Time-dependent models of magnetized pair plasmas

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SUMMARY
A numerical code has been developed to study the time evolution of electron–positron plasmas. The code solves in a self-consistent manner kinetic equations describing the effects of Compton scattering, two-photon pair production, pair annihilation, cooling of pairs via Coulomb scattering, e–e bremsstrahlung, and synchrotron radiation. The kinetic equations are derived under the approximation of homogeneous and isotropic particle distributions following the discussion in Coppi & Blandford. Both stationary (equilibrium) and time-varying output radiation spectra have been computed. Good qualitative agreement with previous calculations is found, except where the differences are attributable to the improved treatment of the microphysics. These differences can be substantial. In magnetized plasmas, the self-absorption turnover frequency is found to vary weakly with the model input parameters. In particular, for mono-energetic injection at energy $\gamma_{\text{inj}}$, the turnover frequency $\nu_{\text{t}}$ is $\sim 3 \times 10^{13} U_{10}^{1/3} \gamma_{\text{inj}}^{-1/3}$ Hz, where $U$ is the smaller of either the magnetic or photon energy density (measured in units of $10^4$ erg cm$^{-3}$). This may be relevant to the spectra of radio-quiet AGN. Also, the spectral index of the inverse Compton scattered radiation can differ significantly from the associated synchrotron radiation spectral index. (In fact, the equilibrium photon and pair distributions are often not well described by power laws.) Varying the energy and particle inputs to the pair plasma gives rise to many different types of spectral variability. The response of the plasma depends sensitively on both the current state of the plasma and the details of the changes in particle injection. Using time variability as a diagnostic (e.g., to determine the relevance of the models considered here) may thus prove difficult. A possible signature, however, is the response to a significant decrease in the injection of energetic pairs. If the initial Thomson optical depth is of order unity or more, the photon spectrum decays from the high-energy end downwards (lower frequencies lag higher frequencies). The decay of the continuum usually uncovers a prominent, long-lived annihilation feature.

Key words: plasmas – radiation mechanisms: miscellaneous – radiative transfer – galaxies: active – X-rays: galaxies.

1 INTRODUCTION

As noted by Jelley (1966) and Herterich (1974), significant electron–positron pair production is likely to occur in a compact energetic source such as the central engine of an active galactic nucleus (AGN). The resulting ‘pair plasma’ can radically alter the emergent spectrum of radiation that must pass through it. Energetic pairs in the plasma can

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Compton upscatter an initially soft (e.g., IR, UV) spectrum to X-ray and gamma-ray energies, while cool pairs can add an annihilation feature at $\sim 0.5$ MeV (e.g., as seen in the Galactic Centre). The plasma may also effectively reprocess gamma-rays into X-rays (see Svensson 1987), disguising the true nature of the source. Finally, the plasma, with its characteristic (non-zero) response times, will play a large role in determining the observed source variability.

Early attempts to describe pair plasmas (e.g., Bisnovatyi-Kogan, Zel’dovich & Sunyaev 1971) made the simplifying assumption that the pairs were in thermal equilibrium, i.e.,

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that their distribution functions could be characterized by a
(relativistic) Maxwellian of fixed temperature. The first
detailed investigations concentrated on understanding the
effects of pair-photon Compton scattering. These culminated
in the work by Sunyaev & Titarchuk (1980), who studied
'Comptonization' in plasmas with low pair temperatures and
large Thomson optical depths using the Kompaneets (1957)
equation. Their results were verified in Monte Carlo simula-
tions (e.g., Lorentz 1981; Górecki & Wilczewski 1984), and
extended to mildly relativistic pair temperatures by Zdziarski
(1985). See also Guilbert (1981, 1986) for a treatment of
Comptonization based on a kinetic equation approach.

A more complete description of a pair plasma and
photon-pair interactions, however, must also treat the
processes of pair production and annihilation. Moreover,
since Compton scattering and these two processes alter
the number and energy of the pairs in the thermal distribu-
tion, one cannot realistically assume a fixed pair distribution.

Much work has thus been devoted to self-consistently
determining the combined effects of these processes in a
thermal plasma. The most advanced treatments to date are
probably those of Guilbert & Stepney (1985) and Kusunose
(1987), which are based on a time-dependent, kinetic
equation code and attempt to solve the radiative transfer
equations explicitly. [For previous treatments, see Lightman
(1982), Araki & Lightman (1983) and Svensson (1982) for
analytical approaches to the problem, and Zdziarski (1984,
1985) for the numerical approaches.]

Unfortunately, there is reason to believe that the pairs in
a pair plasma are not in thermal equilibrium. The assumption
for example, that pair annihilation balances pair production
in a thermal pair plasma leads to a maximum source lumino-
sity which appears to be exceeded in observed AGN (e.g.,
Araki & Lightman 1983). More importantly, thermalization
times for mildly relativistic pairs are likely to be quite long
compared with other time-scales in the problem (see Stepney
1983). The pair distribution is thus likely to have a signifi-
cant non-thermal tail. This tail is often approximated by a
power law. However, one can show that a power law does not
even satisfy the stationary plasma kinetic equations which
include only Compton scattering, pair annihilation, and pair
production. Significant deviations from power-law behaviour
can be expected, especially if additional processes such as
synchrotron self-absorption are included. To solve the pair
plasma problem satisfactorily, one therefore needs a method
for handling arbitrary photon and pair distribution functions.
In addition, since pair plasmas are likely to live in variable
sources, whatever method is employed should be able to
trace the evolution of the particle distributions in time.

Two approaches have been employed thus far. The first,
using the technique described in Pozdnyakov, Sobol' &
Sunyaev (1977), is a Monte Carlo approach where individual
particles are followed as they undergo interactions inside the
source. The main advantage of such a scheme is that it is
easily used to model the radiative transfer well. However,
such a scheme typically suffers from relatively poor photon
statistics at high frequencies (even when variance-reducing
techniques like photon splitting are used), and does not lend
itself to time-dependent calculations. For examples of the
Monte Carlo calculations performed to date, see Stern
(1985), Novikov & Stern (1986), and Stern (1988). The
second approach involves solving the relevant kinetic
equations. Following the time evolution of the system in such
an approach is straightforward and photon statistics are not
an issue. The major problems facing this method are: (i) the
resulting integro-differential equations can be quite cumber-
some as well as 'stiff' (i.e., there are widely differing time-
scales in the problem); (ii) only simple prescriptions for
radiative transfer (such as an escape probability formalism)
can be implemented easily on present-day computers. For
examples of kinetic equation treatments, see Fabian et al.
(1986) (henceforth referred to as FBGPC), Ghisellini (1987a,
b), Lightman & Zdziarski (1987) (henceforth referred to as
LZ), and Svensson (1987).

In this paper, we have chosen to follow the kinetic
equation approach. The code employed for the work pre-
sented here is most similar to that of FBGPC. Like them, we
'discretize' the kinetic equations, placing particles in bins of
finite energy. We also make similar assumptions about the
isotropy and spatial uniformity of the pair and photon
distributions. Our particular code, however, is distinguished
from previous codes in that we have significantly improved
the treatment of microphysical processes (see Coppi &
Blandford 1990, hereafter Paper I) and added the effects of
synchrotron radiation, electron thermalization (Coulomb
cooling), and e⁻e bremsstrahlung.

After discussing in Section 2 the physical details of the
model considered, along with some numerical 'details' of the
code, we proceed in Section 3 to consider the case of a
stationary pair plasma with no magnetic field, comparing our
results with those obtained in previous work. In Section 4, we
consider the effects of adding a magnetic field. Finally, in
Section 5 we consider the time behaviour of the system.

2 MODEL DESCRIPTION

2.1 Microphysics, physical assumptions and definitions

In the present version of the code, we consider the standard
'pair' processes of Compton scattering (γe⁻→γe), pair annihi-
lation (e⁺e⁻→γγ), and pair production (γγ→e⁺e⁻). To these
we add the processes of e⁻e bremsstrahlung (ee⁻→ee),
Coulomb 'cooling' (ee⁻→ee), and synchrotron radiation
(eB→eBγ). (See Svensson 1986 for a general discussion of
which processes are relevant.) All these processes are described
by kinetic equations derived under the assumption of
homogeneous and isotropic particle distributions, as well
as of tangled, relatively weak magnetic fields. We present
here a discussion of only those terms not already discussed in
Paper I. In what follows, we refer only to the pair
distribution N(γ), since we assume (as do LZ and FBGPC) that
the electron and positron distributions are identical, i.e.,
N_e(1-γ) = N_e(γ). Also, all energies will be measured in units of
m_e c². N(γ) is thus the number density of pairs with energy
γm_e c², and n(x) will be the number density of photons with energy
x m_e c².

Our treatment of synchrotron radiation tries to avoid
some of the problems inherent in past treatments, e.g.,
Ghisellini (1987a). For the synchrotron emissivity, we have
taken the relativistic limit averaged over an isotropic pitch
angle distribution, i.e.,

\[ p(\nu, \gamma) = \frac{\nu}{m_e c^2} \left( \frac{\nu}{\nu_c} \right)^{\alpha - 2} \sin \alpha \int_{\nu/\nu_c}^\infty dx K_{5/3}(x) \]  

(2.1)
where \( v_c = (3/4\pi)^{1/2} (eB/m_e c) \). (See Crusius & Schlickeiser 1986 for a more detailed discussion.) This is a good approximation down to energies \( \gamma \sim 3 \) and avoids the problems associated with delta-function approximations (e.g., premature cut-offs in the spectra, artificial 'spiky' spectral features — see the discussion in Paper I on Compton scattering). Because a simple cut-off approximation to the synchrotron cooling rate misses many important effects (see Ghisellini 1987b), we have used the Foked–Planck formalism of McCray (1969). Note that this treatment of synchrotron radiation is essentially the same as employed in de Kool, Begelman & Sikora (1989) and Ghisellini, Guilbert & Svensson (1988). Our code, however, makes no assumptions about the size of the radiation field energy density (i.e., the importance of Compton scattering), as the Compton and synchrotron terms appear on the same footing in the pair kinetic equations. Note also that Ghisellini et al. (1988) have included non-relativistic corrections to their equations, which for certain parameter regimes can affect the very low-energy (\( \gamma \sim 2 \)) pair distribution. The McCray (1969) equations are strictly valid only in the limit \( \gamma \gg 1 \). These corrections, however, do not appear to affect significantly the photon spectra presented here.

In modelling particle escape, we follow LZ and FBGPC. Pairs are assumed to be trapped (e.g., by a weak magnetic field), and there is no pair escape term in our kinetic equations. Photon escape is modelled using a simple escape probability, i.e., by a term in the photon kinetic equations of the form

\[
\hat{n}_{ce}(x) = -\frac{c}{R} n(x)[1 + \tau_{KN} f(x)]^{-1},
\]

(2.2)

where

\[
f(x) = \begin{cases} 
1 & \text{for } x \leq 0.1, \\
(1-x)/0.9 & \text{for } 0.1 < x < 1, \\
0 & \text{for } x > 1,
\end{cases}
\]

(2.3)

and

\[
\tau_{KN}(\theta, x) = 2 \langle \alpha_{KN} \rangle \gamma N_T
\]

(2.4)

as in LZ, equation (21a). Here, \( N_T \) is the number density of cool, thermal pairs, \( \langle \alpha_{KN} \rangle \) represents an average over the thermal particle distribution, \( \beta \) is the electron velocity in units of the speed of light, \( \alpha_{KN} \) is the Klein–Nishina cross-section, and \( R \) is the source radius. The function \( f(x) \) is a correction factor that accounts for the fact that the photon energy \( x \) increases, forward scatterings predominate, reducing the photon escape time. Under this approximation, photons with energy \( x > 1 \) are assumed to be free-stream out of the source with a characteristic escape time given by \( R/c \), the light-crossing time. This approximation appears to reproduce well the results of Monte Carlo calculations which deal with the radiative transfer problem correctly (Zdziarski, private communication).

