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Stellar population explosions in proto-elliptical galaxies

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Summary. A model is proposed for the formation of elliptical galaxies by a hierarchical sequence of mergers among subsystems. Stars are assumed to form in bursts induced by the collisions. The mergers build up systems of increasing total mass, with an increasing mass fraction in stars and increasing metallicities of stars and residual gas. The empirical correlation between metallicity and mass for elliptical galaxies can be explained if the efficiency of star formation increases with the mass of the merging systems, as is plausible dynamically. Star formation in bursts eventually ceases, either because there are no more mergers or because the gas has become too hot to cool between collisions. A disk may then form by accretion of diffuse outlying gas after the violent mergers have stopped, whereas an elliptical galaxy remains if the residual gas is swept away before forming a disk. The model thus accounts for the relative paucity of disk galaxies in dense clusters and for metal-rich intracluster gas; it also predicts that the spheroidal components of spiral and S0 galaxies are identical to elliptical galaxies. The extended time during which stars can form in growing ellipticals means that primeval galaxies may not be spectacular objects at large redshifts; on the other hand, some of the very blue galaxies seen at moderate redshifts could be ellipticals still forming by mergers with bursts of star formation.

An appendix gives a simple theoretical prediction of how the efficiency of star formation in bursts might depend on the mass of the system, as suggested by the results of 3-dimensional collapse simulations.

1 Introduction

The formation of galaxies has usually been viewed as involving the collapse of large gas clouds, whose masses are essentially those of the final galaxies formed. However, recent discussions have emphasized that a detailed picture of galaxy formation should include the effects of non-uniform internal structure and of violent interactions and mergers among subsystems (e.g. Toomre 1977; Larson 1977). In this paper, we suggest that such interactions are important not only for the dynamics but also in driving star formation and chemical evolution, and we show by a simple model how this hypothesis can account for the correlation between mean metallicity and mass of elliptical galaxies.

Evidence pointing toward such a picture includes the following. Within a massive protogalactic cloud the Jeans mass may be only $\sim 10^7$–$10^9 M_\odot$ (Larson 1969), so that the matter is probably highly clumped on scales much smaller than the final galactic mass; thus most of the star formation may occur in dense subsystems that later merge as the collapse proceeds (Toomre 1977, p. 420; Larson 1977). Fall & Rees (1977) and Rees (1977) have pointed out that, if small stellar systems combined at an early epoch to form a galaxy, only those of globular cluster size could have survived to the present, tidal disruption and internal relaxation having destroyed the clumping on larger and smaller scales respectively; direct evidence for such a galaxy formation process would therefore have been almost obliterated. Some indirect consequences could nevertheless be important. For example, Searle (1977) has noted that the metallicity distribution of globular clusters can be understood if the halo stars of our Galaxy formed in many independent small subsystems; and Silk (1978) has suggested that coalescence of protogalaxy fragments may account for the luminosity function of ellipticals in rich clusters, the increase of metallicity with mass, and an enriched intergalactic medium. (White & Rees (1978) have shown that the latter features can also be explained in a model where dark halos undergo mergers, and galaxies form from gas in their transient potential wells.) Collisions between galaxies appear to have been common enough to have affected a substantial fraction of all systems in a Hubble time (Toomre 1977; Vorontsov-Velyaminov 1977), so mergers could be important throughout the lives of galaxies. For example, it has been suggested that cD galaxies grow continually by accretion of surrounding cluster members (Ostriker & Hausman 1977; Hausman & Ostriker 1978) and that many ellipticals result from collisions between disk galaxies (Toomre 1977).

The model proposed in this paper is an extension of the preceding ideas, differing from most of them in that continuing star formation is envisioned to play a central role. We postulate that elliptical galaxies are built by successive mergers between systems of comparable size, and that bursts of star formation are caused by the mergers. Evidence that mergers can induce rapid star formation is provided by the fact that many interacting galaxies have *UBV* colours suggestive of recent bursts (Larson & Tinsley 1978). The model begins with the coalescence of small gas clouds, and star formation at each stage causes the consumption of some residual gas and an increase in the mean metallicities of stars and gas. This model can reproduce the empirical metallicity–mass relation of ellipticals if the efficiency of star formation increases with the mass of the merged system, as is plausible dynamically. Eventually star formation in bursts stops, either because the residual gas becomes too hot to form stars and is swept away, or because there are no more mergers. Some of the ellipticals formed in this way may become the spheroidal bulges of spirals and S0s, if diffuse outlying matter is accreted gradually into a disk with relatively slow star formation, as in earlier collapse models (Larson 1976a; Tinsley & Larson 1978).

We emphasize that in this model most large galaxies are not built simply out of smaller units that could be identified with present-day galaxies. The subsystems that merge to form a given galaxy are generally more gas-rich than any observed galaxy, and most of the stars in the resulting galaxy are formed subsequently from the residual gas. Therefore only a small fraction of a given galaxy's halo population would be represented in the halos of much smaller galaxies. The present model thus answers, at least to some extent, an objection to merger models raised by van den Bergh (1975), who pointed out several significant differences between the halo populations of the Galaxy and the Magellanic Clouds. Differences are expected in the present picture, since most of the galactic halo stars formed in mergers of subsystems more massive than the Magellanic Clouds.

