclosed by the inner edge of the disk.

b) Suppose that the central object is a star-cluster (of roughly uniform density) consisting of Solar-type stars. What is the upper limit on the time-scale for direct collisions among stars in this star cluster?

Problem 6: At what rate, in units of Eddington, does the SMBH in our MW, which has a mass of $M_{\rm BH} = 3 \times 10^6 M_{\odot}$, have to accrete matter, in order for its bolometric flux to be equal to that of the full moon? Here you may use that the full moon is about 400.000 times fainter than the Sun, and that the distance to the galactic center is 8 kpc. Ignore extinction, and use MBW, other books, or the internet for any other quantity you may need.

Problem 7: Student X, who has just finished Astro 530 with Prof. van den Bosch, attends a conference on dynamics on a beautiful tropical island in the Pacific. The first lecture of the first day has a distinguished observational astronomer from Harvard present evidence for a supermassive black hole in an elliptical in Virgo. The speaker shows a series of HST images of the elliptical, which reveal a small nuclear disk of ionized gas. The projected major and minor axes of the gas disk are 2.0 and 1.0 arcsec, respectively, and the observed, radial flux density distribution drops off as R^{-1}. Next the speaker shows a high resolution spectrum of the nuclear gas disk taken through a circular aperture with a diameter of 2 arcsec, which reveals the Hα line in emission. The LOSVD is Gaussian and has a FWHM of a staggering 600km/s! The speaker argues that the high velocity is due to gas on circular motion around a SMBH. Based on the FWHM and the size of the disk, the speaker argues for a BH mass of $M_{\rm BH} \simeq 2 \times 10^8 M_\odot$. Without hesitation, student X shouts out that this cannot be the entire story. "In fact", student X continues, "if there is indeed a black hole present, than the motion of the gas must largely be turbulent". Explain why student X is correct.

Deadline: Dec 1, 2010

Grading:

Problem 1: 15 pt
Problem 2a: 10 pt
Problem 2b: 10 pt
Problem 2c: 15 pt
Problem 2d: 15 pt
Problem 3a: 15 pt
Problem 3b: 20 pt
Problem 4a: 10 pt
Problem 4b: 5 pt
Problem 4c: 5 pt
Problem 5a: 5 pt
Problem 5b: 10 pt
Problem 6: 15 pt
Problem 7: 15 pt

TOTAL : 165 pt
