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# The FU Orionis mechanism

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**Summary.** The large but temporary increase in radius accompanying the recurrent flare-ups of the FU Ori stars can be produced only by strong transient heating of a thin outer layer of the star. Simple composite polytropic models show that the observed changes in radius and luminosity can be accounted for if the outer 0.5 per cent of the mass gains a thermal energy comparable to the kinetic energy it would have in rapid rotation. The models also imply that the masses of the FU Ori stars are near $1\,M_\odot$. The most plausible heating mechanism is an instability due to rapid rotation, and it is suggested that the secular instability of a convective rotating star to bar-like deformations is responsible; this will produce strong shocks, turbulence, and heating of the outermost layers of the star. Matter will also be ejected, and this could account for much of the mass and angular momentum lost from young stars. The most luminous T Tauri stars may have experienced recent FU Ori flare-ups and may still be a magnitude or more brighter than normal.

## 1. Introduction

The most remarkable unexplained phenomenon of early stellar evolution is the abrupt brightening by ~ 5 mag shown by the FU Orionis stars; the properties of these stars and their relation to early stellar evolution have been thoroughly discussed by Herbig (1977). The FU Ori stars are all very young objects still closely associated with the dark clouds from which they formed, and in at least one case, V1057 Cyg, the pre-flare-up star was apparently a normal T Tauri star. The two best studied examples, FU Ori and V1057 Cyg, both brightened by nearly 6 mag over about a year and then began to fade gradually; at the same time, their radii and effective temperatures also increased markedly and then began to decline. A third object, V1515 Cyg, appears to be behaving similarly but on a much longer timescale. A fourth and much fainter FU Ori star has been discovered by Elias (1978), but little is yet known about its properties. Herbig (1977) has argued from the observed number of FU Ori stars that the FU Ori phenomenon is recurrent and must occur at least once every $10^4$ yr in most T Tau stars.

Other characteristics of the FU Ori stars which are probably relevant to an understanding of this phenomenon include broad absorption lines attributable to rapid rotation, composite spectra indicating a range of photospheric temperatures (see also Mould *et al.* 1978; Elias 1978), and blueshifted absorption lines produced by matter leaving the star with velocities of several hundred $\mathrm{km\,s^{-1}}$. It seems clear, especially from the increase in radius and surface temperature, that the flare-up must involve an intrinsic large change in the structure of the star; since the flare-up is a recurrent phenomenon, this change must be a temporary and presumably a superficial one, involving only the outer part of the star. Welin (1978) has defended the alternative possibility that the flare-up is due to the dispersal of a dust shell, but this possibility now seems difficult to reconcile with the data. The FU Ori phenomenon may have important implications for our understanding of early stellar evolution: for example, recurrent FU Ori flare-ups may account for some of the scatter of the T Tau stars in the HR diagram (Herbig 1977) and may explain why some of them have larger radii and luminosities than expected from collapse models (Bodenheimer & Black 1978). Also, the observed occurrence of mass ejection from the FU Ori stars suggests that the FU Ori phenomenon may play an important role in the loss of mass and angular momentum from young stars.

Some possible theories of the FU Ori phenomenon, and observational constraints on these theories, have been discussed by Herbig (1977). Additional theoretical constraints on possible flare-up mechanisms will be considered in Section 2 of this paper, and a mechanism involving an instability of a rapidly rotating contracting star will be suggested. In Section 3, simple composite polytropic models reproducing the properties of the post-flare-up FU Ori stars will be derived and used to estimate the mass of the outer zone involved in the flare-up, the amount of energy required, and the total masses of the FU Ori stars. Further discussion of the flare-up mechanism and its implications for early stellar evolution will be given in Section 4.

## 2 Possible mechanisms

The most fundamental aspect of the FU Ori phenomenon requiring explanation by any theory is the large increase in radius accompanying the flare-up; in the best studied case, V1057 Cyg, the radius increased from $\sim 4$ to $\sim 16 R_\odot$ while the luminosity increased from $\sim 8$ to $\sim 1300 L_\odot$ (Herbig 1977). This expansion occurred over a period of about one year and therefore took place quasi-statically, not as a dynamical phenomenon. Moreover, it is clear from the photometric stability and the quasi-normal spectra of the post-flare-up FU Ori stars that they are basically stable stars in hydrostatic equilibrium. The radius of a hydrostatic star can increase by a large factor only if there is a large increase in the specific entropy of at least the outer part of the star (Larson 1974); thus the FU Ori phenomenon must involve strong heating and expansion of at least an outer envelope of the star. The fact that the outermost layers expand by as much as a factor of 4 in radius indicates that they must receive an amount of heat energy comparable to their gravitational binding energy before the flare-up.

The total energy involved in the flare-up of V1057 Cyg can be estimated from the light curve given by Herbig (1977). The visible energy emitted between 1969 and 1976 was approximately $5 \times 10^{44}$ ergs, and a rough extrapolation of the light curve suggests that $\approx 10^{45}$ ergs will have been emitted by the time the star fades back to its original luminosity. FU Ori is fading much more slowly than V1057 Cyg and therefore its flare-up energy is more difficult to estimate, but it is probably about an order of magnitude larger, or $\approx 10^{46}$ ergs. In either case the flare-up energy is small compared with the total binding energy of the star, which is about $10^{48}$ ergs; thus the flare-up cannot involve a change in the structure of the entire star, but must represent a superficial disturbance of only the outermost one per cent or less of the stellar mass. The mass of the heated and expanded envelope can be estimated more quantitatively by assuming that the energy radiated by V1057 Cyg between 1971 and 1976 was supplied by gravitational contraction of the envelope, which shrank in radius by slightly more than $2R_\odot$ during this period (Herbig 1977). If the energy lost per unit mass is constant throughout the envelope, and if the mass of the star is $\sim 1.2 M_\odot$ (see Section 3), the fractional envelope mass found in this way is $\sim 6 \times 10^{-3}$.