The plan of the paper is as follows. The model for formation of ellipticals is described and worked out for a simple illustrative case in Section 2. Section 3 considers further aspects and implications of the merger picture, including the cutoff of star formation due to excessive heating of the gas in collisions (Section 3.1), the formation of disks and the relations between galaxy morphology and environment (Section 3.2), enrichment of the intergalactic medium (IGM) by swept-out gas (Section 3.3), and some predictions for the appearance of galaxies at large redshifts (Section 3.4). The main results are summarized in Section 4. An appendix gives a simple theoretical prediction of how the efficiency of star formation in bursts may increase with the mass of the system.

2 The proposed model

2.1 MERGERS AND STAR FORMATION

We assume for simplicity that matter is initially clumped into gas clouds of equal mass M_0, which may be the Jeans mass at some epoch before any star formation has taken place. We also assume that subsequent mergers occur only between subsystems of equal mass, so that a galaxy is built up by a hierarchy of mergers in which the masses of the subsystems increase by a factor of 2 at each stage. The mass of the subsystem or galaxy resulting after n stages is therefore

$$M_n = 2^n M_0. \tag{1}$$

The initial mass M_0 is likely to vary according to the density of each region, but the results are very insensitive to its value. A realistic model would of course allow for mergers among unequal systems, but equation (1) provides a simple illustration of the main effects.

A similar hierarchical coalescence model has been suggested by White & Rees (1978) for the formation of massive invisible halos of galaxies. In the present paper we are concerned only with the visible parts of galaxies, but our discussion would be unaltered if the processes we consider were taking place within extended potential wells produced by dark halos. Throughout this paper, the masses referred to are the masses of ordinary stars and gas, excluding any dark halo.

It may be noted that the validity of the hierarchical coalescence picture does not necessarily require that the dynamical or free-fall time be a monotonically increasing function of system size, so that small systems can collapse before large ones. This is because, even after collapse has occurred on a large scale, the result is likely to be the formation of many small condensations or subsystems, and it is only after these have had time to interact and merge that a larger galaxy-like system can be said to have formed. If the cross-sections are small and/or the relative velocities are large, the time required for interactions between subsystems to cause coalescence can be much longer than the free-fall time, as in the case of the formation of cD galaxies in large clusters.

The predicted relation between metallicity and mass of elliptical galaxies depends crucially on the fraction of residual gas converted to stars in each merger, i.e. on the efficiency of star formation as a function of the mass of the system. We have sought a star-formation 'law' that leads to a metallicity–mass relation approximating a power-law, a form suggested by the linearity of the observed colour–magnitude relation (Visvanathan & Sandage 1977). Trials with a variety of models show that this requirement is met only if the efficiency of star formation increases with mass, that is, if successive mergers cause increasing fractions of the residual gas to be converted to stars. In the Appendix it is shown to be plausible theoretically that the efficiency of star formation should increase with the mass of the system.

As an example, we assume that the efficiency of star formation varies as a power p of the total mass of the system. The amount of star formation in each merger is then given by

$$\frac{\text{mass of stars formed}}{\text{mass of gas present}} \propto (\text{total mass})^p$$

or

$$\Delta M_s/M_g = (M/M_e)^p, \tag{2}$$

where the parameter M_e is chosen to give the correct scale for the resulting metallicity–mass relation (see below). It can be shown from equations (1) and (2) that after the nth merger the stellar mass M_{sn} and the gas mass M_{gn} are given by the recurrence relations

$$M_{sn} - 2M_{s,\,n-1} = 2M_{g,\,n-1}(M_n/M_e)^p = 2^{1+np}M_{g,\,n-1}(M_0/M_e)^p, \tag{3}$$

and

$$M_{gn} = 2M_{g,\,n-1}[1 - 2^{np}(M_0/M_e)^p]. \tag{4}$$

The gas fraction μ_n is given by

$$\mu_n = M_{gn}/M_n. \tag{5}$$

The initial values are $M_{g0} = M_0, M_{s0} = 0, \mu_0 = 1$.

The first three columns of Table 1 give the solutions to these equations for M_n and M_{sn}, with $p = 1/3$ (see below), $M_0 = 1.2 \times 10^7 M_\odot$, and $M_e = 3.2 \times 10^{12} M_\odot$; M_{gn} can be obtained from these values using $M_{gn} = M_n - M_{sn}$.

Table 1. Properties of a sequence of mergers with star formation.

Merger no, n	Log $M_n/M_\odot$	Log $M_{sn}/M_\odot$	Log Z_{gn}/y	Log Z_{sn}/y	Log R (pc)	Min. log $t_{\rm dyn}$ (yr)	Max. log $t_{\rm cool}$ (yr)
0	7.08	$-\infty$	$-\infty$	$-\infty$	–	–	–
2	7.68	6.33	−1.35	−1.65	(3.2)	(8.2)	–
4	8.29	7.33	−0.93	−1.24	(3.4)	(8.2)	–
6	8.89	8.20	−0.63	−0.95	(3.6)	(8.2)	4.7
8	9.49	9.02	−0.38	−0.71	(3.8)	(8.1)	4.9
10	10.09	9.80	−0.14	−0.50	4.0	8.1	4.7
12	10.69	10.54	+0.09	−0.31	4.2	8.2	5.7
14	11.30	11.24	+0.33	−0.15	4.5	8.3	6.7
16	11.898	11.888	+0.58	−0.04	4.8	8.4	8.1
17	12.199	12.197	+0.73	−0.01	4.9	8.5	9.1

