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On the Formation of Massive Stars and the Upper Limit of Stellar Masses

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Recent studies of the dynamical behavior of massive pulsationally unstable stars have shown that there is no reason to expect an upper mass limit on the basis of pulsational instability. However, stars of mass greater than about 60 $M_{\odot}$ do not appear to be present either in our galaxy or in the LMC. In this paper we show on the basis of recent dynamical studies of the collapse of protostars that there are several effects which make it increasingly difficult to form stars of larger and larger mass, and which set an upper limit of the order of 50 to 60 $M_{\odot}$ on the mass of a star which can form out of the present interstellar medium. The strongest limit on the mass of a star is provided by the formation of an H II region in the collapsing protostellar envelope surrounding a forming star; this occurs at a mass of the order of 25—60 $M_{\odot}$, depending on the initial conditions. For stars containing little or no heavy elements, however, much larger masses may be possible.

Key words: star formation — massive stars

1. Introduction

The pulsational stability of massive main sequence stars was first studied by Ledoux (1941), and later with more accurate models by Schwarzschild and Härm (1959). On the basis of a linear stability analysis, they concluded that a star with a mass greater than about 60—65 $M_{\odot}$ would be pulsationally unstable and that the pulsations could grow sufficiently rapidly to completely disrupt the star. If this occurs, one should not expect to observe any stars with masses greater than about 65 $M_{\odot}$. More recent work by Stothers and Simon (1970) and Ziebarth (1970) shows that when more accurate opacities are used in the analysis, the critical mass for pulsational instability is increased to a value more like 90 $M_{\odot}$, depending somewhat on the composition.

The observational upper limit on stellar masses is rather uncertain, but there appears to be no convincing evidence for stellar masses much larger than about 60 $M_{\odot}$. According to Batten (1968), the largest reliably determined masses for spectroscopic binaries are about 35 $M_{\odot}$. Sahade (1962) obtained a mass of ~ 64 $M_{\odot}$ for HD 47129, but this is subject to considerable uncertainty, owing to the anomalous characteristics of this system. Stothers and Simon (1968) conclude that the observational data on O stars and supergiants in our galaxy and in the Magellanic Clouds are consistent with an upper mass limit of the order of 60 $M_{\odot}$, except for a few objects of uncertain interpretation which could be more massive. It would appear, then, that the data are consistent with the suggestion of Schwarzschild and Härm (1959) that stellar masses are limited by the occurrence of pulsational disruption for stars with mass greater than the critical value for pulsational instability.

Recently, however, Appenzeller (1970a, b), Ziebarth (1970), and Talbot (1971a, b) have carried out detailed non-linear pulsation calculations for massive, pulsationally unstable stars, and they have found that the pulsations reach a limiting amplitude and have a much less disruptive effect on the star than the linear theory would predict. Ziebarth (1970) finds that no mass is lost from a model with 100 $M_{\odot}$, whereas Talbot (1971b) finds a small mass loss rate of about 3×10^{-5} $M_{\odot}$/yr for a 100 $M_{\odot}$ star, and Appenzeller obtains a mass loss of about $4 \times 10^{-5} M_{\odot}$/yr for a model of 130 $M_{\odot}$. Considering that the pulsation amplitude is likely if anything to have been overestimated (Ziebarth, 1970), and that a massive star rapidly becomes more stable as a result of its evolution, it appears unlikely that these stars will lose a large fraction of their mass during their main sequence lifetimes ($\sim 2 \times 10^{6}$ yr). Even for larger masses up to several hundred solar masses, the mass loss time scales estimated by Appenzeller (1970b) and Talbot (1971b) are of the order of 10^{6} years, which is comparable with the main sequence lifetimes of these objects. Thus it appears that pulsational mass loss is insufficient to completely obliterate very massive stars during their main sequence lifetime, and we are left with no theoretical reason why such very massive stars should not be observed.

It is the purpose of this paper to consider the implications for massive stars of recent dynamical studies of star formation, and to show that there are several effects which make it increasingly difficult to form stars of larger and larger mass and which make it unlikely that stars of mass greater than about 50–60 $M_\odot$ could ever be formed at all. In Section 2 we briefly summarize the relevant aspects of the collapse of a protostar, and in the succeeding sections we discuss the various effects of interest, roughly in order of increasing importance.

