Calculations of three-dimensional collapse and fragmentation

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Summary. Calculations of the fragmentation of an isothermally collapsing cloud have been carried out using a method that follows the motion of individual fluid particles and includes pressure and viscosity forces between neighbouring particles. In a cloud or region whose mass is comparable to the Jeans mass, a highly condensed core forms, surrounded by a diffuse envelope that continues to accrete on to the core; in the presence of rotation, the inner part of the envelope becomes essentially an accretion disc. If the mass exceeds the Jeans mass, several such accreting cores are formed, the number being comparable to the initial number of Jeans masses in the cloud. Binary systems and hierarchical multiple systems are frequently obtained. The mass of the largest object formed is independent of the Jeans mass but depends on the angular momentum and viscosity of the cloud, and is essentially the maximum mass accretable by a single object. The resulting mass spectrum may be determined by the development of a hierarchy of accreting objects of different sizes, such that each object has several smaller ones associated with it. The hypothesis of a self-similar accretion hierarchy predicts a power-law mass spectrum, which in the limit of inefficient accretion has an exponent $x = 1$, in reasonable agreement with observations.

1 Introduction

Many calculations of the gravitational collapse of gas clouds have been made assuming spherical or axial symmetry, and these calculations have yielded important insights into the nature of gravitational collapse and the processes of protostellar evolution (e.g. Larson 1977, 1978a). However, such calculations are limited to following the evolution of individual collapsing protostars from assumed initial conditions; the more difficult problem of understanding the formation and the initial properties of protostars has seen little progress, and the appropriate choice of initial conditions remains an important uncertainty in the theory of protostellar evolution. A related problem of wide interest is that of understanding the mass spectrum or 'initial mass function' (IMF) with which stars are formed; this requires an understanding of fragmentation processes in collapsing clouds, which can only be gained

from three-dimensional collapse calculations that are not restricted to spherical or axial symmetry.

In a cloud of given temperature and density, the minimum mass that any fragment or condensation must have to be gravitationally bound is given by the Jeans mass. Since the Jeans mass decreases with increasing density, it is possible for smaller and smaller bound condensations to form when a cloud collapses and its density rises. A widely-adopted hypothesis (Hoyle 1953) has been that the fragmentation of a collapsing cloud proceeds via the successive subdivision of the cloud into smaller and smaller units as the density increases; if each such subdivision can be treated as a random event with a probability of about 1/2, this process can generate a mass spectrum resembling the observed one (Larson 1973). However, this simple picture ignores the dynamics of collapse and fragmentation, which may well be of dominant importance. For example, fragments may collide and coalesce with each other, and this may play an important role in determining their final mass spectrum (Arny & Weissman 1973); also, some fragments may be tidally disrupted by encounters with denser mass concentrations in the cloud.

Perhaps even more important for fragmentation is the very non-uniform nature of gravitational collapse, as demonstrated by the previous calculations with spherical or axial symmetry; these calculations always show the runaway growth of any density fluctuations, whether they are present initially or are propagated inward from the boundary. In the spherical case the result is the formation of a dense stellar core with a diffuse infalling envelope that is subsequently accreted by it. If similar behaviour occurs in the three-dimensional case, a collapsing cloud might be expected to develop a very lumpy structure, with dense cores embedded in a more diffuse medium and acting as condensation nuclei for the remaining gas. The formation of dense cores will favour the survival of the condensations against tidal disruption or coalescence, but it will also inhibit further fragmentation, since most of the mass may remain in diffuse gas whose dynamics is dominated by the gravitational attraction of the dense cores; as a result, instead of forming more condensations, much of the diffuse gas may be accreted by the cores that have already formed. If stars gain most of their mass by accretion, the resulting mass spectrum may be largely determined by the way in which the diffuse matter is apportioned among the various accreting cores in a collapsing cloud.

Thus, a three-dimensional calculation of fragmentation and star-formation processes should be able to follow not only the formation and orbital motions of dense protostellar cores, but also the continuing accretion of diffuse material by them. Section 2 outlines a finite-particle scheme for simulating three-dimensional gas dynamics which has been developed with these requirements in mind. Some results of calculations made with this method are described in Section 3, and the possible implications of these results for the stellar mass spectrum are discussed in Sections 4 and 5.

2 Methods and assumptions

At present, standard Eulerian grid techniques for three-dimensional gas dynamics are limited to fairly coarse grids, and cannot follow very far the development of small, dense condensations; nor are they generally well suited to following the orbital motions of such condensations, or the continuing accretion of material by them. An alternative approach which seems better suited to the present problem is to follow individually the motions of a number of representative 'fluid particles', or compressible gas elements of finite size and mass, much as in an *n*-body calculation. Condensations are then represented by dense subgroupings of particles, and there is in principle no limit to how small and dense such a condensation can

become; also, orbital motions and accretion processes are directly representable by particle motions. Such a finite-particle scheme for simulating gas dynamics has been described in detail and tested by Larson (1978b); the basic features of this method are summarized here.

The first requirement for a finite-particle scheme is to include, in addition to inverse-square gravity forces between all pairs of particles, some representation of the pressure forces that act between neighbouring pairs of particles. To estimate the pressure force between neighbouring gas elements, consider two adjacent gas elements with density ρ and isothermal sound speed c, and suppose that they have sizes comparable to their separation r and interact via a pressure $P = \rho c^2$ over an interface of area $A \sim r^2$. The repulsive force between them is then $PA \sim \rho c^2 r^2$, and since the mass of each element is $\sim \rho r^3$, the repulsive acceleration is $a_r \sim c^2/r$. In the present calculations it has been assumed that c is constant, i.e. that the collapse is isothermal; this is a reasonable approximation during the early stages of protostellar evolution. The pressure acceleration between neighbouring particles can then be written

$$a_r = C^2/r, \tag{1}$$

where C is a constant related to the sound speed c by a factor of order unity depending on the number of interacting neighbours per particle (Larson 1978b). In most of the present calculations, pressure forces have been assumed to act only between nearest neighbouring particles, but in some cases more complicated schemes with more interacting neighbours per particle have been tried, with similar results.

In simulating star formation processes it is also necessary to include some form of dissipation, since bound condensations cannot form unless the kinetic energy of collapse motions is somehow dissipated. Moreover, if dense stellar cores are to form in a cloud having a realistic amount of angular momentum, some form of viscosity is required to transport angular momentum outward in protostellar condensations. In models of spherical collapse, the velocity of infall becomes highly supersonic and the required dissipation is produced by an accretion shock at the surface of the stellar core; thus it might be expected that shock fronts will also be important in the three-dimensional case, although they will probably have a much less symmetrical and more irregular structure. The momentum transfer produced by the propagation of shock fronts may also provide a source of viscosity that is important for the transfer of angular momentum in collapsing protostars. Thus it is important to include the effects of shock fronts in the calculations.

