Mon. Not. R. astr. Soc. (1984) **210**, 763–777

Black hole remnants in globular clusters

Richard B. Larson *Yale University Observatory, 260 Whitney Avenue, PO Box 6666, New Haven, CT 06511, USA*

Accepted 1984 March 18. Received 1984 March 12; in original form 1983 December 29

Summary. If a globular cluster contains remnants much more massive than the visible stars, these remnants will dominate the cluster core, and will rapidly attain a nearly singular density distribution. Because of the tendency toward equipartition of energy, the lighter visible stars will remain much less centrally concentrated. Simple two-component models show that the observed light profiles and velocity dispersions of typical globular cluster cores are reproduced if the density of heavy remnants varies as $r^{-2.5}$ at small radii, and if a representative mass for the heavy remnants is $\sim 2.5\,M_\odot$. The cusped light profile of M15 can be explained with a somewhat smaller representative mass of $\sim 1.7\,M_\odot$ for the heavy remnants. The required fractional mass in heavy remnants varies from about 1 to 5 per cent, and correlates inversely with the slope of the main-sequence luminosity function. A possible, but not unique, set of assumptions that can account for the remnant properties in a typical case is that the initial mass function (IMF) is the same as that of the solar neighbourhood, and that the massive stars leave remnants whose mass is ~ 15 per cent of the initial stellar mass. The extremes of small and large fractional content of heavy remnants are represented by 47 Tuc and ω Cen, and since both of these clusters have very long relaxation times and thus presumably have not lost many stars, this difference requires either a difference in the IMF or a difference in the amount of mass lost from massive stars.

1 Introduction

A long-standing puzzle in the dynamical theory of globular clusters concerns the phenomenon of 'core collapse', which is predicted to occur in most clusters but is not clearly observed (King 1975). Many theories and simulations of the evolution of globular clusters have found that, within a finite time generally shorter than the cluster age, the core of a model cluster undergoes a runaway evolution toward a singular density distribution of the form $\varrho \propto r^{-2-n}$, where $0.2 \lesssim n \lesssim 0.5$ (e.g. Hénon 1961; Larson 1970; Spitzer 1975; Cohn 1980). A runaway increase in central density is found even when stellar mass loss is taken into account (Angeletti & Giannone 1980). No evidence for the predicted singular density distribution is seen in the observed light profiles of most globular clusters, which become nearly flat at the centre, and in only a few cases is there evidence for a modest central cusp in the light profile (Djorgovski & King 1984).

If a cluster contains a significant range of stellar masses, its structure depends on the mass spectrum of the stars and stellar remnants present, since objects of larger mass are more strongly concentrated toward the centre. In particular, if the cluster contains remnants more massive than the visible stars, these remnants will be strongly concentrated in the cluster core and will affect the distribution of visible stars there (Illingworth & King 1977; Gunn & Griffin 1979). An important effect is that the approach to equipartition of energy tends to cause the visible stars to be excluded from any region dominated by massive remnants. Da Costa (1977) concluded that the cores of globular clusters must in fact be dominated by massive remnants; otherwise, models like those of Da Costa & Freeman (1976) fitted to the star counts of Da Costa (1982) in the outer parts of several clusters predict central surface brightnesses that are much too high. This discrepancy is removed if a few per cent of the total mass is assumed to be in remnants of mass $1.2 M_\odot$; these 'massive remnants' then concentrate strongly in the cluster core, and the less massive visible stars are driven outward, resulting in a lower central surface brightness.

If the cores of globular clusters are dominated by heavy remnants, then core collapse, if it occurs, will involve primarily these unseen remnants. The reason why the light profiles of globular clusters generally show no evidence for core collapse may then be that the visible stars are largely excluded from a collapsed core of heavy remnants. Inagaki & Lynden-Bell (1983) have noted, for example, that if a collapsed core is dominated by neutron stars, the visible stars will have a flatter density profile which will be in better accord with observations. In this paper we consider the possibility that even more massive remnants are present in globular cluster cores, and that they have a nearly singular density distribution like that predicted theoretically. If the typical mass of these remnants is as large as $\sim 2.5 M_\odot$, the predicted core light profile becomes flat enough near the centre to yield good agreement with the observations (Section 3). There is no astrophysical reason why black hole remnants of this mass or even larger should not exist, since globular clusters must once have contained massive stars, and a likely fate for at least some massive stars is collapse to black holes. Evidence that black holes exist in metal-poor systems is provided by the discovery of a black hole candidate of mass $\gtrsim 9 M_\odot$ in the Large Magellanic Cloud (Cowley *et al.* 1983). If such objects exist in globular clusters, they would rapidly sink to the centre (if they had not already been formed there; Larson 1982), and the two most massive ones would soon form a dominant central binary (Aarseth 1973). The cluster could then continue to exist for a long time in a 'post-collapse' state in which the central massive binary steadily shrinks and gives energy to the rest of the cluster, which gradually expands.

In most respects, this picture was foreseen by Hénon (1961, 1975), who developed post-collapse models of globular clusters with singular density distributions and unspecified central energy sources. A collapsed core of heavy remnants with a massive central binary would provide a plausible basis for Hénon's post-collapse models. Also, such a model would remove the problem pointed out by Lightman (1982), who showed that if all globular clusters were now in a pre-collapse state, a very special distribution of initial conditions would be required.

In the present picture the observed core of a globular cluster is the region dominated by heavy remnants, and therefore its size depends on the total amount of mass in heavy remnants and hence on the amount of mass in the progenitor stars of the remnants. Thus, from the light profiles of globular clusters it is possible to derive some constraints on the upper part of the initial stellar mass function (IMF). Da Costa (1982) found large differences in the slopes of the main-sequence luminosity functions in several globular clusters; if these differences partly reflect differences in the IMF (Freeman 1977), there should be corresponding differences in the relative amount of mass in heavy remnants. Differences in the expected sense are in fact found, and they provide additional evidence for variations in at least the present mass spectra in globular clusters (Section 5).

2 Models

To derive some simple models for remnant-dominated globular cluster cores, we assume first that the density distribution of the remnants at small radii is asymptotically of the form $\varrho \propto r^{-2-n}$, which corresponds to $v^2 \propto r^{-n}$ for the radial variation of the mean squared velocity. We also adopt the following further simplifying assumptions: (1) the velocity distribution is everywhere isotropic, and (2) the velocity dispersion of each mass group has a dependence on radius given by $v^2 \propto r^{-n}$ at all radii, where the exponent n is the same for all mass groups. These assumptions, which are intended to apply only to the cores of globular clusters, are suggested by the results of numerical simulations which show approximately a power-law dependence of velocity dispersion on radius in the core region (e.g. Spitzer & Hart 1971; Hénon 1972). The velocity distribution is always predicted to be isotropic throughout the core, and if more than one mass group is present, the velocity dispersion profiles of different mass groups can be approximated by power laws of the same slope (Spitzer & Hart 1971).