2. The Collapse of a Protostar

In recent years a number of authors have made numerical calculations of the dynamical behavior of collapsing interstellar clouds (Penston, 1966; Bodenheimer and Sweigart, 1968; Bodenheimer, 1968; Hunter, 1969; Larson, 1969a; Disney *et al.*, 1969; Narita *et al.*, 1970.) A fundamental result which emerges from all of these studies, despite the wide range in the conditions assumed, is that the collapse is always extremely non-homologous, and the cloud always develops a strong central condensation. Since the free-fall time varies as $\varrho^{-1/2}$, it becomes much shorter near the center of the cloud than in the outer part, and as a result the central part of the cloud is able to collapse into a stellar object of very small mass long before the bulk of the cloud has had time to collapse into the center.

Larson (1969a) found that in this way a stellar core or "embryo star" is formed at the center of the collapsing cloud, after a time which is essentially the initial free-fall time t_f of the cloud. Thereafter, the central embryo star steadily grows in mass as the remaining protostellar material falls into it, the time scale for this accretion process being of the order of another free-fall time. From the results of these calculations, it is found that, after the formation of the central stellar object, a further time interval of about 0.3 t_f is required for 50 percent of the total protostellar material to be accreted; 80 percent of the mass is accreted after about 0.9 t_f, and 99 percent after about 2.5 t_f. By this time, the central object is essentially an ordinary star, although it may still be obscured by the dust in the remnant protostellar cloud.

The free-fall time $t_f = (3\pi/32\, G\varrho)^{1/2}$ for the initial protostellar cloud can be estimated if we assume that the radius, mass, and temperature of the cloud are related by the Jeans criterion in the form adopted by Larson (1969a):

$$R = 0.41 \frac{GM}{\mathscr{R}T} \tag{1}$$

where $\mathscr{R} = 3.36 \times 10^7$ erg g^{-1} °K^{-1} if all the hydrogen is in molecular form. (Strictly speaking, Eq. (1) gives only an upper limit for the radius of a gravitationally unstable cloud, so that R could in fact be somewhat smaller than this value. Also, if the cloud collapses under the influence of an external pressure, its radius may decrease by as much as a factor of 2 before the stellar core is formed, as was found in one of the examples studied by Larson (1969a). We shall therefore keep in mind the possibility that the cloud radius may be smaller than predicted by Eq. (1) by perhaps a factor of 2.) For illustration we shall assume a temperature of 20 °K, as was found by Hunter (1969) for the central regions of massive collapsing clouds; temperatures of this order have also been observed in a cold hydrogen cloud by Riegel and Jennings (1969). With this assumption, the free-fall time obtained from Eq. (1) is $7.0 \times 10^4\, (M/M_\odot)$ years for a protostar of mass M. This is also approximately the time required for a star of mass M to form by accretion at the center of a collapsing protostellar cloud.

Important consequences follow from the fact that the accretion time scale increases with increasing mass. First, for masses greater than about 6 $M_\odot$, the accretion time becomes greater than the pre-main sequence contraction time for the central stellar object; under these circumstances, the central star will reach the main sequence before the infall of material has stopped, and it will then move up the main sequence as it increases in mass.

Secondly, we notice that if the protostellar mass becomes larger than about 60 $M_\odot$, the accretion time exceeds the total evolutionary lifetime of a star of this mass (Stothers, 1966). Thus, if a protostar of mass much greater than 60 $M_\odot$ were to begin collapsing into a central stellar object, the star would go through its entire evolution and end up as a supernova before the accretion of material could be completed. Therefore, there is an upper limit to the mass which a star can acquire by accretion during the time available, and this upper limit is of the order of 60 $M_\odot$. This limit is, however, rather sensitive to the initial conditions; if the initial temperature of the cloud is 40 °K instead of 20 °K, for example, or if (equivalently) the radius of the cloud is a factor of 2 smaller than predicted by Eq. (1), the limiting mass increases to approximately 120 $M_\odot$.

Thus we see, from a consideration only of the time scales involved, that there is an upper limit to the mass which a star can acquire, quite apart from other effects which may intervene to halt the accretion of material before this limit is reached. In the following sections we shall show that there are several ways in which the effects of the radiation field of the central stellar object can tend to halt or reverse the collapse when the central object becomes very luminous.

Since the following discussions will require a knowledge of the density distribution in the infalling cloud, we indicate here how this may be estimated, given the mass and radius of the cloud. During the later phases of the collapse, when half or more of the protostellar material has been accreted on the central stellar object, the mass of the system is essentially all either in the central star or in the outermost part of the protostellar cloud; there is very little mass in the intervening region, which may cover several orders of magnitude in radius. The mass flux $4\pi r^2 \varrho u$ must therefore be very nearly independent of r throughout this region. Since the material is essentially in free fall, the infall velocity u varies as $r^{-1/2}$; therefore the density distribution must have the form

$$\varrho = \varrho_0 \, r^{-3/2} \,. \tag{2}$$

In the detailed calculations of Larson (1969a), this density law was always found to be accurately satisfied in the innermost part of the collapsing cloud. In the outermost part of the cloud, the density distribution is still approximately of the form $\varrho \propto r^{-2}$, as was found for the initial isothermal phase of the collapse; however, as the collapse proceeds, the region in which Eq. (2) applies grows steadily in extent, and eventually occupies almost the whole cloud.