The momentum transfer associated with the propagation of a shock front between neighbouring gas elements can be estimated by considering two particles of separation r that are approaching each other with relative velocity u. If $u \gtrsim c$, a shock front must soon form between the two particles, overtaking one of them in a time $\sim r/u$ and imparting to it a velocity increment $\sim u$ away from the other. This produces an average repulsive acceleration $a_r \sim u/(r/u) = u^2/r$ between the two particles. Since a shock front forms only if the particles are approaching each other, i.e. if $u_r \equiv \dot{r} < 0$, the average repulsive acceleration due to the propagation of shocks between neighbouring particles can be written

$$a_r = \begin{cases} Qu_r^2/r, & u_r < 0 \\ 0, & u_r > 0 \end{cases} \tag{2}$$

where Q is a constant of order unity. Equation (2) is identical in form to the 'artificial-viscosity' term frequently used in numerical gas dynamics to represent shock fronts, and it has the same effect of smoothing out a shock front over a macroscopic 'dissipation length' comparable to the particle spacing. It is found that Q cannot be much smaller than unity if the amount of dissipation is to be sufficient to allow strongly bound condensations to form

within a free-fall time; reasonable results are obtained with $0.25 \lesssim Q \lesssim 1.0$, and the present calculations have all been made with $Q = 0.5$.

Equation (2) provides only a crude macroscopically-smoothed representation of the viscosity associated with shock fronts, and does not explicitly take account of other sources of viscosity such as turbulence and magnetic fields that might also be important in a collapsing cloud. Thus the major uncertainty in the present calculations concerns how well equation (2) represents the real viscosity in a collapsing cloud. Clearly a better understanding of the detailed physics of cloud collapse will be required before any reliable estimate of the total viscosity can be made. However, considering all the possible sources of viscosity, it is perhaps unlikely that the actual viscosity is much less than the 'artificial viscosity' provided by equation (2).

A number of tests of this scheme (Larson 1978b) show that it is capable of representing both the static pressure and the period of radial oscillation of a bounded gas sphere with an accuracy of the order of 5 per cent for a system of 100–150 particles. A more stringent test is to compare the isothermal collapse of such a system with known solutions obtained from conventional hydrodynamic codes with spherical symmetry. The particle system develops a centrally-peaked density distribution of similar form with $\rho \propto r^{-2}$, and the critical temperature required to prevent collapse agrees with the previous determinations to an accuracy of ≈ 20 per cent. The method is also capable of simulating the formation of an accretion disc with a thickness as small as ≈ 10 per cent of its diameter. The rate at which the particles in the disc spiral into the central object provides an empirical estimate of the Reynolds number $\mathscr{R}$ (i.e. the ratio of inertial to viscous forces) by comparison with the theory of viscous accretion discs (Lynden-Bell & Pringle 1974); this comparison yields $\mathscr{R} \approx 50$, which corresponds to a spiralling-in time of the order of 5–10 orbital periods.

3 Results

The present calculations have all started from the simplest initial configuration, namely a roughly uniform and uniformly rotating sphere of gas. In most cases the initial particle positions and velocities have been generated by scattering 100 or 150 particles randomly in a spherical volume whose diameter is 0.9 times the width of Figs 1–6, and giving the system a solid-body rotation but no random motions. The main parameters that have been varied are the isothermal sound speed, the initial angular velocity, and the initial random particle positions. Schemes for distributing the particles in a smoother than random fashion have also been tried, but this seems mainly to increase the overall symmetry of the system during early stages of evolution; its later evolution and final properties are hardly altered. No boundary has been imposed on the system of particles; test calculations show that a fixed boundary has in any case no important effect on the interior evolution of the system.

To avoid the necessity of using for some particles timesteps that are exceedingly short and continue to decrease indefinitely, the particles have been merged when they come closer together than a 'capture radius' which is 1/200 of the initial radius of the system, or about one-third of the diameter of the smallest dots in Figs 1–6. This procedure appears to be justified in that the particles which are merged in this way are nearly always tightly bound together in dense, rapidly-contracting condensations. However, any information about what might happen on smaller scales is clearly lost when the particles are merged; for example, any smaller structures such as close binary systems that might form with sizes smaller than the capture radius would not be 'resolved' as separate objects in these calculations. There are also occasional spurious mergers between particles that are not closely bound, but these are not important for the results described below.
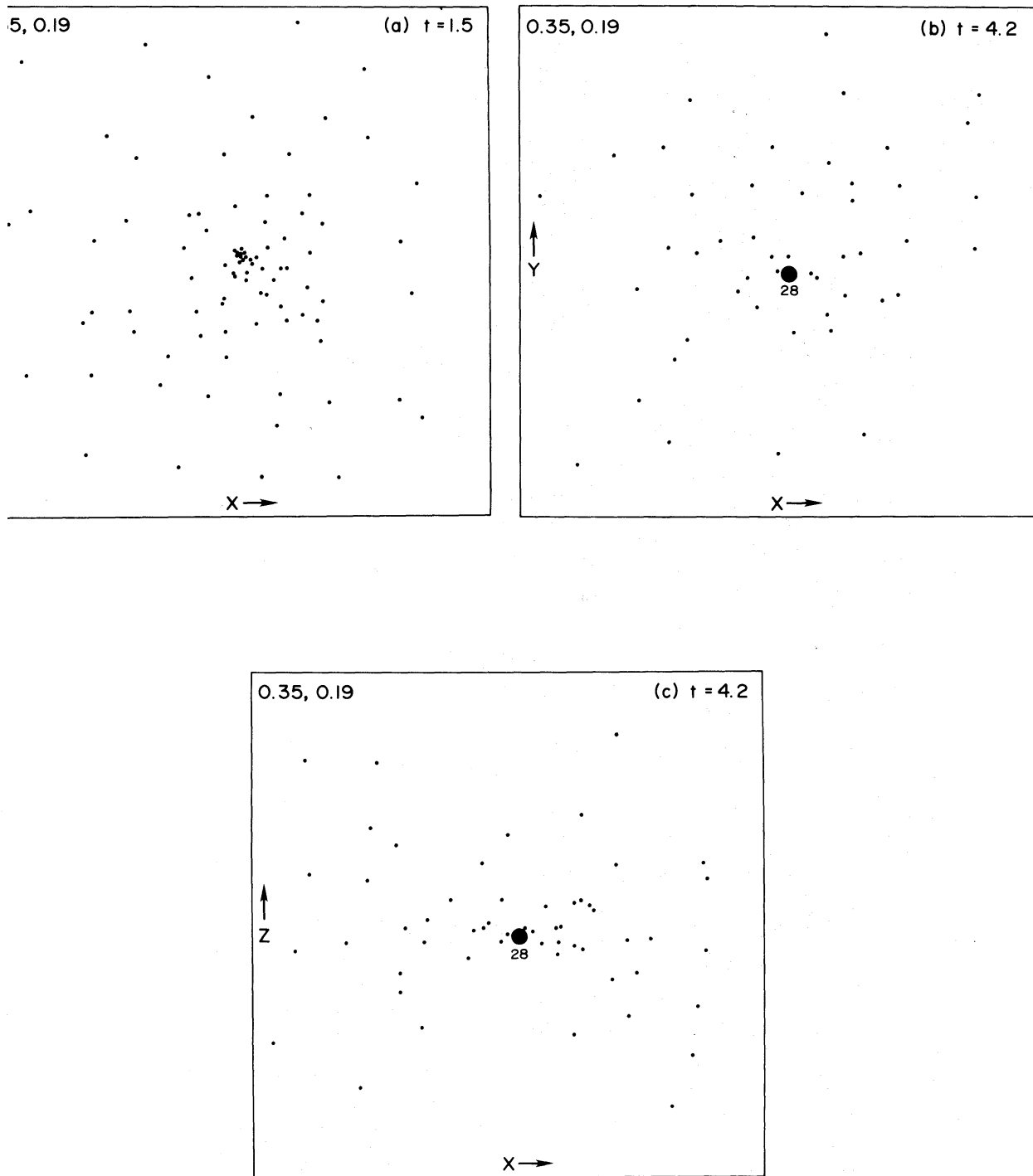
Figure 1. (a) The distribution of particles in a system of 100 particles after 1.5 free-fall times, projected on to the equatorial (*XY*) plane. The initial ratios of thermal energy and rotational energy to gravitational energy are respectively 0.35 and 0.19; in this and all subsequent diagrams these parameters are indicated in the upper-left corner, and the elapsed time in units of the free-fall time is indicated in the upper right. (b) The distribution of particles projected on to the equatorial plane after 4.2 free-fall times. Here 28 particles have coalesced into a single object at the centre. In this and all susbsequent diagrams coalesced objects are represented by large dots whose area is proportional to the number of particles contained, which is indicated beside the dots. (c) The distribution of particles after 4.2 free-fall times projected on to a perpendicular (*XZ*) plane in which the rotation axis is vertical and the equatorial plane horizontal.