If the FU Ori phenomenon recurs every $\sim 10^4$ yr for a period of $\sim 10^6$ yr, which may be a typical duration for the T Tau phase of evolution, the total energy emitted in all of the flare-ups is $\approx 10^{47}$–$10^{48}$ ergs. The only nuclear energy source important in young stars is deuterium burning, suggested by Dopita (1978) to be involved in the FU Ori phenomenon; it can produce a total of $\sim 2 \times 10^{47}$ ergs if all of the deuterium in the star is burned. This might be marginally adequate to power the flare-ups, but is is difficult to see why deuterium burning should occur only in a sequence of bursts rather than steadily, or why the energy should be deposited only in a thin surface layer of the star. If nuclear burning cannot explain the flare-ups, their energy must ultimately have a gravitational origin; this may include energy of infalling matter, as well as energy stored in magnetic fields or rotation.

One possibility mentioned by Herbig (1977) and Dopita (1978) is that the flare-up may be caused by the infall of a Jupiter-sized object; this would release about $10^{45}$ ergs, the energy required to explain the flare-up of V1057 Cyg. However, the kinetic energy of such an object would probably be converted into heat within less than a day, rather than gradually over many months as required; moreover, most of the energy would be deposited not in the surface layers but deep in the star where it would have relatively little effect. It also seems implausible that a typical T Tau star would accrete as many as 100 Jupiters over a period of $10^6$ yr. A more promising mechanism involving accretion might be a recurrent instability of a massive accretion disk (B. Paczynski, see Trimble 1976) which causes a Jovian mass of material to be dumped on to the star within about a year; a possible instability of this sort has been described by Elmegreen (1978). There is, however, no evidence that the FU Ori stars, or most T Tau stars, are surrounded at close distances by such large masses of material in disks, or are presently accreting matter (Herbig 1977). The possibility that mass transfer in a binary system may be involved has been mentioned by Elias (1978) and by Mould *et al.* (1978), but seems unlikely since the constancy of the radial velocities of the FU Ori stars implies that any binary system must have a separation of at least 10 AU (Mould *et al.* 1978).

If interaction with external matter is thus eliminated as a plausible cause of the FU Ori phenomenon, we are left with processes that are intrinsic to the star and derive their energy from slow gravitational contraction. One possibility is that a strong magnetic field is built up during contraction and that a large-scale magnetohydrodynamic instability periodically converts part of the magnetic energy into heat; however, no such instability is presently known, so this explanation must remain speculative. The only remaining possibility is then that gravitational energy is converted into rotational energy during contraction, and that an instability associated with rapid rotation periodically converts rotational kinetic energy into heat in the outermost layers of the star. The importance of rotation is suggested by the fact that both FU Ori and V1057 Cyg are rotating fast enough ($\sim 50$ and $\sim 70\,\mathrm{km\,s^{-1}}$, respectively) for rotational flattening to be significant. The effects of rotation on these stars must have been even more important before they flared up and increased in radius; in fact, rotational velocities as large as $150\,\mathrm{km\,s^{-1}}$ have been observed for some T Tau stars by Kuhi (1978). Mould *et al.* (1978) suggest that rotational flattening may account for the composite spectra of the FU Ori stars, and they further suggest that the flare-ups may be due to successive 'structural readjustments' of a rapidly rotating contracting star.

It is not difficult to identify an instability which could cause periodic flare-ups of a rapidly rotating contracting star. The various instabilities of differential rotation that can occur in axisymmetric stars are probably not relevant, since they are probably self-limiting and act essentially to maintain a stable distribution of angular momentum, without producing any catastrophic effects (Spiegel & Zahn 1970). This leaves the instabilities to bar-like deformations studied by Ostriker & Bodenheimer (1973); this type of instability must almost certainly occur eventually in any realistic, differentially rotating contracting star, if other effects causing a rapid loss of angular momentum do not intervene. For a convective star, in which turbulent viscosity can dissipate energy and redistribute angular momentum, the relevant instability is the secular instability corresponding to the transition from a Maclaurin spheroid to a Jacobi ellipsoid; this occurs when the ratio $t = |T/W|$ of rotational kinetic energy to gravitational energy exceeds about 0.14.

If $t$ rises above 0.14 in a viscous star, the deviation of the star from an axisymmetric shape increases rapidly, and severe distortions of its outer layers and strong perturbations on a state of circular rotation become important. This is shown by recent three-dimensional numerical simulations of rapidly rotating polytropes: for example, Gingold & Monaghan (1978) find that with $t = 0.16$ a strong bar-like distortion develops, the bar being bent at the ends rather like a barred spiral galaxy (see their Fig. 3). Similar results, with perhaps even more extreme 'barred spiral' shapes, have been found in a similar calculation by Lucy (1977). Such severe distortions of the outer laters of a rapidly rotating star will almost certainly cause strong shocks and turbulence which will convert a significant fraction of the rotational kinetic energy of these layers into heat. Thus strong heating and expansion of the outer layers of the star can be expected.

The numerical calculations by Lucy (1977) and Gingold & Monaghan (1978, 1979) did not take dissipative heating effects into account, even though viscosity in some form was present in these simulations and Lucy remarks that its physical effect would have been to cause heating of the outer layers of the star. Therefore these studies could not have predicted heating and expansion of the outer layers of a strongly deformed rotating star; instead both studies found, in some but not all cases, fission into two or more objects. However, even in the absence of heating effects, the occurrence of fission is not conclusively established by these calculations, especially for convective stars that become secularly unstable when $t > 0.14$. Gingold & Monaghan (1978) found that for $0.17 \lesssim t \lesssim 0.22$ their model stars did not fission but became 'pear-shaped', threw off some mass, and then reverted to a more stable bar-like shape with $t \sim 0.17$. Fission was found only for models with much larger values of $t$ (which could not be attained by a slowly contracting star), and only for the unrealistic polytropic index $n = 0.5$. For the more realistic case of $n = 1.5$, the result is not fission but the ejection of a stream of diffuse matter (Gingold & Monaghan 1979). It seems even less likely that fission would occur if the effects of heating and expansion were taken into account in a physically self-consistent way.

