1. Derive the crossing time, the collision time, and the 2-body relaxation time for the following 2 systems, and then draw specific conclusions about the dynamical state of each system.
   a. a star cluster with N=10,000 stars each with mass $=1 \, \text{M}_{\odot}$ and radius $=1 \, \text{R}_{\odot}$, cluster core radius of 3 pc, and velocity dispersion of $\sigma = 0.5 \, \text{km/s}$. Assume that gravitational focusing increases the collision cross section by a factor of 100. The age of the star cluster is 3 Gyr. (In this case the “particles” are stars.)
   b. a galaxy cluster with N=100 galaxies each with mass $=10^{12} \, \text{M}_{\odot}$ and radius $=20 \, \text{kpc}$, cluster radius of 1 Mpc, and velocity dispersion of $\sigma = 700 \, \text{km/s}$. Assume that gravitational focusing increases the collision cross section by a factor of 10. The age of the galaxy cluster is 10 Gyr. (In this case the “particles” are galaxies.)

   NOTE: The velocity dispersions $\sigma$ given above are a measure of the random motions of stars (or galaxies) in the direction along the line-of-sight. Assume that the random motions are isotropic, in which case the typical speed of a star (or galaxy) is $v_{\text{typ}} = \sqrt{3} \, \sigma$.

2. The Carina dwarf spheroidal galaxy has a velocity dispersion $\sigma$ four times less than that at the center of the globular cluster $\omega$ Centauri, while Carina’s core radius (where the surface brightness falls to half its central value) is 80 times greater. Use the virial theorem to derive the mass ratio of Carina to $\omega$ Centauri. Derive the ratio of mass-to-luminosity ratios $M/L$ of Carina to $\omega$ Centauri, using the luminosities of these systems given in Tables 2.3 and 4.1. This is evidence that dwarf galaxies contain large amounts of dark matter.

3. Find the average density for the Milky Way galaxy, assuming a mass of $2\times10^{11} \, \text{M}_{\odot}$ and radius of 10 kpc, and for the Sculptor dwarf galaxy, assuming a mass of $3\times10^{7} \, \text{M}_{\odot}$ and radius of 2 kpc. Calculate the ratio of densities for the 2 galaxies. The virial theorem tells us that, if a galaxy of stars collapses from rest, then, after it has come to equilibrium, it will be eight times denser than at the start. Derive the free-fall times (in years) for the proto-Milky Way and the proto-Sculptor galaxy, using Equation 3.23. An important conclusion from the calculations in this problem is that massive galaxies are expected to form first in the universe, and lower mass galaxies are expected to form later!