

# NON-ISOTHERMAL GRAVOTURBULENT FRAGMENTATION

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## Abstract

The thermodynamic state of star-forming gas determines its fragmentation behavior and thus plays a crucial role in determining the stellar initial mass function (IMF). We address the issue by studying the effects of a piecewise polytropic equation of state on the formation of stellar clusters in turbulent, self-gravitating molecular clouds using three-dimensional, smoothed particle hydrodynamics simulations. In these simulations stars form via a process we call gravoturbulent fragmentation, i.e., gravitational fragmentation of turbulent gas. To approximate the results of published predictions of the thermal behavior of collapsing clouds, we increase the polytropic exponent  $\gamma$  from 0.7 to 1.1 at a critical density  $n_c$ , which we estimated to be  $2.5 \times 10^5 \text{ cm}^{-3}$ . The change of thermodynamic state at  $n_c$  selects a characteristic mass scale for fragmentation  $M_{\text{ch}}$ , which we relate to the peak of the observed IMF. A simple scaling argument based on the Jeans mass  $M_J$  at the critical density  $n_c$  leads to  $M_{\text{ch}} \propto n_c^{-0.95}$ . We perform simulations with  $4.3 \times 10^4 \text{ cm}^{-3} < n_c < 4.3 \times 10^7 \text{ cm}^{-3}$  to test this scaling argument. Our simulations qualitatively support this hypothesis, but we find a weaker density dependence of  $M_{\text{ch}} \propto n_c^{-0.5 \pm 0.1}$ . We also investigate the influence of additional environmental parameters on the IMF. We consider variations in the turbulent driving scheme, and consistently find  $M_J$  is decreasing with increasing  $n_c$ . Our investigation generally supports the idea that the distribution of stellar masses depends on the thermodynamic state of the star-forming gas. The thermodynamic state of interstellar gas is a result of the balance between heating and cooling processes, which in turn are determined by fundamental atomic and molecular physics and by chemical abundances.

## Introduction

One of the fundamental unsolved problems in astronomy is the origin of the stellar mass spectrum, the so-called initial mass function (IMF). Observations suggest that there is a characteristic mass  $M_{\text{ch}}$  for stars in the solar vicinity. The IMF peaks at this characteristic mass which is typically a few tenths of a solar mass. The IMF has a nearly power-law form for larger masses and declines rapidly towards smaller masses (Scale 1998; Kroupa 2002; Chabrier 2003). Scalo, 1998 Kroupa, 2002 Chabrier, 2003

Although the IMF has been derived from vastly different regions, from the solar vicinity to dense clusters of newly formed stars, the basic features seem to be strikingly universal to all determinations (Kroupa 2001). Initial conditions in star forming regions can vary considerably. If the IMF depends on the initial conditions, there would thus be no reason for it to be universal. Therefore a derivation of the characteristic stellar mass that is based on fundamental atomic and molecular physics would be more consistent. Kroupa, 2001 Larson, 2003 Mac Low and Klessen, 2004 Larson, 1981 Fleck, 1982 Padoan, 1995 Padoan et al., 1997 Klessen et al., 1998; Klessen et al., 2000 Klessen, 2001a Padoan and Nordlund, 2002 Schmeja and Klessen, 2004 Jappsen and Klessen, 2004 Bodenheimer, 1995 Papaloizou and Lin, 1995 Lin and Papaloizou, 1996

There are many ways to approach the formation of stars and star clusters from a theoretical point of view. In particular models that connect stellar birth to the turbulent motions ubiquitously observed in Galactic molecular clouds have become increasingly popular in recent years. See, e.g., the reviews by Larson (2003) and Mac Low & Klessen (2004). The interplay between turbulent motion and self-gravity of the cloud leads to a process we call gravoturbulent fragmentation: Supersonic turbulence generates strong density fluctuations with gravity taking over in the densest and most massive regions (e.g., Larson 1981; Fleck 1982; Padoan 1995; Padoan et al. 1997; Klessen et al. 1998, 2000; Klessen 2001; Padoan & Nordlund 2002). Once gas clumps become gravitationally unstable, collapse sets in. The central density increases and soon individual or whole clusters of protostellar objects form and grow in mass via accretion from their infalling envelopes. The mass accretion rates vary with time and are strongly influenced by the cluster environment (Klessen 2001). Schmeja & Klessen (2004) showed that a sharp peak in the mass accretion occurs shortly after the formation of the protostellar core. There is a positive correlation between the value of the peak accretion rate and the final mass of the protostar.