Thus it appears most likely that, at least for convective stars that become non-axisymmetric for $t > 0.14$ and highly distorted for $t > 0.16$, the effect of a rotational instability is to generate shocks and turbulence and hence to cause heating and expansion of the outer layers of the star, rather than to cause fission. The enhanced turbulence generated in this way will tend to transfer angular momentum outward in a differentially rotating star, thus keeping the surface layers rotating rapidly (and perhaps strongly distorted) as the star expands. Eventually, however, continuing expansion will slow the rotation to the point where the instability is quenched; the star will then revert to an axisymmetric shape, and heating will cease. Subsequently the star will gradually cool, shrink, and spin up, until its rotational velocity once again becomes sufficient to cause a bar-forming instability, and another flare-up occurs. The FU Ori phenomenon may thus result from repeated non-axisymmetric instabilities in a rapidly rotating T Tau star which is still in the predominantly convective (Hayashi) stage of evolution. Fission of contracting stars, if it occurs at all, is probably confined to the radiative phase of pre-main-sequence evolution when, because of the absence of viscosity, the secular instability is presumably not relevant and the dynamical instability that occurs for $t > 0.26$ can come into play.

If the numerical simulations of Lucy (1977) and Gingold & Monaghan (1978, 1979) indicate correctly the gross dynamical behaviour of rapidly spinning stars, the mass ejection that both studies find for all sufficiently rapidly rotating models may explain the 'shells' of ejected matter observed spectroscopically in the FU Ori stars. The suggested instability mechanism may then be able to account for most of the distinctive properties of the FU Ori stars: a rapid rise and slow decline in radius, luminosity, and effective temperature; recurrence of the flare-up many times during the evolution of a typical T Tau star; a composite spectrum with rotationally broadened lines; and 'shells' of ejected matter. Because of the complexity of all the processes involved, the flare-up phenomenon cannot easily be modelled in detail; therefore, in the following section we consider instead whether simple spherical polytropic models with heated outer zones can reproduce quantitatively the observed changes in radius and luminosity, and we estimate the values required for the parameters in such models.

## 3 Simple models

Prior to its flare-up the best studied FU Ori star, V1057 Cyg, had a radius and luminosity of about $4R_\odot$ and $8L_\odot$ respectively, and its spectral type was probably about K0 (Herbig 1977); these properties are consistent with those of a star of approximately solar mass on the lower part of its Hayashi track. They are also consistent with the properties predicted by collapse calculations for the immediate post-collapse state of a star with a mass in the range $\sim 1$–$2M_\odot$ (Larson 1977; Bodenheimer & Black 1978). The mass of V1057 Cyg can be estimated from its surface gravity, which in 1971 was $\log g = 2.0 \pm 0.3$, according to Grasdalen (1973); combining this with the radius of $16.6R_\odot$ estimated by Herbig (1977), we obtain a mass of $1.0M_\odot$, with an accuracy of a factor of 2. An upper limit of $2M_\odot$ on the mass of V1057 Cyg is set by its pre-flare-up luminosity of $\sim 8L_\odot$, since a star of greater mass never becomes this faint at any stage of hydrostatic evolution (Iben 1965; Larson 1972). (The mass of $8M_\odot$ derived by Grasdalen for V1057 Cyg from its position in the HR diagram, assuming that it is a normal pre-main-sequence star after the flare-up, is not tenable since the observations now clearly contradict this assumption.) In the following models, masses of 1.0 and $1.5M_\odot$ have been adopted; for this range of masses, the pre-flare-up state of V1057 Cyg must have been almost completely convective in structure, and so can be represented by a polytrope of index $n = 1.5$.

Since the post-flare-up FU Ori stars are clearly still hydrostatic stars, and since their spectra indicate effective temperatures considerably higher than that of the Hayashi track, they must be at least partly radiative in structure. Only the outermost part of the star can change significantly in structure during the flare-up, so the post-flare-up star must consist of a nearly unaltered interior with polytropic index $n = 1.5$, together with a radiative envelope in which $n > 1.5$. (The need for a larger polytropic index to represent the post-flare-up star was noted by Pismis (1971) for models with a single polytropic index.) Thus the outer layers of the star must have been heated sufficiently during the flare-up for radiative energy transport to become dominant over convection; the high post-flare-up luminosity can then be understood as resulting from the increased radiative flux through the outer layers of the star.

Examination of the radiative zones of the models of Larson (1972) shows that they are generally well approximated by polytropes with $n = 3.5$. This is a result of the approximate power-law dependence of the opacity on density and temperature, as can be shown analytically for a radiative envelope of negligible mass. If the equation of state is written $P = \rho \mathscr{R} T$ and the opacity is approximated by $\kappa = \kappa_0 \rho^{\alpha} T^{-\beta}$, and if $L/M$ is constant throughout the envelope (which is a good approximation since both $L$ and $M$ are nearly constant in the outermost part of a star), the equations of stellar structure with zero boundary conditions for $\rho$ and $T$ yield

$$T^{3+\beta} = \frac{4+\alpha+\beta}{1+\alpha} \left[\frac{3\kappa_0 L \mathscr{R}}{16\pi acGM}\right] \rho^{1+\alpha}. \qquad (1)$$

Since the quantity in square brackets is constant, this is a polytropic temperature–density relation with $n = (3+\beta)/(1+\alpha)$. Standard opacity tables (e.g. Cox & Tabor 1976) can be represented reasonably well for the most relevant densities and temperatures by the approximation $\kappa \sim 1.6 \times 10^{24} \rho^{6/7} T^{-3.5}$ (cgs), for which $\alpha = 6/7$, $\beta = 3.5$ and $n = 3.5$; this formula is accurate to about a factor of 2 in the relevant range of density and temperature, and a factor of 1.5 over most of this range. Putting $\kappa_0 = 1.6 \times 10^{24}$, $\alpha = 6/7$ and $\beta = 3.5$, and using the polytropic relation $P = K\rho^{1+1/n}$ with $n = 3.5$, equation (1) can be written in the form

$$L = \frac{16\pi acGM}{13.5\,\kappa_0} \mathscr{R}^{-7.5} K^{6.5} \simeq 8 \times 10^{-63} (M/M_{\odot}) K^{6.5}, \qquad (2)$$

which gives the luminosity $L$ of a star with an outer radiative zone as a function of the stellar mass $M$ and the polytropic constant $K$ of the radiative zone.