If the cloud is initially nearly supported against collapse by thermal pressure and centrifugal force, the collapse proceeds very much as in the spherical case with the development of a sharply centrally-peaked density distribution. An example of this is illustrated in Fig. 1(a), which shows the structure of a system of 100 particles, as projected on to the equatorial plane, after 1.5 free-fall times; the initial thermal and rotational energies assumed in this case are respectively 0.35 and 0.19 times the initial gravitational energy. The dense clump of particles seen in the centre of Fig. 1(a) rapidly collapses into a single coalesced object, which continues to grow in mass by accreting more particles. Fig. 1(b) and (c) shows the structure of the system after 4.2 free-fall times, when the central object has accreted 28 particles; Fig. 1(b) is again a projection along the rotation axis, while Fig. 1(c) is a view perpendicular to this axis, showing that the surrounding envelope is flattened towards the equatorial plane. The innermost part of the envelope is particularly flattened, and has become essentially an accretion disc, within which particles slowly spiral into the central object.

It is noteworthy that only a single density peak and a single condensed object form in this case; no tendency for subdivision into smaller condensations is observed, as would have been predicted by the classical hierarchical fragmentation model. This is partly because of the non-homologous nature of the collapse, which leads to a strongly centrally-peaked density distribution, and partly because of the action of viscosity in allowing an appreciable fraction of the matter to collect together into a single object at the centre. Clearly these effects will limit the amount of fragmentation that can take place in a collapsing cloud; it may even be that the number of condensed objects that form is limited by the number of density peaks that develop early in the collapse, a possibility which is supported by the further results described below.

As the temperature and initial Jeans mass are reduced, the collapse becomes less regular and symmetrical and more condensations appear, the number of condensations increasing with the number of Jeans masses in the initial cloud. For example, when the Jeans mass is half the total mass, the typical result is the formation of a binary system of two condensations orbiting around each other. An example is shown in Fig. 2, which gives results for a system of 150 particles in which the initial thermal and rotational energies are respectively 0.25 and 0.30 times the initial gravitational energy. A projection on the equatorial plane is again shown, and the direction of rotation is counterclockwise. At first the collapse proceeds approximately as before with the formation of a single condensed object near the centre, but the remaining envelope is now less symmetrical and develops a transient 'spiral arm' structure (Fig. 2(a)). Part of the 'spiral arm' then becomes gravitationally unstable and forms a second condensation orbiting around the first, as shown in Fig. 2(b). (Here another small satellite has also formed much closer to the first object, but it soon spirals in and merges with it because of gravitational drag.) Eventually the second main condensation accretes almost as much mass as the first, and the result is a binary system of two nearly equal objects, as seen in Fig. 2(c).