Protostellar collapse is also accompanied by a substantial loss of specific angular momentum, even in the absence of magnetic fields (Jappsen & Klessen 2004). Still, most of the matter that falls in will assemble in a protostellar disk. It is then transported inward by viscous and possibly gravitational torques

(e.g., Bodenheimer 1995; Papaloizou & Lin 1995; Lin & Papaloizou 1996). If low angular momentum material is accreted, the disk is stable and most of the material ends up in the central star. In this case, the disk simply acts as a buffer and smooths eventual accretion spikes. It will not delay or prevent the mass growth of the central star by much. However, if material with large specific angular momentum is accreted, then the mass load onto the disk is likely to be faster than inward transport. The disk grows large and may become gravitationally unstable and fragment. This may lead to the formation of a binary or higher-order multiple system (Bodenheimer et al. 2000; Fromang et al. 2002).

However, most current results are based on models that do not treat thermal physics in detail. Typically, a simple isothermal equation of state (EOS) is used. The true nature of the EOS, thus, remains a major theoretical problem in understanding the fragmentation properties of molecular clouds. Li et al., 2003

Recently Li et al. (2003) conducted a systematic study of the effects of a varying polytropic exponent on gravoturbulent fragmentation. Their results showed that  $\gamma$  determines how strongly self-gravitating gas fragments. They found that the degree of fragmentation decreases with increasing polytropic exponent in the range  $0.2 < \gamma < 1.4$  although the total amount of mass in collapsed cores appears to remain roughly constant through this range. These findings suggest that the IMF might be quite sensitive to the thermal physics. However in their computations,  $\gamma$  was left strictly constant in each case. Here we study the effects of using a piecewise polytropic equation of state with a polytropic exponent that changes at a certain critical density  $\rho_c$  from  $\gamma_1$  to  $\gamma_2$ :

$$\begin{aligned} P &= K_1 \rho^{\gamma_1} & \rho \leq \rho_c \\ P &= K_2 \rho^{\gamma_2} & \rho > \rho_c \end{aligned} \quad (1)$$

where  $K_1$  and  $K_2$  are constants,  $P$  is the thermal pressure, and  $\rho$  is the gas density. We investigate if a change in  $\gamma$  determines the characteristic mass of the gas clump spectrum and thus, possibly, the turn-over mass of the IMF. Mac Low and Ossenkopf, 2000; Ossenkopf et al., 2001; Ossenkopf and Mac Low, 2002 Klessen et al., 2004 Vázquez-Semadeni et al., 2005 Benson and Myers, 1989; Myers et al., 1991

### Thermal properties of star-forming clouds

Gravity in galactic molecular clouds is initially expected to be opposed mainly by a combination of supersonic turbulence and magnetic fields (Mac Low & Klessen 2004). The velocity structure in the clouds is always observed to be dominated by large-scale modes (Mac Low & Ossenkopf 2000; Ossenkopf et al. 2001; Ossenkopf & Mac Low 2002). In order to maintain

turbulence for some global dynamical timescales and to compensate for gravitational contraction of the cloud as a whole, kinetic energy input from external sources seems to be required. Star formation then takes place in molecular cloud regions which are characterized by local dissipation of turbulence and loss of magnetic flux, eventually leaving thermal pressure as the main force resisting gravity in the small dense prestellar cloud cores that actually build up the stars (Klessen et al. 2004; Vázquez-Semadeni et al. 2005). In agreement with this expectation, observed prestellar cores typically show a rough balance between gravity and thermal pressure (Benson & Myers 1989; Myers et al. 1991). Therefore the thermal properties of the dense star-forming regions of molecular clouds must play an important role in determining how these clouds collapse and fragment into stars. Koyama and Inutsuka, 2000 Larson, 1985; Larson, 2005; Spaans and Silk, 2000

Observational and theoretical studies of the thermal properties of collapsing clouds both indicate that at densities below  $10^{-18} \text{ g cm}^{-3}$ , roughly corresponding to a number density of  $n = 2.5 \times 10^5 \text{ cm}^{-3}$ , the temperature decreases with increasing density. This is due to the strong dependence of molecular cooling rates on density (Koyama & Inutsuka 2000). Therefore, the polytropic exponent is below unity in this density regime. At densities above  $10^{-18} \text{ g cm}^{-3}$ , the gas becomes thermally coupled to the dust grains, which then control the temperature by far-infrared thermal emission. The balance between compressional heating and thermal cooling by dust causes the temperature to increase again slowly with increasing density. Thus the temperature-density relation can be approximated with  $\gamma$  above unity (Larson 1985, 2005; Spaans & Silk 2000). Changing from a value below unity to a value above unity results in a minimum temperature at the critical density. As shown by Li et al. (2003), gas fragments efficiently for  $\gamma < 1.0$  and less efficiently for higher  $\gamma$ . Thus, the Jeans mass at the critical density defines a characteristic mass for fragmentation, which may be related to the peak of the IMF.