In order to construct simple models, we assume that the post-flare-up FU Ori star can be represented by an unchanged polytropic interior with $n = 1.5$ and an envelope of constant polytropic index $n = 3.5$ which is joined to the core by a discontinuous jump in specific entropy (and hence in density and temperature) at the interface. Such a discontinuous outward increase in specific entropy is almost certainly unrealistic in detail because it corresponds to a discontinuity in the heat input as a function of radius, but until the dynamics of the FU Ori phenomenon is better understood it will be difficult to construct models that are much more realistic. Another strong simplifying assumption in the present models is the neglect of any effects of rotation on the structure of the star. A less serious assumption is that the structure of the core is not affected by the heating and expansion of the envelope; in reality the reduction in pressure at the bottom of the envelope due to the expansion of the envelope will cause some expansion of the core as well, in order to maintain pressure balance. However, the amount of core expansion will be small, since the core–envelope interface is close to the surface of the initial star anyway (typically within 10 per cent in radius), and both the initial and final pressure at this interface are small compared to the central pressure. To check that the neglect of core expansion does not cause serious errors in the results, some fully self-consistent models with pressure balance at the core–envelope interface have been calculated for selected cases in which these errors were expected to be largest. The fully self-consistent models have radii and luminosities that are smaller than those of models with no core expansion by amounts up to ~ 20 and ~ 40 per cent respectively; in most cases the errors are smaller than this. Such errors would have an insignificant effect on the rather coarse model grids to be derived below, and would not alter any conclusions drawn from them.

If the structure of the pre-flare-up star is given, a model for the post-flare-up star is fixed by the mass $\Delta M$ and the polytropic constant $K$ of its radiative envelope. The luminosity of the post-flare-up star then depends on $K$ through equation (2), while the radius depends on both $\Delta M$ and $K$. A related quantity of significance for the flare-up mechanism is the energy $\Delta E$ gained by the envelope during the flare-up, which also depends on $\Delta M$ and $K$. In order to relate these quantities, it is convenient to express the mass $\Delta M$ and the energy $E$ of the envelope in terms of its polytropic constant $K$ and inner and outer radii $r_i$ and $R$. For a polytropic envelope with negligible mass, the equation of hydrostatic equilibrium with zero boundary pressure yields for the dependence of density and temperature on radius

$$\rho = \left[\frac{GM}{(n+1)K}\left(\frac{1}{r}-\frac{1}{R}\right)\right]^n, \quad \mathscr{R}T = \frac{GM}{(n+1)}\left(\frac{1}{r}-\frac{1}{R}\right). \tag{3}$$

Using these relations, the mass and energy of the envelope can be calculated from

$$\Delta M = \int_{r_i}^{R} 4\pi r^2 \rho dr, \quad E = \int_{r_i}^{R} 4\pi r^2 \rho \left(\frac{3}{2}\mathscr{R}T - \frac{GM}{r}\right) dr. \tag{4}$$

The results can be expressed in the form

$$\Delta M = 4\pi \left[\frac{GM}{(n+1)K}\right]^n R^{3-n} I_n(R/r_i) \tag{5}$$

$$\frac{E}{\Delta M} = -\frac{1}{2}\frac{GM}{(n+1)R}\left[3 + (2n-1)\frac{J_n(R/r_i)}{I_n(R/r_i)}\right] \tag{6}$$

where the integrals

$$I_n(x) = \int_1^x y^{-4}(y-1)^n dy, \quad J_n(x) = \int_1^x y^{-3}(y-1)^n dy \tag{7}$$

have been evaluated numerically and tabulated for both $n = 1.5$ and $n = 3.5$. These integrals can also be evaluated in closed form, and analytic expressions for them are given in the Appendix.

Equations (2), (5) and (6) have been used to construct grids of post-flare-up models in the HR diagram, using as parameters the envelope mass $\Delta M$ and the energy difference per unit mass $\Delta E/\Delta M$ between the final ($n = 3.5$) and initial ($n = 1.5$) states of the envelope. If the star initially has a radius $R_0$ and a polytropic constant $K_0$, equation (5) with $R = R_0$, $K = K_0$, and $n = 1.5$ determines the inner radius $r_i$ of an envelope of specified mass $\Delta M$, and equation (6) gives the initial energy $E_0$ of the envelope. If an increment $\Delta E$ is then added to the initial energy $E_0$, equation (6) with $n = 3.5$ and $E = E_0 + \Delta E$ determines the radius $R$ of the post-flare-up star, and equation (5) determines the new polytropic constant $K$ of the envelope; equation (2) finally gives the post-flare-up luminosity $L$. For the present models we have assumed an initial radius $R_0 = 4R_\odot$, as estimated by Herbig (1977) for the pre-flare-up state of V1057 Cyg, and we have considered masses of $1.0M_\odot$ and $1.5M_\odot$.