To deal with particle injection/acceleration, we adopt the simplified description of LZ and FBGPC, i.e., we assume particles are injected uniformly into the interaction region from some external source(s). Under this assumption, particle injection can be described by two functions, \( S(x, t) \) and \( Q(y, t) \), which do not depend on the pair and photon distributions present in the interaction region. [Feedback mechanisms such as 'pair loading' (see Done & Fabian 1989) will thus not be considered here.] \( S(x, t) \) is the rate of injection (per unit volume) of photons of energy \( x \) at time \( t \), and \( Q(y, t) \) is the analogous injection rate of pairs at energy \( y \) and time \( t \). We will consider here functions of the form \( S(x, t) = S(x) f_s(t) \) and \( Q(y, t) = Q(y) f_p(t) \). Motivated by the desire to model pair plasmas which might be found in AGN, we take the photon injection function \( S(x) \) to be a black body with temperature \( \theta_{inj} = kT_{inj}/m_e c^2 \sim 10^{-5} \), the sort of spectrum that might be radiated by a black hole accretion disc. The pair injection function \( Q(y) \) is taken to be either mono-energetic at some injection energy \( \gamma_{min} \times 10^{-3} \) [i.e., \( Q(y) \propto \delta(y - \gamma_{min}) \)], or a power law of index \( \Gamma \) [i.e., \( Q(y) \propto y^{-\Gamma} \)] extending from some \( \gamma_{min} \) to \( \gamma_{max} \sim 10^{-5} \) as might be produced in shock particle acceleration. The time behaviour of the injection functions considered here will be mostly discontinuous, i.e., changes in particle injection are impulsive — the case studied by FBGPC.

Having specified the form of the injection functions, their normalizations remain to be specified. As discussed in FBGPC, this is most conveniently done by specifying the dimensionless 'compactness parameters' (luminosities) \( L_s \) and \( L_e \), where

\[
L_s = \frac{L_s \sigma_T c}{R m_e c^2} = \frac{4 \pi R^2 \sigma_T}{3 c} \int xS(x) \, dx,
\]

(2.5)

and

\[
L_e = \frac{L_e \sigma_T c}{R m_e c^2} = \frac{8 \pi R^2 \sigma_T}{3 c} \int yQ(y) \, dy.
\]

(2.6)

Here, \( R \) is the characteristic size of the interaction region (thought to be \( \sim 10^{14} - 10^{15} \) cm in AGN),

\[
L_s = \frac{4 \pi}{3} R^2 m_e c^2 \int_0^\infty xS(x) \, dx
\]

is the injected (soft) photon luminosity, and

\[
L_e = \frac{8 \pi}{3} R^2 m_e c^2 \int_1^{\infty} \gamma Q(y) \, dy
\]

is the total injected pair luminosity. Note that these definitions do not match exactly the ones given in either FBGPC or LZ. Like LZ (and unlike FBGPC), we take the interaction region to be spherical with radius \( R \), hence the overall factors of \( 4 \pi/3 \) in the definitions. Unlike LZ, however, we inject pairs, not just electrons. An extra factor of 2 is thus required in our definition of \( L_s \), and the factor of \( (y - 1) \) in the LZ definition (equation 2a) must be replaced by \( y \), since the rest mass of the injected pairs can be effectively transferred to the radiation field via pair annihilation.

A convenient way of normalizing output spectra is similarly given by using the escaping luminosity compactness parameter

\[
l(x) = \frac{4 \pi}{3} R^2 \alpha_e x n_{esc}(x).
\]

(2.7)

When the computed models reach an equilibrium (stationary) state, the total injected energy flux must equal the total

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escaping energy flux. We thus have
\[ l_{\text{tot}} = \int_{x_{\text{min}}}^{x_{\text{max}}} l(x) \, dx = L_e + L_s. \]
The quantity \( x l(x) \) plotted in the figures is a dimensionless
escaping luminosity per logarithmic energy interval, directly
proportional to the more usual \( \nu F(\nu) \), the flux per logarith-
mic frequency interval. For use in the discussion which
follows, we also define the following diagnostic quantities.

(i) The optical depth to Thomson scattering,
\[ \tau_T = 2N_t \sigma_T R, \]
where \( N_t \) is the number density of thermal pairs and \( R \)
is the source radius.
(ii) An analogous optical depth for Compton scattering
off non-thermal pairs,
\[ \tau_C = 2R \tau_T \int_{\gamma_{\text{min}}}^{\gamma_{\text{max}} } N(\gamma) \, d\gamma. \]
(iii) The equilibrium temperature of the thermal pair
distribution,
\[ \theta = kT_{\text{pairs}} / m_e c^2. \]
(iv) The ‘optical depth’ to photon–photon pair produc-
tion
\[ \tau_{\gamma\gamma}(x) = \frac{R}{c} \int_{x_{\text{min}}}^{x_{\text{max}}} n(x) R_{\text{pp}}(x, x_1) \, dx, \]
where \( R_{\text{pp}}(x, x_1) \) is the pair production rate for photons of
energy \( x \) annihilating with photons of energy \( x_1 \).

2.2 Numerical approach
The kinetic equations are solved using a simple first-order
difference scheme. We have found this adequate except when
dealing with synchrotron reabsorption and the evolution of
the pair distribution. For this we have used a second-order
monotonicity scheme with upwind differencing (see Norman
& Winkler 1986). The particle distributions are discretized,
with a resolution of 20 bins per decade of energy. The
photon bins span 12 decades of energy, from \( x_{\text{min}} = 10^{-9} \)
to \( x_{\text{max}} = 10^3 \), while the pair bins span three decades of energy,
from \( \gamma_{\text{min}} = 1 \) to \( \gamma_{\text{max}} = 10^3 \). The first pair bin contains
the thermal pairs with a variable temperature \( \theta \). The number
\( \langle N_{\gamma} \rangle \) and total energy \( \langle E_{\gamma} \rangle \) of the pairs in this thermal bin are
explicitly followed, with the temperature being derived from
these quantities. Since processes other than Compton
scattering can exchange energy with thermal pairs, this
temperature need not be the ‘Compton’ temperature (e.g., as
assumed in LZ) at which Compton heating balances
Compton cooling. A useful approximation for \( \theta \) in terms of
\( u = E_{\gamma}/N_{\gamma} \) is
\[ \theta = \frac{4}{45} \left[ \frac{15u - 27}{8} + \left( \frac{15u - 27}{64} \right)^2 \frac{45}{2} \frac{1}{1 - u} \right]^{1/2}. \]
The approximation becomes exact in the limits \( u \to 1, \infty \) and
is accurate to 5 per cent.

To allow easy vectorization and obtain reasonable execu-
tion times, we have pre-computed and stored in tables as
many quantities as possible. Memory constraints required
the use of approximate final particle energy distributions for
the processes of pair production and annihilation. These
‘two-moment’ approximations have been discussed exten-
sively in Paper I. For less complicated runs (e.g., those shown
in Fig. 1), the exact scattered particle distribution for the
process of non-thermal Compton scattering has been used.
Unless explicitly stated, though, it should be assumed that a
two-moment approximation for Compton scattering was also
used.

The ‘stiffness’ problem referred to in the Introduction
occurs mainly in the evolution of the pair distribution. (High-
energy pairs typically cool on time-scales much shorter than
any other time-scale in the problem.) The stiffness is handled
by making the ‘stationary’ approximation used in FBGFC
and LZ, i.e., assuming that
\[ N(\gamma) \approx \frac{\int_{\gamma'}^{\gamma_{\text{max}}} [Q(\gamma') + P(\gamma') \gamma' d\gamma'}{\gamma(\gamma)}, \]
where \( Q(\gamma) \) is the rate of injection of pairs of energy \( \gamma \) by
the external source, \( P(\gamma) \) is the rate at which pairs of energy \( \gamma \)
are created via pair production, and \( \gamma \) is the cooling rate for
pairs at energy \( \gamma \) due to Compton scattering, Coulomb
cooling and synchrotron radiation. Note that for this
approximation to be valid at energy \( \gamma \), not only must the
cooling time \( \tau(\gamma) \) be significantly shorter than any other
time-scale (e.g., the time-scale for the photon distribution to
time). But the evolution of the pair distribution at the
energy must be well described by the equation
\[ \frac{dN}{dt}(\gamma) = \frac{d}{d\gamma} \left[ \gamma N(\gamma) \right] + P(\gamma) + Q(\gamma). \]
This will not be the case for energies \( \gamma \) where synchrotron
self-absorption effects are non-negligible, or where \( \Delta \gamma / \gamma \),
the fractional change in energy after one Compton scattering,
is not much less than unity (e.g., when most scatterings are in
the Klein–Nishina regime). As pointed out in Blumenthal
(1971) and Zdziarski (1988), errors of order unity can be
incurred if the stationary approximation is used in such
cases. Our code contains explicit checks for these conditions
and switches the approximation off when its use becomes
inappropriate.

If one is not interested in the time evolution of the system,
then one can avoid the stiffness altogether by solving directly
for the stationary particle distributions. For use in problems not
involving synchrotron self-absorption, we have modified one
version of our code to converge on the stationary solution
iteratively. The method is similar to that described in LZ
and uses the fact that the stationary equation for a bin \( N_i \) has
the general form
\[ 0 = -f(N_i) + g(N_i, N_{i+1}) + h(N_{i-1}) + Q(i), \]
where \( f, g, h \) are (usually algebraic) functions describing,
respectively, the rate at which particles are scattered/annihilated
out of bin \( i \), the rate at which particles in bin \( i \) are
scattered back into bin \( i \), and the rate at which particles in
other bins are scattered into bin \( i \). \( Q(i) \) represents the rate at which particles are created/injected into bin \( i \). Taking the bins \( i \neq j \) \( (N_{i,j}) \) to be fixed, one can then solve for \( N_i \). Inserting the newly found bin values back into the kinetic equations and solving again, one obtains a mapping \( N_i \rightarrow N_i^{+1} \) which, when repeated, converges to the stationary value of \( N_i \). For example, when only Compton scattering is being considered, this is equivalent to adding up the `orders’ of Compton scattering. The method appears quite stable provided the various bin values are solved for in the right order. Our code first does one iteration on the photon bins assuming the pair distribution is fixed. The code then iterates repeatedly on the pair bins until the pair distribution is in equilibrium with the photon distribution. The entire process is repeated until the fractional change \( (|N_i^{+1} - N_i^0|/N_i^0) \) in all photon bins is less than some specified amount. We note that, for synchrotron self-absorption, the functions \( f \), \( g \), and \( h \) contain derivatives of the pair distribution. This leads to numerical instabilities in our simple implementation of the method described.

3 UNMAGNETIZED EQUILIBRIUM PLASMAS

There have been extensive discussions of equilibrium (stationary) pair plasmas with no magnetic fields in past work (e.g., FBGPC; Svensson 1986, 1987; LZ; Ghisellini 1987a, b). To ascertain the impact of the improved microphysics employed here, we have repeated these calculations over a wide range of compactness parameters, \( 0.01 \leq l_e \leq 1000 \). The results obtained are qualitatively similar and do not alter the most important conclusions of previous work. (Hence we will not repeat those discussions here.) The quantitative differences, however, are not so negligible (e.g., see Fig. 1) and are ascribable to the differences in the microphysics which we now discuss.

