Galaxies  Astro 530
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Galaxy Rotation Curves & the Tully-Fisher Relation
rotation curve **amplitude** and **steepness** correlates with galaxy mass

<table>
<thead>
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<th>Galaxy</th>
<th>Rotation Curve</th>
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<td>UGC 2885</td>
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<td>F583-1</td>
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<td>D631-7</td>
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Flat rotation curves continue to occur in quite small systems ($V_{\text{flat}} \sim 20 \text{ km/s}$)
Tully-Fisher relation

Traditional Tully-Fisher relation:
Good correlation between galaxy’s stellar luminosity (absolute magnitude) and its maximum rotation velocity (or linewidth, corrected for inclination)

Importance of TF relation:
1. Method of determining distance
2. Tight correlation between stellar luminosity and maximum rotation velocity has important implications for galaxy structure & formation (implies poorly-understood coordination between dark matter and baryonic matter)

Tully & Fisher 1977, AA, 54, 661
why Tully-Fisher relation can be used to determine distance...

one quantity is distance dependent and the other is distance independent

one of the things (velocity) doesn’t depend on distance -- you measure it directly & don’t need to know the distance to put it on the plot

the other thing (luminosity/absolute magnitude) does depend on distance -- you need to know the distance to put it on the plot

Tully & Fisher 1977, AA, 54, 661
Using TF to determine L and distance measure $v_{\text{max}}$ inclination, flux

1. Measure rotation speed.
1b. Correct for inclination

- Measure maximum rotation speed $v_{\text{max}}$ or global HI linewidth $\Delta V$ or $W_{20}$

3. Determine luminosity from rotation speed.

$V_{\text{max},i}$ rotation speed (km/s) corrected for inclination
Using TF to determine L and distance measure $v_{\text{max}}$ inclination, flux

- **Measure** maximum rotation speed $v_{\text{max}}$ or global HI linewidth $\Delta V$ or $W_{20}$

- **Measure** inclination $i$ (to correct observed doppler velocity to galactocentric rotation velocity)

$$v_{\text{max},i} = \frac{v_{\text{max}}}{\sin i}$$

1. Measure rotation speed.
1b. Correct for inclination

3. Determine luminosity from rotation speed.

2. Plot rotation speed on horizontal axis.
Using TF to determine L and distance measure $v_{\text{max}}$ inclination, flux

- **Measure** maximum rotation speed $v_{\text{max}}$ or global HI linewidth $\Delta V$ or $W_{20}$
- **Measure** inclination $i$ (to correct observed doppler velocity to galactocentric rotation velocity)
  \[ v_{\text{max},i} = \frac{v_{\text{max}}}{\sin i} \]
- **Predict** $M$ (or luminosity $L$) from known T-F relation

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1. Measure rotation speed.
2. Plot rotation speed on horizontal axis.
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1b. Correct for inclination.
Using TF to determine L and distance measure $v_{\text{max}}$ inclination, flux

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- **Predict** $M$ (or luminosity $L$) from known T-F relation
- **Measure** $m$ (or flux $f$)
Using TF to determine $L$ and distance

measure $v_{max}$ inclination, flux

- **Measure** maximum rotation speed $v_{max}$ or global HI linewidth $\Delta V$ or $W_{20}$
- **Measure** inclination $i$ (to correct observed doppler velocity to galactocentric rotation velocity)
  $$v_{max,i} = \frac{v_{max}}{\sin i}$$
- **Predict** $M$ (or luminosity $L$) from known T-F relation
- **Measure** $m$ (or flux $f$)
- **Determine distance** from
  $$d \text{ (pc)} = 10^{[(m-M)/5] + 1}$$
  or
  $$d = \left[\frac{L}{4\pi f}\right]^{1/2}$$
Measuring $v_{\text{max}}$ by modeling HI velocity field: the observed velocity field.