2.2 CHEMICAL EVOLUTION

The massive stars formed in each merger eject new metals that enrich the residual gas and subsequent generations of stars. We assume that new metals are mixed instantaneously and uniformly throughout the gas, and that the initial mass function of stars is invariant. It can then be shown that chemical evolution follows the usual 'simple' model, provided that mergers are always between identical systems. Therefore the metallicity of the gas after the nth merger is given by Searle & Sargent's (1972) equation,

$$Z_{gn} = y \ln \mu_n^{-1}, \tag{6}$$

where y is the yield, i.e. the mass of metals ejected per unit mass of gas converted to stars. Since the total mass of metals ejected from stars up to the nth stage is yM_{sn}, and the gas contains a mass $Z_{gn}M_{gn}$ of these metals, the average metallicity of stars is given by

$$Z_{sn}M_{sn} = yM_{sn} - Z_{gn}M_{gn}. \tag{7}$$

From equations (6) and (7), we have then

$$Z_{sn} = y\left[1 - \frac{\mu_n}{1-\mu_n}\ln \mu_n^{-1}\right]. \tag{8}$$

At early stages, when $M_s \ll M$, equations (3)–(8) lead to the asymptotic power-law relation $Z_s \propto M_s^{p/(1+p)}$. For comparison, the colour–magnitude relation for elliptical galaxies has been tentatively calibrated as a metallicity–mass relation of the form $Z_s \propto M_s^{0.25}$ (Tinsley 1978b). This would be predicted if $p = 1/3$, i.e. if the efficiency of star formation in mergers varies as $M^{1/3}$, so we adopt this star-formation 'law' in the present model. A possible theoretical justification of this law is given in the Appendix. Only a few details of the model would need to be changed if the colour–magnitude relation or its calibration were revised.* Although the specific model given here is for mergers of equal subsystems at each stage, very similar results are obtained in models with unequal subsystems provided that one assumes an equivalent relation between the total mass after merger and the fraction of added gas that is turned into stars. For example, the limiting case of continuous accretion of metal-free gas, with a star-formation law $dM_s/dM \propto M^{1/3}$, leads to a relation between Z_s and M_s that is almost identical to that derived here.

Fig. 1 shows the metallicities of stars and gas as a function of the total mass in stars at any stage, as calculated from the above equations with $M_0 = 1.2 \times 10^7 M_\odot$, $M_e = 3.2 \times 10^{12} M_\odot$, and $y = 0.02$. The results are also included in Table 1. The gas is always more metal-rich than the average for stars, since its metallicity increases monotonically with time. Since we picture all ellipticals forming by similar merger sequences that end with different total masses, this figure also gives the predicted metallicities of ellipticals as a function of mass. The stellar systems have the required approximately power-law relation between mean metallicity and mass, with a turnover toward the asymptotic value $Z_s \rightarrow y$ given by equations (2)–(7) as $M_s \rightarrow M \rightarrow M_e$ and $\mu \rightarrow 0$. The cutoff and dashed lines shown at $10^{12} M_\odot$ represent possible effects of cessation of star formation, as discussed in Section 3.

The progressive enrichment occurring during mergers might also produce metallicity gradients in elliptical galaxies, since the cool gas that forms new stars at each stage is likely to be more centrally condensed than the older, less metal-rich stars, as in the gas-dynamical collapse models of Larson (1974a). If so, the present picture predicts that the radial abundance gradients in elliptical galaxies should mimic the abundance variations among ellipticals of different masses. Empirical evidence on this point has been ambiguous, but in a recent review Faber (1977) concludes that the same composition parameters seem to be involved in the two cases. Gradients in galactic disks could of course have a different character, because stars with much longer lifetimes can contribute to the enrichment.

Further aspects of chemical evolution will be considered in the next section, in discussing the possible fates for gas left over from the formation of elliptical galaxies.

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* The calibration in Tinsley (1978b) was based on the slope of the $(B-V, M_V)$ relation from Sandage (1972). Visvanathan & Sandage's (1977) data for $b-V$ colours might correspond to a flatter $(B-V, M_V)$ relation (Sandage, private communication), which would imply a smaller power than 0.25 for the (Z_s, M_s) relation.