For illustration, we consider the point in time when half of the total protostellar mass has been accreted on the central star. At this time, we find from the calculations reported by Larson (1969a) that the density distribution is in fact approximately of the form (2) throughout the infalling cloud, with the constant ϱ_0 given by

$$\varrho_0 \cong 0.10 \, M R^{-3/2} \tag{3}$$

where M is the mass of the central star and R is the radius of the protostellar cloud. If we use Eq. (1) to relate the cloud radius R to the total protostellar mass $2\,M$, and if we assume a temperature of 20 °K as before, we obtain

$$\varrho_0 \cong 3.1 \times 10^6 \, (M/M_\odot)^{-1/2} \, \mathrm{g\,cm^{-3/2}} \,. \tag{4}$$

If the radius of the protostellar cloud is decreased by a factor of 2, the numerical coefficient in Eq. (4) increases by a factor of $2^{3/2}$ to 8.7×10^6. Using Eqs. (2) and (4), we can now estimate the density at any point in the collapsing protostellar envelope of a star of mass M.

In this paper we shall, as before, assume spherical symmetry throughout, and neglect such effects as rotation and magnetic fields; thus we are again discussing a rather idealized model. It is worth noting, however, that if massive stars are to form at all, then rotation cannot be very important, since rapid rotation will tend to produce fragmentation into smaller stars.

3. Radiative Heating of the Protostellar Material

After a stellar object has formed at the center of a protostellar cloud, the stellar luminosity, converted into infrared radiation by the dust in the opaque inner part of the cloud, may significantly heat up the outer part of the cloud, where most of the mass still resides. If the outer layers of the cloud become heated to temperatures much higher than the initial temperature, pressure forces may become large enough to overbalance gravity and halt or reverse the collapse of these outer layers. Because of the rapid increase of luminosity with mass, this effect may be expected to become more important with increasing mass.

Estimating the temperature of the outer part of the protostellar cloud is a rather intricate problem, since a variety of heating and cooling effects are involved. For illustration, however, we shall adopt the somewhat arbitrary assumption that the temperature of the gas is the same as that of the dust grains, which strictly speaking is justified only in the denser inner part of the cloud. Since the outer part of the cloud is optically thin at infrared wavelengths, the temperature of the dust grains can be estimated by the usual procedure of equating the energy absorbed from the central radiation source to the amount emitted from the surface of a grain, assuming emission in accordance with Kirchhoff's law. In performing this calculation, we shall assume, as was done by Larson (1969b), that the wavelength dependence of the dust opacity is given by $\varkappa_\lambda \propto \lambda^{-3/2}$; this will also serve as a very rough interpolating formula for the opacity data given by Kellman and Gaustad (1969). The temperature of a grain at distance r from a source having a luminosity L and a blackbody spectrum with temperature T_s is then given by

$$T = \left(\frac{L}{16\pi\sigma r^2}\right)^{\frac{2}{11}} T_s^{\frac{3}{11}} . \tag{5}$$

Putting $P = \varrho \mathscr{R} T$ and assuming $\varrho \propto r^{-3/2}$, we can now calculate the ratio of pressure to gravity forces as

$$\left|\frac{\text{pressure}}{\text{gravity}}\right| = \left|\frac{1}{\varrho}\frac{\partial P/\partial r}{GM/r^2}\right| = \frac{41}{22}\frac{\mathscr{R}}{GM}\left(\frac{L}{16\pi\sigma}\right)^{\frac{2}{11}} T_s^{\frac{3}{11}} r^{\frac{7}{11}} . \tag{6}$$

We note that pressure will be unimportant in the inner part of the cloud, but its importance increases with increasing r. We now evaluate expression (6) at the outer edge of the cloud, using Eq. (1) with $T = 20$ °K to estimate the radius of the cloud. For T_s we adopt a value of 500 °K, which is typical of the infrared emission from a protostar during the later stages of the collapse. Since the luminosity L enters only to a small power, we assume that the central stellar object is a main sequence star whose luminosity may be approximated by $(L/L_\odot) \cong (M/M_\odot)^{3.3}$ (Allen, 1963). With these assumptions, we finally obtain

$$\left|\frac{\text{pressure}}{\text{gravity}}\right|_{r=R} \cong 0.54 (M/M_\odot)^{0.24} . \tag{7}$$

From Eq. (7) we see that the radiative heating effect is not likely to be of major importance for masses less than about 10 $M_\odot$, but it may become important for larger masses; for $M = 60\, M_\odot$, for example, the ratio (7) becomes as large as ~ 1.5, so that in this case the collapse of the outermost layers of the protostellar cloud may be halted or reversed. In view of the uncertainties, however, and the very weak dependence of expression (7) in the mass, this argument does not yield any very well defined upper limit on stellar masses.