An important parameter of the models is then the exponent n. Lynden-Bell & Eggleton (1980) showed that in a single-mass system the value of n at the time of core collapse must be between 0 and 0.5, and all of the numerical results for single-component systems fall in this range, a recent determination (Cohn 1980) being $n=0.23$. In multi-component systems, the value of n for individual mass groups will be larger than the value for the system as a whole, since the average mass decreases outward and the average velocity dispersion therefore decreases less rapidly with radius than the velocity dispersions of individual mass groups. The detailed Monte Carlo results of Spitzer & Hart (1971) show the expected tendency for the velocity dispersion profiles of individual mass groups to be steeper in multi-component models than in single-component models.The values of n found in these simulations are in the range $\sim$0.2–0.5, and they tend to increase with time (see also Hénon 1972).

Recent simplified analytical theories for the post-collapse evolution of single-mass systems (e.g Inagaki & Lynden-Bell 1983; Heggie 1984) have predicted the eventual growth at the centre of a singular isothermal region with $n=0$. For multiple-mass systems, the post-collapse structure must be more complex, and probably remains more centrally concentrated with a value of $n>0$. N-body models of clusters, which may be the best guide, tend to develop radial profiles of density and velocity dispersion that are considerably steeper than isothermal (e.g. Aarseth 1968).

Models have been calculated for $n=0$, $n=0.25$ and $n=0.5$, but only the models with $n=0.5$ were found to yield satisfactory agreement with the observations in all respects. Models with $n=0$ and $n=0.25$ can provide equally good fits to the observed core light profiles, but they predict central velocity dispersions that are systematically too small, and they do not explain the two high-velocity stars found by Gunn & Griffin (1979) in M3, which are satisfactorily accounted for by models with $n=0.5$ (Section 3). Hereafter only models with $n=0.5$ will be considered.

A final simplifying assumption in the present models is the use of only two mass groups, one representing the heavy remnants and the other representing the visible stars and low-mass remnants. Such simple two-component models cannot be expected to represent in detail the structure of globular cluster cores, but they should at least serve to indicate approximately how large a population of heavy remnants is required. A second dimensionless model parameter is then the ratio v_1^2/v_2^2 of the mean squared velocities of the light and heavy objects, which by assumption is independent of radius. If complete equipartition of energy holds, then $v_1^2/v_2^2=m_2/m_1$, where m_1 and m_2 are masses of the light and heavy objects; otherwise,

1984MNRAS.210..763L

$v_1^2/v_2^2 < m_2/m_1$. Results will be presented for three models, Models A, B and C, in which v_1^2/v_2^2 is equal to 2, 3 and 6 respectively.

The models have been calculated by integrating the equation of hydrostatic equilibrium $d(\varrho v^2)/\varrho\,dr = -GM(r)/r^2$ outward for each mass group, starting from the power-law solutions for $\varrho(r)$ that apply at small radii where the heavier objects are completely dominant. For each model, the projected surface density $\mu(r)$ of the lighter objects has been calculated as a function of radius, and the resulting profiles have been characterized by two parameters: (1) the core radius r_c, defined as the radius at which the surface density $\mu(r)$ falls to half its value at a radius of one-tenth r_c, and (2) μ_0, the surface density at a projected radius of $0.1\,r_c$.

The results are presented in dimensionless form in Figs 1 and 2. Fig. 1 shows the surface density profiles, plotted as $\log(\mu/\mu_0)$ versus $\log(r/r_c)$, for the lighter objects in Models A, B and C, and Fig. 2 shows the spatial density distributions of both the light objects (solid curves) and the heavy objects (dashed curves) in Models B and C, normalized by μ_0/r_c in each case. The density distribution of the light objects follows a power law of relatively shallow slope at small radii, and if v_1^2/v_2^2 is as large as 6 (as in Model C), this slope becomes zero at small radii. The heavy objects have much steeper density profiles, and essentially all of their mass is contained within the core radius r_c. In units of $\mu_0 r_c^2$, the total mass M_2 in heavy remnants in Models A, B and C is equal to 11.4, 1.02 and 0.24 respectively. Also of interest for comparisons with observations is the velocity dispersion of the light objects at radius r_c; in units of $(G\mu_0 r_c)^{1/2}$, this is equal to 2.06, 1.07 and 0.85 in Models A, B and C.

3 Application to globular clusters

In this section we compare these models with data for six well-studied globular clusters for which detailed surface brightness profiles have been given by Da Costa (1977, 1979); these are 47 Tuc (NGC 104), NGC 6752, 6397, M92 (NGC 6341), M3 (NGC 5272), and ω Cen (NGC 5139). Some basic properties of these clusters are listed in Table 1. For the first three clusters, star counts extending to faint magnitudes are available (Da Costa 1982), as are detailed multi-component

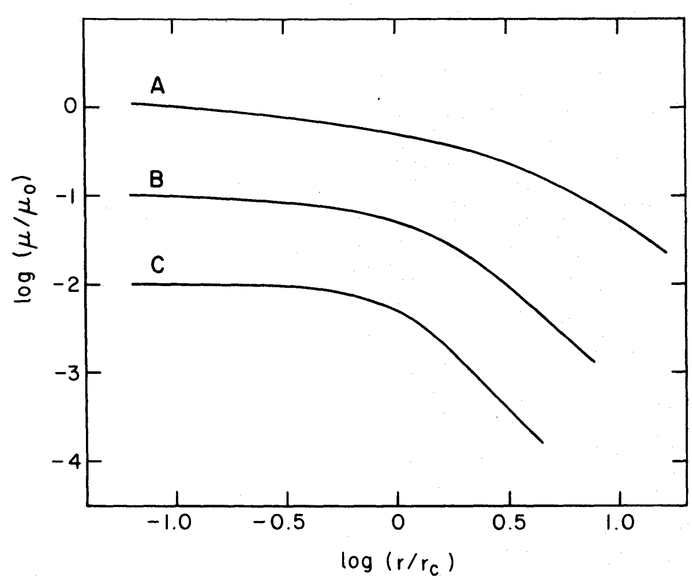

Figure 1. Dimensionless core surface-density profiles for Models A, B and C, which have $v_1^2/v_2^2=2$, 3 and 6 respectively. Successive profiles are displaced by one unit in $\log(\mu/\mu_0)$ for clarity. The core radius r_c and the reference surface density μ_0 are defined such that the surface density at radius r_c is one-half the surface density μ_0 at a radius of one-tenth r_c.