NGC 3198
THINGS
Walter+2008
Measuring $v_{\text{max}}$ by modelling HI velocity field: the observed HI data cube (channel maps)

$\text{NGC 3198}$

THINGS

$\text{Walter+2008}$

$v=844 \text{ km/s}$

$v=823 \text{ km/s}$

$v=492 \text{ km/s}$
Measuring $v_{\text{max}}$ by modelling HI velocity field

- Observed velocity field
- Model velocity field
- Residual velocity field

- Major axis position-velocity diagram
- Minor axis position-velocity diagram

- Optical image
- Rotation curve

- 21cm image
- Inclination
- Position angle

- Global spectrum

Spiral NGC 3198
THINGS
Walter+2008
HI spatial-velocity diagram & rotation curve for spiral NGC 3198

HI gas with velocities higher than $V_{\text{rot}}$ due to turbulence/disruption/spiral arm streaming/outflows etc.

ROTATION CURVE (uncorrected for inclination)

200 400

THINGS Walter+2008
Measuring the width of the HI profile

Most HI profiles are double-horned since rotation curves are \( \sim \)flat(-ish) and there is generally lots of HI gas in the outer parts of galaxies where the rotation curve is \( \sim \)flat(-ish).

Integrated HI spectral line profile

**\( W_{50} \):** HI linewidth measured at 50% of peak HI intensity

**\( W_{20} \):** HI linewidth measured at 20% of peak HI intensity

**\( W_{20} \) only a little larger than \( W_{50} \) but generally better indicator of \( V_{\text{max}} \)**
Measure global linewidth (usually HI) or maximum rotation velocity (from optical Hα, CO, HI rotation curves)

How does the HI linewidth relate to the maximum rotation speed?

$$\Delta v_i = \frac{(W_{20} - v'_{corr})}{\sin i} = 2 \frac{v_{\text{max}}}{\sin i} = 2 \frac{v_{\text{max},i}}{\sin i}$$

All of these quantities are distance-independent!!

$W_{20}$ is HI linewidth measured at 20% of peak HI intensity

$v'_{corr}$ correction for non-circular motions (e.g., turbulence, spiral arm streaming); $v'_{corr} \sim 3.6 \sigma = 3.6(10 \text{ km/s}) \sim 36 \text{ km/s} (\text{amount of correction is debated})$

$v_{\text{max}}$ maximum observed rotation speed (uncorrected for inclination)

$v_{\text{max},i}$ maximum observed rotation speed (corrected for inclination)
Measure inclination of galaxy

i.) Model velocity field

or...

ii.) Measure axial ratio $b/a$ of outer isophotes of NIR/optical image

$$\cos i = \frac{(b/a)}{1 - q^2}$$

if intrinsic axial ratio $= q$ (when galaxy edge-on) for galaxies with bulge or thicker disks
Velocity field modeling: error patterns

The type of error pattern reveals which parameter has an error.
Scatter and slope of TF are of great interest

**Scatter** -- accuracy as distance indicator, galaxy structure & evolution

**slope** -- galaxy structure & evolution
TF in different bands

least scatter in I-band
... WHY?

different edge-on galaxy sample
Karachentsev+2002
How good a distance indicator is TF?

B band scatter $1\sigma \sim 0.6$ mag $\rightarrow$ $\sim 30\%$ in distance
I band scatter $1\sigma \sim 0.3$ mag $\rightarrow$ $\sim 15\%$ in distance
K band scatter $1\sigma \sim 0.5$ mag $\rightarrow$ $\sim 25\%$ in distance
How good a distance indicator is TF?

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dust extinction and stellar population ($M_*/L$) variations contribute to scatter
Measure apparent magnitude (flux)

some people try to correct for dust extinction within galaxy and from the Milky Way, but corrections are uncertain (smaller corrections in NIR than optical)
How good a distance indictor is TF?