3.1 Compton scattering by thermal pairs

As discussed in Paper I, the inclusion of dispersion in the scattered photon distribution is necessary to generate modestly accurate spectra. The scheme employed here has been checked against published Monte Carlo simulations (e.g., Górecki & Wilezowski 1984) for \( \theta = kT_{\text{gas}}/m_e c^2 \) up to \( r \approx 10 \) and agrees reasonably well. The errors are dominated by the uncertainties in the radiative transfer. The LZ escape probability employed here attempts to mock up a spherical source with soft photon injection that varies radially as \( \sin(kr)/kr \) (see equation 8 of Sunyaev & Titarchuk 1980; \( r \) is distance from the centre of the source, and \( k = 2\pi N_T \sigma_T / c^2 \)). In practice, however, the approximation does not quite succeed and the results obtained generally fall somewhere

![Figure 1. Comparison of results with those of the Lightman & Zdziarski (1987) code.](image-url)
between those for a sphere with central injection and those for a sphere with uniform injection. When the diffusion approximation holds \((\tau_r \gg 1, \theta \ll 1)\) and the exact escape probability is known (e.g., Sunyaev & Titarchuk 1980), the results agree to better than \(\pm 20\) per cent, except at the highest scattered energies where the spectrum is underestimated (see discussion in Paper I). Our results appear consistent with an alternative diffusive scheme employed by LZ.

### 3.2 Compton scattering by non-thermal pairs

The present code also improves upon its predecessors in its treatment of non-thermal Comptonization. The effects of the various approximations employed in past work (e.g., not using the Klein–Nishina cross-section, and ignoring ‘dispersion’) have been discussed in Paper I. These effects are most noticeable in ‘photon-starved’ \((l_\gamma \ll l_s)\) models where a photon is likely to undergo multiple scatterings off non-thermal pairs. We note that the approximations made can significantly affect both the photon and pair distributions. In particular, the use of the cooling approximation,

\[
N(\gamma) = -\int \frac{Q(\gamma)}{\gamma} \, \mathrm{d}y, \tag{3.1}
\]

in the photon-starved regime can give poor results (disregard the order unity).

### 3.3 Pair annihilation

In models without direct pair escape, pair annihilation is the process which allows the pair distribution to reach equilibrium, converting pairs into photons which may then escape the source. An accurate treatment of pair annihilation is therefore crucial to determining the equilibrium pair distribution and thus the final output photon spectrum. While dealing adequately with the annihilation of thermal pairs, previous treatments (e.g., FBGPC; LZ) have ignored the possibility of non-thermal pairs annihilating with themselves or with the thermal distribution. When Compton cooling is fast (e.g., large \(l_s\)), the injected pairs are brought into the thermal distribution in a time much shorter than the timescale to annihilate \((\sim RT/c\tau_r)\), and non-thermal pair annihilation is indeed unimportant. However, in a photon-starved model (small \(l_s\)), the Compton cooling times at low \(\gamma\) can be quite long, causing pairs to ‘pile up’ there. Including non-thermal pair annihilation in this case can alter the equilibrium photon spectrum significantly. A crude parameter for gauging the importance of non-thermal pair annihilation is given by the value of \(\tau_n/(l_\gamma + l_s)\). [When \(\tau_n/(l_\gamma + l_s) \ll 1\), non-thermal pair annihilation is usually ignorable.]

Fig. 2 shows an example of what happens when \(\tau_n/(l_\gamma + l_s)\) is not small. When non-thermal pair annihilation is turned off, the result is a visibly different output spectrum. The entire spectrum, not just the region around the annihilation line, is affected. To gauge the importance of non-thermal annihilation, note that, in the case without Coulomb cooling, well over half the total annihilation luminosity comes from non-thermal pairs annihilating with either thermal pairs or other non-thermal pairs. Neglecting non-thermal pair annihilation can therefore be a dangerous assumption. Note, however, that the importance of non-thermal annihilation can be considerably lessened by the presence of a significant Coulomb cooling rate (see below). One should also remember that the usual estimate for \(\tau_r\) obtained by balancing the total pair injection rate against the thermal pair annihilation rate breaks down when non-thermal annihilation is important. Significant numbers of cooling pairs may annihilate before joining the thermal distribution. Mention should also be made of the treatment of the ‘annihilation line’ at \(x \approx 1\). As in Fig. 2, it may be quite broad and also asymmetric (if the contribution from non-thermal pairs is important). Approximating it by a delta-function (e.g., LZ; FBGPC) is often not justified. (In our code, we generate the thermal annihilation spectrum using the approximation of Svensson (1983), which appears to be quite good.)

### 3.4 Pair production

As is evident from the series of models shown in Fig. 1, some of the most visible disagreement with previous results (in particular those of LZ) arises when the process of pair production becomes significant. As explained in Paper I, the approximation used in Svensson (1987) and LZ can overestimate \(\tau_p\) by up to factors of order 2. Also, the fact that it includes no dispersion may lead to significant errors in the shape of \(P(\gamma)\), the energy distribution of ‘created’ pairs. These two effects can lead to a serious underestimate of the \(\gamma\)-ray spectrum and a distortion of the X-ray spectrum (since the distribution of the non-thermal pairs which upscatter soft photons depends on \(P(\gamma)\)). Note, however, that when the plasma is pair dominated \((\tau_{\gamma\gamma} \gg 1\) for \(x \approx 1\), \(P(\gamma) \gg Q(\gamma))\), the pair distribution and the photon distribution for \(x < 1\) become relatively insensitive to even large discrepancies in \(\tau_{\gamma\gamma}\) (To first order, the equilibrium high-energy photon distribution simply readjusts itself to keep everything else the same.) Hence the surprisingly good – typically much better than a factor 2 – agreement with the results of LZ in quantities such as \(\tau_r\) and the pair yield (e.g., Table 1).

### 3.5 Coulomb cooling

In the present version of the code, we make use of the treatment of Coulomb scattering presented in Paper I, i.e., we simplify matters somewhat by considering only Coulomb collisions between non-thermal and thermal pairs (a well-justified approximation as long as \((\int \mathrm{d}y' N(\gamma') N_\gamma) \ll \tau_r\) for \(\gamma \approx 10 - \text{usually the case in models considered here} and \text{treatment of the effects of these collisions as a cooling term,} \]

\[
\gamma_{\text{cool}}(\gamma) \propto 2RT \ln \Lambda \tau_r, \tag{3.1}
\]

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Figure 2. Effects of including Coulomb cooling and non-thermal pair annihilation in a photon-starved ($l_i \ll l_e$) model. The pair distribution shown in (b) is the sum of the thermal and non-thermal components.

\[ \dot{\gamma}_{\text{other}} = \dot{\gamma}_{\text{comp}} \sim \frac{c}{\pi R} \gamma^2 (l_i + l_e). \quad (3.2) \]

Coulomb cooling should therefore not be neglected for

\[ \gamma < \gamma_c \left( \frac{3\pi}{2} \ln \Lambda \frac{\tau_i}{(l_i + l_e)} \right)^{1/2}. \quad (3.3) \]

(In equation 3.3, $\ln \Lambda$ is the usual Coulomb logarithm, typically $\sim 15$--25 for the models considered here.)

As an example of where Coulomb cooling has non-negligible effects, consider the model of Fig. 2 (for which the test described above gives a $\gamma_c = 50$). Turning on Coulomb cooling has two main consequences. First, Coulomb cooling allows injected pairs to transfer a non-negligible fraction of their injected energy directly to the thermal pairs (instead of losing all of it via Compton scattering to the radiation field). This tends to raise the equilibrium temperature of the thermal pair distribution (increasing it by a factor 5 in the case of Fig. 2). Coulomb cooling also prevents the accumulation of pairs near $\gamma = 1$ that would otherwise occur when $\dot{\gamma}_{\text{comp}}$ becomes small, sweeping the pairs into the thermal distribution (see Fig. 2b). As a side effect, this additional cooling, just like a large Compton cooling rate, prevents a significant amount of annihilation by cooling pairs while they are in their non-thermal state. To summarize, then, even though Coulomb scattering conserves total pair number and energy, the pair distributions calculated with and without Coulomb cooling can be quite different in the region near $\gamma = 1$. As seen in Fig. 2(a), this difference is also reflected in the equilibrium photon distributions, especially in the region $\sim 1$. Coulomb cooling should not be ignored in models with $l_i/l_e \gg 1$, $l_i \leq l_e$.

### 3.6 Bremsstrahlung

Our code is also capable of (crudely) dealing with e-e bremsstrahlung, another process not considered in previous treatments which could have a significant impact. (See Paper I for a description of the method employed.) In models of the type considered here, bremsstrahlung serves two functions: (i) as an additional source of photons; (ii) as another pair cooling mechanism. Unlike Coulomb scattering, however, e-e bremsstrahlung really is a 'higher order' process and for most choices of parameters may indeed be ignored. This is true of the models presented in the figures. It is not true,
Table 1. Comparison with models of Lightman and Zdziarski (1987).

\[
\begin{array}{ccccccc}
\ell_p & \frac{P_Y}{Y} & \tau_p & \tau_C & \theta & \sigma_{2-10} \\
1 & 1.7 \times 10^{-3} & 0.047 & 1000 & 1.1 \times 10^{-3} & 7.9 \times 10^{-3} & 0.637 \\
L & 1.6 \times 10^{-3} & 0.062 & 1000 & 1.0 \times 10^{-3} & 1.0 \times 10^{-2} & 0.655 \\
10 & 0.023 & 0.02 & 63.2 & 7.0 \times 10^{-3} & 5.7 \times 10^{-3} & 0.863 \\
L & 0.025 & 0.025 & 38.9 & 6.9 \times 10^{-3} & 6.0 \times 10^{-3} & 1.03 \\
100 & 0.057 & 0.33 & 4.5 & 1.1 \times 10^{-2} & 2.2 \times 10^{-3} & 0.979 \\
L & 0.099 & 0.35 & 3.7 & 1.1 \times 10^{-2} & 2.5 \times 10^{-3} & 1.14 \\
1000 & 1.2 & 1.24 & 1.4 & 6.2 \times 10^{-3} & 6.2 \times 10^{-4} & 1.42 \\
L & 0.14 & 1.34 & 0.7 & 5.1 \times 10^{-3} & 5.3 \times 10^{-4} & 1.61 \\
\end{array}
\]

All models were computed assuming mono-energetic pair injection at \( \gamma_{inj} = 10^3 \) and blackbody soft photon injection at \( \theta_{inj} = 1.07 \times 10^{-1} \). The exact energy distribution of Compton scattered photons was used in our calculations. Key to symbols: \( P_Y \) is the 'pair yield' \( \int dy P(y)/\int y Q(y) dy \) (e.g., see LZ). \( P(y) \) is the creation rate of pairs of energy \( y \) by two-photon pair production; \( Q(y) \) is the external pair injection rate. \( \alpha^* \) is the photon energy \( x \) where \( \tau_p(x) = 1 \). \( \alpha_{2-10} \) is the best-fitting spectral index \( (F \propto \nu^{-\alpha}) \) to the 2–10 keV X-ray band. \( \ell_p, \tau_p, \tau_C, \) and \( \theta \) are as defined in the text (see Section 2.1).