As the temperature is further reduced, increasingly irregular and lumpy structures are obtained. Most of the condensations collapse to form separate objects, but some are tidally disrupted and some collide and merge with other condensations. For example, Fig. 3 shows the results for a system of 150 particles in which the Jeans mass is initially about one-third of the total mass. About four condensations begin to appear early in the collapse (Fig. 3(a)), but two of them soon merge and only three main condensations survive, forming a hierarchical triple system in which two objects are relatively close together (Fig. 3(b)). Subsequently all three objects grow considerably in mass by accreting more particles, but the hierarchical triple structure is preserved, as shown in Fig. 3(c) after 4.3 free-fall times.
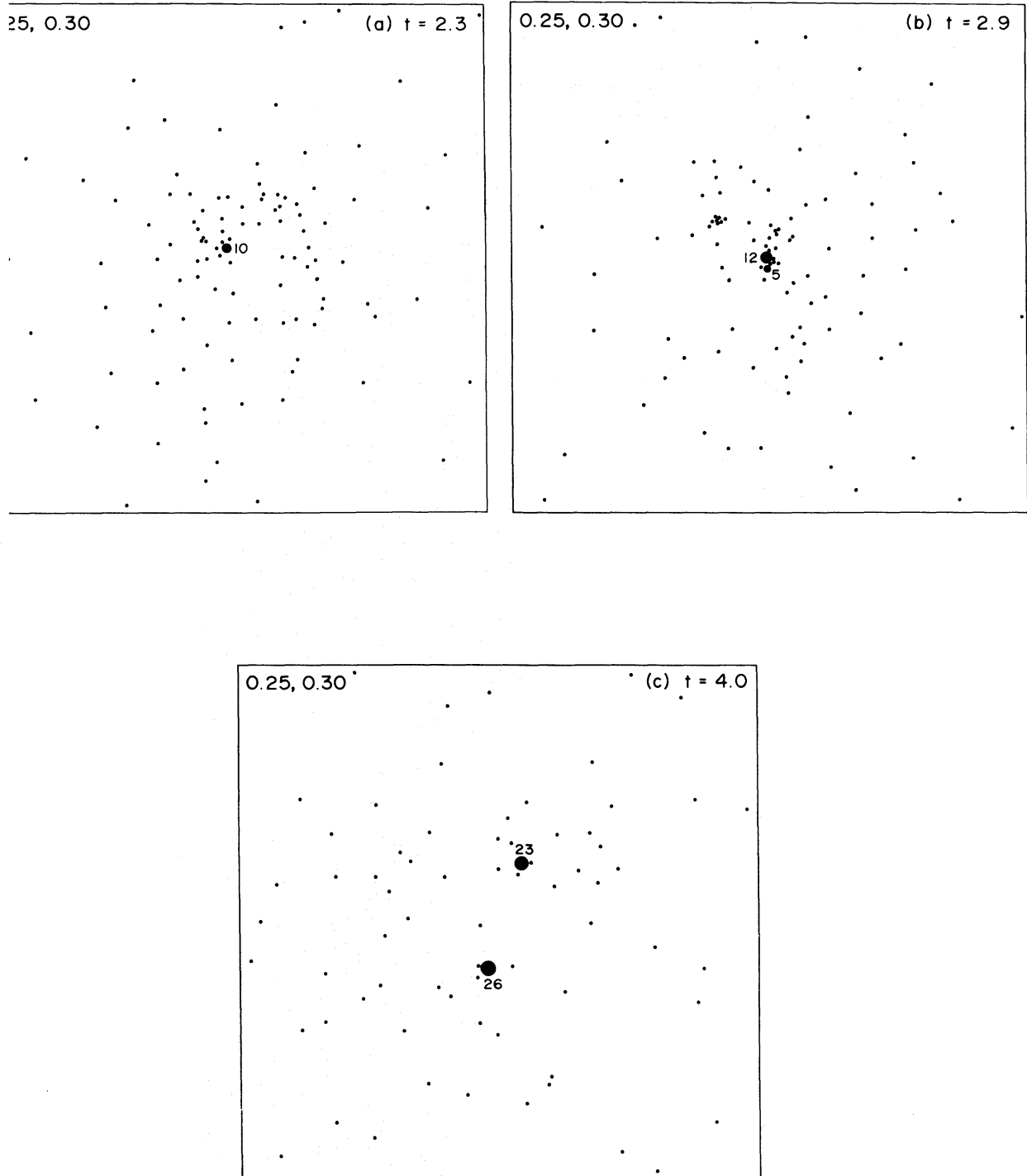
Figure 2. (a) The distribution of a system of 150 particles after 2.3 free-fall times; here and in all subsequent diagrams the particles are shown projected on to the equatorial plane. In all cases the direction of rotation is counterclockwise. The initial ratios of thermal and rotational energy to gravitational energy are here 0.25 and 0.30. (b) The particle distribution after 2.9 free-fall times. Note that the two objects of masses 12 and 5 units rapidly spiral together and merge to form the largest object in the resulting binary system. (c) The particle distribution after 4.0 free-fall times.

Fig. 4 shows results for a system with a still lower temperature, and with a somewhat smaller initial angular velocity than the previous cases; the effect of the lower angular velocity is that the system initially collapses to a more compact configuration, as seen in Fig. 4(a), but the number of condensations formed is hardly affected. The resulting system (Fig. 4(b)) contains six objects and is dominated by two binary pairs, with two smaller peripheral objects. Continuing toward lower temperatures and larger numbers of condensations, Fig. 5 shows results for a system in which the initial Jeans mass is only about one-tenth of the total mass. The collapse is now very irregular, and many condensations appear
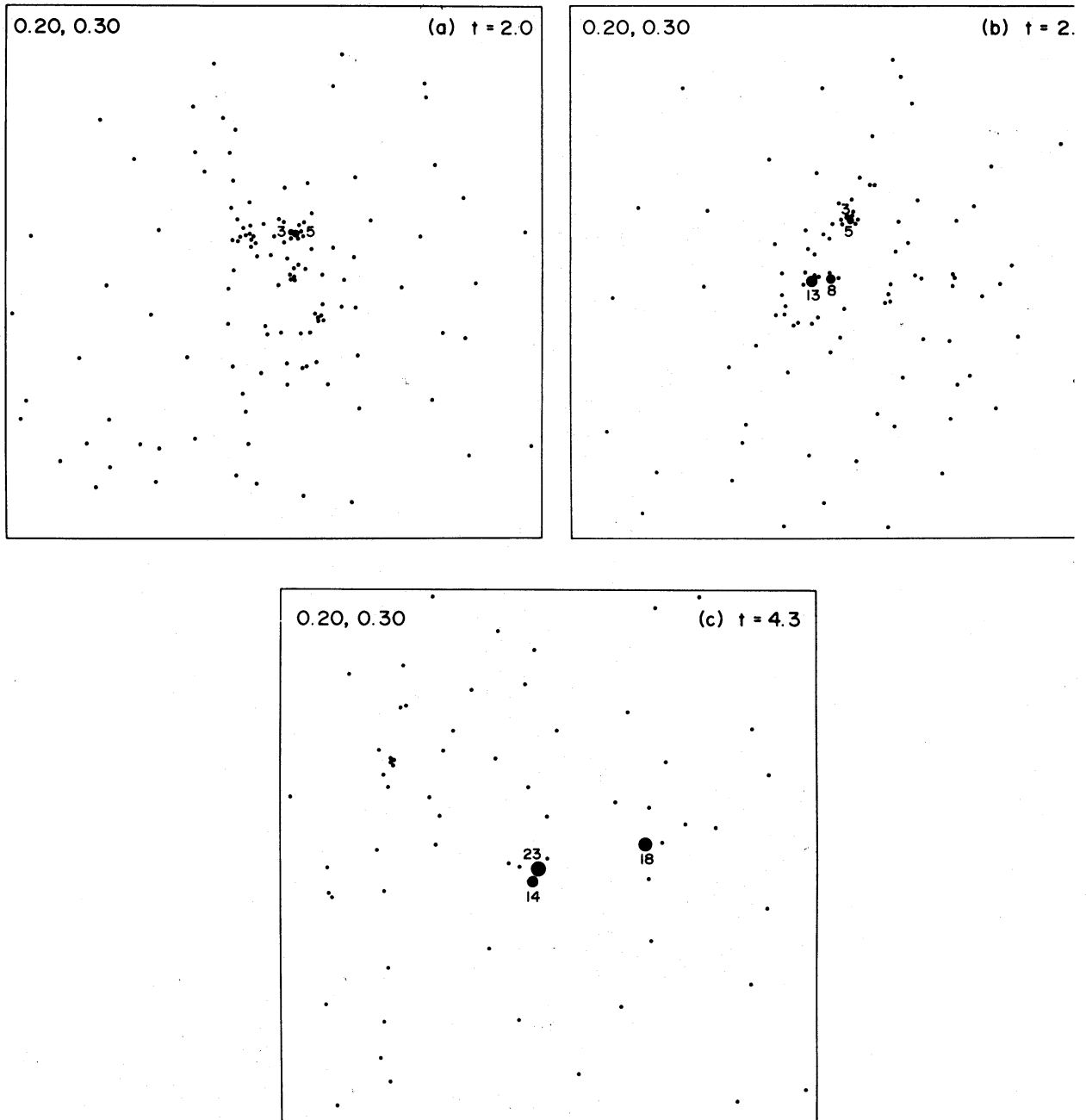