4. Radiation Pressure Effects

If the luminosity of the central stellar object becomes high enough, radiation pressure acting on the dust grains in the collapsing envelope may become important enough to slow down or prevent further accretion of material on the central stellar object, once a certain luminosity is reached. At protostellar densities the frictional coupling between gas and dust is sufficiently strong that forces acting on the dust grains are effectively transmitted to the gas. Estimation of the radiation pressure effect is, however, subject to considerable uncertainty, since the composition and the optical properties of the dust are not well known.

Because the dust grains evaporate very quickly when heated above their evaporation temperature, the innermost part of the collapsing cloud will be evacuated of dust grains, and there will be a dust shell whose inner edge is located at the point where the grains evaporate. Since the dust opacity is highest at visible wavelengths, radiation pressure first becomes important at the inner edge of this shell, where the visible radiation from the central star is absorbed directly. During most of the accretion process the dust shell is highly opaque at visible wavelengths, and therefore all of the stellar radiation is absorbed in a relatively thin layer at the inner edge of the dust shell. The radiation pressure exerted at this point will be able to halt the infall of material if the outward momentum flux of the radiation field balances the inward momentum flux of the infalling matter, i.e. if the radiation pressure $L/4\pi r^2 c$ balances the dynamical pressure ϱu^2. (Since the material is in free fall and $u^2 = 2GM/r$, this is equivalent to the condition that the radiation pressure must be able to support a shell of mass $8\pi r^3 \varrho$ at distance r from a central star of mass M.[1])

To evaluate the relative importance of radiation pressure and dynamical pressure, we use Eqs. (2) and (4) to estimate the density at any point in the cloud, and we set $u^2 = 2GM/r$. We then obtain for the ratio of radiation pressure to dynamical pressure at the inner edge of an opaque shell

$$\left|\frac{\text{radiation pressure}}{\text{dynamical pressure}}\right| = \frac{L/4\pi r^2 c}{\varrho u^2} \cong 1.3 \times 10^{-11} \frac{L/L_\odot}{(M/M_\odot)^{1/2}} r^{1/2} . \tag{8}$$

To estimate the radius r of the inner edge of the dust shell we make use of the solution obtained by Larson (1969b) for the temperature distribution in the optically thick inner part of a protostellar cloud. This was derived by assuming $\varrho = \varrho_0 r^{-n}$ and $\varkappa_\lambda = \varkappa_0 \lambda^{-p}$, and integrating the diffusion equation to obtain an analytic solution for $T(r)$ in the optically thick region. With $n = p = 3/2$, the result is

$$T = 4.1 \times 10^{14} (\varkappa_0 \varrho_0)^{2/5} (L/L_\odot)^{2/5} r^{-1} . \tag{9}$$

If we assume that the grains evaporate at a temperature of ~ 1500 °K, as would be the case for silicate grains, and if we adopt $\varkappa_0 = 7 \times 10^{-5}$ as was

[1]) This neglects the possibility that radiation may be scattered back from the dust shell rather than absorbed; in this case, the scattered radiation will increase the radiation pressure on the shell (see Faulkner, 1970).

done by Larson (1969b), we obtain for the inner edge of the dust shell

$$r_{\text{shell}} \cong 2.4 \times 10^{12} \frac{(L/L_\odot)^{2/5}}{(M/M_\odot)^{1/5}} \text{ cm} . \tag{10}$$

Finally, if we substitute this into Eq. (8) we obtain at the inner edge of the dust shell

$$\left|\frac{\text{radiation pressure}}{\text{dynamical pressure}}\right| \cong 2 \times 10^{-5} \frac{(L/L_\odot)^{6/5}}{(M/M_\odot)^{3/5}} . \tag{11}$$

This ratio increases rapidly with increasing mass, and becomes equal to unity for a mass of about 20 $M_\odot$, corresponding to a luminosity of about $4 \times 10^4 L_\odot$. The numerical coefficient in Eq. (11) is, however, uncertain by at least a factor of 3, which produces an uncertainty of at least a factor of 1.5 in the mass at which radiation pressure becomes important.