However, for extremely photon-starved \( (\ell_p \ll 1) \) models, where the thermal pairs may reach a relativistic temperature \( \theta \sim \Gamma \). This case will be examined in more detail elsewhere (see Zdziarski, Coppi & Lamb 1990). We note that the contribution to the overall bremsstrahlung spectrum from non-thermal pairs can be quite significant. Like the contribution of non-thermal pairs to the annihilation spectrum, it should not be ignored.

4 STATIONARY MODELS WITH MAGNETIC FIELDS

We now consider the effects of adding a magnetic field to our model of a pair plasma. As discussed in Section 2, our treatment of synchrotron radiation makes the assumptions of isotropic pitch angle and radiated photon distributions. This requires that the magnetic fields under consideration be relatively weak and tangled. In view of this, we shall restrict our attention here to field strengths \( B_0 \leq 1000 \) G, values thought to be typical of what is found in AGN central engines. For such relatively low field strengths, synchrotron radiation represents another source of soft (IR–UV) photons which can replace or augment the external soft photon source. Thus, since the cooling rate for energetic \( (\gamma \gg 1) \) pairs Compton scattering off a soft photon field has...
(to first order) the same energy dependence as the cooling rate due to synchrotron emission, the spectra produced in such models will not differ grossly from those seen in 'unmagnetized' models. [This is in contrast to some of the cases examined by Ghisellini (1987a, b), where the maximum synchrotron photon energy $\gamma_{\text{max}}$ may exceed unity, and synchrotron radiation provides a source of hard photons which may pair-produce immediately, without intermediate upscatterings.]

It is easiest to begin by examining models with only pair injection and no external photon source (i.e., models with $l_{\gamma} = 0$), as such models contain most of the new physics and are easier to analyse. Note that in such models, the equilibrium radiation field is completely determined by the pair injection function $Q(\gamma)$, the source region radius $R$, and the magnetic field strength $B_{0}$. (Because of the inclusion of synchrotron self-absorption, $R$ does not scale out of the problem as it does in 'unmagnetized' models.) To help understand the spectra characteristic of such models, we consider two limiting cases: (i) the 'synchrotron-dominated' regime where the Compton upscattering of photons can be treated as a perturbation; and (ii) the 'Compton-dominated' regime where synchrotron effects may instead be treated as a perturbation. These two regimes may be distinguished through the

use of the dimensionless energy 'densities'

$$\tilde{U}_{\gamma} = \frac{R\alpha_{\gamma}}{m_{e}c^{3}} U_{\gamma}, \quad \tilde{U}_{\text{rad}} = \frac{R\alpha_{\gamma}}{m_{e}c^{3}} U_{\text{rad}}.$$  (4.1)

Here $U_{\gamma} = B_{0}^{2}/8\pi$ is the magnetic energy density, and $U_{\text{rad}} = m_{e}c^{2} \int x n(x) \, dx$ is the radiation energy density. Expressed in terms of these quantities, the cooling rate for pairs at energies high enough that synchrotron self-absorption may be neglected is given by

$$\dot{\gamma} = \frac{4c}{3R} \gamma^{2} \tilde{U}_{\gamma} (1 + \tilde{U}_{\text{rad}}/\tilde{U}_{\gamma}).$$  (4.2)

Thus, for $\tilde{U}_{\text{rad}}/\tilde{U}_{\gamma} \gg 1$, Compton scattering dominates the pair cooling, while for $\tilde{U}_{\text{rad}}/\tilde{U}_{\gamma} \ll 1$, synchrotron emission dominates. At kinetic equilibrium, energy conservation demands $\tilde{U}_{\text{rad}} \sim \gamma$, and we thus see that 'Compton-dominated' models correspond to models with input parameters such that $l_{\gamma}/\tilde{U}_{\gamma} \gg 1$, while 'synchrotron-dominated' models correspond to models with $l_{\gamma}/\tilde{U}_{\gamma} \ll 1$. Examples of the spectra produced in these limiting cases (as well as for some intermediate values of $l_{\gamma}/\tilde{U}_{\gamma}$) can be seen in Figs 3–7.

---

**Figure 3.** A 'Compton-Dominated' ($l_{\gamma} \gg U_{\gamma}$) model and its corresponding unmagnetized model. Note the similarity of the two spectra.
4.1 ‘Compton-dominated’ models

For models with $l_s \gg \dot{U}_B$, Compton cooling dominates synchrotron cooling (and heating via reabsorption) for all pair energies $\gamma$. Thus, the shape of the equilibrium pair distribution is quite similar to that seen in the unmagnetized models of the previous section (where the pair distribution is well described by a power law at most energies). One would therefore expect to see an upscattered photon distribution that also resembles those obtained in the unmagnetized case. In fact, for $x \gtrsim x_{\text{max}} \sim (\gamma_{\text{max}} \sim 10^4) B/c$, the output spectrum is very close to that produced in an unmagnetized model with the same $l_s$ and $l_e = l_{\text{synch}}$, e.g., see Fig. 3. Here, $l_{\text{synch}}$ is a synchrotron ‘compactness parameter’ analogous to $l_e$, i.e.,

$$l_{\text{synch}} = \frac{4 \pi R^2 \tau_T}{3c} \int x n_{\text{synch}}(x) \, dx,$$

where $n_{\text{synch}}(x)$ is the rate of synchrotron emission per unit volume of photons of energy $x$. A crude estimate shows that (to within factors of order unity) $l_{\text{synch}} \sim \dot{U}_B$. Compton-dominated models thus correspond to unmagnetized models with $l_s \ll l_e$, and once $l_{\text{synch}}$ has been determined accurately, the analysis used to understand the unmagnetized case carries through essentially unchanged to the Compton-dominated case. Note an important characteristic of such ($l_s \ll l_e$, $\dot{U}_B \ll \dot{U}_F$) models, namely the flat X-ray spectrum ($\alpha_x \simeq 0.5$ with flux $F_x \propto \nu^{-\alpha_x}$) composed of several ‘orders’ of Compton scattering. (For a discussion of such spectra, see the case where $\tau_c \sim 1$ in LZ and Ghisellini 1987b.) This is contrary to Ghisellini (1989) – to what one might expect from the analysis of Bonometto & Rees (1971), which would predict a spectral index $\sim 1$ (e.g., as in the pair-dominated cases in Fig. 1). Unfortunately, since several orders of Compton scattering contribute to the photon spectrum, Klein–Nishina corrections cannot be neglected and the analysis of such models is somewhat complicated. (Hence, constructing a simple and accurate estimate for $l_{\text{synch}}$ is difficult.)

The differences between Compton-dominated and unmagnetized spectra reflect the different origins of the soft photons in the models (non-thermal versus assumed thermal). As the synchrotron spectrum radiated by power-law electrons does not much resemble a Planck distribution,
the agreement between the models is not very good for energies $x < x_{\text{max}}$. In particular, the photon distribution due to synchrotron radiation is typically much broader than the corresponding blackbody distribution, a fact which is also reflected in the upscattered photon distributions. (In general, when dealing with synchrotron emission, the finite width of the emitted photon distribution can never be neglected.) Note also that the synchrotron portion of a spectrum in the Compton-dominated case does not look much like the `conventional' spectrum obtained from radiating power-law electrons. The effects of thermal upscattering and multiple (non-thermal) Compton scattering (important in the $l_\gamma, l_{\text{synch}} \ll l_e$ regimes) can significantly modify the part of the spectrum due to optically thin synchrotron emission, i.e., the result will not be a simple power law of index $(p-1)/2$ where $p$ is the power-law index of the pairs. Also, while the strongly self-absorbed part of the spectrum is well described by the usual power law of index 2.5, there is typically no sharp self-absorption `break' (turnover) marking the change-over from optically thin to optically thick synchrotron emission (it is smeared away).

### 4.2 'Synchrotron-dominated' models

In contrast to the Compton-dominated case, a synchrotron-dominated model cannot be conveniently related to an unmagnetized model with similar equilibrium pair and photon distributions. The reasons for this can be found by examining the (now non-negligible) effects of synchrotron emission/reabsorption on the equilibrium pair distribution. The most important of these is the `suppression' of the non-thermal pair distribution. At energies high enough that self-absorption effects can be neglected, the addition of a large synchrotron cooling rate to the usual Compton cooling rate means that $N(\gamma)$ at such energies is always smaller in the synchrotron-dominated model than in a pure Compton (unmagnetized) model with $l_\gamma = l_{\text{synch}}$ and the same $l_e$. This suppression has several consequences. With fewer non-thermal pairs to upscatter soft photons, the high-energy (X- and $\gamma$-ray) end of the photon distribution is correspondingly suppressed. (Note for example Figs 4 and 5, where more and more of the emitted power is shifted to the synchrotron emission peak as $U_B/l_e$ increases.) With fewer high-
energy photons present, pair production is also suppressed, and the pair yields obtained in synchrotron-dominated models can be much lower than would be expected from the corresponding pure Compton models. Synchrotron-dominated models therefore tend to have low values of \( \tau_T \) and weak, if not invisible, annihilation lines. The suppression of the non-thermal pair distribution also means that typical values of \( \tau_C \) (the optical depth to non-thermal Compton scattering) are much less than unity and secondary Compton scattering is consequently unimportant. Thus, unlike the Compton-dominated case, the part of the soft photon distribution due to optically thin synchrotron emission is well described by the conventional power law of index \((p-1)/2\). [For these models, \( p = \Gamma + 1 \) where the pair injection function \( Q(\gamma) \propto \gamma^{-\Gamma} \).] To first order, the upscattered photon distribution is also described by a power law of the same index. Note, however, that the non-negligible width of the soft (synchrotron) photon distribution often causes the upscattered photon distribution to deviate from the predicted power law well before the maximum upscattered frequency (\( \sim 4/3 \gamma_{\text{max}}^2 \)) is reached. Examples of the spectral shapes that may be produced by a synchrotron-dominated model are found in Figs 4–6. Note the effect of varying \( \Gamma \) in Fig. 6.

At the low-energy end of the pair distribution, synchrotron self-absorption effects become important and the deviations from what might be expected in the pure Compton case are even more marked. As noted by Rees (1967a) and McCray (1969), a power-law distribution is in general not a stable equilibrium solution when the process of synchrotron radiation dominates the evolution of the pair distribution. In particular, there will be some energy, \( \gamma_e \), for which heating from the reabsorption of photons becomes important, and for \( \gamma < \gamma_e \) this heating will actually dominate (slightly) over the cooling (e.g., see de Kool et al. 1989). The energy \( \gamma_e \) is of order \((3x_0/4x_0)^{1/2} \), where \( x_0 = B/B_\odot, x_0 = h\nu/m_ec^2 \), and \( \nu_e \) is the frequency where self-absorption becomes important—see below. This causes the formation of a Maxwellian-like distribution (e.g., see Fig. 5b) at energies lower than \( \gamma - \gamma_e \) and is referred to as the 'synchrotron boiler' effect in Ghisellini et al. (1988).