Figure 3. (a) The distribution of a system of 150 particles after 2.0 free-fall times; the initial ratios of thermal and rotational energy to gravitational energy are here 0.20 and 0.30. (b) The particle distribution after 2.6 free-fall times, showing the development of a triple system. (d) The particle distribution after 4.3 free-fall times.

(Fig. 5(a)); most of them survive and become separate condensed objects (Fig. 5(b)), and a tendency for the formation of binary subsystems is again evident. There is possibly also a tendency for the most massive objects to form near the centre, and for the peripheral objects to be less massive.

Finally, Fig. 6(a) and (b) presents results for two calculations made with the lowest assumed temperature, and with a variant of the numerical scheme which employs a larger number of interacting neighbours per particle. In the previous calculations the average number of interacting neighbours per particle was always about 1.5, but here it is initially
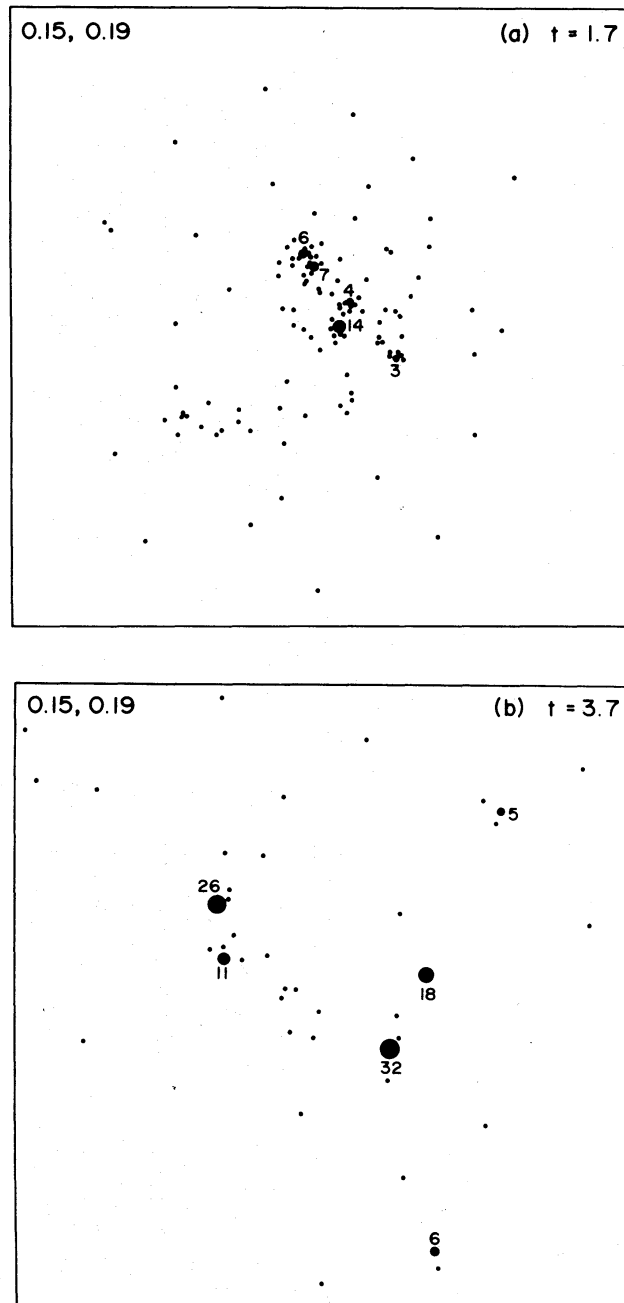

Figure 4. (a) The distribution of a system of 150 particles after 1.7 free-fall times; initial ratios of thermal and rotational energy to gravitational energy are 0.15 and 0.19. Note that the latter number is smaller than in Figs 2, 3, 5 or 6. (b) The particle distribution after 3.7 free-fall times; note that the system is dominated by two massive binaries.

about 3.8 and later decreases to about 2.4 as the collapse proceeds and the density distribution becomes more non-uniform. This simulates a moderate drop in temperature during the collapse, yielding a thermal energy about 0.075 times the initial gravitational energy, which corresponds to an initial Jeans mass about one-sixteenth of the total mass. With this variant of the numerical scheme, a higher proportion of low-mass objects is formed, and a larger fraction of the cloud is accreted by the condensed objects; the latter difference may be attributable to a higher effective viscosity (see below). The cases illustrated in Fig. 6(a) and (b) differ only in the use of a different set of random numbers for the initial particle positions. Although the results differ considerably in detail, they share some basic features
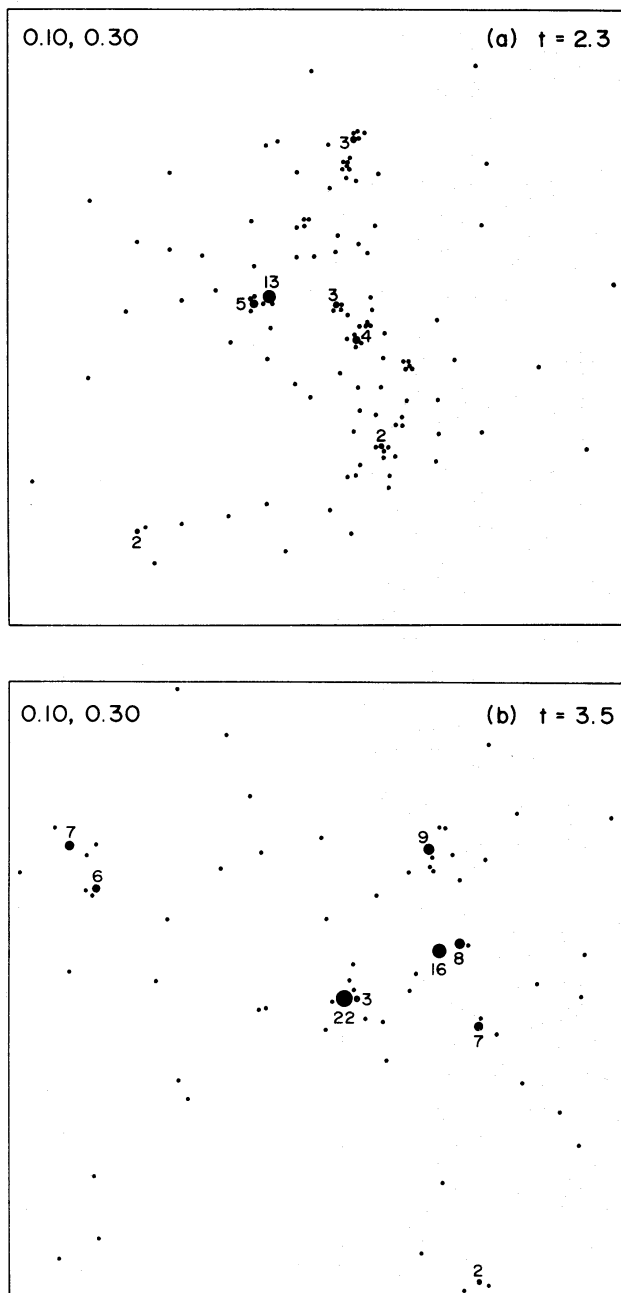