The resulting model grids are shown in Figs 1 and 2 for masses of $1.0M_\odot$ and $1.5M_\odot$ respectively. The grid parameters are the fractional envelope mass $\Delta M/M$, indicated by the solid curves, and the energy increment per unit mass $\Delta E/\Delta M$ (in units of $GM/R_0$), indicated by the dashed lines. Also shown are dotted curves giving the timescale $\tau = \Delta E/L$ for post-flare-up cooling of the envelope, assuming that the energy $\Delta E$ is radiated away at a rate $L$. For comparison, the approximate observed locations in the HR diagram of V1057 Cyg in 1971 and 1976 are shown by the crosses, and the locations of FU Ori in 1939 and 1963 are shown by the circles, all based on data given by Herbig (1977). The grids are extended to temperatures as low as the Hayashi track, which is shown for a star of $1M_\odot$

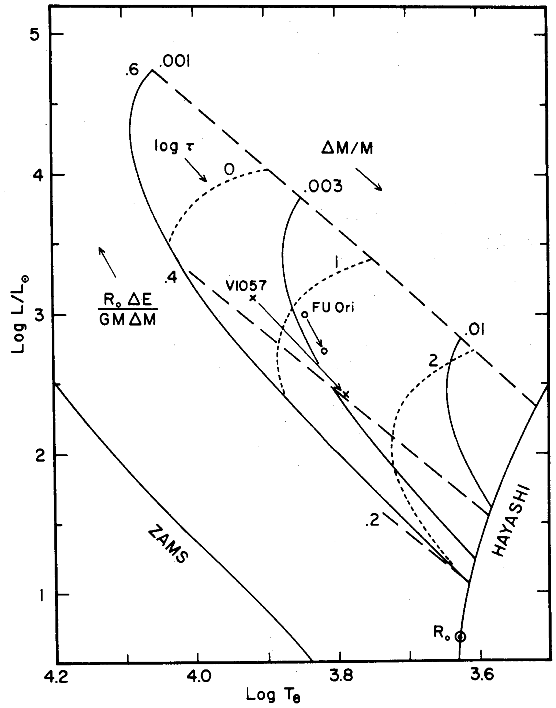


**Figure 1.** A grid in the HR diagram of post-flare-up models for a star of mass $M = 1 M_\odot$ and initial radius $R_0 = 4 R_\odot$. The parameters of the grid are the fractional envelope mass $\Delta M/M$, which is constant along the solid curves, and the energy increment per unit mass $\Delta E/\Delta M$ in units of $GM/R_0$, which is constant along the dashed lines. The dotted curves give the fading timescale $\tau = \Delta E/L$ in years. The approximate positions of V1057 Cyg in 1971 and 1976 are indicated by the crosses, and the positions of FU Ori in 1939 and 1963 are indicated by the circles. The zero-age main-sequence and the approximate position of the Hayashi track for a star of $1 M_\odot$ are also shown.

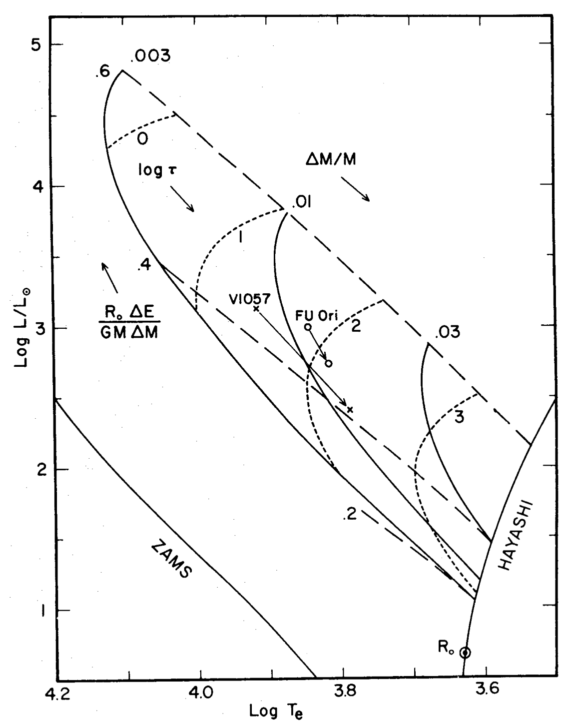


**Figure 2.** Same as Fig. 1 but for $M = 1.5 M_\odot$.

(Iben 1965). Near this locus the assumption of a fully radiative envelope breaks down and a surface convection zone becomes important. In the limit where the heating of the envelope is not sufficient to create a significant radiative zone, the post-flare-up star will remain on or near the Hayashi track with only a moderate increase in luminosity.

The models show that the post-flare-up radius depends primarily on the energy per unit mass $\Delta E/\Delta M$ added to the envelope, and only weakly on the envelope mass $\Delta M$; thus the dashed lines in Figs 1 and 2 are almost lines of constant radius. The post-flare-up radii of V1057 Cyg and FU Ori correspond to values of $\Delta E/\Delta M$ between 0.45 and 0.50 times the initial gravitational energy per unit mass at the stellar surface, $GM/R_0$. This is almost the same as the kinetic energy per unit mass $GM/2R_0$ of material rotating at the Keplerian or 'breakup' velocity at the surface of the star.

The luminosity and temperature of the post-flare-up star depend not only on $\Delta E/\Delta M$ but also on the envelope mass $\Delta M$, increasing with decreasing $\Delta M$. This is because in an envelope of given size the density and opacity decrease with decreasing mass. The luminosity also increases strongly with increasing stellar mass $M$; as a result the fractional envelope mass $\Delta M/M$ required to produce a given luminosity increases with increasing $M$. If $M = 1.0\,M_\odot$, the post-flare-up luminosities of V1057 Cyg and FU Ori correspond to $\Delta M/M \sim 0.003$ (see Fig. 1), whereas if $M = 1.5\,M_\odot$, the required value is $\Delta M/M \sim 0.01$ (see Fig. 2). These values are of the same order as the fractional envelope of mass $\Delta M/M \sim 0.006$ estimated in Section 2 from the luminosity and rate of contraction of V1057 Cyg, and this agreement supports the assumption that the post-flare-up luminosity is supplied by envelope contraction. These small values of $\Delta M/M$ also justify the assumption of a small envelope mass that was used in the models.