As can be seen in Fig. 5(b), however, Compton cooling off the synchrotron photons suppresses this tendency of low-energy pairs to form a Maxwellian. As \( U_{\text{rad}}/U_\beta \) increases, the peak of the Maxwellian-like shape gets pushed to lower and lower energies. In Fig. 5(b), not much of a Maxwellian remains by \( \beta = 10 \). Consequently, we would argue that the synchrotron boiler is not a widely applicable method for
creating thermal distributions with \( \theta \geq 1 \). Rather, it will only be of interest in the limited parameter space \( L_s \leq 1 \). Note also in Fig. 5(b) that Coulomb cooling tends to destroy the Maxwellian shape, primarily at the low-energy end. One might not expect this, since \( \tau_\gamma \) in these models is quite small, and the corresponding Coulomb cooling rates are also small. However, as noted above, for \( \eta < \gamma_i \), heating from reabsorption almost exactly cancels the cooling from emission. A small additional cooling rate (Compton or Coulomb) can therefore have a very important effect. As a final observation, note the ‘disjointed’ power-law shape taken on by the pair distribution as \( U_{\text{rad}} \) increases. This appears to be in accord with the predictions of de Kool et al. (1989).

The possibly large deviation from power-law behaviour at energies \( \gamma < \gamma_i \) is also reflected in the photon distribution. An ‘excess’ of pairs at these energies should produce a corresponding excess of (upscattered) photons at energies \( x_{\gamma, \text{max}} \leq x \leq 4/3 \gamma_i^2 x_{\gamma, \text{max}} \). This can be clearly seen in Fig. 5(a). A non power-law pair distribution at energies \( \gamma < \gamma_i \) (i.e., for which self-absorption is important) will also give a strongly self-absorbed part of the photon spectrum that has a spectral index different from the usual value of 2.5 obtained with a power-law pair distribution. In fact, as \( \tilde{U}_p \) becomes much greater than \( L_s \) and the pair distribution becomes more and more a Maxwellian, the spectral index (for frequencies \( \eta < \nu_i \)) approaches a limiting value of 2.0 (\( F_\nu \propto \nu^2 \)), the value for a strongly self-absorbed photon distribution in equilibrium with a thermal pair distribution. The spectral index thus varies continuously with \( \tilde{U}_p/L_s \), going from 2.5 (\( \tilde{U}_p \ll L_s \)) to 2.0 (\( \tilde{U}_p \gg L_s \)). The detailed dependence on \( \tilde{U}_p/L_s \) appears to be quite complicated, and we have not found an easy way of extracting the effects of Compton scattering from those of synchrotron emission/absorption. We remind the reader that discontinuities or abrupt changes in slope in the pair distribution (e.g., as seen in Fig. 5b) may give rise to spectral indices (over a limited range of energies) that exceed 2.5. See de Kool & Begelman (1989) for some detailed examples of this.

In contrast to Compton-dominated models, synchrotron-dominated models tend to show a fairly sharp break in their spectra at \( \nu_i \), the frequency at which \( R \alpha_s \sim 1 \) and the synchrotron absorption of photons becomes important (\( \alpha _s \) is the synchrotron absorption coefficient). One can often make a simple estimate for this ‘turnover’ frequency, \( \nu_i \). In the

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Figure 7. Model with both external photon injection and a magnetic field. Note the variation of the near-infrared slope with the minimum pair injection energy, $\gamma_{\text{min}}$.

The simplest case of mono-energetic pair injection, taking $N(\gamma \geq \gamma) = \int Q(\gamma) \, d\gamma/\dot{\gamma}$ (with $\dot{\gamma}$ as given in equation 4.2) and using the approximation for the absorption coefficient discussed in McCray (1969), we have

$$\nu_s \approx 3.2 \times 10^{13} \left( \frac{U_\gamma U_{\text{rad}}}{U_\beta + U_{\text{rad}}} \right)^{1/3} \gamma_{\text{min}}^{-1/3} R_4^{1/3} \text{Hz},$$  \hfill (4.3)$$

where again $U_{\text{rad}} \approx (3/4 \pi) I_\nu$. This estimate should be valid even in the transition regime $l_\gamma \sim U_\beta$ and appears to work reasonably well, as can be seen by examining the spectra shown in Figs 4–7. Note the form this expression takes when $U_{\text{rad}}/U_\beta$ is very small and $U_{\text{rad}}$ and $U_\beta$ are used instead of $U_{\text{rad}}$ and $U_\gamma$:

$$\nu_s \approx 3 \times 10^{13} U_{\text{rad}}^{1/3} \gamma_{\text{min}}^{1/3},$$  \hfill (4.4)$$

where $U_{\text{rad}}$ is measured in units of $10^4$ erg cm$^{-3}$. The magnetic field and radius of the source drop out completely (an interesting result if $U_{\text{rad}}$ scales as $M$, the mass of the central black hole). The important observation to draw from these expressions (and the models shown) is that the turnover frequency is relatively insensitive to the input model parameters. Thus, if the models discussed prove relevant to real AGN, one might expect to find a turnover always in the neighbourhood of $\sim 10^{13}$ Hz.

4.3 Models with external soft photon injection

Nothing very remarkable happens when an external photon source is added (e.g., as in Figs 7, 9a and 10a). Although the detailed variation of equilibrium spectra with the injection parameter $I_\nu$ is hard to estimate analytically, the limiting case of $I_\nu \gg U_\beta$ is clear. In this limit, $I_\nu$ is also $I_{\gamma(\text{synch})}$ and pairs cool primarily by Compton scattering off the soft externally injected photons rather than by synchrotron emission or Compton scattering off synchrotron photons. The equilibrium pair distribution is thus independent of $U_\beta$ and, for $x > x_{\text{max}}$, one obtains spectra identical to those obtained with an unmagnetized model with the same $l_\gamma$ and $l_\nu$. The synchrotron part of the spectrum ($x < x_{\text{max}}$) may then be treated as a perturbation, determined by considering the spectrum radiated by a fixed pair distribution. As above, note that although they originate from the same pair distributions, the X-ray and optically thin synchrotron portions of the
spectrum are not identical in shape. In particular, the spectral indices may be rather different.

One particular model with differing indices that may be of interest is obtained by taking $l_{\nu} \sim \nu_{\mathrm{IR}}$, $B_{\nu} \lesssim 1000$ G and injecting pairs in a steep power law of index $\Gamma \sim 2.4$. As noted in Zdziarski (1986), the resulting spectrum (e.g., Fig. 7) often resembles the canonical (radio-quiet) AGN spectrum with $\alpha_{\mathrm{IR}} \sim 1.1$, $\alpha_{\nu} \sim 0.7$, and a sharp break/turnover at $\sim 10^{17}$ Hz. (Here $\alpha_{\mathrm{IR}}$ and $\alpha_{\nu}$ are the near-infrared and X-ray spectral indices.) The ‘blue bump’ is provided by externally injected photons (e.g., from an accretion disc corona), and the infrared is synchrotron emission from pairs. Note one ‘feature’ of this model: varying the minimum injection energy $\gamma_{\mathrm{min}}$ causes the infrared spectral index to vary, and can make it less than unity) without perturbing the X-ray portion of the spectrum very much. (See for example Fig. 7 and the values of $\alpha_{\nu}$ in Table 2.) One could thus have a ‘universal’ power-law injection index $\sim 2.4$ for the pairs, and explain observed variations in $\alpha_{\mathrm{IR}}/\alpha_{\nu}$ as variations in $\gamma_{\mathrm{min}}$. Such models also need not predict a strong correlation between $\alpha_{\nu}$ and $\alpha_{\mathrm{IR}}$. The non-thermal (synchrotron) hypothesis for the origin of the infrared continuum should therefore not be dismissed out of hand.

Unfortunately, the sort of model just discussed may have several difficulties matching observations. These mainly reflect the fact that the infrared and X-ray photons originate from the same pairs (source region). Based on limited experimentation, it may not be possible to obtain the correct ratios of the luminosities in the IR, UV, and X-ray (2–10 keV) bands and still have reasonable values of $\alpha_{\mathrm{IR}}$ and $\alpha_{\nu}$. More importantly, one would expect variations in pair injection to show up both in the infrared and the X-rays, i.e., infrared and X-ray time variability should be about the same. This is typically not observed (but see the next section).

5 TIME VARIATION

In this section, we investigate the time-response of the model pair-plasma system to impulsive (step-function) changes in the particle injection rates – such as might occur in a bursting compact source. Such variations reveal most of the dynamical effects at work in a pair plasma and are the easiest to understand. (This is also the case examined in FBGCP.) We will not perform any specific spectral analyses, as in Done & Fabian (1989), since the detailed answers are likely to be quite sensitive to the model assumptions, in particular to the source geometry and the treatment of assumed radiative transfer (which determine important quantities such as the time required for a photon to escape).

5.1 Relaxation time-scales

We begin by examining the various relaxation time-scales for the particle distribution in our model. The more complex, ‘collective’ time behaviour seen in the following section may be understood as the interplay of these time-scales.

Consider first the non-thermal pair distribution. When pair injection is changed, the relevant time-scale for the pair distribution (at energy $\gamma$) to respond is the cooling time, $\tau_{\text{cool}} = \gamma/\Gamma$. Typically, this cooling time is much shorter than any other time-scale in the problem and, as a rule of thumb, the non-thermal pair distribution ‘equilibrates’ much faster.

![Table 2](image)

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<th>Fig. 2: All Processes</th>
<th>$l_{\nu}/l_{\nu}$</th>
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<th>$\tau_F$</th>
<th>$\tau_C$</th>
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The remaining column headings are as defined in Table 1. 

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than the photon distribution. (Hence the validity of the stationary approximation discussed in Section 2.2.) Exceptions to this rule may occur, however, in 'photon-starved' models with $t_e \ll \tau_T$ (Compton cooling is slow at low $\gamma$) or synchrotron-dominated models at energies where the emitted synchrotron radiation is strongly self-absorbed. When only photon injection is changed, variations in the pair distribution will be slower and occur on the time-scales for the cooling and pair production rates (i.e., the soft photon distribution) to change. Note one semantic point. After the initial rapid response to a change in pair injection, the non-thermal pair distribution is essentially 'locked' on to the more slowly varying photon distribution. Thus the time-scale for the non-thermal pair distribution to approach its final, stationary value is really the longer photon-response time-scale.

The thermal pair distribution and the corresponding annihilation line (whose luminosity is $\propto \tau_T^{-1}$) generally vary much more slowly than the non-thermal distribution and can often be the most sluggish components of the pair plasma. Consider first the response to an increase in particle injection. Exactly how long the thermal distribution takes to build up depends on the total pair injection rate (external injection + pair production). When the stationary value of $\tau_T$ predicted from the new injection parameters is less than unity (e.g., for $l_p, l \lesssim 10$), the total pair injection rate is small. The number of thermal pairs along with the annihilation line intensity can thus take many light-crossing times, $t_{\text{cross}} = R/c$, to build up to their stationary values. When, on the other hand, the production rate of pairs is large and the predicted value of $\tau_T$ exceeds unity, the number of thermal pairs can build up quite fast, on a time $t_{\text{cross}}$. In either case, note that there will always be a lag with respect to the rest of the photon spectrum (in particular the gamma-rays) as the photon distribution must first build up before pair production can increase significantly.