We conclude that, subject to the uncertainties, radiation pressure can begin to impede the collapse of the inner part of the protostellar cloud for stellar masses greater than about 20 $M_\odot$. We note, however, that while further accretion of matter will be impeded, it will not necessarily be cut off altogether; if the infalling material piles up near the inner edge of the dust shell and reaches densities much greater than that given by Eq. (2), gravity will again become dominant over radiation pressure and more material can fall into the star. It seems likely, however, that in such a situation the flow would be unstable and that further accretion of matter would occur in an irregular fashion.

There is a second way in which radiation pressure effects can become important for massive protostars: the infrared radiation emitted from the inner part of the protostellar cloud may exert an appreciable pressure on the outer parts of the cloud. The radiative force per unit mass is given by $\varkappa L/4\pi r^2 c$; comparing this with the gravitational force per unit mass, we have

$$\left|\frac{\text{radiation pressure}}{\text{gravity}}\right| = \frac{\varkappa L/4\pi r^2 c}{GM/r^2} = 7.8 \times 10^{-5} \varkappa \frac{L/L_\odot}{M/M_\odot} \tag{12}$$

where $\varkappa$ is the opacity due to dust grains, averaged over the spectrum of the illuminating radiation. Here a good knowledge of the dust opacity is essential, and since this is not yet available, we can only indicate the possible importance of this effect. If we somewhat arbitrarily take the formula $\varkappa_\lambda = 7 \times 10^{-5} \lambda^{-3/2}$ cm^2 g^{-1} used earlier, and substitute $\lambda = 6$ microns, the wavelength of the peak of a 500 °K blackbody spectrum, we obtain $\varkappa \sim 5$ cm^2 g^{-1}. Substituting this into Eq. (12), we find that radiation pressure and gravity are in balance for a mass of about 25 $M_\odot$ and a corresponding luminosity of about $7 \times 10^4 L_\odot$. Alternatively, if we consider the mineral grains discussed by Gaustad (1963), for which $\varkappa_\lambda \cong 20\, Q_\lambda$, and if we estimate $Q_\lambda \sim 0.05$ on the basis of data given by Kellman and Gaustad (1969), we obtain $\varkappa \sim 1$ cm^2 g^{-1}; the critical mass then increases to a value more like 90 $M_\odot$, corresponding to $L \sim 10^6 L_\odot$. Clearly the importance of radiation pressure in the outer part of the protostellar cloud is quite uncertain, but it could well be an important effect in limiting stellar masses if the dust opacity is high enough.

5. Formation of an H II Region

The factor which appears to limit most decisively the mass which a star can attain is the formation and rapid growth of an H II region in the protostellar material surrounding a newly formed massive star. As soon as the outer layers of the protostellar cloud become ionized, the temperature rises by at least two orders of magnitude and the pressure jumps by an even larger factor, so that further infall of material is immediately halted and the remaining protostellar material expands back into the interstellar medium. (The innermost part of the cloud will of course continue to collapse despite the high temperature, but this will involve only a negligible fraction of the mass of the cloud.)

Since the pre-main sequence contraction time for a massive star ($\approx 10^4$ years) is much shorter than the accretion time scale ($\approx 10^6$ years), we can assume that the stellar object which forms at the center of a massive protostellar cloud quickly reaches the main sequence and then moves up the main sequence as it continues to grow in mass. Suppose now that at some stage the central star becomes hot enough to ionize the hydrogen in a small region near the center of the protostellar cloud. Since pressure forces in the innermost part of the collapsing cloud are insignificant compared with gravity, even for temperatures of 10^4 °K, the density distribution in the ionized region is still given by Eq. (2). (We neglect for the moment the possible effects of dust grains, but we shall show below that the dust is probably not important for the development of the H II region.)

In a normal H II region the ionization structure is determined by the balance between radiative ionizations and recombinations. In the present situation these processes do not exactly balance, because we have un-ionized material continually falling into the ionized region, becoming ionized, and then being accreted on the central star. However, the rate at which ionizations and recombinations are occurring in the ionized region is several orders of magnitude higher than the rate at which neutral atoms are entering it; this is readily seen from the fact that the flux of ionizing photons from a hot O star is of the order of $10^{49}\,\mathrm{s}^{-1}$, whereas the accretion rate is less than 10^{45} atoms s^{-1}. Thus we can still make the usual assumption of the equality of ionization and recombination rates. For the same reason, the time required to form the H II region is insignificant compared with the time scale for the infall of material, so that at any stage we can assume that we have an equilibrium H II region of the type first studied by Strömgren; the essential difference is that here we assume not a uniform density distribution, but one of the form given by Eq. (2).