B band scatter $1\sigma \sim 0.6$ mag -> $\sim 30\%$ in distance
I band scatter $1\sigma \sim 0.3$ mag -> $\sim 15\%$ in distance
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dust extinction and stellar population $(M_*/L)$ variations contribute to scatter

Smallest scatter in NIR ($0.9-2\mu m$)
worse in optical ($0.4-0.7\mu m$) due to dust and large $M_*/L$ variations
worse in mid-IR ($3-5\mu m$) due to large $M_*/L$ variations
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**dust extinction** and **stellar population ($M_*/L$) variations** contribute to scatter

Smallest scatter in NIR (0.9-2µm)
worse in optical (0.4-0.7µm) due to dust and large $M_*/L$ variations
worse in mid-IR (3-5µm) due to large $M_*/L$ variations

I band (0.9µm) is “sweet spot” for minimizing effects of young stellar populations on TF. MS OBA stars produce lots of light in the optical but less in NIR and mid-IR. TPAGB stars (thermally pulsating asymptotic giant branch stars) become important at 3-5µm, less important at 1µm.
TF is currently the best distance indicator for spiral galaxies

- Tully Fisher (spirals) ~15%
- “fundamental plane” (ellipticals) ~15%
- surface brightness fluctuations (SBF) (ellipticals) ~3% (Mei+2007)

15% distance errors not good enough to measure relative distances to galaxies within groups or clusters
using TF to measure distances to clusters

since cluster galaxies all at similar distances, can see TF relation *even with measured fluxes* (apparent magnitudes)

while individual galaxies have ~15% TF distance errors, having many cluster galaxies means that the average cluster distance can be measured to ~15/N^{1/2} %
Scatter and slope of TF are of great interest

Scatter -- accuracy as distance indicator, galaxy structure & evolution

slope -- galaxy structure & evolution
TF relation yields $L \sim v_{\text{max}}^4$

$$M_H = c - s \log v_{\text{max}}$$
$$2.5 \log L_H = c_1 + s \log v_{\text{max}}$$
$$\log L_H^{2.5} = c_1 + s \log v_{\text{max}}$$
$$L_H^{2.5} = 10^{c_1} v_{\text{max}}^s$$
$$L_H \sim v_{\text{max}}^{s/2.5}$$
$$L_H \sim v_{\text{max}}^b$$

$b \approx 3.5$ in $B$ band
$b \approx 4.0$ in $I,J,H$ bands

$\rightarrow L \propto v_{\text{max}}^4$

c = intercept or ‘zero-point’
s = slope (for TF in magnitudes)
b = slope (for TF in luminosity)
b = $s/2.5$

Difference in slope comes from the color-luminosity or color-mass relation: TF is less steep at B since low mass/lum galaxies are bluer on average, since they are both more metal poor and younger (larger fraction of young stars, which emit lots of light in the optical)
TF relation yields $L \sim v_{\text{max}}^4$

M_H = c - s \log v_{\text{max}}$

$2.5 \log L_H = c_1 + s \log v_{\text{max}}$

$\log L_H^{2.5} = c_1 + s \log v_{\text{max}}$

$L_H^{2.5} = 10^{c_1} v_{\text{max}}^s$

$L_H \sim v_{\text{max}}^{s/2.5}$

$L_H \sim v_{\text{max}}^b$

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\[ c = \text{intercept or 'zero-point'} \]
\[ s = \text{slope (for TF in magnitudes)} \]
\[ b = \text{slope (for TF in luminosity)} \]
\[ b = s/2.5 \]

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\[ \text{NOTE: not all studies find } b=4 \]
\[ \text{range of values found } b \approx 3-4 \]
The slope of the TF relation depends on how you measure velocities.

You can get slopes between 3.0-4.3.
4 different versions of TF relation

Tightest “TF relation” is with baryonic mass (stars & gas)

There is tight correlation between a galaxy’s baryonic mass and its non-baryonic mass & mass distribution

Baryonic masses of galaxies correlate tightly with rotation velocity but not radius/size

McGaugh 2005b PRL
2 galaxies with same total stellar luminosity
one is high surface brightness (compact stellar distribution)
other is low surface brightness (diffuse stellar distribution)
TF relation & rotation curves vs. surface brightness

The TF relation doesn’t depend on central surface brightness!