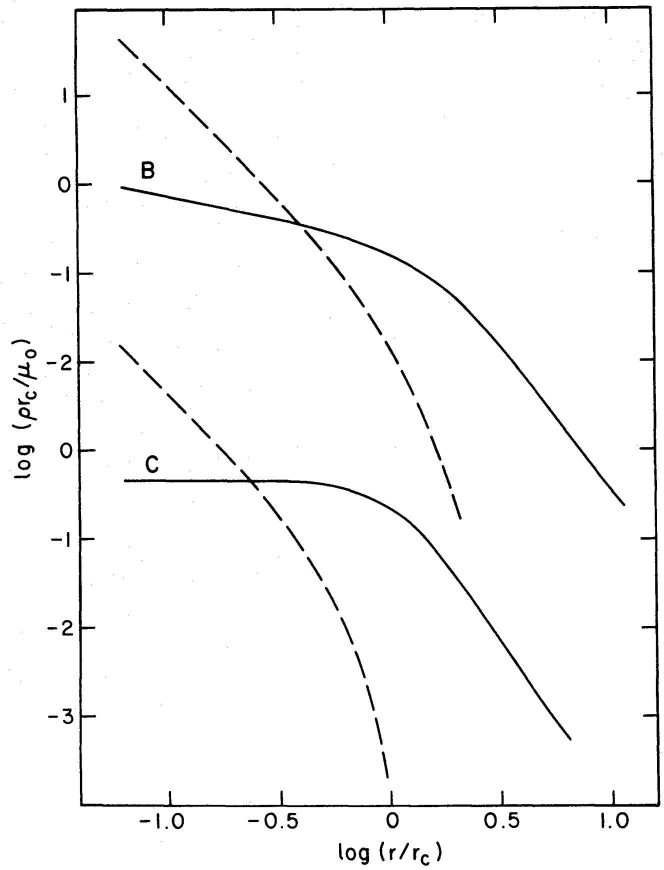

Figure 2. Spatial density distributions, normalized by μ_0/r_c, for the light objects (solid curves) and the heavy objects (dashed curves) in the core regions of Models B and C.

models fitted to these star counts (Da Costa 1977). The masses given in Table 1 for 47 Tuc, NGC 6752 and 6397 are those of Da Costa's models with standard assumptions about remnants. These models have mass-to-light ratios M/L_V of 2.6, 1.4 and 1.1 respectively; these values reflect the progressively smaller observed slopes of the main-sequence luminosity functions in these clusters. The corresponding values of the slope x of the mass function $dN/d \log m \propto m^{-x}$ in these models are listed in Table 1. For the remaining clusters, models based on detailed star counts are not available. Since M92 and M3 have main-sequence luminosity functions that are nearly as flat as that of NGC 6397 (see below, Fig. 5), their masses have been estimated from their absolute magnitudes (Harris & Racine 1979) by assuming that they have M/L_V=1.2. For ω Cen, the results obtained below suggest that this cluster has an even higher proportion of massive stars than the others, and a value for M/L_V of 1.1 has been arbitrarily assumed. Values of the concentration parameter $\log(r_t/r_c)$ are also listed in Table 1 for each cluster, where r_t is the total

Table 1. Cluster parameters.

Cluster	$M(M_\odot)$	$\log(r_t/r_c)$	t_{rh}(yr)	x
47 Tuc	1.0 (+6)	2.04	2 (+10)	2.4
NGC 6752	1.2 (+5)	1.79	2 (+9)	1.0
NGC 6397	4.4 (+4)	1.70	4 (+8)	0.6
M92	1.6 (+5)	1.71	2 (+9)	
M3	3.0 (+5)	1.77	1 (+10)	∿ 0.7
ω Cen	1.2 (+6)	1.22	3 (+10)	

radius and r_c is the core radius as given by Da Costa (1977, 1979). Table 1 also gives for each cluster the reference relaxation time t_{rh} defined by Spitzer & Hart (1971); the first three values are from Da Costa (1977), and the remaining ones are average values obtained by scaling the values for NGC 6752 and 6397 by a factor $M^{1/2}(r_c r_t)^{3/4}$.

M3 has been modelled in detail by Oort & van Herk (1959), Da Costa & Freeman (1976), and Gunn & Griffin (1979), but the available star counts for this cluster (Sandage 1954, 1957) do not extend much below the main-sequence turnoff. Gunn & Griffin (1979) note that Sandage's (1957) luminosity function has a shallow slope corresponding to $x{\sim}0.7$ (as quoted in Table 1) near the limit of the counts, but they regard this result as very uncertain and prefer models with $x{\sim}2.0$ and $M/L_V{\sim}2.1$. However, these models are not constrained by detailed faint star counts, so they do not determine x or M/L_V as well as do the models of Da Costa for 47 Tuc, NGC 6752 and 6397.

The observed surface-brightness profiles of the cores of these six clusters have been compared with the surface-density profiles of the lighter objects in Models A, B and C shown in Fig. 1. Models A and C are found to be too extreme to fit any of the observed profiles; the predicted profile of Model A has too little curvature, and that of Model C has too much curvature. Model B, however, provides an acceptable fit to all of the observed profiles, and an excellent fit to some. The fit is essentially perfect for M3 and the largest deviations are for 47 Tuc, which would be

