Galaxies  Astro 530
Prof. Jeff Kenney

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Galaxy Rotation Curves
The radial distribution of starlight in spiral galaxy disks is roughly exponential.

Q: Suppose all the galaxy’s mass is in this exponential disk. What rotation curve would be expected, if the mass is proportional to the light?
Rotation curves can be used to estimate the masses and mass distributions in galaxies

\[ v_{\text{circ}}(R) = \left[ R \frac{d\phi}{dR} \right]^{\frac{1}{2}} \]

from Newton’s Laws & Poisson’s equation

\[ \phi = \text{gravitational potential} \]

\( v_{\text{circ}} \) is *theoretically calculated* circular speed (of test particle), best defined for *spherically symmetric mass distribution*. 
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In case of \textit{spherically symmetric mass distribution only}:

\[ v_{\text{circ}}(R) = \left[ \frac{GM(R)}{R} \right]^{1/2} \]

\( M(R) \) = enclosed mass within radius \( R \)

If you can determine \( v_{\text{circ}}(R) \) then you can know \( M(R) \)
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If mass distribution not spherically symmetric, then \( v_{\text{circ}} \) also depends on exterior mass and details of mass distribution!
How does theoretical $V_{\text{circ}}$ relate to observed $V_{\text{rot}}$?

$$v_{\text{circ}}(R) = v_{\text{rot}}(R) + v_{\text{corr}}(R)$$

$v_{\text{circ}}$ is \textit{theoretically calculated} circular speed

$v_{\text{rot}}$ is \textit{observationally measured} rotation velocity

$v_{\text{corr}}$ is correction for velocity dispersion/disordered or random motions/”asymmetric drift” (all same thing)

\textit{Most gas and stars do not have pure circular orbits, but also have non-circular components to their motions}

$v_{\text{corr}}$ is typically:

Small for gas ($V_{\text{rot}} \gg \sigma$)

Medium for stellar disks ($V_{\text{rot}} > \sigma$)

Large for stellar bulges, E’s ($V_{\text{rot}} < \sigma$ or $V_{\text{rot}} << \sigma$)

\textit{Notation on $V_{\text{rot}}$ and $V_{\text{circ}}$ is not consistent; and some people sloppy -- BEWARE!}
The radial distribution of starlight in spiral galaxy disks is roughly exponential

Q: Suppose all the galaxy’s mass is in this exponential disk. What rotation curve would be expected, if the mass is proportional to the light?
solar system rotation curve

2 things are shown in plot
points: observations
curve: theory i.e.
good agreement! \( v_{\text{rot}} = \sqrt{\frac{GM_{\odot}}{r}} \)

mass distribution very close
to point mass in center

if orbits obey \( v_{\text{rot}} \sim r^{-1/2} \), they are called “keplerian orbits” and indicate nearly all mass
is at radii less than \( r \)
this will be an important point for galaxies....
Rotation curve of exponential disk

Figure 2-17. The circular-speed curves of: an exponential disk (full curve); a point with the same total mass (dotted curve); the spherical body for which $M(r)$ is given by equation (2-170) (dashed curve).

Binney & Tremaine 1987

A: flattened exponential disk
B: spherical distribution with same interior mass as exponential disk
C: point mass with same total mass as exponential disk
Rotation curve of exponential disk

B+C have similar velocities at $R>4R_{\text{disk}}$

Keplerian behavior ($V\sim R^{-1/2}$)
so little additional mass
beyond $R>4R_{\text{disk}}$ for A,B,C

A peaks at $R=2R_{\text{disk}}$
A $\sim 15\%$ higher than B (at peak)

A slower than B at $R<1R_{\text{disk}}$
→ WHY?

A faster than B at $R>1R_{\text{disk}}$
→ WHY?

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A slower than B at $R<1R_{\text{disk}}$
$\rightarrow$ Slowed by exterior mass

A faster than B at $R>1R_{\text{disk}}$
$\rightarrow$ sped by interior mass concentrated in disk

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Starlight and HI gas in spiral galaxy

Rotation curve in this spiral galaxy

DISTRIBUTION OF DARK MATTER IN NGC 3198

Rotation curve expected from exponential disk

NGC 3198 Walter+2008
Starlight and HI gas in spiral galaxy

Rotation curve in this spiral galaxy

DISTRIBUTION OF DARK MATTER IN NGC 3198

observed (total)

Dark matter

Rotation curve expected from exponential disk

Visible matter (stars & gas)

Dark matter dominates in outer parts
Both baryonic matter and dark matter are centrally concentrated in galaxies, but **baryonic matter is more concentrated**
baryonic matter is centrally concentrated in galaxies  
(~1-15% of total mass)

Dark matter dominates in outer parts  (~85-99% of total mass)
even if we know the expected *shape* of the rotation curve of the stars only, from the stellar light distribution, how do we know the mass of those stars?

