ASTRO 530: Problem Set 2

Problem 1: The main-sequence lifetime of stars scales with their zero-age main-sequence (ZAMS) masses roughly as $\tau_{\rm MS} \propto M^{-3}$. Under the assumption that all stars burn roughly the same mass fraction of their hydrogen into helium while at the main sequence, give a simple estimate of the value of $\alpha \equiv {\rm dln}L/{\rm dln}M$.

Problem 2: The gravitational potential energy can in general be written as

$$U = -\alpha \frac{GM^2}{r_{\rm e}}$$

with $r_{\rm e}$ the effective radius. Derive the value of α for a uniform sphere.

Problem 3: The surface photometry of a galaxy that appears circular on the sky reveals a surface brightness profile $I(R) \propto R^{-\gamma}$. Assuming that the galaxy is intrinsically spherical, the three-dimensional luminosity density $\nu(r) \propto r^{-\alpha}$. What is the relation between α and γ?

Problem 4: The luminosity function of galaxies is well-fit by a Schechter function

$$\Phi(L){\rm d}L = \Phi^* \left(\frac{L}{L^*}\right)^{\alpha+1} \exp\left[-\frac{L}{L^*}\right] \frac{{\rm d}L}{L}$$

Derive the corresponding LF in magnitudes, i.e., give $\Phi(\mathcal{M})\,{\rm d}\mathcal{M}$, where $\mathcal{M} = -2.5\log(L/L_\odot) + \mathcal{M}_\odot$.

Problem 5: The Poisson equation relates the gravitational potential, Φ, of an object to its density distribution, ρ, and is given by

$$\nabla^2\Phi = 4\pi G\rho$$

a) Show that in the case of spherical symmetry this implies

$$\frac{\mathrm{d}\Phi}{\mathrm{d}r} = \frac{GM(r)}{r^2}$$

where $M(r)$ is the mass enclosed by radius r.

b) Suppose that the gravitational potential is filled with some hot gas in hydrostatic equilibrium, which has temperature and gas density profiles of $T(r)$ and $\rho_{\mathrm{gas}}(r)$, respectively. Under the assumption that the gas obeys the ideal gas law, show that one can infer the enclosed mass of the halo from

$$M(r) = -\frac{k_{\mathrm{B}}T(r)\,r}{\mu m_{\mathrm{p}}G}\left[\frac{\mathrm{dln}\rho_{\mathrm{gas}}}{\mathrm{dln}r} + \frac{\mathrm{dln}T}{\mathrm{dln}r}\right]$$

where μ is the mean molecular weight in units of the proton mass m_{p}.

c) What is the mean molecular weight μ for a fully ionized, primordial (i.e., zero metallicity) gas with a helium mass fraction of $Y = 0.25$?

d) For an imaginary, spherical cluster, X-ray observations have indicated that its X-ray emitting gas has a temperature of 3×10^7K, and a gas density profile $\rho_{\mathrm{gas}} \propto r^{-2}$. What is the mass of the halo within $1.5h^{-1}$Mpc under the assumption that the gas is fully ionized, and has zero metallicity with a helium mass fraction of $Y = 0.25$.

Problem 6: In an Einstein-de Sitter (EdS) universe ($\Omega_{\mathrm{m}} = 1$, $\Omega_\Lambda = 0$), two identical sources, 1 and 2, have an SED $L_\nu \propto \nu^{-\alpha}$, and are located at $z_1 = 1.0$ and $z_2 = 2.0$, respectively. For what value of α do both sources have the same apparent magnitudes.

(Hint: Use Eqs. [3.13], [3.75] and [3.104] and take k-corrections into account).

Useful Numbers

$k_{\mathrm{B}} = 1.381 \times 10^{-16}\,\mathrm{erg\,K^{-1}}$
$m_{\mathrm{p}} = 1.673 \times 10^{-24}\,\mathrm{g}$
$G = 4.3 \times 10^{-9}\,(\mathrm{km/s})^2\,\mathrm{M_\odot^{-1}\,Mpc}$

Deadline: Oct 15, 2010

Grading:

Problem 1: 5 pt
Problem 2: 10 pt
Problem 3: 5 pt
Problem 4: 10 pt
Problem 5a: 5 pt
Problem 5b: 15 pt
Problem 5c: 20 pt
Problem 5d: 5 pt
Problem 6 : 25 pt

TOTAL : 100 pt