Figure 5. (a) The distribution of a system of 150 particles after 2.3 free-fall times; initial ratios of thermal and rotational energy to gravitational energy are 0.10 and 0.30. (b) The particle distribution after 3.5 free-fall times.

in common and illustrate particularly well the previously noted tendency for the frequent occurrence of binary and multiple subsystems. Another trend suggested by these results, particularly Fig. 6(a), is the possible development of a hierarchy of subsystems with similar structures on different scales.

4 Implications for the mass spectrum of fragmentation

As has been already noted, the use of a finite capture radius means that these calculations can provide information only about the distribution of matter on scales larger than about
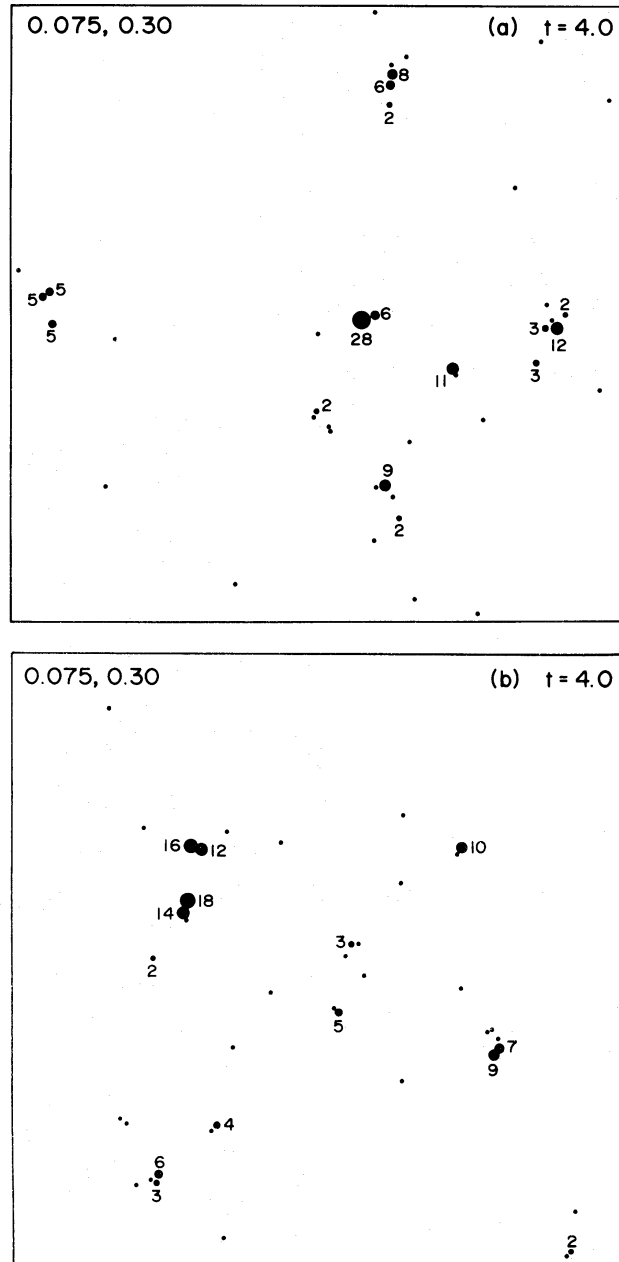

Figure 6. (a) The final distribution of a system of 150 particles after 4.0 free-fall times; initial ratios of thermal and rotational energy to gravitational energy are ~0.075 and 0.30. This case differs from those previously illustrated in that the calculation employs a larger number of interacting neighbours per particle. (b) Same as (a) except for a different set of random numbers for the initial particle positions.

the size of the dots in Figs 1–6 (which might correspond to $\approx 10^{-2}$ pc in a real collapsing cloud). By analogy with the large-scale behaviour, subsystems smaller than this size will almost certainly form, so many of the present 'condensed objects' may actually represent close binary or multiple systems. Furthermore, because of the limited number of particles used in the simulations, accurate results are possible only for a limited range of masses. For these and other reasons, the mass spectrum of the resulting objects may not be very meaningful in detail, and greater attention should be given to the general features and trends shown by the results.

Accordingly, we consider first two quantities related to the mass spectrum that are directly given by the results, and that appear to be relatively independent of the numerical details: these are (1) the total number of objects formed, and (2) the mass of the largest object after a fixed time taken as 4.0 free-fall times. The dependence of these quantities on the initial thermal and rotational energies of the system is summarized in Tables 1 and 2, which are based on a large number of calculations in which these parameters have been varied systematically. In the tables, α and β are the ratios of the initial thermal and rotational energies to the initial gravitational energy, and $M/M_J = (2\alpha)^{-3/2}$ is approximately the number of Jeans masses in the initial cloud. Most entries are averages of several calculations made with different initial particle positions or with other differences of computational detail, except for the number of interacting neighbours per particle which has been kept constant at about 1.5 in all cases.

An important result evident in Table 1 is that the average number of objects formed does not depend significantly on β, i.e. on the initial rotational energy, but increases

Table 1. Average number of objects formed[*].