Consider now the response (relaxation) time when particle injection is reduced. If only (soft) photon injection is reduced, nothing significant happens until the soft photon distribution has had time to decrease, i.e., only after a time $t_{\text{ann}}$ (computed using the initial value of $\tau_T$) has passed. The thermal pairs will then approach equilibrium on the same time-scale as for the soft photon distribution ($t_{\text{ann}}$ computed using the final value of $\tau_T$). If pair injection is reduced, however, the response can be much more dramatic. Typically, the number of high-energy photons drops rapidly (see below), causing a corresponding decrease in the total pair production rate. With the number of incoming (cooled) pairs significantly reduced, the thermal pair distribution is then free to annihilate away on the time-scale $t_{\text{ann}} \sim 1/t_T$ (the time required for half the pairs to annihilate). An initial $\tau_T \gg 1$ can therefore disappear in a time $\ll R/c$. Note, though, that once $\tau_T$ drops below unity, $t_{\text{ann}}$ increases and the annihilation line becomes a persistent feature, still visible, in the case where particle injection is completely turned off, long after all other photons have escaped the source. (This behaviour is to be contrasted with that in models with pair escape. There the annihilation line and thermal pairs will be gone in a few characteristic pair escape times.)

The relaxation times of the photon distribution remain to be discussed. In the simplest case where the old and new injection parameters are such that $\tau_T$ is always less than unity, the photon distribution behaves in a uniform manner, building up or decaying at all energies on a time-scale $\sim t_{\text{cross}}$ (the escape time for photons in this case). Whenever $\tau_T$ starts out or becomes greater than unity, however, one must make a distinction between high ($x \gtrsim 1$) and low ($x \lesssim 1$) energy photons. Low-energy photons are 'trapped' in the source by the thermal pairs and respond on the longer time-scale $t_{\text{esc}} \sim (1 + \tau_T/3)/t_{\text{cross}}$ (the characteristic soft photon escape time). Note that the value of $\tau_T$ may change significantly during the evolution of the soft photon distribution. The soft photon distribution relaxes to its new stationary value after a few $t_{\text{esc}}$, computed using the final value of $\tau_T$ (see FBGPC).

Because of Klein–Nishina effects, high-energy photons are not trapped significantly by thermal pairs and in principle should always respond on the time-scale $t_{\text{cross}}$. High-energy photons, however, do feel the effects of downscattering by the thermal pairs (which effectively acts as an absorption term) as well as of annihilation off soft photons (a true absorption) and a varying injection (upscattering) rate due to changes in the non-thermal pair and soft photon distributions. This has two consequences. When soft photon injection is varied or pair injection is increased, the high-energy photon distribution, like the non-thermal pair distribution, becomes (after some initial transient behaviour) 'locked' on to the soft photon distribution and follows its evolution. The 'relevant' relaxation time-scale is then the soft photon time-scale $t_{\text{esc}}$. When pair injection is significantly decreased, however, the high-energy end of the non-thermal pair distribution drops rapidly in response, as does the corresponding upscattering rate of high-energy photons. The high-energy photon distribution can then decay away on the (often) much shorter time-scale $t_{\text{esc}}(x)$, which varies with increasing photon energy $x$. Consequently, higher energies tend to lead lower energies in responding to a decrease in pair injection.

The preceding discussion has ignored the fact that, in a 'real' source, changes in particle injection cannot take place simultaneously across the source, but will be spread out over a time $\gtrsim t_{\text{cross}}$ (e.g., imagine a shock wave propagating across the source that accelerates pairs behind it). The evolution of features on times $< t_{\text{cross}}$ seen in some of the output spectra discussed below is therefore artificial and reflects the crude radiative transfer employed here and in past treatments. For example, although locally $\tau_T$ (and the annihilation line luminosity) may drop significantly in a time much less than $t_{\text{cross}}$, this cannot happen globally in a time less than $t_{\text{cross}}$. Effects such as the sudden release of trapped photons (see FBGPC and below), which depends critically on the Thomson opacity disappearing in less than $t_{\text{cross}}$, may then be much less dramatic (or not seen at all) and may vary with the particular source geometry (i.e., the details of radiative transfer). The observed time behaviour of a source will be given by the local (since every region of the source is taken to be identical) responses of the model system considered here, smeared over a light-crossing time $R/c$. (The use of a photon escape probability with minimum escape time $t_{\text{cross}}$ does do this to some extent.) Strictly speaking, then, the results of the present code are valid only when injection changes are slow enough (i.e., take place over times $\gg t_{\text{cross}}$) that the system has time to keep itself 'homogenized'. Regardless of the exact details, however, it is clear that the presence of a pair plasma can greatly alter the observed time
variability of a source, especially when the value of $\tau_T$ exceeds unity. A pair plasma acts like a capacitor, storing up particles and energy, and smearing out the response of the system to changes in injection.

5.2 Sample results

We now turn to some specific examples. Consider first the ‘easiest’ case to understand (Fig. 8a), one in which the injection parameters are always low enough that the value of $\tau_T$ always remains much less than unity. Following the arguments above, $\tau_{\infty}$ will always be $\sim t_{\text{cross}}$, and the photon distribution, except the annihilation line, should respond to a change in injection on a time-scale $t_{\text{cross}}$. The (normalized) escaping luminosity in the various wavebands shown should then go as $l(t)\approx 1-e^{-t}$ for $0 < t < 12$ (in response to particle injection being turned on at $t=0$) and as $l(t)\approx e^{-t/\tau_{\infty}}$ for $12 < t < \infty$ (in response to injection being turned off at $t=12$). The exception is the annihilation line intensity which, because the total pair creation/injection rate is relatively small and $\tau_T < 1$, should take many $t_{\text{cross}}$ to build up and decay away. Comparing with Fig. 8(a), this does not appear a bad first approximation. (Indeed it works very well for the UV luminosity, since the only processes significantly affecting the soft photon distribution are photon escape and external photon injection.) However, the escaping $\gamma$-ray luminosity always leads the UV luminosity slightly, while the X-ray luminosity significantly lags the UV luminosity. The annihilation line intensity also continues to increase even after all particle injection is turned off. The explanation for this behaviour is more subtle than one might have at first guessed.

Consider, for example, the fast rise of the $\gamma$-ray luminosity relative to the UV luminosity. Since the 2–10 MeV gamma-rays tracked in Fig. 8(a) are primarily upscattered soft photons, the ‘injection’ rate of $\gamma$-rays goes approximately as the number of soft photons times the number of high-energy pairs. Assume now that the pair distribution is fixed. The injection of $\gamma$-rays then will not turn on until the soft photon number has had time to build up, i.e., one might expect the rise in the $\gamma$-rays to lag slightly the rise in soft (UV) photons. The pair distribution is definitely not fixed in time, however.

![Figure 8](image.png)

**Figure 8.** Luminosity response of unmagnetized models to impulsive changes in pair injection. (a) A case where the injection parameters are such that $\tau_T \ll 1$ at all times. (b) The opposite case, where the increase in particle injection causes $\tau_T$ to significantly exceed unity. The various frequency bands referred to in this and the following figures are defined as follows: (IR): 90 000–30 000 Å; (Opt): 9000–3000 Å; (UV): 900–300 Å; (X-ray): 2–10 keV; (Gamma-Ray): 2–10 MeV; and (Annihilation Line): the total annihilation luminosity, $l_{\text{ann}} = (4\pi/3)\int R_\tau \delta n(x) \, dx$, including the contribution from non-thermal pairs.
At early times ($\lesssim t_{\text{cross}}$) when $U_{\text{rad}}$ is still small, the cooling times are long and the high-energy pair distribution is significantly enhanced over its stationary value. In the example shown, this larger number of higher pairs more than compensates for the initially small number of soft photons available to be upscattered. In fact, the $\gamma$-ray luminosity actually overshoots its equilibrium value (very) slightly. The decrease in the number of high-energy pairs with increasing $U_{\text{rad}}$ eventually wins out over the partially compensating increase in soft photon number.

Given that the X-rays are also primarily upscattered soft photons, one might expect them to show the same behaviour. However, the pairs responsible for upscattering soft (UV) photons to X-ray (2–10 keV) energies are $\sim 30$ times lower in energy than those responsible for the $\gamma$-rays. Their cooling times are significantly longer ($\propto \gamma^2$), and at early times when $U_{\text{rad}}$ is small, they can be a non-negligible fraction of the photon response time-scale $\sim t_{\text{esc}}$. Consequently, the 'turn on' of the X-rays is delayed by about 0.2–0.3$t_{\text{cross}}$ until sufficient numbers of injected pairs have cooled down to build up the pair distribution at $\gamma \sim 10$. Also, although the maximum value of $\tau_\gamma$ is $\ll 1$, the effects of pair production on this medium-energy ($\gamma \sim 10–30$) range of the pair distribution cannot be neglected. The additional source of pairs represented by pair production leads to an enhancement by a factor $\sim 3$ in the number of these pairs and the upscattering rate of photons to X-ray energies. Pair production does not turn on fully, however, until the $\gamma$- and hard X-ray distributions have had time to build up, i.e., only after a few $t_{\text{esc}}$. (This is reflected in the annihilation line intensity, a useful indicator of how the total pair production rate is behaving.) This then is the main cause of the observed X-ray to UV 'lag' when injection is turned on.

The fact that pair production dominates external injection at pair energies responsible for the upscattered X-rays also explains the X-ray to UV lag when all particle injection is cut. Significant pair production ceases only when most gamma-rays have escaped the source region. For this model, where $\tau_\gamma \ll 1$ and $\tau_{\gamma\gamma} \ll 1$ for $x \lesssim 300$, this happens on a time-scale $\sim t_{\text{cross}}$. (Note, though, that the effects of pair production on the $\gamma$-rays are not completely negligible and cause the $\gamma$-ray luminosity to decrease slightly faster than the UV luminosity.) Thus, the pair distribution at these energies is maintained by pair production for a few $t_{\text{cross}}$. This, and the fact that the soft photon number decays on a time-scale also $\sim t_{\text{cross}}$, mean that the injection (upscattering) of X-rays does not stop completely until after a few $t_{\text{cross}}$. Hence the 'slow' decay of the X-ray luminosity. A significant pair production
rate after all particle injection is stopped also explains the behaviour of the annihilation line. Only when the total pair production rate drops below the annihilation rate (\(\sim \tau_P\)) does its intensity begin to decrease. Note, then, that even in a model which is neither pair-saturated nor Thomson-thick, the process of pair production can have a non-negligible effect on the time evolution.

Turn now to a case (Fig. 8b) where the injection parameters are boosted from initial values in which the pair plasma is in a moderately pair-saturated and Thomson-thick regime (at \(t = 0, L_p = 100, L_e = 250\), and the state of the plasma is the equilibrium one shown in Fig. 1) to values (\(L_p = 1000, L_e = 2500\)) which put the pair plasma into a very pair-saturated and Thomson-thick (\(\tau_T \sim 12\)) regime. (In the ‘pair-saturated’ regime, pair production is very important and the creation of pairs by pair production dominates the external injection of pairs at most energies – see Svensson 1987.) In contrast to the previous case, the ‘transient’ time behaviour is now quite dramatic. One noticeable feature is the burst of gamma-ray luminosity that occurs just after injection is increased. For \(\sim \tau_{\text{cross}}\), until increasing downscattering by thermal pairs and annihilation of lower energy photons (pair production) can catch up, the spectrum looks much harder than one might predict from the stationary states before and after the increase in injection. The rapid increase in the number of gamma-rays causes a corresponding jump in the total pair production rate. Note how rapidly (compared to Fig. 8a) the annihilation line builds up. Also note that the X-ray luminosity actually leads to UV luminosity in building up. Here there is no significant wait for pair production to turn on and all pair cooling times are \(\ll \tau_{\text{cross}}\) (hence no wait for injected pairs to cool and build up the pair distribution in response to the increase in injection).