To obtain a first approximation to the size of the H II region, we start with the following equation (Spitzer, 1968, Eq. 4–54):

$$\frac{dS(r)}{dr} = -\,4\pi r^2\, x^2\, n_{\mathrm{H}}^2\, \alpha \tag{13}$$

where $S(r)$ is the number of ionizing photons flowing outward across a shell of radius r, x is the degree of ionization (here assumed equal to unity throughout the H II region), n_{H} the number density of hydrogen atoms, and $\alpha \cong 2.6 \times 10^{-13}\,\mathrm{cm}^3\,\mathrm{s}^{-1}$ is the recombination coefficient for recombinations to the second and higher levels. Equation (13) assumes that the H II region is optically thin at wavelengths longward of the Lyman limit; in the present situation, however, we are dealing with densities that are much higher than are normally encountered in H II regions (up to $\sim 10^{-12}\,\mathrm{g\,cm}^{-3}$), and it is possible that the material may become opaque to Balmer continuum radiation, for example. If this happens, significant photoionization will occur from the second level as well as from the ground state of the hydrogen atom, and the extent of the ionized region will be greater than that predicted from Eq. (13). Since this effect is difficult to estimate, we shall continue to use Eq. (13), with the understanding that it provides a lower limit to the size of the H II region and therefore an upper limit on the stellar mass at which the growth of an H II region cuts off further mass accretion.

If we substitute $n_{\mathrm{H}} = n_0\, r^{-3/2}$ into Eq. (13), we can now estimate the radius of the H II region by integrating Eq. (13) from the surface of the star ($r = R_s$) out to the point where $S(r)$ drops to zero, which we take as the radius $R_{\mathrm{H\,II}}$ of the H II region. The integration is elementary and yields

$$R_{\mathrm{H\,II}} = R_s \exp\left\{\frac{S(R_s)}{4\pi\alpha n_0^2}\right\}. \tag{14}$$

Thus the size of the H II region depends exponentially on $S(R_s)$, the rate at which Lyman continuum photons are emitted by the central star, and the H II region can be expected to grow rapidly once S reaches a certain critical value.

To determine the conditions required to ionize the whole protostellar cloud, we first use Eq. (1) to estimate $R_{\mathrm{H\,II}}/R_s \sim 10^7$ (the value of this ratio is not critical, since it enters only logarithmically); equation (14) then yields

$$\frac{S}{4\pi\alpha n_0^2} = \ln\left(\frac{R_{\mathrm{H\,II}}}{R_s}\right) \cong 16\,. \tag{15}$$

If we use Eq. (4) for ϱ_0 and put $n_0 = 4.2 \times 10^{23}\,\varrho_0$, as would be appropriate for a hydrogen abundance of $X = 0.70$, we obtain

$$n_0 \cong 1.3 \times 10^{30}\,(M/M_\odot)^{-1/2}\,.$$

It should be noted, however, that the value of n_0 is sensitive to the radius R of the protostellar cloud, being proportional to $R^{-3/2}$; this is important, since n_0 appears squared in Eq. (15). To show this dependence explicitly, we define a "standard" radius R_1 as the value of R calculated from Eq. (1) with $\mathscr{R} = 3.36 \times 10^7\,\mathrm{erg\,g^{-1}\,{}^\circ K^{-1}}$ and $T = 20\,{}^\circ\mathrm{K}$. For a value of R different from R_1, n_0 is then given by the more general expression

$$n_0 \cong 1.3 \times 10^{30}\,(M/M_\odot)^{-1/2}\,(R/R_1)^{-3/2}\,. \tag{16}$$

Substituting this expression for n_0 into Eq. (15) we obtain

$$(M/M_\odot)\, S \cong 9 \times 10^{49}\,(R_1/R)^3\ \mathrm{s}^{-1} \tag{17}$$

as the condition required for ionization of the whole protostellar cloud.

Using data given by Hjellming (1968), we obtain the following approximate values for the quantity $(M/M_\odot)\,S$ as a function of the stellar mass and main sequence spectral type:

$M/M_\odot$	*Spectrum*	$(M/M_\odot)\,S$ (s^{-1})
20	O 8.5	2.5 (+49)
25	O 7.5	9.0 (+49)
30	O 6.5	2.2 (+50)
45	O 5.5	8.6 (+50)
60	O 4.5	2.1 (+51)

Comparing these values with Eq. (17), we see that if $R = R_1$, the maximum mass which a star can attain before the protostellar cloud becomes ionized is about 25 $M_\odot$. A larger mass is possible if the cloud radius R is somewhat smaller than R_1; if $R = R_1/2$, the maximum mass increases to $\sim 43\, M_\odot$, and if $R = R_1/3$, a maximum mass of $\sim 60\, M_\odot$ is obtained.