If the FU Ori stars had the same structure in detail as these models, their post-flare-up evolution would follow a model sequence with constant $\Delta M$ but decreasing $\Delta E$, i.e. one of the solid curves in Figs 1 and 2. However, if there is in reality a smooth outward increase of specific entropy in the star, rather than a discontinuous jump, the region which is approximately polytropic with $n = 3.5$ will extend farther inward as the envelope cools, and the effective envelope mass $\Delta M$ will increase. The direction of evolution in the HR diagram will then be intermediate between a locus of constant $\Delta M$ and a locus of constant $\Delta E/\Delta M$. The directions of evolution of V1057 Cyg and FU Ori indicated by the arrows in Figs 1 and 2 are not very accurately determined, especially for FU Ori, but they at least appear to be consistent with this prediction. Thus the basic properties of the FU Ori stars seem, at least qualitatively, to be consistent with the present simplified models.

### 3.1 MASSES OF THE FU ORI STARS

The predicted post-flare-up cooling timescale $\tau$ depends strongly on all of the parameters of the models, including the initial radius $R_0$ and the total mass $M$. The models shown in Figs 1 and 2 are strictly applicable only to V1057 Cyg, since the assumed $R_0$ of $4\,R_\odot$ is that estimated for V1057 Cyg. Comparison of the rate of post-flare-up evolution of V1057 Cyg with the timescales given by the dotted curves in Figs 1 and 2 suggests that a mass of $1.0\,M_\odot$ is more appropriate than $1.5\,M_\odot$. To the extent that the models apply in detail, this comparison can be made more quantitatively as follows. Examination of the calculated model sequences indicates that the time required for a model in the relevant part of the HR diagram to fade by 1 mag is about $0.2\,\tau$. V1057 Cyg took about three years to fade by 1 mag after reaching maximum brightness, so the value of $\tau$ for this star is about 15 yr; interpolating between the dotted curves in Figs 1 and 2 then yields an estimated mass of about $1.2\,M_\odot$. This agrees closely with the mass of $1.0\,M_\odot$ estimated from the surface gravity of V1057 Cyg

(Section 2); the present value may even be more accurate, in view of the strong sensitivity of $\tau$ to the stellar mass.

For FU Ori the time required to fade by 1 mag will probably be about 50 yr, implying $\tau \sim 250$ yr. However, Figs 1 and 2 cannot be used to estimate the mass in this case because FU Ori was initially about 1 mag fainter and hence was smaller in radius than V1057 Cyg, and the effect of this is to reduce the estimated mass. Assuming an initial radius of $2.5\,R_\odot$, we estimate from a less complete set of models with different initial radii that the mass required to explain the rate of fading of FU Ori is about $1.3\,M_\odot$. Thus FU Ori appears to be an object which has about the same mass as V1057 Cyg but is at a later stage of evolution (or, perhaps, has formed under different initial conditions).

The third object studied by Herbig, V1515 Cyg (see also Gottlieb & Liller 1978), has shown a smaller ($\sim 4$ mag) and more gradual ($\sim 5$–$30$ yr) brightening than V1057 Cyg or FU Ori, and has not yet clearly begun to fade, so no similar mass estimate is possible. The pre-flare-up luminosity of V1515 Cyg was about $7\,L_\odot$, only slightly fainter than V1057 Cyg, and in 1976 the luminosity and effective temperature of V1515 Cyg were nearly the same as those of V1057 Cyg. The apparently much longer timescale of V1515 Cyg may then indicate a mass closer to the upper limit of $2\,M_\odot$ set by its pre-flare-up luminosity. It may be significant that none of the known FU Ori stars appear to have masses greater than $2\,M_\odot$, since this is about the largest mass for which a significant convective phase of pre-main-sequence evolution is predicted by the collapse models of Larson (1972).

## 4 Discussion and implications

### 4.1 THE HEATING MECHANISM

According to the above models, the FU Ori stars can be interpreted as stars of approximately solar mass, initially on the lower part of their Hayashi tracks, that have experienced strong transient heating of an outer zone containing less than one per cent of the stellar mass. The heat energy required is approximately equal to the kinetic energy that this zone would have if the star were rotating rapidly, and since both FU Ori and V1057 Cyg are in fact rotating rapidly and must have been spinning even faster before the flare-up, the required energy can plausibly be supplied by rotation. The models also require the heating to be strongly concentrated in the surface layers of the star, since if it is spread over more than a few per cent of the stellar mass, the stellar surface temperature will not rise above that of the Hayashi track. This concentration of heating in the outer layers would be expected if the heating results from a rotational instability, since the numerical simulations of Lucy (1977) and Gingold & Monaghan (1978) show that the rotational distortions are most severe in the surface layers of the star. Another effect tending to make the heating effect increase outward in the star is that, since the temperature in the star decreases outward, the Mach number and the entropy generated by shocks and turbulence increase outward (see below).

The models of Section 3 make no prediction of the light curve or even of the timescale for the rise in luminosity. The observed rise time may reflect in part the time required for radiative redistribution of energy to establish radiative equilibrium throughout the envelope following rapid heating. An example of such a radiative readjustment process, which has about the right timescale for the FU Ori phenomenon, is the rapid approach to radiative equilibrium in some of the protostellar cores of Larson (1972). In these models, energy is transported outward within the core by a 'luminosity wave' that progressively heats and expands layers at larger radii, and causes a rapid rise in luminosity when it approaches the surface of the core. The possible relevance of this phenomenon to the FU Ori stars was suggested by Grasdalen (1973). However, such a process cannot by itself account for the FU Ori flare-ups because the predicted increase in radius and luminosity is too small, and because it is not recurrent. It does not appear possible to construct even *ad hoc* models in which an increase in radius by as much as a factor of 4 can be caused by radiative redistribution of energy, but an increase by a factor of 2 could conceivably be caused in this way; thus radiative readjustment could play a role in the approach to maximum luminosity, or in the small post-maximum fluctuations of FU Ori.