Rotation curves of HSB and LSB spirals with *same stellar luminosity and same $v_{\text{max}}$*!

Tully-fisher relation vs. surface brightness

Different symbols are different central surface brightnesses

The TF relation doesn’t depend on central surface brightness!

McGaugh & deBloq

Solid symbols are galaxies binned by surface brightness: $\mu_0 < 22$ (filled stars); $22 < \mu_0 < 23$ (filled squares); $23 < \mu_0 < 24$ (filled triangles); and $\mu_0 > 24$ (filled circles). Clearly, galaxies fall on the same Tully-Fisher relation irrespective of surface brightness.
2 galaxies with same dark matter distributions, very different baryon distributions, same stellar luminosity and same $v_{\text{max}}$!

**FIG. 1** (color online). The rotation curves of two galaxies, NGC 2403 and UGC 128, of similar mass but very different size. The contributions of the baryonic components (stars plus gas: solid lines) are very different even though the dark matter halos (dashed lines) are similar. Lines for the baryons and dark halo of NGC 2403 end with its velocity data at 20 kpc while those of UGC 128 continue to 40 kpc. Arrows mark the radius $R_p$ where the peak of the baryonic rotation occurs.

McGaugh 2005 PRL
Tully-fisher mysteries

Why does $v_{\text{max}}^4 \propto L$?

Why do LSB galaxies obey the TF relation?

Let’s predict what relation we might expect between $L$, $v_{\text{max}}$ for disk galaxies ...
Tully-fisher mysteries

Why does $v_{\text{max}}^4 \propto L$?
Why do LSB galaxies obey the TF relation?

Let’s predict what relation we might expect between $L$, $v_{\text{max}}$ for disk galaxies ...
Assume: virial theorem, exponential disk

Result:

$v_{\text{max}}^4 \propto Y_{\text{OR}}^2 I_0 L$ \text{ is predicted}
$v_{\text{max}}^4 \propto L$ \text{ is observed}

$\Rightarrow Y_{\text{OR}}^2 I_0 \approx \text{constant}$ \text{ but why should this be??}

$Y_{\text{OR}} = \text{total mass-to-light ratio}$
within observable radius $R_{\text{OR}} = xR_d$
$I_0 = \text{central disk surface brightness}$
Implication of TF relation not depending on surface brightness

correlation of central surface brightness $I_0$ (or $\mu_0$ in magnitudes) and mass-to-light ratio $Y_{OR}$ needed to have all galaxies obey TF relation: Higher M/L in LSB galaxies

\[ Y_{OR}^2 I_0 \approx \text{constant} \]
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McGaugh 2005 PRL
As baryons infall toward center of halo, DM halo should get compressed

adiabatic infall in spherical halo model

Rotation curve and mass model of the luminous high surface brightness galaxy NGC 2903. The rotation curve data are plotted as circles with error bars. The contribution to the rotation by the baryonic component (stars plus gas) is denoted by squares. Large squares are for the modeled mass-to-light ratio specified in Table 1. Also shown as small squares are the limiting cases of the minimum (gas only, with $\gamma = 0$) and maximum disk. The dashed line shows the adiabatically formed exponential disk used to approximate the observed baryon distribution. The solid line is the total rotation due to disk plus compressed halo. The primordial NFW halo (with parameters given in Table 1) is shown by the dotted line, and the compressed halo is shown by the dot-dashed line.
**Summary of key points on TF relation**

Tightest TF-type relation is with baryonic mass

\[ (M_{\text{baryon}} - v_{\text{max}}) \text{ better than } M_{\text{star}} - v_{\text{max}} \text{ better than } L_{\text{star}} - v_{\text{max}} \]

\[ M_{\text{baryon}} \sim v_{\text{max}}^4 \]

TF relation is a statement that **galaxies obey virial theorem**, but that **something in addition to the virial theorem is important in determining spiral galaxy structure**

\[ Y_{\text{OR}}^2 I_0 \approx \text{constant} \]

TF relation provides tight constraints on how baryonic and dark matter get radially rearranged during galaxy formation and evolution