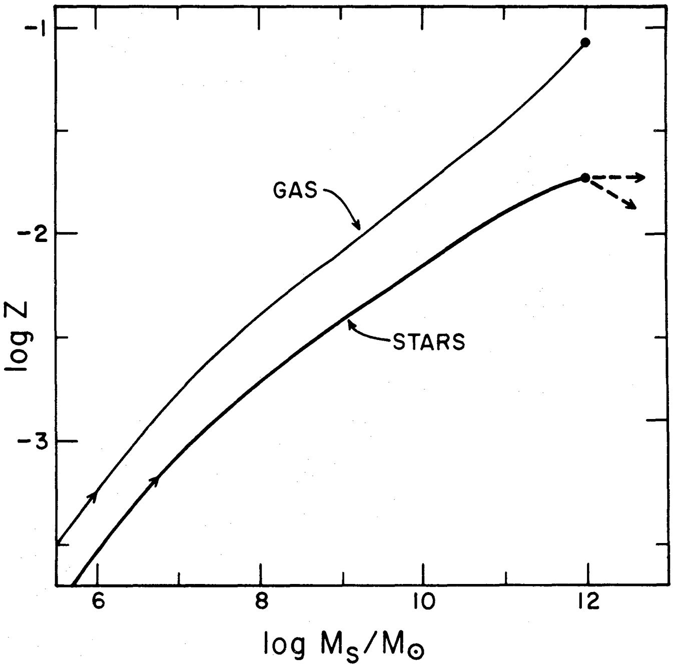

Figure 1. Metallicities of stars and gas as a function of the total mass of stars, in the model of Section 2 for the formation of elliptical galaxies. This plot represents both the chemical evolution of a given system growing by mergers, and the metallicities of stars and residual gas of elliptical galaxies with different final masses. The filled circles show the point beyond which there will be little star formation in mergers, because the gas cannot cool sufficiently between collisions; arrows indicate possible outcomes of further mergers without star formation (Section 3.1).

3 Further results and implications

3.1 CESSATION OF STAR FORMATION

Collisions between galaxies tend to shock-heat the gas to approximately the virial temperature of the merged systems, and eventually the gas may become so hot that it cannot cool enough to form stars before the next collision, especially if it is not already very clumpy. Core velocity-dispersions of elliptical galaxies are observed to increase with luminosity (Faber & Jackson 1976; Sargent *et al.* 1977), so presumably the virial temperature increases as mergers proceed, and excess heating becomes an increasing hazard to star formation. This effect is enhanced by the declining fractional gas mass, so it is unlikely that the gas will ever be completely turned into stars in successive mergers.

We have estimated the earliest stage at which heating could intervene, by calculating the maximal cooling time and comparing it with the minimal time between collisions at each stage. The cooling time t_{cool} is calculated on the assumptions that the gas is heated to the virial temperature of the merged system and is uniformly dispersed throughout its volume, and the dynamical time t_{dyn} is taken to be the free-fall time. Radii of systems of different mass are estimated from Oemler's (1976) 'maximum radii' and 'total magnitudes' of elliptical galaxies, using $H_0 = 50\ \text{km s}^{-1}\ \text{Mpc}^{-1}$. The cooling rate, taken from Cox & Tucker (1969), is dominated by the contribution of heavy elements, whose abundances are given by the values of Z_{g} in Section 2.2. Table 1 gives the radii and times found in this way. (Radii and values of t_{dyn} are shown in brackets for systems with $R < 10$ kpc because of the paucity of data in that range, and cooling times are given only for systems with virial temperatures above 10^4 K.) It is seen that $t_{\text{cool}} \ll t_{\text{dyn}}$ for small galaxies, implying that heating does not prevent star formation in such systems, but the ratio $t_{\mathrm{cool}}/t_{\mathrm{dyn}}$ increases with mass until the two times become equal at $10^{12} M_{\odot}$. This critical mass happens to be nearly the same as Rees & Ostriker's (1977) and Silk's (1977) values for self-gravitating hydrogen-helium gas clouds, because the enhancement of cooling by metals in our models is compensated by the smaller gas fraction; slightly larger critical masses were found by White & Rees (1978) for gas initially at the virial temperature of a heavy halo. The cooling cutoff in our model would have occurred sooner for a smaller value of M_e, so the continued increase of colour index with luminosity up to nearly the brightest cluster galaxies (e.g. Visvanathan & Sandage 1977) suggests that M_e should not be smaller than $10^{12} M_{\odot}$. Similarly, raising M_e would give a greater critical mass, but if $M_e > 10^{13.5} M_{\odot}$, the predicted metallicity of giant elliptical galaxies becomes implausibly low.

There may not be a complete cutoff in star formation at the above critical point, since some cool gas may survive in dense clumps and some mergers may take longer than the free-fall time, but mergers beyond $\sim 10^{12} M_{\odot}$ are likely to result in much less efficient star formation than occurred earlier. The residual gas (which is 1 per cent of the total mass at $10^{12} M_{\odot}$, in the numerical example of Table 1) will become hotter with each additional merger, until it is eventually dispersed into an ambient IGM (see below, Section 3.3).

Mergers that are not accompanied by star formation lead to galaxies with a smaller metallicity for their mass than those found according to the model in Section 2. In Fig. 1, the critical mass of $10^{12} M_{\odot}$ is marked by a filled circle, and the dashed arrows indicate possible results of further mergers with stellar systems of the same mass (horizontal arrow) or a smaller mass (sloping arrow). The latter is the process proposed for the formation of cD galaxies by Hausman & Ostriker (1978). A flattening, or even a turnover, of the colour–magnitude relation is thus predicted for the most massive galaxies, and this effect can be seen in several sets of data (e.g. Faber 1973; Visvanathan & Sandage 1977). However, a similar effect would be predicted even if there were no cutoff to star formation in mergers, since the value of Z_s must tend asymptotically to the yield y as the gas is consumed; a flattening of the colour–magnitude relation is thus not an unambiguous indication that cD galaxies form by purely stellar-dynamical processes. It is also predicted in collapse models with supernova-driven winds (Larson 1974b; Tinsley 1978b), and must occur in any model, since the limit $Z_s \rightarrow y$ as $M_g \rightarrow 0$ is predicted by the model-independent equation (7) above.