α	M/M_J	$\beta = 0.30$	$\beta = 0.19$	$\beta = 0.11$
0.35	1.7	1.0	1.0	2.0
0.30	2.2	2.1	2.2	2.0
0.25	2.8	2.9	2.7	3.2
0.20	4.0	3.6	3.9	3.7
0.15	6.1	4.8	5.6	6.0
0.10	11	8.6	7.4	7.0
0.07	19	12	10	

[*] α = initial ratio of thermal energy to gravitational energy, M/M_J = number of Jeans masses in initial cloud, β = initial ratio of rotational energy to gravitation energy.

systematically with decreasing α, i.e. with decreasing temperature and Jeans mass; moreover, the number of objects formed agrees closely with the initial number of Jeans masses in the cloud, except at the lowest temperatures where the Jeans mass becomes very small. This close agreement is probably fortuitous, but it is consistent with the fact that many of the condensations appear to form during early stages of the collapse; also, the subsequent destruction of some of the condensations is largely compensated by the formation of new ones from left-over gas, so that the total number of condensations remains nearly constant and closely related to the initial Jeans mass. The independence of this number on the initial angular velocity of the cloud appears to result from another approximate cancellation of effects: if the angular velocity is reduced, more condensations are formed because of the higher mean density attained by the cloud, but more of them are destroyed by interactions because of their close proximity.

Table 2. Percentage of total mass in the largest object*.

α	$\beta = 0.30$	$\beta = 0.19$	$\beta = 0.11$
0.35	14	25	32
0.30	17	26	36
0.25	17	24	37
0.20	16	24	34
0.15	19	22	34
0.10	17	25	43
0.07	16	25	

*α = initial ratio of thermal energy to gravitational energy,
β = initial ratio of rotational energy to gravitational energy.

Table 2 shows a second important result, which is that the mass of the largest object formed does not depend on the temperature or Jeans mass of the cloud, but increases systematically with decreasing angular momentum. This is because when the angular momentum is decreased the cloud becomes more condensed, and a larger fraction of its mass can be accreted in a given time by the objects that form in it, particularly the largest one. Thus there is evidently a second mass parameter, in addition to the Jeans mass, that is relevant in determining the masses of the condensed objects that can form in a collapsing cloud; this is the maximum mass that can be accreted by a single object in a given time, as limited by the angular momentum and viscosity of the gas. Table 2 shows that this maximum accretable mass does not depend on the Jeans mass or the number of smaller objects formed. Although no systematic calculations have been made to test the dependence of these results on viscosity, several calculations made with different values of the parameter Q in equation (2) show a tendency for the mass of the condensed objects to increase with increasing Q, i.e. with increasing viscosity.

A rough analytical estimate can be made of how the maximum accretable mass should depend on the viscosity ν, or equivalently on the Reynolds number $\mathscr{R}$ in the collapsing cloud. In a viscous accretion envelope or disc, the time required for matter at radius r to spiral into the central object is $t \sim r^2/\nu$ (Lynden-Bell & Pringle 1974). Thus as time progresses, material spirals in from larger and larger radii, and at any time t there is a zone of radius $r_a \sim (\nu t)^{1/2}$ within which most of the matter has already been accreted by the central object. If the collapse initially produces approximately an isothermal mass distribution with $m \propto r$ in the accretion envelope, the accreted mass will grow with time according to $m_a \propto r_a \propto (\nu t)^{1/2}$, or $m_a \propto \mathscr{R}^{-1/2} t^{1/2}$. The latter proportionality becomes an approximate equality if m_a is expressed in units of the total mass and t is in units of the free-fall time. Thus, if $\mathscr{R} \approx 50$, the fractional mass that can be accreted in one free-fall time is $\approx \mathscr{R}^{-1/2}$, or about 0.15; this is in good agreement with the results in Table 2, which refer to somewhat longer times. Even the dependence on β can be qualitatively understood in this way, since with a smaller β the accretion envelope becomes more compact and has a shorter free-fall time. Thus the efficiency of accretion processes and hence the masses of the stars that ultimately form must depend on the viscosity of the cloud, and it is clearly important to understand better the sources of viscosity in collapsing clouds.

We have so far considered only the most massive object, but most of the objects that form evidently do not experience such favourable conditions for growth and end up with smaller masses. The final mass spectrum then depends on the relative numbers of objects that succeed in accreting various amounts of mass. In the present calculations, the Reynolds number and hence the efficiency of accretion do not appear to vary greatly for different objects; instead, the formation of objects with different masses appears to result from the

formation of 'accretion domains' of various sizes, each with a dominant object that accretes a fraction of order $\mathscr{R}^{-1/2}$ of the matter in its domain. The accretion domains sometimes appear to have a hierarchical arrangement; for example, if the accretion envelope or disc around a large object is cool enough, it may become gravitationally unstable and form several smaller satellite objects (as in Fig. 2), each with its own smaller accretion domain around it. Some of these satellites may in turn develop still smaller satellites of their own, formed by similar processes acting on smaller scales; a hierarchical arrangement of this sort is seen for example in Fig. 6(a). This suggests that a collapsing cloud may tend to develop a hierarchy of accreting objects and associated accretion zones having similar structures on different scales. A simple model based on the hypothesis of a self-similar accretion hierarchy and predicting a power-law mass spectrum is discussed in the following section.

The mass of the smallest condensations that can form depends of course on the Jeans mass, and the calculations show that more and more objects of smaller and smaller mass form as the Jeans mass is reduced. The mass of the smallest object is actually somewhat less than the initial Jeans mass; this is because the Jeans mass decreases in the densest parts of the collapsing cloud, and because the envelope of a collapsing condensation is usually not completely accreted by the core. In the present calculations, the mass of the smallest objects formed is usually about one-fifth of the initial Jeans mass.

5 A simple model for the mass spectrum

The present calculations suggest that the final mass spectrum results at least in part from the development of a hierarchy of accreting objects and associated accretion zones of various sizes. In each such zone a fraction $\approx \mathscr{R}^{-1/2}$ of the mass may be accreted by the largest object, while the material which has too much angular momentum to join it may condense instead into several smaller satellite objects. For example, in a collapsing cloud there might develop near the centre a single most massive object whose 'accretion domain' is the whole cloud; around it there might be several less massive objects which accrete matter from smaller regions around them, and so on. The numerical results suggest that similar processes occur on different scales in the hierarchy, although it is difficult to verify this quantitatively because of the small number of particles involved. As long as the collapse remains iso-thermal, it is at least possible that the processes and structures on different scales are strictly similar and differ only by scale factors, if the temperature is the only important physical variable determining the behaviour of the gas. It is therefore of interest to consider the simple hypothesis that the collapse establishes a self-similar accretion hierarchy with similar structure on different scales.

The type of mass spectrum produced by a self-similar accretion hierarchy can be illustrated by a simple geometrical construction (Mandelbrot 1977), which is given here only to illustrate the idea and is not intended as a literal description of real collapsing clouds. Suppose that a collapsing cloud is represented by a equilateral triangle, and suppose that the largest object that forms in it is represented by the inverted triangle constructed by joining the midpoints of its sides. This leaves three smaller corner triangles, each similar to the original triangle but having one-half the linear dimension and one-quarter of the area. The same construction is then repeated for each of these three smaller triangles, and so on. After $n + 1$ steps, the number of inverted triangles (condensed objects) of the smallest size is $N = 3^n$, and the area (mass) of each one is $m = 4^{-n}$ times the mass of the largest one. Combining these expressions, we have $N = m^{-\log 3/\log 4} = m^{-0.79}$. Since there is a constant logarithmic mass interval between different levels of the hierarchy, N is also proportional

to the number of objects per unit logarithmic mass interval, $dN/d \log m$. Thus we can write

$$dN/d \log m \propto m^{-0.79},$$

which is a power-law mass spectrum with $x = 0.79$ in the usual notation. For comparison, the Salpeter mass spectrum has $x = 1.35$, and a representative value of x in stellar systems is $x \sim 1$ (Tinsley 1978).