Thus, in order to account for the existence of stars more massive than about 25 $M_\odot$ we require one of the following possibilities: (1) the protostellar cloud must be somewhat more compressed than is required by the Jeans criterion (Eq. 1), or (2) the initial value of $\mathscr{R}T$ must be larger than the value 6.72×10^8 erg g^{-1} which we have assumed. Possibility (1) can arise if the total amount of mass involved in the collapse is much larger than that of the star which is formed; in this case the central part of the collapsing cloud will be somewhat more compressed than would be estimated from Eq. (1). In fact, we find from the results of Larson (1969a) that if the mass of the central star is only one quarter of the total protostellar mass (rather than one half, as previously assumed), the value of ϱ_0 is increased by a factor of about 2 over that given by Eq. (4); if the mass of the central star is one tenth of the total mass, ϱ_0 is increased by a factor of about 3. The maximum factor by which ϱ_0 can be increased in this way appears to be about 4. The limiting stellar masses obtained when ϱ_0 is increased by factors 2, 3, and 4 are about 35 $M_\odot$, 44 $M_\odot$, and 53 $M_\odot$, respectively.

The initial value of $\mathscr{R}T$ can be larger than we have assumed if the initial temperature is higher than the 20 °K we have assumed, or if the hydrogen is not all in molecular form, which would increase the gas constant $\mathscr{R}$. In the extreme case where all of the hydrogen is in atomic form, $\mathscr{R}$ is increased by a factor of 1.8, and the cloud radius R is decreased by the same factor; this would increase the limiting stellar mass from 25 $M_\odot$ to about 39 $M_\odot$. Also, it is possible that turbulent motions may contribute to the kinetic energy content of the cloud; this would increase the effective value of $\mathscr{R}T$, which would again have the effect of increasing the limiting mass.

Since the possibilities mentioned above are reasonable ones, particularly for large, low density clouds, we conclude that it is possible with reasonable assumptions concerning the initial conditions to account for the existence of stars with masses up to perhaps 50–60 $M_\odot$. Stars much more massive than this cannot form, however, unless the initial conditions are substantially different from those we have considered, which seems unlikely; we conclude that such objects could form only under exceptional circumstances.

We have up to now neglected any possible effects of dust grains on the growth of the H II region. This makes no difference until the H II region reaches the inner edge of the dust shell; beyond this point, however, the growth of the H II region may be inhibited if the grains persist and absorb ionizing photons. From data given by Mathews (1969), however, it appears likely that the grains will be rapidly destroyed by sputtering as soon as the H II region comes in contact with them, at least in the dense inner part of the cloud. Thus the growth of the H II region will probably not be much affected by the existence of dust grains (cf. also Mathews, 1969). (We note also that as dust is cleared away from the central part of the protostellar cloud, radiation pressure has a better chance to be effective in driving away the remaining dusty material, since the relative importance of radiation pressure increases with increasing r (cf. Eq. 8).)

6. Formation of Extreme Population II Stars

We note that the foregoing discussions apply only to the formation of Population I stars under conditions presently encountered in the interstellar medium. As we have already emphasized, the maximum stellar mass depends strongly on the assumed initial temperature, and if the temperature is higher than the values we have considered, higher limiting masses will result. Higher initial temperatures would in fact be expected if the protostellar material contains a smaller abundance of heavy elements, since the heavy elements provide the cooling agents. Thus some of the first Population II stars may well have been formed with masses considerably above the upper limit which we have found for Population I stars

As an extreme case, suppose that the protostellar material contains no heavy elements at all; then it cannot cool below about 10^4 °K. In this case the only one of our arguments for a limiting mass which is still clearly applicable is the time scale argument mentioned in Section 2. If we consider a protostellar cloud consisting of neutral atomic hydrogen at a temperature of 10^4 °K, the free-fall time is about 2.6 $(M/M_\odot)$ years. Assuming that the lifetime of a very massive star is $\sim 10^6$ years (Wagoner, 1969), we find, equating the free fall time to the stellar lifetime, that the maximum possible stellar mass is of the order of $4 \times 10^5\, M_\odot$. (This happens to be about the same as the maximum mass which can be supported by hydrogen burning against a relativistic collapse.) We estimate that an H II region would form at a mass of roughly $10^4 - 10^5\ M_\odot$, but this may not make much difference to the collapse, since the material is already at a temperature of 10^4 °K anyway.