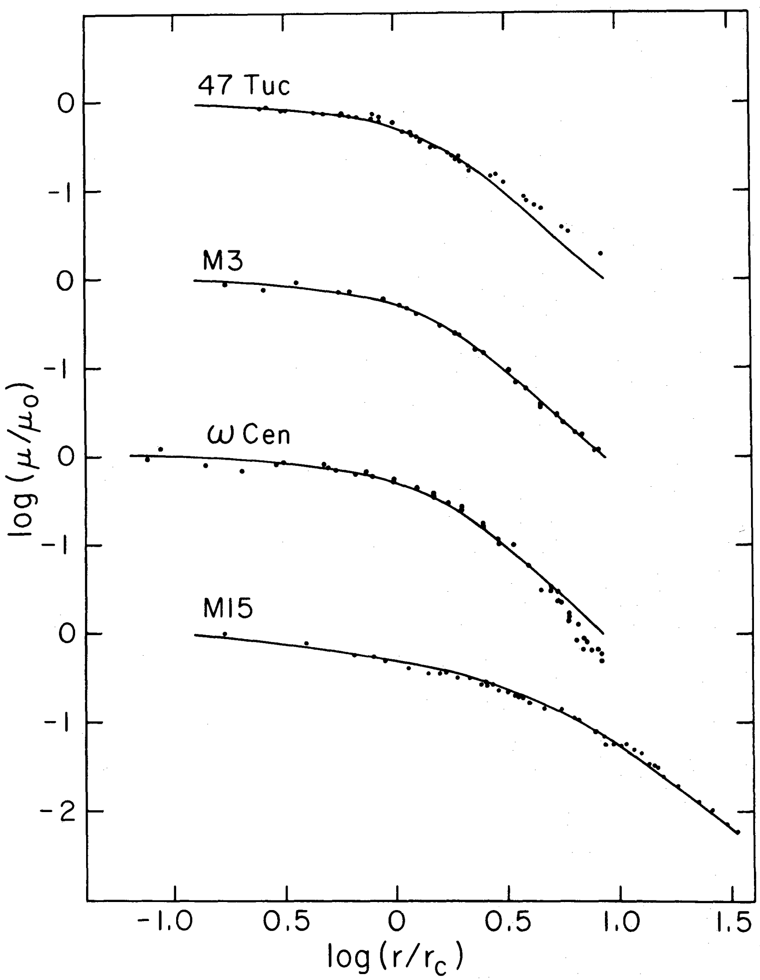

Figure 3. Fits of models to the observed V light profiles of several clusters, illustrating some of the best (M3, M15) and worst (47 Tuc, ω Cen) cases. For 47 Tuc, M3 and ω Cen the predicted profile of Model B is fitted to the data of Da Costa (1977, 1979); the parameters for these fits are given in Table 2. For M15, the predicted profile of Model A is fitted to the data of Newell & O'Neil (1978), as described in Section 6. In all cases, the conversion between surface brightness and surface density is based on an assumed M/L_V of 0.6 for the lighter objects in the models.

better fitted by a model intermediate between A and B, and ω Cen, which would be better fitted by a model intermediate between B and C. In view of the simplifications of the models, these differences may not be very meaningful, so we have chosen to fit all of the observed light profiles with Model B. Examples of the best and worst of these fits are shown in Fig. 3, which shows fits of Model B to 47 Tuc, M3 and ω Cen.

The values of r_c and ω_0 derived from these fits are given in Table 2. The cluster distances and reddenings used have been taken from Harris & Racine (1979). To convert the core surface-brightness profiles into surface-density profiles, it is also necessary to know the mass-to-light ratio of the visible stars and low-mass remnants in the core of each cluster. As the best available estimate of this quantity for 47 Tuc, NGC6752 and 6397, we have adopted the central M/L_V values of Da Costa's models for these clusters, which are 0.6, 0.6 and 0.8 respectively. For M3, the (possibly less reliable) models of Da Costa & Freeman (1976) and Gunn & Griffin (1979) predict somewhat smaller projected central M/L_V values of 0.4 and 0.45 respectively. We have arbitrarily assumed that in M92, M3 and ω Cen the mass-to-light ratio of the light objects in the cluster core is 0.6, a value that should be accurate within a factor of 1.5.

Table 2. Model fits for Model B.

Cluster	r_c (pc)	μ_0 ($M_\odot$/pc^2)	M_2 ($M_\odot$)	M_2/M
47 Tuc	0.46	43000	9300	0.0090
NGC 6752	0.55	8400	2600	0.023
NGC 6397	0.47	7200	1600	0.037
M92	0.55	14000	4300	0.027
M3	1.10	6800	8400	0.028
ω Cen	3.0	6400	60000	0.050

From the values of r_c and μ_0 listed in Table 2, the total mass M_2 of the heavy remnants in each cluster has been calculated from $M_2=1.02\,\mu_0 r_c^2$, and the fractional mass in heavy remnants M_2/M has been calculated using the cluster masses given in Table 1. The estimated uncertainty in M_2 arising from the profile fitting procedure is of the order of 10 per cent or less; thus the variation by a factor of ~5 in the values of M_2/M listed in Table 2 is highly significant. Moreover, the relative values of M_2/M are essentially independent of which model is used to fit the data; for example, a model with $v_1^2/v_2^2=3$ and $n=0.25$ yields values of M_2/M that are almost exactly half those given in Table 2. This because the relative values of M_2/M depend basically on the ratios of core size to total cluster size, as can be seen from the close inverse correlation between the values of M_2/M in Table 2 and the cluster concentration parameters in Table 1. Thus, in the present interpretation, the concentration parameter of a cluster is determined by the fractional content of heavy remnants, and does not represent a stage of evolution toward a singular density distribution.

It is relevant to inquire whether there has been time for core collapse to have occurred in all of the clusters considered, as has been assumed. The time required for core collapse to occur in standard models is very sensitive to the assumed stellar mass spectrum, and varies from about $15t_{\rm rh}$ in the case of equal masses to a value about 10 times smaller than this for a realistic mass spectrum (Hénon 1975). Thus, from the values of $t_{\rm rh}$ listed in Table 1, we conclude that there has been ample time for core collapse to occur in NGC 6752, 6397 and M92, and that core collapse has probably occurred in M3, while it is not clear that this has been the case in the two most massive clusters, 47 Tuc and ω Cen. However, since the initial conditions for globular clusters are not known, it cannot be ruled out that core collapse has occurred even in these massive clusters. It is possible, for example, that the most massive stars are formed in a dense central subcluster (Larson 1982); in this case, globular clusters might contain condensed cores of massive objects already at the time of formation.

The validity of the present models is supported by some further comparisons with observations. For most of the clusters studied, the velocity dispersions predicted by the models can be compared with observed velocity dispersions. The results of this comparison are given in Table 3, which lists for each cluster the typical projected radius of the stars observed, the measured velocity dispersion with its uncertainty, and the projected velocity dispersion calculated from the model at that radius. The data have been taken from Illingworth (1976) for 47 Tuc, Da Costa *et al.* (1977) for NGC 6397, Wilson & Coffeen (1954) for M92, Gunn & Griffin (1979) for M3, and Harding (1965) for ω Cen. The agreement is generally quite satisfactory, with no tendency for the predicted values to be systematically too high or too low. This is in contrast to Da Costa's (1977) 'standard' models, which predict central velocity dispersions that are two low. Thus the inclusion of a condensed core of heavy remnants as in Model B completely removes this discrepancy between predicted and observed velocity dispersions. In particular, the high central velocity dispersion of 47 Tuc provides evidence that even this massive cluster contains a condensed core of heavy remnants.

Table 3. Velocity dispersions.