Smaller ellipticals may sometimes have lower metallicities than predicted by our model, if, like the galaxies above $10^{12} M_{\odot}$ in Fig. 1, they have gained mass partly by mergers after their gas was swept away. This may account in part for the observed scatter of ellipticals below the mean colour–magnitude locus, although as noted by Faber (1977) and Burstein (1977), bluer colours could also be due to residual star formation in some cases.

3.2 DISK GALAXIES AND THE ENVIRONMENT

In the collapse models of Larson (1976a), a galaxy acquires a disk if it has residual outlying gas that dissipates its energy and settles into a plane on a longer time-scale than that for the formation of the spheroidal component. Disks can form similarly in the present scheme, if violent mergers between comparably massive systems stop occurring and there are still either residual gas or outlying gas-rich small subsystems that continue to be accreted. Gradual accretion of outlying material may still be occurring in present-day spirals, either as continuing infall of diffuse gas (Larson 1972; Saar & Einasto 1977) or as the accretion of small satellites like the Magellanic Clouds (Tremaine 1976). S0 galaxies can then be produced as in the usual picture by the later sweeping of gas out of the disks of spirals (Gunn & Gott 1972).

This model predicts that the spheroidal bulges of spiral and S0 galaxies should be essentially identical to elliptical galaxies of the same mass. There is some evidence in support of this prediction. For example, the nuclear stellar populations of M31 and the Galaxy closely resemble those of ellipticals (Faber 1973; Whitford 1978); the central regions of S0s follow nearly the same mean colour–magnitude relation as ellipticals (Visvanathan & Sandage 1977); and the nuclear bulges of spirals show an increase of spectral line strength with bulge size (McClure, private communication).

Disk formation by the accretion process suggested here is less likely to occur in dense clusters of galaxies than in the field or in sparse groups, since the outlying gas will tend to be swept away by the cluster IGM. Thus we can account qualitatively for the smaller proportion of disk galaxies (S + S0) relative to ellipticals in dense clusters than in low-density regions (Oemler 1974; Butcher & Oemler 1978b). (A similar explanation has been suggested by Ostriker (1977), on the basis of Ostriker & Thuan's (1975) model for disk formation from gas shed by halo stars.) Galaxies without any neighbours of comparable size are unlikely to be moving through an IGM that could sweep them, so they can retain their residual gas and form disks. This prediction is borne out by the rarity of isolated ellipticals (Huchra & Thuan 1977). In the most massive systems, the residual gas would be too hot to form stars in a Hubble time, and it could remain in a hot corona; perhaps M87 (Mathews 1978) is an example of this situation.

The phenomenon of hypergalaxies (Saar & Einasto 1977) fits naturally into this scheme: the dwarf companions surrounding large galaxies are simply surviving subsystems that have not yet merged with the main agglomerate. Einasto and coworkers (Chernin, Einasto & Saar 1976; Saar & Einasto 1977) have noted that the closer and/or less massive companion galaxies are ellipticals while the further and/or larger ones are spirals and irregulars. Their explanation in terms of stripping of the former by a corona of gas around the central large spiral is quite similar to the above picture for the formation of different morphological types.

The gas that forms a disk may have been partly enriched by ejecta from the spheroidal bulge, as in gas-dynamical collapse models (Tinsley & Larson 1978) or in Ostriker & Thuan's (1975) model of disk formation. Star formation in the disk will in any case eventually raise the mean metallicity of disk stars toward the yield value. Wide-aperture observations should therefore show disk galaxies to be more metal-rich than ellipticals of the same mass. This may account for the many S0s whose colours are redder than typical ellipticals of the same magnitude (Burstein 1977). Some S0s are bluer than ellipticals of the same magnitude (Faber 1977; Burstein 1977); these could be experiencing residual star formation, or could have been stripped so recently that they have bluer turnoff stars than ellipticals.

Our picture thus places most ellipticals either on or below the 'merger locus' for stellar metallicities in Fig. 1, while disk systems tend to lie above that line because they succeed eventually in converting a greater fraction of their initial gas into stars.

3.3 ORIGIN AND ENRICHMENT OF THE INTERGALACTIC MEDIUM

An essential factor in the formation of an elliptical galaxy, in the present scenario, is the eventual loss to the IGM of its residual metal-enriched gas. The model in Section 2 predicts the mass and metallicity of the gas left over from an elliptical of given final mass, so, using the mass function for galaxies, we can estimate the total amount and metallicity of IGM that they contribute to a cluster. Similar predictions were previously made by Larson & Dinerstein (1975) from models with supernova-driven mass loss; and, following the subsequent discovery of the iron X-ray feature in clusters of galaxies, several authors explained these observations with similar models and results (e.g. Vigroux 1977; Silk 1978; De Young 1978). Such calculations are insensitive to details of the models, because the metallicity of the gas lost is always closely related to the mean stellar metallicity.