If radiative readjustment cannot account for the entire rise in luminosity, the heating process itself must continue to act during at least part of the brightening phase. This could happen if the heating is due to an instability that is not quenched until the stellar radius has increased substantially. If rotational distortions are involved, a relevant timescale for the heating process is the rotation period, which for V1057 Cyg is about 12 sin $i$ days. The number of rotations required for shock heating to produce a sufficient increase in entropy in the outer layers of the star can be roughly estimated as follows. The models indicate that $P/\rho^{5/3}$ must increase during the flare-up by a factor of order 50 in the lower part of the envelope (where most of the envelope mass is located); the required increase is even larger in the outermost part of the envelope. In the lower part of the envelope a shock front with a velocity of, say, $100\,\mathrm{km\,s^{-1}}$ would have a Mach number of about 2 and would raise $P/\rho^{5/3}$ by only a factor of 1.3; therefore it would require about $\ln 50/\ln 1.3 \sim 15$ shocks of this strength to produce the required increase in entropy. This would in turn require about eight rotations of the star, if the bar distortion behaves as a wave pattern rotating with the angular velocity of the bulk of the star (about twice the angular velocity of the surface layers (Bodenheimer & Ostriker 1973)). The predicted rise time would then be of the order of 100 days, which is comparable with the observed rise times of FU Ori and V1057 Cyg.

From similar considerations we can also see why only an outer zone of small mass is strongly heated by a bar-forming instability. At twice as great a depth in the star as considered above, a shock with the same velocity would increase $P/\rho^{5/3}$ by only a factor of 1.05, and even after 15 shocks the increase would be only a factor of 2 and would have relatively little effect. The heating effect will decrease even more rapidly with depth in the star if the velocities induced by the instability decrease with depth, as seems likely. Clearly a full three-dimensional hydrodynamic calculation will be required to establish more quantitatively the dependence of the heating effect on depth within the star.

### 4.2 LOSS OF MASS AND ANGULAR MOMENTUM

In addition to explaining the FU Ori phenomenon, the bar instability may also play an important role in the loss of mass and angular momentum from contracting stars. In fact, it may provide an efficient means of shedding angular momentum and thereby allowing contraction to continue at the expense of only a small amount of mass loss. Both Lucy (1977) and Gingold & Monaghan (1978, 1979) found mass to be ejected from all sufficiently rapidly spinning models, the mass loss being associated with (and presumably caused by) the formation of an elongated object that throws off matter from its ends. The ejected matter trails out into 'spiral arms' (see especially Lucy 1977), and it is clear that gravitational torques must transfer angular momentum outward from the bar to the ejected matter. If the development of a bar-like shape is accompanied by heating and expansion of the outer part of the star, the amount of angular momentum carried off per unit mass is increased, owing to the increase in the radius at which material is thrown off.

If a contracting star periodically loses both mass and angular momentum through such a mechanism, a relation between the mass and the radius of the star can be predicted as follows. The angular momentum of a star of mass $M$ and radius $R$ rotating at the critical velocity for stability can be written

$$J = AG^{1/2} M^{3/2} R^{1/2} \tag{8}$$

where $A$ is a constant of order unity depending on the structure of the star. A possible model for the structure of a convective star rotating at the critical velocity for stability is provided by one of the models studied by Bodenheimer & Ostriker (1973), for which $n = n' = 1.5$ and $|T/W| = 0.135$ (see their Fig. 5); for this model $A = 0.20$. If the radius, mass and angular momentum of the star change by small amounts $\Delta R$, $\Delta M$ and $\Delta J$, and if $A$ does not change, equation (8) implies

$$\frac{\Delta J}{J} = \frac{3}{2}\frac{\Delta M}{M} + \frac{1}{2}\frac{\Delta R}{R}. \tag{9}$$

If an angular momentum $\Delta J$ is carried away by a mass $\Delta M$ which is ejected with a velocity proportional to the Keplerian velocity $(GM/R)^{1/2}$ at the equator of the star, we can write

$$\Delta J = B(GMR)^{1/2} \Delta M \tag{10}$$

where $(GMR)^{1/2}$ is the angular momentum per unit mass corresponding to rotation at the Keplerian velocity, and $B$ is a constant of order unity depending on the velocity of ejection. From equations (8), (9) and (10) we then obtain for $A = 0.20$

$$\frac{\Delta R}{R} = \left(\frac{2B}{A} - 3\right)\frac{\Delta M}{M} = (10B - 3)\frac{\Delta M}{M}. \tag{11}$$

The constant $B$ can be determined quantitatively only from a detailed three-dimensional hydrodynamical calculation taking into account heating effects, but a rough guess can be made by assuming that matter is ejected into a parabolic orbit from the post-flare-up radius of $4R$; then $B = 2^{3/2}$. Equation (11) then gives

$$\frac{\Delta M}{M} = 0.04\,\frac{\Delta R}{R}, \text{ or } M \propto R^{0.04}. \tag{12}$$

Equation (12) implies that for each factor of 2 decrease in radius, a fractional mass loss of only 0.03 is required to carry away the excess angular momentum. This is well below the upper limit on pre-main-sequence mass loss that is implied if the energy driving the mass loss comes from gravitational contraction, as is the case here; according to Weidenschilling (1978), if the mass varies with radius according to

$$M \propto R^{\eta},$$

then the theoretical upper limit on $\eta$ for a convective star is 0.23. The values of $\eta$ deduced by Weidenschilling (1978) from the 'observed' rates of mass loss of T Tau stars range from 0.003 to 0.15, with a median of about 0.03 that is almost the same as the value given by equation (12). Thus, although the mechanisms involved are presumably quite different, the sporadic mass loss associated with FU Ori flare-ups may be comparable in net effect to the more continuous mass loss inferred from the spectra of the T Tau stars. Since only some T Tau stars show spectroscopic evidence of mass loss, it is possible that the rotational instability mechanism provides the dominant mode of loss of both mass and angular momentum for contracting pre-main-sequence stars, at least during the convective phase of evolution.

If impulsive mass loss is an important phenomenon of early stellar evolution, it will play a significant and perhaps dominant role in the interactions between T Tau stars and surrounding interstellar matter. Dopita (1978) has suggested that at least some of the Herbig–Haro objects may be produced by mass ejection from FU Ori stars, appearing as luminous shock fronts where the ejected matter collides with ambient gas. Other phenomena such as maser sources may also sometimes result from collisions between ejected material and ambient gas, and holes in dark clouds may sometimes be created by FU Ori mass ejection.