Thus there appears to be a possibility that, if the galaxy was formed out of material containing essentially no heavy elements, the very earliest stars to form may have been very massive objects with masses up to $\approx 10^5\ M_\odot$. The possibility of very large masses is also suggested by the fact that the high temperature of 10^4 °K makes fragmentation into small masses difficult. If such massive objects did in fact form, then presumably their nuclear evolution and eventual explosion could have been instrumental in enriching the interstellar medium in heavy elements (Wagoner, 1969).[2])

7. Conclusions

We now summarize the various theoretical arguments for the existence of an upper limit to the mass with which a Population I star can be formed. The time scale argument of Section 2 suggests a maximum possible mass of the order of $60-120\ M_\odot$, depending on the initial conditions. A consideration of radiative heating of the protostellar material does not yield a well defined maximum mass, but suggests that for masses higher than about $10\ M_\odot$ it may become gradually more difficult to form stars of larger and larger mass. Radiation pressure acting on the inner part of the protostellar cloud will begin to impede further infall of material for masses greater than about $20\ M_\odot$, although further accretion need not be brought to a halt. Radiation pressure acting on the outer part of the protostellar cloud may become sufficient to halt the collapse at a mass of the order of $50\ M_\odot$, although this number is uncertain by at least a factor of 2. The effect which most decisively limits the mass of a star is the growth of an H II region in the protostellar cloud, which occurs at a mass of the order of $25-60\ M_\odot$, depending on the initial conditions.

Considering all the effects which have been discussed, we feel that, with reasonable assumptions for the initial conditions, the largest mass with which a Population I star can form out of the present interstellar medium is of the order of $50-60\ M_\odot$. It seems unlikely that stars more massive than this could form, except perhaps in exceptional circumstances. This conclusion does not apply to extreme Population II stars, for which much larger masses may be possible.

[2]) We have learnt from Dr. P. J. E. Peebles (private communication) that at sufficiently high densities, cooling below 10^4 °K can be produced by molecular hydrogen formed by the reactions $e + \mathrm{H} \to \mathrm{H}^- + \gamma$, $\mathrm{H}^- + \mathrm{H} \to \mathrm{H}_2 + e$. The conclusions of this section may therefore be somewhat modified, in the direction of lower masses.

References

Allen, C. W. 1963, Astrophysical Quantities. London, Athlone Press, 2nd ed.

Appenzeller, I. 1970a, *Astr. Astrophys.* **5**, 355.

Appenzeller, I. 1970b, *Astr. Astrophys.* **9**, 216.

Batten, A. H. 1968, *J. R. astr. Soc. Can.* **62**, 192 (A).

Bodenheimer, P. 1968, *Astrophys. J.* **153**, 483.

Bodenheimer, P., Sweigart, A. 1968, *Astrophys. J.* **152**, 515.

Disney, M. J., McNally, D., Wright, A. E. 1969, *Mon. Not. R. astr. Soc.* **146**, 123.

Faulkner, D. J. 1970, *Astrophys. J.* **162**, 513.

Gaustad, J. E. 1963, *Astrophys. J.* **138**, 1050.

Hjellming, R. M. 1968, *Astrophys. J.* **154**, 533.

Hunter, J. H. 1969, *Mon. Not. R. astr. Soc.* **142**, 473.

Kellman, S. A., Gaustad, J. E. 1969, *Astrophys. J.* **157**, 1465.

Larson, R. B. 1969a, *Mon. Not. R. astr. Soc.* **145**, 271.

Larson, R. B. 1969b, *Mon. Not. R. astr. Soc.* **145**, 297.

Ledoux, P. 1941, *Astrophys. J.* **94**, 537.

Narita, S., Nakano, T., Hayashi, C. 1970, *Progress theor. Phys.* **43**, 942.

Mathews, W. G. 1969, *Astrophys. J.* **157**, 583.

Penston, M. V. 1966, *R. Obs. Bull. Number* **117**, 299.

Reigel, K., Jennings, M. 1969, *Astrophys. J.* **157**, 563.

Sahade, J. 1962, Symposium on Stellar Evolution, La Plata National University at La Plata, p. 185.

Schwarzschild, M., Härm, R. 1959, *Astrophys. J.* **129**, 637.

Spitzer, L. 1968, Diffuse Matter in Space, New York, Wiley-Interscience.

Stothers, R. 1966, *Astrophys. J.* **144**, 959.

Stothers, R., Simon, N. R. 1968, *Astrophys. J.* **152**, 233.

Stothers, R., Simon, N. R. 1970, *Astrophys. J.* **160**, 1019.

Talbot, R. J. 1971a, *Astrophys. J.* **163**, 17.

Talbot, R. J. 1971b, *Astrophys. J.* **165**, 121.

Wagoner, R. V. 1969, *A. Rev. Astr. Astrophys.* **7**, 553.

Ziebarth, K. 1970, *Astrophys. J.* **162**, 947.

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