1. Suppose you observe magnitudes and colors for a star cluster like 47 Tucanae (in SG Fig 2.14). but you have a systematic error of 0.2 mag in the absolute magnitudes (and therefore luminosities) (your derived absolute magnitudes are 0.2 mag brighter than the real values; there are no systematic errors in the B-V colors). How much will this systematic absolute magnitude error change estimates of age and metallicity? To do this, just make rough estimates from the plots in the bottom panels of SG Fig 2.14 (also shown with added labeling in Slides from Lecture 5).

a.) For the age effect, use the region just to the right of the main sequence turn-off (the sub-giant branch) where the isochrones are maximally spread in the vertical direction. A vertical shift of 0.2 magnitudes in the sub-giant branch corresponds to how big a change in age?

b.) For the metallicity effect, compare the bottom of the main sequences of 47 Tuc and M92. A vertical shift in the main sequence of 0.2 magnitudes corresponds to how big a change in metallicity? [HINT: Pick some color, and see at what magnitude the main sequence is at that color in the clusters of 2 different metallicities.]

c.) Discuss how might you use the color-magnitude diagrams to distinguish between the effects of age, metallicity, and systematic errors in absolute magnitudes. [Lecture 5].

2. The Milky Way Galaxy’s luminosity is about $2 \times 10^{10} L_{\odot}$. Makin the very rough approximation that it is a sphere 5 kpc in radius, use SG equation 1.3 to show that, if it radiated as a blackbody, $T_{\text{eff}} = 5$ K. Near the Sun, starlight heats interstellar dust to an average temperature of 15-20K. Why is it higher than 5K? [based on SG 1.12] [on dust heating] [Lectures 2 & 6].
3. Dust grains in the ISM absorb radiation, which heats them up, causing them to radiate.

a.) Calculate the equilibrium temperature of a large dust grain located 1 pc from an O star with \( L = 10^6 L_{\text{sun}} \). In equilibrium, the rate of incoming energy balances the rate of outgoing energy. Note that a grain of radius \( r_g \) absorbs starlight over an area \( \pi r_g^2 \), but emits from its whole surface. Assume the dust grain radiates as a perfect blackbody (in reality it acts as a modified blackbody).

b.) At what wavelength will the emission from this dust grain peak?

c.) How far from the star would the dust have to be to reach an average temperature of 150 K, so that it radiates at 20 \( \mu \text{m} \)?

d.) The galaxy M82 is undergoing a burst of star formation. Use SG Fig 2.24 to learn the wavelength at which it emits most of its energy, and then show that its large dust grains have \( T \sim 50 \text{ K} \). [Lectures 2 & 6].

4. Near the Sun, the diffuse interstellar gas has a density of about one atom \( \text{cm}^{-3} \). Show that you would need to compress a cube of gas 30 km on a side into \( 1 \text{ cm}^3 \) to bring it to Earth’s normal atmospheric density and pressure (6\( \times 10^{23} \) atoms in 22.4 liters: a cube 10 cm on a side has a volume of one liter). Interstellar gas is about \( 10^{10} \) times more rarefied than a good laboratory vacuum, which is itself \( \sim 10^8 \) times less dense than Earth’s atmosphere.

Assume that each dust grain is a sphere of radius 0.1 \( \mu \text{m} \), and the gas contains one grain for every \( 10^{12} \) hydrogen atoms. Show that, as light travels through a 1 cm layer of the compressed gas in the first part of the problem, about 1% of it will be intercepted. State the simplifying assumption you need to make about the grain distribution to make this calculation. Show that \( \kappa = 0.0084 \text{ cm}^{-1} \), so that a layer about 120 cm thick would transmit a fraction \( 1/e \) of the rays (\( \tau = 1 \)). If the air around you were as dusty as interstellar space you could see for less than a meter, as in the London fogs described by Charles Dickens. [SG Problem 1.11, with corrections & addition] [on dust extinction] [Lectures 4 & 6].