## Numerical Approach

Springel et al., 2001 Benz, 1990; Monaghan, 1992 Bate and Burkert, 1997 Bate et al., 1995 Jappsen et al., 2005 To gain insight into how molecular cloud fragmentation and the characteristic stellar mass may depend on the critical density, we perform a series of smoothed particle hydrodynamics (SPH) calculations of the gravitational fragmentation of supersonically turbulent molecular clouds using the parallel code GADGET by Springel et al. (2001). SPH is a Lagrangian method, where the fluid is represented by an ensemble of particles, and flow quantities are obtained by averaging over an appropriate subset of SPH particles; see Benz (1990) and Monaghan (1992). The method is able to resolve large density contrasts as particles are free to move, and so naturally

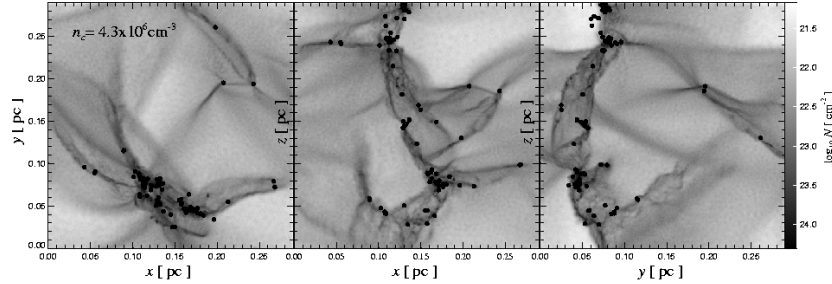


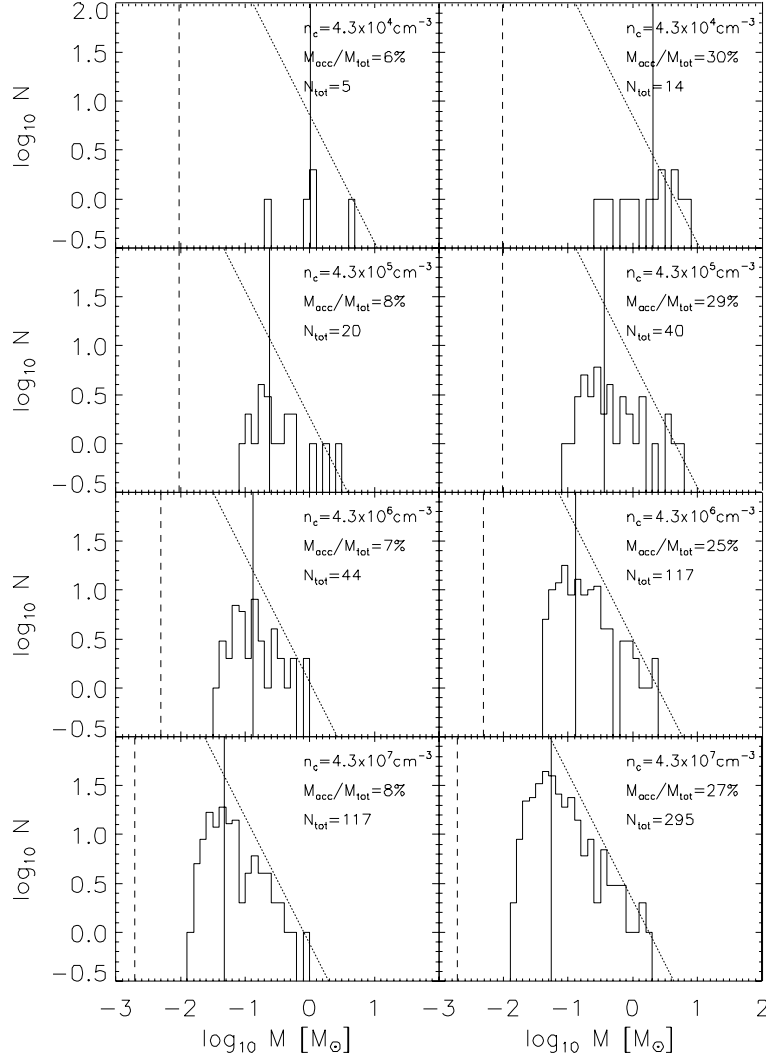
Figure 1. Column density distribution of the gas and location of identified protostellar objects (black circles) using the model with  $n_c = 4.3 \times 10^6 \text{ cm}^{-3}$  where approximately 50% of the gas is accreted. Projections in the  $xy$ -,  $xz$ -, and  $yz$ -plane are shown.