There is, of course, no reason why a real collapsing cloud should be divided up in just this way, with the largest object always accreting one-quarter of the mass in its domain and the remaining material being divided among three smaller objects. It is possible, for example, that accretion processes will often be less efficient, so that a smaller fraction of the mass available is accreted by objects at each level in the hierarchy. In the limiting case where a negligible (but constant) fraction of the mass is accreted by the objects at each level, the same total mass remains available to objects at all levels in the hierarchy, so the same amount of mass condenses into objects in each logarithmic mass interval; this yields $x = 1$. Thus the concept of a self-similar accretion hierarchy predicts, in the limit of inefficient accretion, a reasonable zero-order approximation to the observed IMF in stellar systems.

It is difficult to verify the prediction of a power-law mass spectrum directly from the present calculations because of the limited number of particles and the limited logarithmic mass range represented. However, the results of those calculations that were made with several interacting neighbours per particle are at least not inconsistent with the above predictions. For example, if the results shown in Fig. 6(a) and (b) are combined, the total masses of all the objects in the approximately equal logarithmic mass intervals of 4–7, 8–15 and 16–31 particles are respectively 49, 85 and 62 particles. For comparison, a power-law spectrum with the same total mass and with $x = 0.79$ would predict 56, 65 and 75 particles in these three mass intervals, while a mass spectrum with $x = 1$ would predict 65 particles in each interval. Clearly these very limited results cannot exclude either value of x, or even provide a strong test for the power-law nature of the mass spectrum.

If isothermal collapse establishes a power-law mass spectrum with a slope depending on the efficiency of accretion processes, the mass spectrum must have lower and upper mass limits determined by other factors. In the present results the lower limit depends on the Jeans mass, and is usually about one-fifth of the initial Jeans mass. However, this is not necessarily an absolute lower mass limit, since the Jeans mass might attain still smaller values in small regions of high density that are not accurately modelled in these simulations. For isolated, spherically collapsing condensations, an absolute lower limit to the Jeans mass of the order of $10^{-2} M_\odot$ is set by the effect of a finite opacity when the density becomes high enough (e.g. Low & Lynden-Bell 1976).

An absolute upper mass limit is set by the maximum fraction of the cloud that can be accreted by a single object. The present results suggest that this fraction increases with decreasing cloud angular momentum, but does not depend on the size or temperature of the cloud. Thus the maximum mass should increase with increasing cloud mass and with decreasing angular momentum; that is, the most massive objects should form in the most massive and dense clouds. It is possible, however, that other effects often intervene to halt accretion before a protostellar core attains the maximum accretable mass; then the upper mass limit is set by effects such as radiation pressure, ionization, stellar winds, etc., all of which may become important for core masses greater than $\approx 30–60 M_\odot$ (e.g. Larson 1977). In all cases, the upper mass limit increases with increasing cloud density, since this favours infall and accretion processes over the effects of radiation pressure, etc. Thus, whether the upper mass limit is determined by a maximum accretable mass or by other effects which intervene to halt accretion, it seems likely that the most massive stars will tend to form in the densest

and most massive clouds, i.e. clouds like the 'giant molecular clouds' that are often observed to be associated with H II regions.

6 Conclusions

The present simulations of three-dimensional collapse confirm and strengthen several basic inferences from previous one- and two-dimensional collapse calculations. In all cases, the collapse is highly non-uniform and is characterized by the runaway development of small regions of high density embedded in more diffuse envelopes. For a cloud whose mass is comparable to the Jeans mass, the three-dimensional results are similar to the spherical case and yield a centrally condensed, approximately symmetrical density distribution with a small protostellar core, or at most a few such cores, near the centre. As in the spherical case, the core begins with a very small mass, and subsequently grows considerably in mass by accreting material from the diffuse envelope.

With a finite angular momentum, the gas accreted by the protostellar core forms a flattened rotating envelope or disc, in which matter spirals gradually inward. The likely formation of accretion discs around forming stars is of interest because this would offer an attractive explanation for the origin of the solar system. Since binary companions can sometimes form from secondary condensations in such accretion discs, there is probably a continuity between the formation of planetary systems and the formation of binary systems, the result depending on the mass of the accretion disc.

An important result of the present calculations is that the number of condensations formed in a collapsing cloud (Table 1) is always comparable to the number of Jeans masses initially present, regardless of the temperature or initial angular velocity. Thus the amount of fragmentation that occurs is essentially determined by the initial Jeans mass; this is consistent with the fact that, once well-defined condensations have formed, there is little further subdivision into smaller condensations. This is partly because of the non-uniform nature of the collapse which leads to the formation of highly condensed cores, and partly because of the effect of viscosity in allowing additional diffuse gas to be accreted by these cores. Thus, the classical picture of successive fragmentation of a cloud into a hierarchy of smaller and smaller units does not appear to apply for the present results. Instead, the number of objects formed appears to be determined by the initial conditions, while the final mass spectrum is determined at least in part by accretion processes and the competition between different accreting objects.

The mass of the largest object formed (Table 2) can be much larger than the initial Jeans mass, and in fact does not appear to depend on the Jeans mass at all; instead, it represents the maximum amount of matter that can be accumulated into a single object, as determined by the angular momentum and viscosity of the cloud. The mass spectrum of smaller objects may be determined by the development of a hierarchy of accreting objects and accretion domains of different sizes, such that each object has associated with it several smaller 'satellite' objects that form from material that has too much angular momentum to join it. If all scales of the hierarchy have similar structure, differing only by scale factors, a power-law mass spectrum is predicted; a simple geometrical model predicting such a spectrum was described in Section 5. In the limit of inefficient accretion, the hypothesis of a self-similar accretion hierarchy predicts a power-law mass spectrum with $x = 1$, in reasonable agreement with observations.

While the hypothesis of a self-similar accretion hierarchy remains to be confirmed by more detailed calculations, it seems established by the present results, together with previous calculations of spherical and axisymmetric collapse, that the formation of stars in a collaps-

ing cloud occurs largely as a result of accretion or accumulation processes. Thus the dynamics of accretion processes, such as the formation and evolution of accretion discs, are likely to be of central importance for star formation, and it is important to understand better not only the thermal behaviour of the gas but also the processes responsible for viscosity and dissipation in collapsing clouds.

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