1.) The relationship between volume density $\rho(r)$ and enclosed mass $M(r)$, in the case of spherical symmetry, is:

$$\rho(r) = \frac{1}{(4\pi r^2)} \frac{dM(r)}{dr}$$

a.) The inner parts of typical rotation curves are approximately "solid body" over some range in radius, i.e. $V_{\text{rot}} \sim r$. What volume density distribution $\rho(r)$ does this correspond to? [pretend that all the mass is spherically distributed.]

b.) The outer parts of most rotation curves are approximately "flat", i.e. $V_{\text{rot}} \sim \text{constant}$. What density distribution $\rho(r)$ does this correspond to?

c.) Suppose that most of the mass interior to the solar radius ($R_o = 8.5$ kpc) in the Milky Way were in a spherical dark halo. If the rotation curve is flat, with $V=220$ km s$^{-1}$, what is the volume density of the dark halo in the immediate vicinity of the solar neighborhood?

d.) The density of the local disk is estimated to be 0.18$^{+0.03}_{-0.03}$ $M_{\odot}$ pc$^{-3}$, calculated from observations of the velocity dispersion and stellar density profile in the direction perpendicular to the galactic plane. Most of this local disk mass seems to be in the form of stars. Assuming that it is all in the form of stars, calculate the local surface mass density of stars, assuming that the vertical distribution is well described by an exponential function with a scale height of 300 pc, and the density given above is the midplane density.

e.) Calculate the stellar mass of the disk inside the solar circle, assuming that the radial distribution in the disk is exponential with a scale length of 4 kpc, and using the local surface mass density of stars from part d. The total mass of baryons inside the solar circle is perhaps 1.5 times the stellar mass in the disk, if we account for gas in the disk and stars in the bulge.

f.) Compare the mass of dark matter and the mass of baryons inside the solar circle. Then compare the local density of the dark matter halo with the local (disk) density of baryons. Comment on how these things are consistent with one another.

2. One form of the Tully-Fisher relationship: is $M(H) = 2 \cdot 10 \log V(\text{max})$.

a.) Suppose an Sb galaxy has $i=40$ degrees, $m(H)=15$ mag, and $V_{\text{max}}$ (uncorrected for $i$)=$150$ km s$^{-1}$. Assume that $m(H)$ has been corrected for all extinction effects. What is the distance to the galaxy?
b.) Some people have claimed that the TF relationship has an intrinsic scatter (1 sigma) of only 0.25 mag, while others have claimed that the scatter is as high as 0.7 mag. What are the distance uncertainties (in percent) that correspond to these 2 estimates of the scatter? [HINT: start by writing the expression for the distance in terms of the distance modulus \( \mu = m-M \).]

3.) a.) Derive the total luminosity \( L \) of a disk in terms of the central surface brightness \( I_0 \) and the exponential disk scale length \( R_d \), assuming that the radial surface brightness profile of the disk is described by: \( I(R) = I_0 \exp (-R/R_d) \).

b.) Show that disk galaxies that obey the virial theorem are expected to follow the relation: \( V_{\text{max}}^4 = Y_{\text{OR}}^2 I_0 L \)

using the derived expression for disk luminosity \( L \) from part a as well as these relations:
- \( V_{\text{max}}^2 = GM_{\text{OR}} / R_{\text{OR}} \) (from the scalar virial theorem)
- \( Y_{\text{OR}} = M_{\text{OR}} / L = \text{total mass to light ratio inside the observable radius } R_{\text{OR}} = x R_d \)
- \( M_{\text{OR}} = \text{total mass inside some observable radius } R_{\text{OR}} \).

c.) Given that the observed Tully-Fisher relation corresponds to \( L \approx V_{\text{max}}^4 \), what does this imply about galaxy disk structure?

4.) Images of background galaxies get distorted due to gravitational lensing by foreground galaxies, potentially yielding information on the masses and mass distributions of the foreground galaxies. In order to extract this information (in the case of weak lensing, which is far more common, and can only be done statistically by examining images of many background galaxies), one needs to know the expected distribution of projected galaxy shapes on the sky. This depends on both the distribution of intrinsic shapes of galaxies, and on projection effects. In order to understand the projection effects, you should understand the answer to the question: Are edge-on or face-on disk galaxies more common? In particular, for this problem, answer the following 2 questions:

a.) What fraction of disk galaxies have inclinations less than 5 degrees?

b.) What fraction of disk galaxies have inclinations greater than 85 degrees?

c.) What fraction of disk galaxies have inclinations greater than 45 degrees?

d.) What fraction of disk galaxies have inclinations less than 45 degrees?

Note: \( i=0 \) degrees is face-on, \( i=90 \) degrees is edge-on.
Assume that the spin axis of galaxies point in random directions.