**two possible stellar disk rotation curves**

1. “Maximum-disk” fit to rotation curve
2. Less-than-maximum-disk

\[ \frac{M_\ast}{L_\ast} = \frac{1}{2} \frac{M_\ast}{L_\ast} \text{(maximum disk)} \]

both are plausible .... how do we know which \( \frac{M_\ast}{L_\ast} \) is correct? how do we know \( \frac{M_\ast}{L_\ast} \) doesn’t vary with radius?
Q: How do we know that the “dark” matter in outer galaxies is not stars or gas?
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Q: How do we know that the M/L of the stars in a galaxy doesn’t increase with radius to explain the ~flat rotation curves?

Q: How can we measure the mass of stars in a disk?
HR diagram of stars in Solar Neighborhood

how much does the M/L ratio vary for stars?
measuring mass of a stellar disk

stars move vertically in disk under the influence of gravity from the disk.

can measure surface mass density $\Sigma(r)$ of disk if we can measure scale height $h_z$ of disk and vertical velocity dispersion $\sigma_z$ of stars in disk

$$\sigma_z^2 = \langle v_z^2 \rangle \cong 2 \pi G \Sigma h_z$$

this from Newton's Laws plus Liouville's theorem (conservation of phase space density)
for locally isothermal self-gravitating sheet (disk)

\[
\frac{\partial F_z}{\partial z} = -4\pi G \rho
\]

**Poisson’s equation** (Newton’s Laws)

\[
\frac{\partial \rho}{\partial z} = \frac{\rho F_z}{\langle v_z^2 \rangle}
\]

**Liouville’s equation** – one of Jeans equations (which comes from Collisionless Boltzmann Equation) in cylindrical coordinates, assuming isothermal disk i.e. \( \langle v_z \rangle = \) constant with \( z \)

\[
F_z = \text{gravitational force in } z
\]

\[
\sigma_z = \left( \langle v_z^2 \rangle \right)^{1/2} = \frac{1}{N} \sum_{i=1}^{N} \left( V_{z_i} - \bar{V}_z \right)^2 \right)^{1/2}
\]

\[
= \text{velocity dispersion in } z
\]

\[
\rho(z) = \rho_0 \sech^2 \left( \frac{z}{2z_e} \right)
\]

\[
\text{with } 2z_e = \left( \langle v_z^2 \rangle \right)^{1/2} \left( \frac{2\pi G \rho_0}{\langle v_z^2 \rangle} \right)^{1/2}
\]

\[
\langle v_z^2 \rangle = 8\pi G \rho_0 z_e^2
\]

\[
= 2\pi G (4\rho_0 z_e) z_e
\]

\[
= 2\pi G \Sigma z_e
\]

\[
\text{where } \Sigma = \int \rho(z) dz = 4\rho_0 z_e
\]

\[
z_e = h_z = \text{vertical scale height of disk}
\]

\[
\Sigma = \text{mass surface density of disk}
\]
How to estimate the mass to light ratio of disks

measure $\sigma_z$ and $h_z$ of the disks, determine $\Sigma$ from:

$$\sigma_z^2=\langle v_z^2 \rangle = A \pi G \Sigma h_z$$

a. $A = 2$ if $\rho = \rho_o \sech^2 (z/h_z)$
b. $A = 1.7$ if $\rho = \rho_o \sech (z/h_z)$
c. $A = 1.5$ if $\rho = \rho_o \exp (z/h_z)$

vertical scale height $h_z$
vertical velocity dispersion $\sigma_z$
mass surface density $\Sigma$

measure light surface brightness of disk $I(R)$ ($L_{\text{sun}}$ pc$^{-2}$)
estimate mass surface density of disk $\Sigma(R)$ ($M_{\text{sun}}$ pc$^{-2}$)
ratio gives $\Sigma(R)/I(R)$ or $M_*/L(R)$

This has been done in a few galaxies, enough to know that flat rotation curves are not generally due to stellar disks with a strongly varying $M_*/L(R)$
How do we know the $M_*/L$ of the stars in a galaxy?

1. Measure mass surface densities $\Sigma$ and $M_*/L$ ratios of stellar disks from measurements of vertical scale height $z_e$ and vertical stellar velocity dispersion $\sigma_z$

$$\sigma_z^2 = A \pi G \Sigma z_e$$
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2. From galaxy colors/spectroscopy/spectral energy distribution (SED) which give information on stellar population (including $M_*/L$)
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*Can only do this in small number of galaxies*

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*Can do this in lots of galaxies – but need to relate to direct measurements of #1*
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3. Understanding disk structure though galaxy dynamics e.g., whether a bar or strong spiral structure forms in a disk depends on how much of the total mass is in the disk (more disk structure if more mass in disk); also dark matter halo should make bars rotate more slowly (slowed by dynamical friction)

*Gives rough quantitative estimate of disk mass – but not as accurate as #1*
rotation curve **amplitude** and **steepness** correlates with galaxy mass

Flat rotation curves continue to occur in quite small systems ($V_{\text{flat}} \sim 20$ km/s)
Rotation curves vs. galaxy mass

More massive galaxies:
1. Higher $v_{\text{max}}$
   ($v_{\text{max}} = \text{maximum } v_{\text{rot}}$)
2. Steeper $v_{\text{rot}}(R)$

High mass

Low mass
How much do the baryons contribute to the mass of galaxies?