Cluster	<r> (pc)	Observed (km/s)	Model B (km/s)
47 Tuc	0.1	10.5 ± 0.4	11.2
NGC 6397	3.5	3.1 ± 0.7	2.4
M92	1.4	4.4 ± 1.1	4.7
M3	1.9	5.1 ± 0.5	5.0
ω Cen	17	5.7 ± 3.8	6.1

Gunn & Griffin (1979) found two stars in M3 with well-determined radial velocities that deviate from the mean by 17 and 23 km s^{-1}; these stars are almost certainly cluster members, but their velocities are exceedingly improbable in a Maxwellian distribution, and Gunn & Griffin were unable to account for them on the basis of any plausible dynamical process. The present models predict an excess of high-velocity stars relative to a Maxwellian distribution because the velocity dispersion increases inward as $r^{-1/4}$, so that stars passing close to the centre typically have much higher than average velocities, acquired by interaction with the heavy remnants. To estimate the number of such high-velocity stars, we assume that at each radius in the inner core the velocity distribution is locally Maxwellian, and we assume that each orbit passing through this region is populated in equilibrium, so that the number of stars in each part of the orbit is proportional to the time spent in that part of the orbit. Integration over the innermost radius and the energy of the orbits then yields the estimate that in Gunn & Griffin's sample of 111 stars, the expected number of stars with $|v|\geq 17$ km s^{-1} is 1.6, and the expected number with $|v|\geq 23$ km s^{-1} is 0.4. These predictions are quite compatible wih the existence of one star with $|v|=17$ km s^{-1} and one with $|v|=23$ km s^{-1}. Other models, such as ones in which the velocity dispersion varies only as $r^{-1/8}$, fail by a large factor to explain these observations, so this is a sensitive test for the structure of globular cluster cores.

Finally, the predicted radial variation of the projected velocity dispersion can be compared with the results of Gunn & Griffin (1979) for the radial variation of the velocity dispersion in M3. The results of this comparison are shown in dimensionless form in Fig. 4. The data points in this diagram are reproduced from fig. 7 of Gunn & Griffin, with the addition of a second higher value

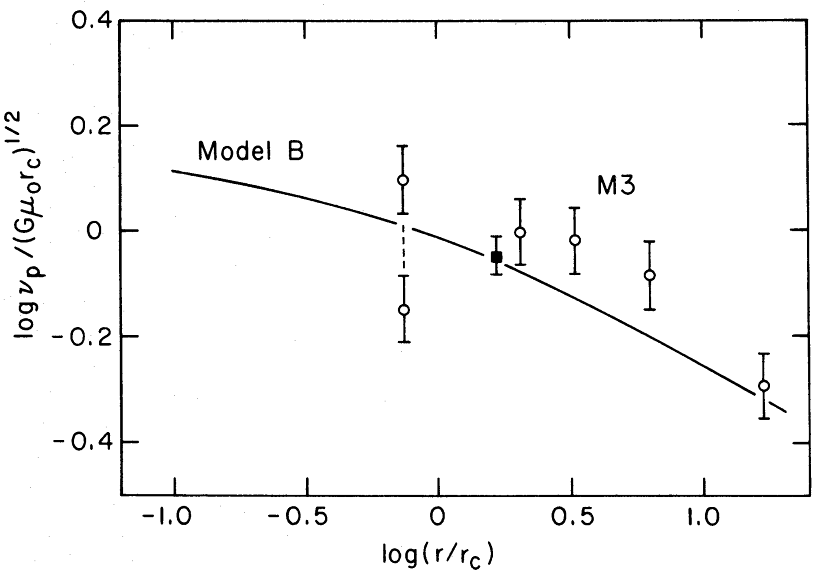

Figure 4. The projected velocity dispersion v_p, averaged along lines-of-sight, plotted in units of $(G\mu_0 r_c)^{1/2}$ versus r/r_c for Model B. The data points for M3 are from fig. 7 of Gunn & Griffin (1979), with the addition of a recalculated value for the innermost radial zone which includes the two high-velocity stars (uppermost point). The filled square is Gunn & Griffin's average for their three innermost zones.

for the velocity dispersion of the innermost radial zone, recalculated including the two high-velocity stars (both located in this zone) which were excluded by Gunn & Griffin. The filled square is Gunn & Griffin's average for their three innermost zones, and is the value listed in Table 3. If the larger of the two velocity dispersions for the innermost zone is taken as the more correct one, the predicted and the observed radial variation of velocity dispersion agree well in form, but the model curve lies systematically below the data by about 0.08 in $\log v_p$. This discrepancy would be removed if the assumed central M/L_V of the light objects in M3 were increased from 0.6 to 0.87, which is close to the value of 0.8 appropriate for Da Costa's (1977) model of NGC 6397. The slope of the predicted curve in Fig. 4 is directly proportional to the parameter n of the models, so that a smaller observed variation of velocity dispersion with radius would imply a smaller value of n, but would not necessarily rule out a collapsed core.

4 Possible implications for the stellar mass spectrum

Since the slope x of the present stellar mass function decreases along the sequence 47 Tuc→NGC 6752→NGC 6397, the fractional mass in heavy remnants might be expected to increase along this sequence, and this is in fact what is found (Table 2). In an effort to see whether the available evidence for the other clusters is at least qualitatively consistent with such a trend, the main-sequence luminosity functions of most of these clusters have been plotted in Fig. 5. The curves for 47 Tuc, NGC 6752 and 6397 have been reproduced from fig. 9 of Da Costa (1982). The curve shown for M92 is a slightly smoothed version of the luminosity function obtained by van den Bergh (1975) for his zones 1, 2 and 3. A very similar luminosity function for M92, not extending quite as faint, was obtained by Sandage & Katem (1983). The luminosity function shown for M3 has been constructed from Sandage's (1954, 1957) star counts as published by Oort & van Herk (1959).

An effort has been made to present data for comparable regions in the various clusters, since the luminosity function may vary with radius. The star counts of Da Costa and van den Bergh all refer to approximately comparable regions with radii between 0.2 and 0.5 times the cluster radius