For the present calculation we have used the numerical results of Table 1, together with a luminosity function from Felten (1977) and a mass–luminosity scaling from Faber & Jackson (1976), normalized to $M/L_B = 7\, M_\odot/L_\odot$ at $M = 10^{11}\, M_\odot$. The results depend on the parameter M_e: greater values of M_e imply less efficient star formation, so for a galaxy of given mass more gas is lost but its metallicity is lower. For the range $10^{12}\, M_\odot < M_e < 10^{13.5}\, M_\odot$ discussed in Section 2, we find that a population of elliptical galaxies loses a gas mass of 0.1 to 0.5 times the total stellar mass, including a mass of metals that is 0.2 to $0.5\, Z_\odot$ times the stellar mass. It is useful to normalize these results to the integrated luminosity of a cluster of galaxies, taking into account that only 1/3 of the galaxies in a typical cD cluster are ellipticals (Oemler 1974), the rest being spirals and S0s with ~ 50 per cent of their luminosity in a bulge or 'elliptical' component (Freeman 1970; Spinrad *et al.* 1978). We then find that ellipticals and spiral bulges contribute a mass of IGM that is (in solar units) about 0.7 to 3 times the luminosity of the cluster, with a metallicity of 0.8 to $1.6\, Z_\odot$. (If the gas lost from bulges of spirals and S0s is incorporated in their disks, these mass estimates should be halved.)

The mass of intergalactic gas derived in theoretical studies of the X-ray emission from the Coma cluster (e.g. Bahcall & Sarazin 1977; Cowie & Perrenod 1978; Perrenod 1978) is very model-dependent. Excluding unrealistic isothermal models with a formally infinite mass, the IGM mass lies in the range 1.5–30 times the cluster luminosity in solar units, allowing $50 < H_0 < 100$, and its iron abundance is within a factor of 2 of solar. The predicted early gas-loss from elliptical galaxies may therefore account for the IGM and for its metal content, if the actual gas mass is at the lower end of the range from cluster models.

Later gas ejection by evolving stars will add about as much IGM again as the initial loss, i.e. a few times the cluster luminosity in solar units (Tinsley 1973), with a slightly lower mean metallicity (Z_s rather than Z_g, as illustrated in Table 1 and Fig. 1). Cowie & Perrenod (1978) and Perrenod (1978) have constructed models for the Coma cluster in which the source of IGM is continuous ejection from stars, at a rate normalized to account for the X-ray emission; the required present IGM mass is 5–15 per cent of the *virial* mass of the cluster (for $50 < H_0 < 100$), corresponding to ~ 10–25 times its luminosity. As those authors conclude, such models are implausible because stellar ejection probably provides too little gas by an order of magnitude.[*] The later sweeping of spirals to form S0s probably makes an even smaller contribution, since spiral galaxies have typically less than 10 per cent of their mass in gas.

Early gas loss from elliptical galaxies (including those destined to become bulges of spirals) therefore appears as the most significant galactic source of IGM. The possibility that galaxies do not provide enough IGM, which is suggested by some models of cluster X-ray emission, is interesting since it would imply a significant amount of leftover gas that was never incorporated into galaxies.

3.4 THE APPEARANCE OF DISTANT YOUNG GALAXIES

The present model of galaxy formation leads to some novel predictions for the appearance of young galaxies at large redshifts. Differences from earlier predictions arise mainly because

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[*] Cowie & Perrenod (1978) and Perrenod (1978) suggest that gas loss from a virial mass of ordinary halo stars, as discussed by Ostriker & Thuan (1975), could supply an IGM of 5–15 per cent of the cluster's virial mass. This seems very implausible, since halos made of ordinary stars (unlike halos of hypothetical hidden mass) would have an M/L comparable to that of elliptical galaxies and, with the virial mass, would provide more intergalactic light than is observed in the Coma cluster (Melnick, White & Hoessel 1977).

the time-scales for mergers can considerably exceed the collapse times of conventional models, so young elliptical galaxies can have less spectacular early bursts of star formation. This is important because the luminosity of a galaxy whose light is dominated by young stars is proportional to the star-formation rate. If mergers occur in times proportional to the free-fall times of the resulting elliptical galaxies (e.g. in a few orbital times), then star formation occurs at an increasing rate right up to the cutoff in each case; the maximum rate reached by a given elliptical is nevertheless smaller than that found in a conventional collapse model.

The search for massive primeval galaxies, usually envisaged as collapsing gas clouds with very rapid star formation (e.g. Partridge & Peebles 1967; Meier 1976; Sunyaev, Tinsley & Meier 1978), becomes more difficult in the present picture. The most massive galaxies are here predicted to form with negligible new star formation as agglomerates of smaller galaxies, the process continuing unspectacularly today in dense clusters. Similarly, it is unlikely that very young ellipticals would contribute significant features in galaxy count–magnitude relations, as previously predicted (Tinsley 1978a), because in the present model their period of active star formation is smeared out over a large redshift range.

A consequence of these changes is that not only disks but also the spheroidal components of some galaxies may remain relatively blue and bright up to recent epochs. Some of the unexpectedly blue galaxies found in Kron's (1977) deep counts and by Butcher & Oemler (1978a) in clusters at redshifts near 0.4 may thus be ellipticals still undergoing energetic mergers and star formation.

Finally, some cosmological observations might be affected by the extreme clumpiness at early times of systems that today are a single large galaxy. Galaxy counts would be affected at faint magnitudes if pieces that are destined to become a single galaxy are counted separately; tests based on the calculations of Tinsley (1978a) show that either more or fewer galaxies than normally predicted could be observed, depending on such parameters as the redshift at which most stars form, and the time-scale for mergers. Another possibility is that the multiplicity of quasar absorption redshifts (e.g. Boroson *et al.* 1978) will be easier to understand if there are intervening clumpy protogalaxies that have not yet merged extensively; and the narrowness of individual absorption lines might be explained if each is formed in a subsystem whose internal velocity dispersion is characteristic of a small galaxy.