Dark matter vs. baryons in massive HSB spirals vs. low mass LSB dwarfs

**Massive high surface brightness spiral**

![Graph of Velocity vs. Radius](image1)

- **DM**
- **Stars**
- **Gas**

**Low mass, low surface brightness dwarf**

![Graph of Velocity vs. Radius](image2)

- **DM**
- **Stars**
- **Gas**

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**Maximum disk fit shown (plausible stellar M/L)**

¾ maximum disk is best estimate for typical HSB disks (McGaugh 2005)

Baryons (disk stars & gas) dominate mass near centers of massive HSB spirals

**Reasonable stellar M/L disk shown**

1/4 maximum disk – best estimate for typical LSB disks – but some MUCH less! (McGaugh 2005)

Dark matter halo dominates mass at all radii, even in center!
How much do the baryons contribute to the mass of galaxies? Dark matter vs. baryons in massive spirals vs. low mass dwarfs

While low mass galaxies are more DM-dominated than high mass galaxies, the highest DM densities occur in massive/HSB disks (higher amplitude in DM part of rotation curve for massive galaxies).

It’s not that low mass galaxies are DM-rich -- they are baryon-poor! Low mass galaxies probably lose baryons during evolution.
Rotation curve of typical dark matter halo

Inner parts: Constant density core $\rho(R) = \text{constant}$ at $R < a_H$
Has $\sim$solid body rotation curve at small radius

Outer parts: Isothermal sphere $\rho(R) \sim R^{-2}$ at $R > a_H$
(has constant velocity dispersion = temperature with radius)
Has $\sim$flat rotation curve at large radius

Rotation curve for Dark matter halo
$\rho(R) \sim 1/(R^2 + a_H^2)$

Density distribution of typical dark matter halo represented by:

$$4\pi G \rho(R) = v_H^2 / (R^2 + a_H^2)$$
Rotation curves tend to become \( \sim \) flat at large radii

If rot curve flat \( V = \text{CNST} \), implies density \( \rho \sim R^{-2} \)
interior mass increases \( M(R) \sim R \)

Mass is only interior mass – rotation curves still rising at last measured point – can’t get total galaxy masses from these rotation curves!

Rubin, Ford & Thonnard 1978
gravitational lensing to measure cluster masses

Gravity of foreground cluster ("lens") bends light rays from background sources (galaxies)
use gravitational lensing to measure galaxy masses

- Gravity of foreground galaxy ("lens") bends light rays from background source
- Total mass of "lens" can be measured from amount of bending
Deflection angle $\alpha$ depends on mass $M$ and distance of closest approach $b$

$$\alpha \approx \frac{4GM}{bc^2} = \frac{2R_S}{b}$$

$R_S = \frac{2GM}{c^2}$ = Schwarzschild radius
Galaxy masses from gravitational lensing

**Advantages of lensing:**

- Don’t need tracer particles in galaxies, can probe potential well beyond location of baryonic tracers (stars & gas) in galaxies
- Doesn’t depend on dynamical state of stuff in galaxy, works if galaxy not in equilibrium & orbits of stars & gas disturbed

**Disadvantages of lensing:**

- Individual galaxies cause weak lensing, with only modest distortions (~1%) to image shapes → must do statistical analysis, requiring 100’s-1000’s of background galaxies (hard)
- Can’t get accurate total mass of individual galaxy from weak lensing, but can get statistical average mass of sample of foreground galaxies by combining weak lensing signals around many foreground galaxies
Galaxy Mass Results from Gravitational Lensing

- masses from lensing roughly agree with those from galaxy kinematics – both reveal that galaxies dominated by dark matter

- “Field galaxies” seem to have $M(R) \sim R$ (i.e., $\sim$flat rotation curves) out to $\sim100$-300 kpc, no Keplerian dropoff clearly seen (McKay et al 2001)

- Cluster galaxies (E&S0) seem to have mass extending out only 10’s of kpcs $\rightarrow$ tidal truncation in clusters (Natarajan et al 2002)

Easier to do lensing studies in clusters, since lensing from the cluster mass amplifies light from background galaxies, making it possible to detect more of them
How do rotation curves behave near nuclei?
How do rotation curves behave near nuclei?

How do central black holes affect rotation curves?
Rotation curves including central black hole

MW: $= 3 \times 10^6 \, M_{\odot}$

M31, N4258: $M_{\text{BH}} = 4 \times 10^7 \, M_{\odot}$

[M87: $M_{\text{BH}} = 3 \times 10^9 \, M_{\odot}$]

Black hole dominates

Milky Way

Galaxy dominates

Sofue & Rubin 2001, ARAA