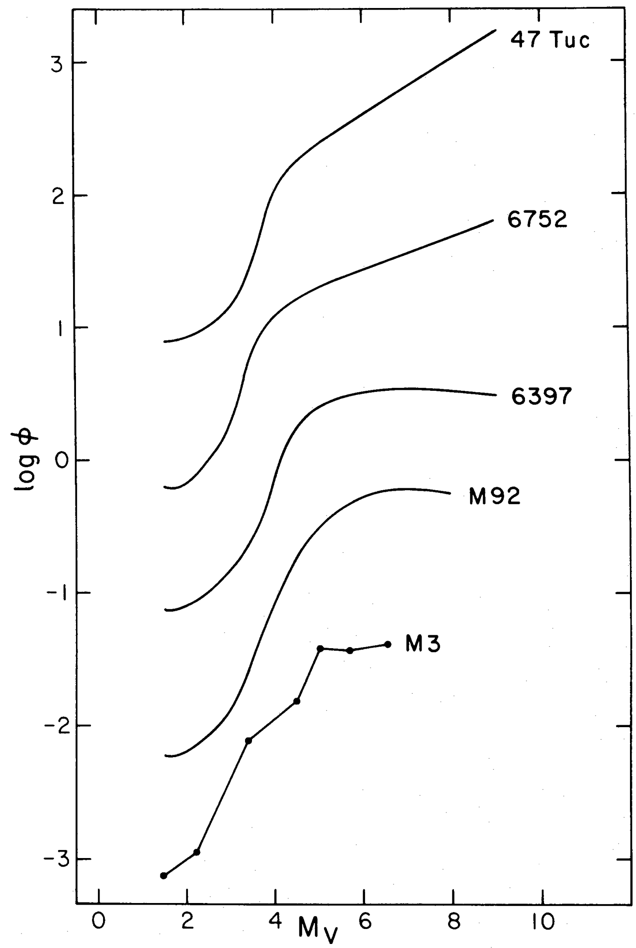

Figure 5. Luminosity functions, arbitrarily normalized, for annular outer zones between about $0.2r_t$ and $0.5r_t$ in 47 Tuc, NGC 6752, 6397, M92 and M3. See text at beginning of Section 4 for explanation.

r_t. For the fainter stars in M3 with $M_V \geqslant 4.5$, counts made at radii between 5 and 8 arcmin (about 0.2 to $0.3r_t$) have been used to construct the luminosity function shown in Fig. 3; for $M_V \leqslant 4.5$, counts made at radii between 2 and 5 arcmin have been used to extend the luminosity function to brighter magnitudes.

It is clear that at faint magnitudes the luminosity function of M92, and with less certainty that of M3, do not rise nearly as steeply as that of 47 Tuc, nor are they as steep as that of NGC 6752. They appear most similar to NGC 6397, although careful comparison suggests that they are both intermediate between NGC 6752 and 6397. The values of M_2/M for M92 and M3 given in Table 2 are also intermediate between those for NGC 6752 and 6397. Thus all five of the clusters in Fig. 5 conform well to an inverse correlation between the slope of the main-sequence luminosity function and the fractional mass in heavy remnants. The least certain case is M3, but even if the parameters preferred by Gunn & Griffin (1979) for M3 were adopted instead, the correlation would still hold, since in this case M3 would have x=2.0 and M_2/M=0.014 and would be intermediate in both properties between 47 Tuc and NGC 6752. This correlation adds strength to both the present interpretaton of the light profiles of globular cluster cores, and to the evidence for significant differences in the present mass functions of globular clusters.

Part of the variation in M_2/M among clusters can be explained just by the known variation in the mass function of the lower main-sequence stars, which is reflected also in the different cluster mass-to-light ratios. This can be seen from the fact that the ratio of heavy remnants to turnoff stars, which is proportional to M_2/L, varies by only about a factor of 1.7 between 47 Tuc and NGC 6397, compared with a factor of 4 variation in M_2/M. However, the variation in either M_2/L or M_2/M is much smaller than would be predicted if power-law mass spectra with the slopes listed in Table 1 were extrapolated to the masses of the remnant progenitors; thus, the initial mass functions of the massive stars in different clusters must have been much more similar than are the present mass functions of the fainter stars.

To draw more quantitative inferences about the IMF of the remnant progenitors, it is necessary to know something about the typical masses of the heavy remnants. If full equipartition of energy were to hold, the ratio of masses of the heavy and light objects in Model B would be 3.0. If the typical mass of the visible stars determining the light profile is $0.8 M_\odot$, a representative mass for the heavy objects would then be $2.4 M_\odot$. However, complete equipartition of energy between the heavy and light objects cannot be attained, because the heavy objects near the centre continue to lose energy to the lighter objects and ultimately to the outer parts of the cluster, and this energy flux implies a finite departure from equipartition. The magnitude of this departure can be estimated by using a formula given by Spitzer (1975) to calculate the rate of transfer of energy from heavy objects to light objects in the cluster core, and equating the result to the outward energy flux in Hénon's (1975) post-collapse models, which is about one-tenth of the total binding energy per reference relaxation time $t_{\rm rh}$. This calculation shows that for the six clusters studied, the mass ratio m_2/m_1 exceeds the equipartition value v_1^2/v_2^2 by an average factor of 1.16. The representative mass of the heavy remnants in Model B then becomes about $2.8 M_\odot$. Somewhat smaller masses could result if the typical mass of the stars determining the light profile is smaller than $0.8 M_\odot$.

These remnants must have been produced by stars of much larger initial mass, since massive stars lose a major part of their mass during late stages of evolution. There is evidence that stars with masses up to $5 M_\odot$ evolve into white dwarfs (Anthony-Twarog 1982); therefore the heavy remnants discussed here must have come from stars considerably more massive than $5 M_\odot$. In the absence of a better knowledge of the amount of mass lost, the present results are consistent with a wide range of initial possibilities, but it is at least not difficult to account for the result that typically ~3 per cent of the cluster mass must be in heavy remnants whose representative mass is, say, $\sim 2.5 M_\odot$. For example, if the IMF is the same as that of the solar neighbourhood as modelled by Miller & Scalo (1979), and if the mass of each remnant is either $0.6 M_\odot$ or 15 per cent of the initial stellar mass, whichever is larger, then 3.1 per cent of the mass remaining at the present time is predicted to be in remnants more massive than $0.85 M_\odot$, and the mass-weighted average mass of these remnants is $2.4 M_\odot$. We note that the Miller-Scalo IMF has a slope of $x \sim 0.85$ in the mass range of the observed stars in globular clusters, and this is not very different from any of the values of x in Table 1 except that for 47 Tuc.

5 Variations in the initial mass function?

Both the available main-sequence star counts and the inferred fractional masses in heavy remnants indicate substantial variations in at least the present mass functions of low-mass stars in globular clusters. Among the clusters for which star count data are available, the extremes of variation are represented by 47 Tuc, which has $x=2.4$ and $M_2/M=0.009$, and NGC 6397, which has $x=0.6$ and $M_2/M=0.037$. On the basis just of the inferred fractional mass in heavy remnants, an even more extreme case than NGC 6397 is ω Cen, which has $M_2/M=0.050$; however, no information is available on the luminosity function of ω Cen.