4 Conclusions

According to Toomre (1977, p. 420), 'It seems almost inconceivable that there wasn't a great deal of merging of sizeable bits and pieces (including quite a few lesser galaxies) early in the career of every major galaxy . . . the few mergers we are still privileged to witness are just the statistical dregs of a once very common process.' Evidence from *UBV* colours that tidal interactions and mergers often induce bursts of star formation in galaxies was found by Larson & Tinsley (1978), who suggested that such bursts might have been responsible for the rapid early star formation required in elliptical galaxies. In the present paper we have suggested a model in which elliptical galaxies form by a hierarchy of mergers of subsystems of increasing size, each merger producing a burst of star formation; and we have shown that the observed metallicity–mass relation for elliptical galaxies follows directly from this picture if the efficiency of star formation in the bursts increases with the mass of the merging subsystems.

The general picture of galaxy formation by mergers or gas accretion accompanied by star formation seems almost inescapable, as it combines features of most of the theoretical models that have been proposed, and allows simple explanations of many observed properties of galaxies. The 'violent relaxation' associated with the merging of stellar systems can produce a smooth halo, while gaseous dissipation causes the remaining gas to form a more condensed and metal-rich core. As long as mergers continue, formation of a permanent disk component is prevented and an elliptical-like stellar system is built up, but if eventually all the available systems are accreted and mergers cease, the leftover gas can settle to a disk. In a dense cluster, mergers may continue up to masses greater than $10^{12} M_\odot$, in which case the shocked gas becomes too hot to form a disk or new stars, and instead becomes part of a hot intergalactic medium. This heating process, together with sweeping of galaxies by the intergalactic gas, can account for both the existence of metal-rich hot gas and the relative paucity of disk galaxies in dense clusters.

Two specific predictions of this picture that can be tested are that the bulge components of spiral galaxies should be identical to elliptical galaxies, and that composition gradients in elliptical galaxies should involve the same parameters as the variations between galaxies of different mass. The latter prediction follows because the same processes that produce chemical enrichment within galaxies also make the larger galaxies more metal-rich. Present data seem to be in accord with these predictions, but more definitive tests would be valuable.

There are important theoretical implications if star formation occurs in bursts induced by violent events, rather than (as in the conventional view) continuously at a rate depending on parameters such as the gas density. In the present picture the most important aspect of star formation is not its rate but its efficiency in bursts, and the most important parameter is not the density but the mass of the interacting subsystems. A star formation efficiency proportional to $M^{1/3}$ can account for the observations, and the plausibility of a star-formation 'law' of this form is supported by the theoretical discussion in the Appendix. Although this functional form must be considered quite tentative, we suggest that this is the type of relation that star formation theories should attempt to justify.

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Appendix: efficiency of star formation in bursts

In Section 2 and in earlier discussions (Larson 1976b; Larson & Tinsley 1978) it was suggested that star formation in proto-elliptical galaxies occurs in bursts triggered by collisions between different parts of the system. Such collisions generate strong shock fronts, and since the shocked gas typically cools rapidly, it is also strongly compressed; if the isothermal sound speed in the cooled gas is c and the collision velocity is u, the gas density is increased by a factor $\sim (u/c)^2$. If u is comparable to the virial velocity of the system, the degree of compression and hence the efficiency of star formation may be expected to increase with increasing virial velocity and therefore with increasing mass of the system. Here we present an estimate of the dependence of the efficiency of star formation on the mass of the system, based in part on the results of the 3-dimensional collapse calculations of Larson (1978).

If two parts of a collapsing system of mass M and radius R collide and produce a compressed layer of gas, the pressure P_1 in this layer can be estimated as the dynamical pressure ρu^2, where $\rho \sim M/R^3$ and $u^2 \sim GM/R$; this yields

$$P_1 \sim \frac{GM^2}{R^4}. \tag{A1}$$

This is also the pressure in a self-gravitating gas layer of surface density M/R^2, so pressures of this order are likely to be produced quite generally, regardless of the details of the dynamics (for example, whether the gas is compressed by a high-velocity impact or by more gradual settling into a disk). If the isothermal sound speed c in the cooled and compressed gas is known, the density of the layer can be estimated from

$$\rho_1 = \frac{P_1}{c^2} \sim \frac{GM^2}{c^2 R^4}, \tag{A2}$$

and its thickness h from

$$h \sim \frac{M}{\rho_1 R^2} \sim \frac{c^2 R^2}{GM}. \tag{A3}$$

Equation (A3) predicts that the ratio h/R for the compressed layer is proportional to the ratio of thermal energy to initial gravitational energy, $\alpha = 2.5\, c^2 R/GM$. This prediction is well satisfied by the results of the 3-dimensional collapse simulations of Larson (1978), which show that the thickness of the disk-like layer resulting from the collapse of a rotating cloud is closely proportional to α; with β (the initial ratio of rotational to gravitational energy) equal to 0.30, they yield $h/R = (0.69 \pm 0.04)\,\alpha$ for $0.10 \leqslant \alpha \leqslant 0.35$ (here R is the initial radius and h is the mean distance of matter from the central plane at the time when dense condensations begin to form).