The burst in pair production and rapid rise in \(\tau_T\) have another interesting consequence. For about 0.5 \(R/c\) after injection is increased, the escaping X-ray and UV luminosities remain approximately constant. This is an example of the sudden ‘trapping/release’ effect (anticorrelation of X-ray luminosity with changes in injection) discussed in FBGPC that occurs whenever \(\tau_T\) can be made to change on a time-scale faster than the photon response time-scale \(\tau_{\text{resp}}\). Another example is the outburst of X-ray and UV luminosity that occurs when particle injection is cut. Because of the large value of \(\tau_{\gamma}\), at most gamma-ray energies, the gamma-ray distribution quickly collapses, causing significant pair production to stop in \(\ll \tau_{\text{cross}}\). The value of \(\tau_T\) then drops to unity on the annihilation time-scale \(1/\tau_T\) (also \(\ll \tau_{\text{cross}}\)), allowing the previously trapped UV/X-ray photons to stream out of the source. (The escaping X-ray luminosity is not enhanced by quite the same factor as the UV luminosity, due to downscattering by the relatively cold thermal pairs.) Note, however, that, in contrast to what appears to be implied in FBGPC, there is nothing special about the X-rays. Lower photon energies (e.g., UV) will show the same (temporary) ‘anticorrelation’ with changes in injection. Also, this anticorrelation of escaping X-ray (UV) luminosity is not a universal behaviour of pair plasmas. Rather extreme changes in injection, from or to states with high \(L_p\) and \(L_e\) are required to obtain it. For mono-energetic pair injection at \(\gamma_{\text{max}} \sim 10^4\) and \(L_p, L_e\), always \(\sim 300\), no such effects are seen in the models considered here. [LZ and Svensson (1987) suggest that typical AGN compactness parameters are in the range 1–30,]

Moreover, in a real source, the value of \(\tau_T\) cannot be made to rapidly build up or drop throughout the source in less than a crossing time (see above). This gives the photon distribution time to respond to the changes in the escape rate and any trapping/release effects may be significantly reduced.

The last example we consider is the response of the ‘AGN-like’ model of Section 4.3 to changes in the pair injection rate (see Figs 9 and 10). Here the boost in pair injection from \(L_p = 3\) to 30 takes the pair plasma from an initial equilibrium state where pair production is unimportant (note the lack of an annihilation feature in Fig. 8a) to one where pair production is moderately important (more so than in the first case examined). Because the maximum value of \(\tau_T = 0.8 < 1\), one might expect behaviour on time-scales similar to those in the first case, where the annihilation line builds up and decays slowly (over several \(\tau_{\text{cross}}\)) and the normalized escaping luminosities in other bands go roughly as

\[
l(t) = 1 - (1 - 1/f) e^{-(t-t_0)}
\]

for injection boosted by a factor \(f\) at \(t = t_0\), and

\[
l(t) = 1/f - (1/f - 1) e^{-(t-t_0)}
\]

for injection reduced by a factor \(f\) at \(t = t_0\). (The factor \(f\) is of course energy dependent.) This is indeed the case. It is interesting to note that an appreciable magnetic field, such as in the Fig. 9 model, does not cause major qualitative changes in the time behaviour (except of course at the strongly self-absorbed frequencies). Besides adding a soft photon source, the main effect of synchrotron radiation in these models is simply to shorten further the pair cooling times. As above, however, it will still prove instructive to examine the deviations from the ‘predicted’ \(l(t)\) behaviour evident in Fig. 9(b).

Consider first the escaping X-ray luminosity. For much the same reasons as discussed above for Fig. 8(a), the X-ray luminosity always leads behind \(l(t)\). When injection is increased, the injection rate of X-rays again does not build up to its full value until after a time \(\sim \tau_{\text{cross}}\). This is the time required for the gamma-ray distribution to build up and the pair production rate to increase (note that the annihilation line, whose growth rate reflects the pair production rate, also does not begin to grow rapidly until after \(\tau_{\text{cross}}\), as well as for the (IR-optical) synchrotron photon distribution to build up. The importance of pair production and the presence of synchrotron photons (which are upscattered to X-ray energies) may be gauged by noting that the X-ray luminosity increases by a factor \(\sim 16\) instead of the factor 10 one might have first guessed. Similarly, when injection is reduced, the X-ray luminosity does not drop off significantly until after a time \(\sim \tau_{\text{cross}}\) has passed (and the X-ray injection rate falls). However, because \(\tau_T \sim 1\), the gamma-rays (and pair production) cut off faster and the effect is not as pronounced as in Fig. 8(a). An additional, though minor, factor contributing to deviations from \(l(t)\) behaviour is the non-negligible Thomson optical depth at \(t \gtrsim 14 R/c\), which traps the X-rays slightly, increasing the soft photon escape time to \(\sim 1.2 \tau_{\text{cross}}\).

Because it is mostly optically thin synchrotron emission from relatively high-energy (\(\gamma \gtrsim 50\)) pairs (for which ‘injection’ of pairs by pair production is not important), the IR luminosity behaves quite differently. Since the injection rate of IR photons goes crudely as \(L_p / U_{\text{rad}}\) i.e., as the number of high-energy pairs, the IR injection rate initially jumps by a factor 10 when \(L_p\) is increased but then quickly drops as \(U_{\text{rad}}\)
builds up in response (on a time \(-t_{\text{conn}}\)). The IR luminosity thus appears to shoot up rapidly at first and then abruptly level off (as \(U_{\text{rad}}\) reaches a stationary value). Also, because of the dropping injection rate (the final rate is only a factor 6, not 10, higher), the IR luminosity finds itself close to its stationary value much sooner than the luminosity at other frequencies (see Fig. 9b). At later times (\(t \approx 14\ R/c\)), the luminosity at IR and lower energies drops slightly because of 'trapping' by thermal pairs, even though \(\tau_{\gamma} < 1\). If \(\tilde{\tau}_{\gamma} > (c/R) t_{\text{esc}}(x)|\bar{n}(x)/\bar{n}(x)|\), the escaping luminosity will decrease, even if \(\bar{n}(x) > 0\). By the time thermal pairs start building up, \(\bar{n}(x \leq \text{IR})\) is small, especially at the strongly self-absorbed frequencies. The \(\approx 20\%\) per cent increase in \(t_{\text{esc}}\), due to \(\tau_{\gamma} = 0.8\) causes a corresponding 20 per cent drop in the escaping IR and far-IR luminosity. When pair injection is reduced at \(t = 22\ R/c\), the IR response is much more straightforward. The high-energy pair distribution (the IR injection rate) quickly drops to its new stationary value and the time response is well described by \(t_{\gamma}\).

For \(l_e = 30\), the optical emission is also dominated by synchrotron radiation. The optical luminosity should thus respond in much the same manner as the IR luminosity. The differences seen in Fig. 9(b) can be accounted for by noting two facts. First, at \(t = 11\ R/c\), the injection rate of optical luminosity from the external source is significantly larger than from optical synchrotron emission. Hence, the optical luminosity does not vary as much as the IR luminosity in response to the changes in pair injection (a factor 4 versus a factor 6). Secondly, the presence of significant numbers of lower energy (IR) photons which can be Compton up-scattered to optical energies partially compensates for the decrease in synchrotron luminosity that accompanies the increase in \(U_{\text{rad}}\). This causes the optical luminosity to level off later than the IR luminosity and prevents the analogue of the IR 'overshoot' (at \(t \approx 14\ R/c\)). Also, when pair injection is reduced, the upscattering of IR photons does not stop right away and this causes the optical luminosity to lag \(l_{\gamma}\) slightly (though not as much as the X-ray luminosity).

In contrast, the escaping gamma-ray luminosity does not deviate much from an \(l_{\gamma}(t)\) curve with \(f = 10\), at least when injection is increased. In this case, the decrease in number of high-energy pairs (roughly \(\propto 1/U_{\text{rad}}\)) is (more) than compensated by the increase in the number of soft photons (\(\propto U_{\text{rad}}\)) that can be upscattered. The good agreement with
$l(t)$ is actually somewhat fortuitous. The ‘overcompensation’ is just balanced by an increased photon–photon annihilation rate. This need not always occur, e.g., as in Fig. 8(b) where the increased photon–photon annihilation rate more than wins out. When pair injection is decreased, the effects of the non-negligible photon–photon annihilation ($r_{\gamma\gamma} \sim 1$) cause the gamma-ray luminosity to drop off on the time-scale $t_{\text{sh}} < t_{\text{grow}}$. The accompanying rapid drop in the total pair production rate prevents the overshot in annihilation line intensity seen in Fig. 8(a).

It is worth remarking on one characteristic of models where the IR is predominantly synchrotron radiation, namely that the IR emission always shows less variability than the X-ray emission. In Figs 9 and 10, for example, the IR luminosity varies by only a factor of 6 in response to a factor of 10 change in pair injection, while the X-ray luminosity varies by a factor of 16. The reasons for this behaviour (see above) may be crudely summarized as follows: when pair injection is changed by some factor $N$, the change in number of high-energy non-thermal pairs and the change in synchrotron (IR) luminosity go roughly as $N$, while the change in the number of IR photons upscattered to X-ray energies goes as $N^2$. The fact that pair production can increase the number of pairs responsible for upscattering photons to X-ray energies, while it does not appreciably change the number of pairs emitting at IR energies, only makes the difference in IR versus X-ray variability larger. Also, since the strongly synchrotron self-absorbed (far-IR) tail is relatively insensitive to changes in input parameters, it will tend to show even less variability (e.g., see Figs 9 and 10).

As a practical consequence of this, non-thermal models of IR and X-ray emission from the same source region (e.g., such as for AGN) are not necessarily doomed if the X-rays show variability that exceeds some threshold factor (say 2) and the IR does not. To illustrate this, we have carried out one simulation along the lines of Done & Fabian (1989). The run was started from an equilibrium model of the type shown in Figs 9 and 10, but with $L_x = 20$ and $L_x = 6$. ($B_0$ is again 300 G.) The pair injection ($L_x$) was then varied up and down by a factor of 2, following the prescription of Done & Fabian (1989). The emergent X-ray and IR luminosities, normalized by their $L_x = 20$ equilibrium values, are then plotted against $L_x$ in Fig. 11. As expected, the IR varies significantly less in amplitude than the X-rays. Also as predicted from above, the IR tends to lead the X-rays in responding to the changes in $L_x$. Note, though, that neither the IR nor X-ray luminosities track $L_x$ very faithfully. The large excursions in X-ray flux seen in Done & Fabian (1989) are not seen here because the
model parameters are not such that the model can hop back and forth between pair-dominated and non-pair-dominated states.

For completeness, we have also included a figure (Fig. 10c) showing the luminosity response when pair injection is decreased gradually over $4\tau_{\text{cross}}$ instead of being abruptly cut. (This presumably avoids the problems with radiative transfer and light crossing time effects mentioned above.) As one might expect, the time behaviour is qualitatively quite similar to that of Fig. 10(b). Note that the gamma-rays, because they respond on the shorter time-scale $\tau_{\text{g}} < \tau_{\text{cross}}$, follow the pair injection most closely. Also, since the gamma-rays and the pair production rate take longer to fall off, the X-rays lag the other frequencies even more and the annihilation line continues to build even after injection is reduced.

5.3 The general remarks

The examples presented are only an illustrative subset of the wide variety of responses possible for a pair plasma. Because of this variety, predicting the response requires a detailed knowledge of the initial state of the plasma and the changes in injection. Alternatively, determining whether an observed variation is consistent with a pair plasma, or inverting the variation to constrain the injection changes, requires (at the least) detailed knowledge of the photon distribution throughout the time over which the variation occurred. Observationally, this translates into having simultaneous multiband observations over extended periods of time. If one has only a few snapshots of the plasma or continuous time information in only one or two bands, one can run into trouble (i.e., there will not be enough information to pose a well-constrained problem).