An important question is to what extent these variations can be explained by differential loss of low-mass stars from clusters during their evolution. Da Costa (1977) argued, on the basis of cluster simulations by Spitzer & Shull (1975) and others showing only a weak mass dependence of the escape rate of low-mass stars, that the differences in x in Table 1 cannot be completely accounted for by differential loss of low-mass stars, so that some variation in the IMF is required. The post-collapse models of Stodolkiewicz (1982) also show a weak mass dependence of the escape rate for low-mass stars, and thus lend support to this conclusion. However, the calculations have not been carried far enough to rule out a substantial loss of low-mass stars from NGC 6397, which has a short reference relaxation time $t_{\rm rh}$ of only 4×10^8 yr; thus the small value of x found for this cluster could still largely reflect a loss of low-mass stars rather than an unusually shallow IMF. Of course, loss of low-mass stars could not account for the anomalously *large* value of x found in 47 Tuc.

Weak evidence for a shallow IMF in some clusters is provided by the fact that M3 appears to have a nearly flat main-sequence luminosity function (Fig.5), despite the fact that it has a relatively long relaxation time of 1×10^{10} yr and therefore should not have lost many low-mass stars. A more complete luminosity function for M3 would be of great importance for confirming or refuting this possibility. Even more interesting would be a main-sequence luminosity function for ω Cen, which has a very long relaxation time of $\sim3\times10^{10}$ yr and thus should not have lost any significant number of low-mass stas. Thus if any large difference were found between the main-sequence luminosity functions of 47 Tuc and ω Cen, it would almost certainly reflect a difference in the IMF. We note that, even apart from any difference in the present main-sequence luminosity functions, the ratio M_2/L shows a difference of about a factor of 2.4 between 47 Tuc and ω Cen, and this implies either that the upper IMF of ω Cen was enhanced by about a factor of 2.4 relative to that of 47 Tuc, or that stellar mass loss was less important in ω Cen.

6 The M15 phenomenon

In addition to the six clusters considered above, an accurate light profile for M15 (NGC 7078) is available (Newell & O'Neil 1978). M15 differs from most well-studied globular clusters in that it shows clear evidence for a central cusp in its light profile; several additional examples of this phenomenon have recently been found by Djorgovski & King (1984). The predicted profile of Model B is too flat near the centre to fit the observations of M15, but a close fit is provided by Model A (see Fig. 3). Assuming that the mass-to-light ratio of M15 is 1.2 and that the central M/L of the lighter objects is 0.6, the fit of Model A to the observed profile yields r_c=0.13 pc, $\mu_0=8.0\times10^4 M_\odot \mathrm{pc}^{-2}$, $M_2=1.5\times10^4 M_\odot$, and M_2/M=0.040. The fractional mass in heavy remnants is thus comparable to the values found for the other clusters. The main distinguishing feature of M15 is then just that the best fitting value of v_1^2/v_2^2 is only about 2/3 as large as is typical, implying that the representative mass of the heavy remnants is also only about 2/3 as large, or $\sim1.7 M_\odot$. This type of model provides a better fit to the observed light profile of M15 than do models with a central massive object, as proposed by Newell, Da Costa & Norris (1976). An interpretation of M15 somewhat similar to that proposed here, involving a core dominated by a similar population of heavy remnants but without a singular density distribution, was suggested by Illingworth & King (1977).

Thus, instead of containing more dark mass than other clusters, M15 may just contain somewhat less massive remnants. One possible origin for such a difference could be that the IMF of M15 was lacking in the most massive stars, say stars more massive than $30 M_\odot$; the mass-weighted average mass of the heavy remnants would then be reduced from 2.4 to $1.7 M_\odot$, assuming again a Miller–Scalo IMF. Alternatively, the most massive stars in M15 may have left fewer or less massive remnants than is typical.

7 X-ray sources in globular clusters

The X-ray sources observed in globular clusters are believed to be binary systems consisting of a neutron star and a main-sequence star, probably formed by tidal capture (van den Heuvel 1980). Part of the evidence for this interpretation is that the X-ray sources have a median angular distance from the centre of the cluster that is about 0.4 times the core radius r_c; from this result it has been inferred that the most probable mass of the X-ray sources is about $2M_\odot$ (Grindlay 1981). Although this conclusion is based on conventional cluster models without singular cores, it is consistent also with the present models for globular cluster cores. The median distance of the heavy objects from the centre in Model B is $0.2r_c$; since the observed X-ray sources all have projected radii equal to or greater than this, they must have masses smaller than the $\sim 2.5M_\odot$ of the heavy remnants in Model B, yet considerably larger than the $\sim 0.8M_\odot$ of the brighter visible stars, and this is consistent with a typical mass of $\sim 2M_\odot$. Conversely, if it is accepted that the X-ray sources are binaries of mass $\sim 2M_\odot$, the above comparison shows that the typical mass of the heavy remnants in most clusters is greater than $2M_\odot$.

Among the clusters considered in this paper, 47 Tuc and M15 contain X-ray sources. These are also the two clusters with the highest central surface brightnesses and the highest central densities of visible stars and low-mass remnants. This result is consistent with the tidal capture hypothesis for the origin of the X-ray binaries, according to which the rate of formation of binaries is proportional to the product of the density of visible stars and the density of neutron stars in the cluster core. Either a relatively small fractional mass in heavy remnants, as in 47 Tuc, or a relatively small average mass for the heavy remnants, as in M15, can lead to a high central density of visible stars, and hence a high probability of forming an X-ray binary.

8 Conclusions

The properties of globular cluster cores can be accounted for if they are dominated by a population of heavy remnants having a 'collapsed' density distribution of the form $\varrho \propto r^{-2.5}$ at small radii. The required typical mass of the heavy remnants in most clusters is in the range ~ 2–$3M_\odot$, and is larger than the typical mass of the X-ray binaries; thus the heavy remnants are probably mostly black holes, although neutron stars and binaries will contribute. In a typical cluster like M3 the required fractional mass in heavy remnants is about 3 per cent. A simple, but not unique, set of assumptions that is compatible with these results is that in a typical cluster the IMF is the same as in the solar neighbourhood, and the massive stars leave remnants whose mass is about 15 per cent of the initial stellar mass.

The inner core of heavy remnants has a binding energy that diverges logarithmically at small radii if $n=0.5$, so it can supply a large amount of energy to the rest of the cluster, leading to the expansion and eventual dispersal of the cluster. The two most massive remnants will form a dominant central binary that absorbs an increasing fraction of the total binding energy. Under the above assumptions, the masses of the two largest remnants will typically be in the range 20–30$M_\odot$, and the entire binding energy of a typical cluster could be contained in such a binary if its separation is ~ 0.03 AU.