If the compressed layer fragments gravitationally into clumps that continue to collapse approximately isothermally, the 3-dimensional calculations suggest that the development of each clump can be at least crudely represented by assuming spherical isothermal collapse from an initial density ρ_1. Then the density distribution in each clump at the time when it develops a dense core is approximately the usual density distribution for spherical isothermal collapse, $\rho \propto r^{-2}$. We now postulate that the amount of matter that finally goes into stars is proportional to the mass which at this time has reached densities higher than some critical density ρ_c; that is, we assume that subsequent dissipative processes are so effective that some fixed fraction of the matter at densities greater than ρ_c is always turned into stars. Since the mass distribution in the clumps is $m(r) \propto r$ or $m(\rho) \propto \rho^{-1/2}$, where $m(\rho)$ is the amount of mass having densities higher than ρ, the efficiency f of star formation then varies as

$$f \propto \frac{m(\rho_c)}{m(\rho_1)} = \left(\frac{\rho_c}{\rho_1}\right)^{-1/2}. \tag{A4}$$

Substituting for ρ_1 from equation (A2), we obtain

$$f \propto \rho_c^{-1/2}\left(\frac{GM^2}{c^2R^4}\right)^{1/2} = \frac{G^{1/2}}{c\rho_c^{1/2}}\,\frac{M}{R^2}. \tag{A5}$$

Thus, if c and ρ_c can be considered to be constant, the efficiency of star formation is predicted to be proportional to the surface density M/R^2 of the system.

The validity of equation (A4) is supported by the 3-dimensional collapse calculations, if f is taken to be proportional to the total mass that goes into the 'condensed objects' in these simulations. The results (not previously published) for the average fraction of the mass that goes into condensed objects are given in Table A1 as functions of the parameters α and β.

Table A1. Percentage of total mass in condensed objects.

α	$\beta = 0.30$	$\beta = 0.19$	$\beta = 0.11$
0.35	14	25	51
0.30	26	43	53
0.25	33	48	69
0.20	39	60	74
0.15	45	66	74
0.10	58	68	84
0.07	72	79	

This fraction increases with decreasing α and with decreasing β, since the thickness of the disk-like layer containing the condensations decreases with decreasing α while its radius decreases with decreasing β, and both effects increase the mean density ρ_1. Fig. A1 shows the fractional mass M_c/M in condensed objects plotted versus the layer density ρ_1, as calculated in units of the initial density from the changes in the average particle coordinates between

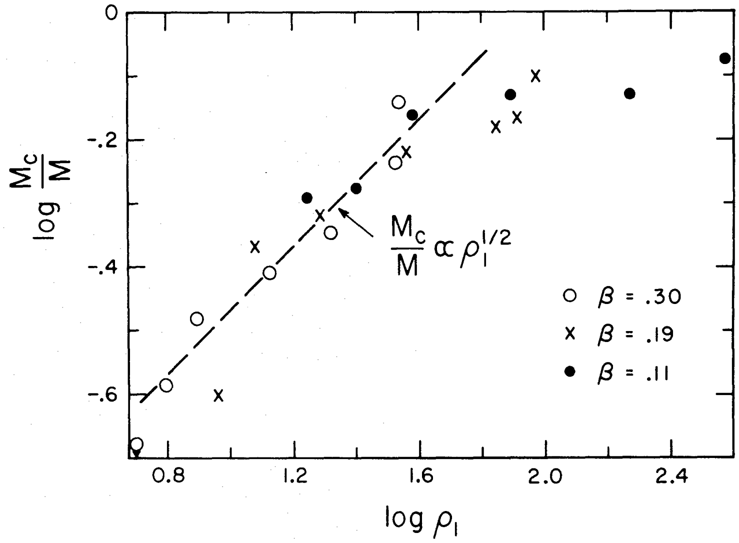

Figure A1. The fractional mass M_c/M in condensed objects plotted versus the density ρ_1 of the compressed layer in which the condensations form, based on data from Table 1 and Larson (1978).

the beginning of the collapse and the time when the condensed objects begin to form. The numerical results follow the predicted relation $M_c/M \propto \rho_1^{1/2}$ over about a factor of 10 in ρ_1, after which M_c/M levels off as it approaches the maximum possible value of 1. This agreement adds some support to the assumptions that the efficiency of condensation processes depends on the intermediate density ρ_1, and that the relative amounts of matter that collapse to various higher densities are approximately the same as for spherical isothermal collapse.

If the above assumptions are accepted, the dependence of the star formation efficiency f on the mass M of the collapsing system can be estimated from equation (A5) if the relation between M and R is known. A simple possibility is that the average density M/R^3 is approximately the same in systems (e.g. protogalaxies) of different size; then equation (A5) predicts

$$f \propto \frac{M}{R^2} \propto M^{1/3}. \tag{A6}$$

This is the same star-formation 'law' that was found in Section 2 to explain Tinsley's (1978b) calibration of the colour–magnitude relation for elliptical galaxies. While this agreement may well be fortuitous, considering the many assumptions that have been made and the uncertainty in the metallicity–mass relation, it does add some theoretical support to the idea that star formation in protogalaxies or colliding galaxies occurs in bursts whose star-forming efficiency is an increasing function of the mass of the system or subsystem in which they occur.