Consider again, for example, the problem of determining whether IR and X-ray emissions are due to pairs in the same source region. One might look for correlations in IR and X-ray luminosities. However, depending on when one observes the model in Figs 9 and 10, for example, one can see a positive correlation ($11 R/c \geq \tau \geq 12 R/c$), almost no correlation ($12 R/c \geq \tau \geq 14 R/c$), or a negative correlation ($\tau \geq 14 R/c$). Also, if $l_{\text{g}}$ is held constant and $l_{\text{e}}$ is increased (a case not shown), the IR luminosity actually drops (since $U_{\text{rad}}$...
is increased) while the X-ray luminosity increases or stays about the same. Thus, unless one has additional information, e.g., that the UV luminosity ($l_\lambda$) did not vary appreciably, it is not at all clear what is expected. Similarly, if one were looking at an outburst resembling that of Fig. 8(b) and noticed the outburst only $\sim 2t_{cross}$ after it really began, one should not be dismayed to find that the gamma-rays show no variability while the X-rays are constantly increasing in intensity. This does not imply that the X-rays and the gamma-rays come from causally distinct regions.

One should also be careful about drawing too many conclusions from hardness ratios (e.g., $L_{\gamma}/L_{\alpha}$) or spectral indices. For known changes in particle injection, definite predictions can be made about how their stationary values should change. Increasing the input luminosity while keeping $l_\lambda/l_\alpha$ constant, for example, generally results in more secondary pair production and gives softer (lower $L_{\gamma}/L_{\alpha}$) spectra. Increasing $l_\lambda/l_\alpha$, on the other hand, increases the number of non-thermal pairs and tends to give harder spectra. Thus, if one does not know how the injection changed, knowing $L_{\gamma}/L_{\alpha}$ before and after does not necessarily provide any useful constraint. Making use of the time-varying hardness ratios is also somewhat tricky. Hardness ratios do not always interpolate monotonically between their initial and final stationary values. (See Fig. 8b where the spectrum is initially very hard and Fig. 10b where the spectrum becomes and remains softer than the final and initial states for $\sim 7t_{cross}$.)

Also, to the extent $l_\lambda(t)$ describes the evolution of the frequencies being looked at (usually the case for X-ray and lower energies, $\tau_r \lesssim 1$), quantities like hardness ratios and spectral indices will not vary significantly from their initial values until $\exp(-t-t_0)\sim 1/|t-1|$. If the reduction in injection, $f_\alpha$, is large, the shape of the spectrum may then look unchanged for several (many) $t_{cross}$s, with all the spectral changes occurring in the last few $t_{cross}$. In short, then, a non-stationary spectrum may look nothing like the final state it asymptotically approaches. The observation of a very steep (soft) spectrum, for example, does not automatically imply a steep pair injection function $\beta(t)$ if the source luminosity turns out to be varying appreciably. One should be careful not to misinterpret a time-varying spectrum as if it were a stationary one.

Yet another way of extracting information from time variability is to cross-correlate in time the measured intensity at two different frequencies. In a non-relativistic thermal pair plasma of fixed temperature and density, where photons are injected at low energies, photons must ‘walk’ their way up to higher (X-ray) energies via multiple Compton scatterings.
Reaching a higher energy on average takes more scatterings and more time. One can thus make a definite prediction: whenever the source luminosity increases, higher frequencies should always lag lower frequencies. However, if one allows the temperature and optical depth to vary self-consistently and admits the possibility of a non-thermal tail, the situation is no longer so clear. In the first case (fixed temperature, density) it is relatively straightforward to write down a Green's function for the evolution in time (which may then be Fourier transformed, etc.). In the second case, however, the ‘transfer function’ itself varies with time in a non-trivial manner. Again, then, unless one knows exactly what parameter regime is being observed, ‘lead/lag’ information may not be very useful. Just by looking at Figs 8–10, it is possible to come up with examples where one frequency can either lead or lag another depending on the initial state of the plasma.

A relatively unexplored area that might prove useful to look at is the time variability of the hard (≥511 keV) gamma-rays (preferably alongside simultaneous measurements at some lower energy). Because it has the shortest response time-scale and (at least in the models considered here) is due to a portion of the pair distribution not directly affected by pair production, the gamma-ray end of the spectrum is probably the best indicator of changes in the ‘first-order’ pair injection spectrum $Q(\gamma)$. Knowing the gamma-ray spectrum when the source is stationary is even more useful, as it seriously constrains the possibilities for $Q(\gamma)$ – e.g., see LZ. If the gamma-ray spectrum shows marked steepening or softening on a time-scale $\sim t_{\text{post}}$, one can be fairly sure a change in pair injection is involved – an important piece of information. (A change in soft photon injection typically results in a much more sluggish response, especially if $\tau_r > 1$ – see Section 5.1.) The gamma-ray response to a decrease in pair injection also has the characteristic that higher energies drop off faster, i.e., higher frequencies lead lower frequencies. [Exactly how much they lead is of course a measure of $\tau_r(x)$.] This behavior is hard to mimic by changes in soft photon injection. Observations of gamma-rays during a sizeable drop in pair injection can also lead to a rather unambiguous indication of pairs in a source. In the models considered, a relatively long-lived annihilation feature almost always emerges from the decaying continuum (e.g., Fig. 10a). Needless to say, finding no significant variability on time-scales shorter than (or at least as short as) the shortest observed X-ray time-scales would place current models in severe difficulty.

Unfortunately, the details of the time behaviors discussed here (e.g., the IR overshoot in Fig. 10b, the release of trapped X-rays in Fig. 8b) often depend sensitively on the relative
values of the various relaxation time-scales. Different microphysics (i.e., approximations) or assumptions about the geometry of the system can lead to different values for these time-scales, and consequently quite different behaviours. This should not be forgotten when interpreting the results of numerical calculations. Although it does not contradict the qualitative results presented in FBGPC, for example, the code discussed here does not reproduce well the detailed time behaviour presented in their figures (even accounting for differences in the definitions of \( l_r \), \( l_e \), etc.). This is not surprising considering the different treatments of the microphysics, e.g., the use of different escape probabilities. Similarly, the results presented here are based on a radiative transfer scheme that does not allow for variations in geometry, anisotropies in particle distributions, or spatial inhomogeneities. They should not be considered 'definitive'.

6 DISCUSSION AND CONCLUSION

Past studies of pair plasmas have suffered from two potentially serious shortcomings: (i) unsophisticated treatments of the radiative transfer, and (ii) overly simple approximations of the basic microphysical processes. We have concentrated on the second of these problems, trying to separate the real physical effects from the artefacts of approximations. The code described here can correctly handle Klein–Nishina scatterings where the photon and electron energies are of the same order. It can also solve self-consistently for the pair and photon distributions over a large range of values of \( U_{\text{rad}}/U_{\text{Ip}} \).

Some general conclusions follow from the work presented here. First, approximations used in earlier work are reliable only in restricted parameter regimes. The Compton scattering approximation of LZG, for example, works well only when a few orders of Compton scattering are important and \( \theta_{\text{rad}} \gamma_{\text{max}} < 1 \). Errors of order unity occur when this is not the case. Some of the approximations for handling pair production have analogous limitations. Contrary to earlier assumptions, Coulomb scattering can indeed be important for non-thermal pairs. In the synchrotron-dominated case, even a small Coulomb cooling rate will modify significantly the low-energy pair distribution. Moreover, interesting parameter regimes (e.g., \( l_r \ll l_e \)) exist where the Coulomb energy exchange times are similar to the other cooling time-scales.
The thermal component of pair distribution can then be quite hot (\( kT/m_e c^2 \sim 1 \)), since not all the energy of the injected pairs is lost to the radiation field. In general, achieving more than order-of-magnitude accuracy appears to require a careful numerical treatment. Analytic approximations, though convenient, have limited validity and accuracy. When fitting data, spectral models based on past approximations could result in bad estimates of source parameters or could lead to the erroneous conclusion that a particular spectrum is not consistent with being produced by a pair plasma.

Secondly, pair-plasma models can reproduce a wide variety of spectral shapes and variability (see the figures in this paper). When the calculation is done self-consistently and details such as the presence of cool pairs are taken into account, the deviations from the spectral shapes produced in older synchrotron self-Compton (SSC) or Compton-upscttering models (e.g., Rees 1967b; Bonometto & Rees 1971) can be quite significant. This should not be forgotten when comparing observations to theory. Power-law pair injection with index \( \Gamma \), for example, does not automatically imply a synchrotron or X-ray power-law index of \( \Gamma/2 \). The portions of the spectrum due to optically thin synchrotron emission and Compton upscattering of soft photons (e.g., the IR and X-rays) can have different spectral indices and still arise from the same pair distribution, and a portion of the spectrum which is strongly synchrotron self-absorbed need not have a spectral index of 2.5. In general, the models considered here appear to have no special spectral index in the 2–10 keV energy range except for \( \alpha_{\gamma-1} \sim 1 \) in very high-compactness (\( l_\gamma \gg l_\nu \)) models. Anything between a spectral index of 0 and 1 can be produced, given the right combination of injection parameters. The fact that objects in certain classes tend to show similar spectral indices (e.g., Seyfert 1 galaxies appear to have \( \alpha_{\gamma-1} = 0.7 \)) thus implies that additional physics (e.g., a reason why \( l_\gamma \) is always a certain fraction of \( l_\nu \)) is still missing from these models. Also, since the combination of parameters giving a particular 2–10 keV spectral index is often not unique, the 2–10 keV spectral index is a poor diagnostic of the state of a pair plasma. To constrain a pair-plasma model, one therefore needs good, simultaneous observations over a wide range of energies. In particular, it would be very useful to set limits on the presence of the annihilation features which are generically produced in the isotropic, unbeamed models considered here.

Thirdly, in considering variability, it is also worth remembering that a pair plasma acts essentially as a low-pass filter, damping out rapid variations. A source size derived from variability measurements may well be an overestimate. More importantly, the response function of this filter depends on both the energy being observed and the current state of the pair plasma. Luminosities measured at different frequencies can therefore vary in different manners (and still be due to the same source). In particular, they may sometimes appear correlated and sometimes anticorrelated, and a comparison of the response at two frequencies need not show a consistent lag or lead of one frequency with respect to the other. In response to a change in injection, a pair plasma may also show hardness ratios that do not interpolate smoothly between those of the initial and final states. One must therefore be careful in using variability information. Pair-plasma models make firm predictions only if the state of the plasma and the changes in particle and energy injection are well constrained. Again, this requires good, simultaneous multiband observations. In this regard, we note that the gamma-ray flux generally tracks changes in pair injection most closely (and thus is a useful diagnostic). If the observed gamma-rays and X-rays originate in the same plasma, the models considered here consequently predict that the gamma-rays should vary on time-scales the same as or shorter than those of the X-rays. Finally, we note that a plasma in a transitional, non-stationary state can have an emergent radiation spectrum which looks different from that of any stationary state and apparently violates constraints derived from stationary models.

In sum, it appears that models based on pair plasmas are sufficiently versatile to accommodate existing observations of the continua of AGN. Until the arrival of detailed (and simultaneous) multiband observations (e.g., by the GRO satellite), it is difficult to see how such models can be convincingly confirmed or ruled out.

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