There are significant differences in the fractional mass in heavy remnants in different clusters; among the systems studied, the extremes are represented by 47 Tuc, which has about 1 per cent of its mass in heavy remnants, and ω Cen, which has about 5 per cent of its mass in heavy remnants. The fractional mass in heavy remnants correlates inversely with the slope of the main-sequence luminosity function, as would be expected if the differences were due to variations in the slope of the initial mass function. However, the inferred variations in the slope of the IMF for stars above the turnoff mass are small compared with the observed variations in the slope of the present main-sequence luminosity function; hence large variations in the IMF, if they occur, must be confined mostly to stars less massive than $\sim 1M_\odot$. Such a result would be consistent with evidence regarding the IMF in young open clusters and associations in the solar neighbourhood, which suggests that large variations can occur in the IMF of low-mass stars, while the IMF for massive stars is less variable (e.g. Scalo 1978; Larson 1982).

It is not yet clear to what extent some of the differences in the present mass functions of globular clusters can be explained just by differential escape of low-mass stars. A suggestion of a real variation in the IMF is provided by the fact that M3 appears to have a flatter main-sequence luminosity function than 47 Tuc, despite having a long relaxation time so that it should not have lost many low-mass stars. A further suggestion is provided by the fact that the ratio M_2/L is about 2.4 times larger for ω Cen than for 47 Tuc, although this difference could also be due to a difference in the amount of mass lost from massive stars. More data on the main-sequence luminosity functons of clusters with long relaxation times such as 47 Tuc, M3 and ω Cen would help to clarify the importance of variations of the IMF in globular clusters.

Acknowledgments

I am indebted to G. S. Da Costa, J. P. Vader and R. J. Zinn for much help and information, and especially to G. S. Da Costa for kindly supplying privately many details about his models for globular clusters.

References

Aarseth, S. J., 1968. *Bull. Astr.,* **3,** 105.
Aarseth, S. J., 1973. *Vistas Astr.,* **15,** 13.
Angeletti, L. & Giannone, P., 1980. *Astr. Astrophys.,* **85,** 113.
Anthony-Twarog, B. J., 1982. *Astrophys. J.,* **225,** 245.
Cohn, H., 1980. *Astrophys. J.,* **242,** 765.
Cowley, A. P., Crampton, D., Hutchings, J. B., Remillard, R. & Penfold, J. E, 1983. *Astrophys. J.,* **272,** 118.
Da Costa, G. S., 1977. *PhD thesis,* Australian National University.
Da Costa, G. S., 1979. *Astr. J.,* **84,** 505.
Da Costa, G. S., 1982. *Astr. J.,* **87,** 990.
Da Costa, G. S. & Freeman, K. C., 1976. *Astrophys. J.,* **206,** 128.
Da Costa, G. S., Freeman, K. C., Kalnajs, A. J., Rodgers, A. W. & Stapinski, T. E., 1977. *Astr. J.,* **82,** 810.
Djorgovski, S. & King, I. R., 1984. *Astrophys. J.,* **277,** L49.
Freeman, K. C., 1977. *The Evolution of Galaxies and Stellar Populations,* p. 133, ed. Tinsley, B. M. & Larson, R. B., Yale University Observatory, New Haven.
Grindlay, J. E., 1981. *X-ray Astronomy with the Einstein Satellite,* p. 79, ed. Giacconi, R., Reidel, Dordrecht, Holland.
Gunn, J. E. & Griffin, R. F., 1979. *Astr. J.,* **84,** 752.
Harding, G. A., 1965. *R. Obs. Bull.,* No. 99.
Harris, W. E. & Racine, R., 1979. *Ann. Rev. Astr. Astrophys.,* **17,** 241.
Heggie, D. C., 1984. *Mon. Not. R. astr. Soc.,* **206,** 179.
Hénon, M., 1961. *Ann. d'Astrophys.,* **24,** 369.
Hénon, M., 1972. *Gravitational N-Body Problem, IAU Colloq. No. 10,* p. 44, ed. Lecar, M., Reidel, Dordrecht, Holland.
Hénon, M., 1975. *Dynamics of Stellar Systems, IAU Symp. No. 69,* p. 133, ed. Hayli, A., Reidel, Dordrecht, Holland.
Illingworth, G. D., 1976. *Astrophys. J.,* **204,** 73.
Illingworth, G. D. & King, I. R., 1977. *Astrophys. J.,* **218,** L109.
Inagaki, S. & Lynden-Bell, D., 1983. *Mon. Not. R. astr. Soc.,* **205,** 913.
King, I. R., 1975. *Dynamics of Stellar Systems, IAU Symp. No. 69,* pp. 99, ed. Hayli, A., Reidel, Dordrecht, Holland.
Larson, R. B., 1970. *Mon. Not. R. astr. Soc.,* **150,** 93.
Larson, R. B., 1982. *Mon. Not. R. astr. Soc.,* **200,** 159.
Lightman, A. P., 1982. *Astrophys. J.,* **263,** L19.
Lynden-Bell, D. & Eggleton, P. P., 1980. *Mon. Not. R. astr. Soc.,* **191,** 483.
Miller, G. E. & Scalo, J. M., 1979. *Astrophys. J. Suppl.,* **41,** 513.

Newell, B., Da Costa, G. S. & Norris, J., 1976. *Astrophys. J.,* **208,** L55.

Newell, B. & O'Neil, E. J., 1978. *Astrophys. J. Suppl.,* **37,** 27.

Oort, J. H. & van Herk, G., 1959. *Bull. astr. Inst. Neth.,* **14,** 299.

Sandage, A. R., 1954. *Astr. J.,* **59,** 162.

Sandage, A. R., 1957. *Astrophys. J.,* **125,** 422.

Sandage, A. R. & Katem, B., 1983. *Astr. J.,* **88,** 1146.

Scalo, J. M., 1978. *Protostars and Planets, IAU Colloq. No. 52,* p. 265, ed. Gehrels, T., University of Arizona Press, Tucson.

Spitzer, L., Jr., 1975. *Dynamics of Stellar Systems, IAU Symp. No. 69,* p. 2, ed. Hayli, A., Reidel, Dordrecht, Holland.

Spitzer, L., Jr. & Hart, M. H., 1971. *Astrophys. J.,* **166,** 483.

Spitzer, L., Jr. & Shull, J. M., 1975. *Astrophys. J.,* **201,** 773.

Stodolkiewicz, J. S., 1982. *Acta Astr.,* **32,** 63.

van den Bergh, S., 1975. *Astrophys. J.,* **201,** 585.

van den Heuvel, E. P. J., 1980. *X-Ray Astronomy,* p. 115, eds Giacconi, R. & Setti, J., Reidel, Dordrecht, Holland.

Wilson, O. C. & Coffeen, M. F., 1954. *Astrophys. J.,* **119,** 197.