### 4.3 DISTRIBUTION OF T TAURI STARS IN THE HR DIAGRAM

If FU Ori flare-ups recur many times in T Tau stars, the stars that have recently experienced flare-ups will be displaced above and to the left of their normal positions in the HR diagram; a few will be shifted as far as the present positions of V1057 Cyg and FU Ori, but most will have evolved back closer to the Hayashi track where the fading timescale is much longer (see Figs 1 and 2). This prediction is qualitatively consistent with the distribution of the brighter T Tau stars in the HR diagram (Cohen & Kuhi 1976, 1979; Rydgren, Strom & Strom 1976). Extrapolating the recent evolution of V1057 Cyg with a timescale that increases as in the models, we estimate that V1057 Cyg will reach the Hayashi track after ~50–100 yr, at which time it will have a radius of $\sim 10 R_\odot$ and a luminosity of $\sim 25 L_\odot$; these properties are comparable to those of the brightest T Tau stars (Cohen & Kuhi 1979). The fading timescale will have increased by about two orders of magnitude, and therefore it will take a few hundred years for the star to fade by another magnitude. The radius and luminosity of FU Ori when it reaches the Hayashi track may be comparable to those of V1057 Cyg, but the timescale will probably be at least an order of magnitude longer. These predicted long fading timescales do not conflict with existing information about the long-term stability of luminous T Tau stars.

Thus some of the more luminous T Tau stars may have experienced FU Ori flare-ups within the past $\sim 10^2$–$10^3$ yr and may still be fading very slowly back to their normal luminosities of less than $10 L_\odot$. If the FU Ori phenomenon recurs every $10^4$ yr, and if the fading timescale of V1057 Cyg is typical, the fraction of T Tau stars that have recently experienced flare-ups and are still more than 1 mag brighter than normal is then about $10^{-2}$; if the timescale of FU Ori is more typical, this fraction may be as large as $\sim 2 \times 10^{-1}$. Thus, at any time perhaps as many as 10 per cent of the T Tau stars that are normally fainter than $10 L_\odot$ will have luminosities of $25 L_\odot$ or higher. More complete statistics on the properties of the T Tau stars (and the FU Ori stars) will be needed to decide whether the luminous T Tau stars can be accounted for in this way, but present data (e.g. Cohen & Kuhi 1979) showing that most T Tau stars have luminosities of only a few $L_\odot$, while relatively few are more luminous than $10 L_\odot$, do not contradict the possibility. If this explanation can be shown to be consistent with the data, the apparent contradiction between the luminosity of only a few $L_\odot$ predicted by collapse models (Bodenheimer & Black 1978) and the observed existence of some T Tau stars more luminous than $10 L_\odot$ may be resolved.

## 5 Conclusions

The observed properties of the FU Ori stars can be plausibly accounted for only by the recurrent heating and expansion of an outer zone containing less than one per cent of the stellar mass. The models of Section 3 suggest that the fractional mass in the heated zone is about $5 \times 10^{-3}$, and that the heat energy required per unit mass is about $0.5\, GM/R_0$, i.e. the kinetic energy of rotation at the 'breakup velocity'. The models also imply that the masses of V1057 Cyg and FU Ori are slightly over one solar mass, which is quite plausible for T Tauri stars similar to the pre-flare-up states of these stars.

A recurrent instability that causes a violent disturbance in the outer layers of the star is required to produce the heating, and the most plausible candidate is the instability of a rapidly rotating star to non-axisymmetric deformations. This possibility is consistent with the observed large rotational velocities and the composite spectra of the FU Ori stars. However, other possibilities such as magnetohydrodynamic phenomena or interactions with external matter cannot be completely eliminated, and could even be responsible for the smaller flare-ups of some T Tau stars suggested by Herbig (1977) to be less spectacular analogs of the FU Ori stars.

If a rotational instability is responsible for the FU Ori phenomenon, both mass and angular momentum will be lost at each flare-up, and this may account for much of the mass and angular momentum lost from young stars. Our understanding of early stellar evolution is otherwise not fundamentally altered by the FU Ori phenomenon, but quantitative interpretations of the distribution of T Tau stars in the HR diagram will be affected. In particular, many of the more luminous T Tau stars may have experienced FU Ori flare-ups some time within the past $\sim 10^2$–$10^3$ yr and may still be slowly fading. This might allow the luminosities of T Tau stars to be understood on the basis of existing collapse models, without necessitating revisions such as a higher initial density (Bodenheimer & Black 1978).

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## Appendix

Analytic expressions for the integrals $I_n(x)$ and $J_n(x)$ defined in equation (7) are given below for $n = 3/2$ and $n = 7/2$.

$$I_{3/2}(x) = \int_1^x \frac{(y-1)^{3/2}}{y^4}\,dy = \frac{1}{8}\arctan(x-1)^{1/2} + (x-1)^{1/2}\left(\frac{1}{3x^3} - \frac{7}{12x^2} + \frac{1}{8x}\right)$$

$$J_{3/2}(x)=\int_1^x \frac{(y-1)^{3/2}}{y^3}\,dy=\frac{3}{4}\arctan(x-1)^{1/2}+(x-1)^{1/2}\left(\frac{1}{2x^2}-\frac{5}{4x}\right)$$

$$I_{7/2}(x)=\int_1^x \frac{(y-1)^{7/2}}{y^4}\,dy=-\frac{35}{8}\arctan(x-1)^{1/2}+(x-1)^{1/2}\left(\frac{1}{3x^3}-\frac{19}{12x^2}+\frac{29}{8x}+2\right)$$

$$J_{7/2}(x)=\int_1^x \frac{(y-1)^{7/2}}{y^3}\,dy=\frac{35}{4}\arctan(x-1)^{1/2}+(x-1)^{1/2}\left(\frac{1}{2x^2}-\frac{13}{4x}-\frac{20}{3}+\frac{2x}{3}\right)$$

I am indebted to the referee for pointing out that these integrals can be evaluated in closed form.