the particle concentration increases in high-density regions. We use the Bate & Burkert (1997) criterion for the resolution limit of our calculations. It is adequate for the problem considered here, where we follow the evolution of highly nonlinear density fluctuations created by supersonic turbulence. We replace the central high-density regions of collapsing gas cores by sink particles (Bate et al. 1995). These particles have the ability to accrete gas from their surroundings while keeping track of the total mass, the linear and angular momentum of the collapsing gas. This enables us to follow the dynamical evolution of the system over many local free-fall timescales (see Fig. 1). Bodenheimer, 1995; Papaloizou and Lin, 1995; Lin and Papaloizou, 1996 Bodenheimer et al., 2000; Fromang et al., 2002

We compute models where the polytropic exponent changes from  $\gamma = 0.7$  to  $\gamma = 1.1$  for critical densities in the range  $4.3 \times 10^4 \text{ cm}^{-3} < n_c < 4.3 \times 10^7 \text{ cm}^{-3}$ . Each simulation starts with a uniform density distribution, and turbulence is driven on large scales, with wave numbers  $k$  in the range  $1 \leq k \leq 2$ . We use the same driving field in all four models. The global free-fall timescale is  $t_{\text{ff}} \approx 10^5 \text{ yr}$ . For further details see Jappsen et al. (2005). Salpeter, 1955 Lada and Lada, 2003 Bonnell et al., 2001 Klessen, 2001b Schmeja and Klessen, 2004 Jappsen and Klessen, 2004

## Dependency of the Characteristic Mass

To illustrate the effects of varying the critical density, we plot in Fig. 2 the resulting mass spectra at different times when the fraction of mass accumulated in protostellar objects has reached approximately stages of 10% and 30%. This range of efficiencies corresponds roughly to the one expected for regions of clustered star formation (Lada & Lada 2003). In the model with a critical density  $n_c$  of  $4.3 \times 10^4 \text{ cm}^{-3}$  the change in  $\gamma$  occurs below the initial mean



*Figure 2.* Mass spectra of protostellar cores for four models with critical densities in the range  $4.3 \times 10^4 \text{ cm}^{-3} < n_c < 4.3 \times 10^7 \text{ cm}^{-3}$ . We show two phases of evolution, when about 10% and 30% of the mass has been accreted onto protostars. The *vertical solid line* shows the median mass of the distribution. The *dotted line* serves as a reference to the Salpeter (1955) slope of the observed IMF at high masses. The *dashed line* indicates our mass resolution limit.

density. The mass spectrum shows a flat distribution with only few, but massive protostellar objects. These reach masses up to  $10 M_\odot$  and the minimum mass is about  $0.3 M_\odot$ . All other models build up a power-law tail towards high masses. This is due to protostellar accretion processes, as more and more gas gets turned into stars (see also, Bonnell et al. 2001; Klessen 2001; Schmeja &

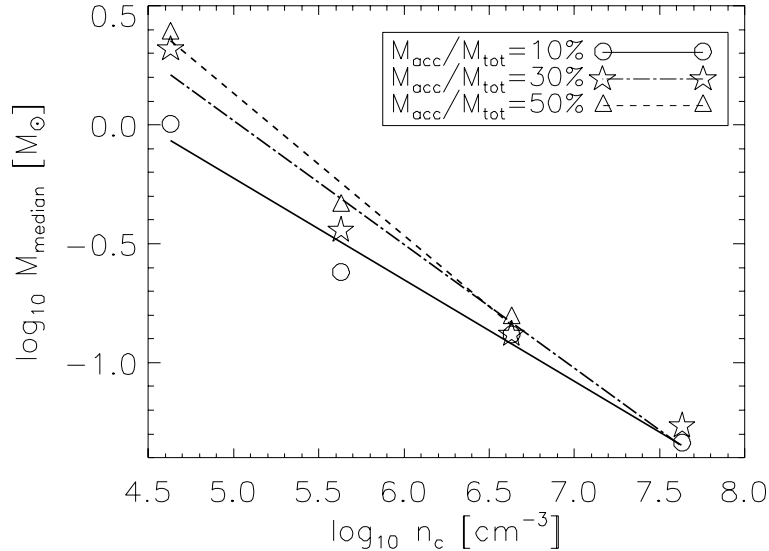


Figure 3. Plot of the median mass of the protostellar cores  $M_{\text{median}}$  versus critical density  $n_c$ . We display results for different ratios of accreted gas mass to total gas mass,  $M_{\text{acc}}/M_{\text{tot}}$ , and fit the data with straight lines. Their slopes take the values  $-0.43 \pm 0.05$  (solid line),  $-0.52 \pm 0.06$  (dashed-dotted line), and  $-0.60 \pm 0.07$  (dashed line), respectively.

