Astro 418/518: Stellar Dynamics

Problem Set #2
Due January 31, 2012

1. The density profile for a single power-law distribution is given by:

$$\rho(r) = \rho_o (\frac{r}{r_o})^{-\alpha}$$

Show that the circular velocity for this generalize density profile is:

$$v_c^2(r) = \frac{4\pi G \rho_o r_o^\alpha}{3-\alpha} r^{2-\alpha}$$

Plot the density and circular velocity as a function of radius for this distribution in the case where $\alpha = 2$ (singular isothermal sphere) and $\alpha = 0$ (homogeneous sphere). Compare (over-plot) these two profiles with a double power-law NFW profile. (Hint: set all constants equal one and compare the relative profiles in a log-log plot).

2. Assume the mass of the Milky Way Galaxy is dominated by dark matter outside the Sun's orbit having a spherical radial density profile which is isothermal (v_c = const.) and cuts off at 10 R_{sun}. Assume the Sun rotation velocity is 220 km s^{-1} at a distance 8 kpc from the Galaxy center.

a. What is the rotation period at the Sun. How many orbits (roughly) since the Galaxy formed?

b. What is the total mass within the Sun's orbit?

c. What is the escape velocity from the Solar neighborhood? Find this by deriving the form of the potential versus radius between R_{sun} and the outermost radius, and then the form of the potential beyond this. Set the potential at infinity to zero. Find the other constant in the potential law by matching the rotation speed between R_{sun} and 10 R_{sun}. Then use the potential at R_{sun} to find v_{esc}.

d. What is the orbital period of a typical dwarf satellite galaxy at 100 kpc (assume circular orbits)? How many orbits since this galaxy formed?

e. What is the escape velocity for this satellite?

3. Derive the circular velocity due to the Plummer potential:

$$\Phi(r) = -\frac{GM}{\sqrt{b^2 + r^2}}$$

where b is a constant and plot (or sketch) the results. Obtain the density profile that gives rise to this potential. Show that M is the mass of the system.