Scale analysis (mathematics)

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Scale analysis is a powerful tool used in the mathematical sciences for the simplification of equations with many terms. First the approximate magnitude of individual terms in the equations is determined. Then some negligibly small terms may be ignored.

Contents

Example: vertical momentum in synoptic-scale meteorology

Consider for example the momentum equation of the Navier–Stokes equations in the vertical coordinate direction of the atmosphere

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}-\frac{u^2+v^2}{R}=-\frac{1}{\varrho}\frac{\partial p}{\partial z}-g+2\Omega u\cos\varphi+\nu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right), \qquad (1)$$

where R is Earth radius, Ω is frequency of rotation of the Earth, g is gravitational acceleration, φ is latitude ρ is density of air and ν is kinematic viscosity of air (we can neglect turbulence in free atmosphere).

In synoptic scale we can expect horizontal velocities about $U = 10^1$ m.s^{-1} and vertical about $W = 10^{-2}$ m.s^{-1}. Horizontal scale is $L = 10^6$ m and vertical scale is $H = 10^4$ m. Typical time scale is $T = L/U = 10^5$ s. Pressure differences in troposphere are $\Delta P = 10^4$ Pa and density of air $\rho = 10^0$ kg·m^{-3}. Other physical properties are approximately:

$R = 6.378 \times 10^6$ m;

$\Omega = 7.292 \times 10^{-5}$ rad·s$^{-1;}$

$\nu = 1.46 \times 10^{-5}$ m^2·s$^{-1;}$

$g = 9.81$ m·s$^{-2.}$

Estimates of the different terms in equation (1) can be made using their scales:

$$\frac{\partial w}{\partial t} \sim \frac{W}{T}$$

$$u\frac{\partial w}{\partial x} \sim U\frac{W}{L} \qquad v\frac{\partial w}{\partial y} \sim U\frac{W}{L} \qquad w\frac{\partial w}{\partial z} \sim W\frac{W}{H}$$

$$\frac{u^2}{R} \sim \frac{U^2}{R} \qquad \frac{v^2}{R} \sim \frac{U^2}{R}$$

$$\frac{1}{\varrho}\frac{\partial p}{\partial z} \sim \frac{1}{\varrho}\frac{\Delta P}{H} \qquad \Omega u \cos\varphi \sim \Omega U$$

$$\nu\frac{\partial^2 w}{\partial x^2} \sim \nu\frac{W}{L^2} \qquad \nu\frac{\partial^2 w}{\partial y^2} \sim \nu\frac{W}{L^2} \qquad \nu\frac{\partial^2 w}{\partial z^2} \sim \nu\frac{W}{H^2}$$

Now we can introduce these scales and their values into equation (1):

$$\frac{10^{-2}}{10^5} + 10\frac{10^{-2}}{10^6} + 10\frac{10^{-2}}{10^6} + 10^{-2}\frac{10^{-2}}{10^4} - \frac{10^2+10^2}{10^6}$$

$$= -\frac{1}{1}\frac{10^4}{10^4} - 10 + 2\times 10^{-4}\times 10 + 10^{-5}\left(\frac{10^{-2}}{10^{12}} + \frac{10^{-2}}{10^{12}} + \frac{10^{-2}}{10^8}\right). \qquad (2)$$

We can see that all terms — except the first and second on the right-hand side — are negligibly small. Thus we can simplify the vertical momentum equation to the hydrostatic equilibrium equation:

$$\frac{1}{\varrho}\frac{\partial p}{\partial z} = -g. \qquad (3)$$

See also

- Approximation

References

- Barenblatt, G. I. (1996). *Scaling, self-similarity, and intermediate asymptotics*. Cambridge University Press. ISBN 0-521-43522-6.
- Tennekes, H.; Lumley, John L. (1972). *A first course in turbulence*. MIT Press, Cambridge, Massachutes. ISBN 0-262-20019-8.

External links

- Scale analysis and Reynolds numbers (http://www.env.leeds.ac.uk/envi2210/notes/node7.html)

The Wikibook *Partial Differential Equations* has a page on the topic of: ***Scale Analysis***

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Categories: Applied mathematics

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