Klessen 2004). The mass spectrum becomes more peaked for higher  $n_c$  and shifts to lower masses.

We find closest correspondence with the observed IMF (Scalo 1998; Kroupa 2002; Chabrier 2003) for a critical density  $n_c$  of  $4.3 \times 10^6 \text{ cm}^{-3}$  and for stages of accretion around 30%. For high masses, the distribution exhibits a (Salpeter 1955) power-law behavior. For masses about the median mass the distribution has a small plateau and then falls off towards smaller masses.

The change of median mass  $M_{\text{median}}$  with critical density  $n_c$  is quantified in Fig. 3. As  $n_c$  increases  $M_{\text{median}}$  decreases. We fit our data with straight lines. The slopes take values between  $-0.4$  and  $-0.6$ .

## Summary

Using SPH simulations we investigate the influence of a piecewise polytropic EOS on fragmentation of molecular clouds. We study the case where the polytropic index  $\gamma$  changes from 0.7 to 1.1 at a critical density  $n_c$ , and consider the range  $4.3 \times 10^4 \text{ cm}^{-3} < n_c < 4.3 \times 10^7 \text{ cm}^{-3}$  around the realistic value to determine the dependence of the mass spectrum on  $n_c$ . Koyama and Inutsuka, 2000

Our investigation supports the idea that the distribution of stellar masses depends, at least in part, on the thermodynamic state of the star-forming gas. If

there is a low-density regime in molecular clouds where temperature  $T$  sinks with increasing density  $\rho$ , followed by a higher-density phase where  $T$  increases with  $\rho$ , fragmentation seems likely to be favored at the transition density where the temperature reaches a minimum. This defines a characteristic mass scale. The thermodynamic state of interstellar gas is a result of the balance between heating and cooling processes, which in turn are determined by fundamental atomic and molecular physics and by chemical abundances. The derivation of a characteristic stellar mass can thus be based on quantities and constants that depend solely on the chemical abundances in a molecular cloud.

A simple scaling argument based on the Jeans mass  $M_J$  at the critical density  $n_c$  leads to  $M_J \propto n_c^{-0.95}$  (see Jappsen et al. 2004). If there is a close relation between the average Jeans mass and the gravoturbulent fragmentation spectrum, a similar relation should hold for the characteristic mass  $M_{\text{ch}}$  of protostellar cores. Our simulations qualitatively support this hypothesis, however, with the weaker density dependency  $M_{\text{ch}} \propto n_c^{-0.5 \pm 0.1}$ .

The density at which  $\gamma$  changes from below unity to above unity selects a characteristic mass scale. Consequently, the peak of the resulting mass spectrum decreases with increasing critical density. This spectrum not only shows a pronounced peak but also a powerlaw tail towards higher masses. Its behavior is thus similar to the observed IMF.

Altogether, supersonic turbulence in self-gravitating molecular gas generates a complex network of interacting filaments. The overall density distribution is highly inhomogeneous. Turbulent compression sweeps up gas in some parts of the cloud, but other regions become rarefied. The fragmentation behavior of the cloud and its ability to form stars depend strongly on the EOS. If collapse sets in, the final mass of a fragment depends not only on the local Jeans criterion, but also on additional processes. For example, protostars grow in mass by accretion from their surrounding material. In turbulent clouds the properties of the gas reservoir are continuously changing. In a dense cluster environment, furthermore, protostars may interact with each other, leading to ejection or mass exchange. These dynamical factors modify the resulting mass spectrum, and may explain why the characteristic stellar mass depends on the EOS more weakly than expected.

We also studied the effects of different turbulent driving fields and of a smaller driving scale. For different realizations of statistically identical large-scale turbulent velocity fields, we consistently find that the characteristic mass decreases with increasing critical mass. However, there are considerable variations. The influence of the natural stochastic fluctuations in the turbulent flow on the resulting median mass is almost as pronounced as the changes of the thermal properties of the gas. Also when inserting turbulent energy at small wavelengths, we see the peak of the mass spectrum decrease with increasing critical density.



The current study using a piecewise polytropic EOS can only serve as a first step. Future work will need to consider a realistic chemical network and radiation transfer processes in gas of varying abundances.

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