

Ratios

Many problems in science can be greatly simplified by forming a ratio between a known system (such as the Earth-Sun system in astronomy) and a system containing the unknown or desired quantities.

Q. How will I know if a problem requires me to form a ratio of 2 equations in order to solve it ?

A. Problems that compare two or more objects (such as the Sun and another star) normally require a ratio to be constructed. Such problems often give quantities as some multiple of a well known quantity (e.g. radius of the Sun) or state that the unknown star is some number of times bigger/smaller than a known quantity or say a star is “like” the sun.

Q. Why should I use ratios if the problem can be solved by inserting all the numbers in standard units (m, kg ,s) ?

A. Three reasons:

Firstly: universal constants such as π , σ , h or G cancel when a ratio is formed.

Secondly: putting in numerical values directly tends to make it hard to know if an answer is correct, as one big number looks much like another big number. But if an answer is written in terms of earth/solar units then the meaning is much clearer,

For example; the radius of the star Fred is 6.96×10^{12} cm , what does this mean ?

Equivalently the radius of Fred is $100 R_{\odot}$. This means Fred is 100 times bigger than the Sun.

Thirdly: less emphasis is placed on using a calculator – so less chance to make a mistake. Often after the ratio is formed the problem can be done without a calculator.

Solving problems by using ratios is an important technique that can be applied to problems in all sciences. In astronomy it is particularly useful.

Why ?

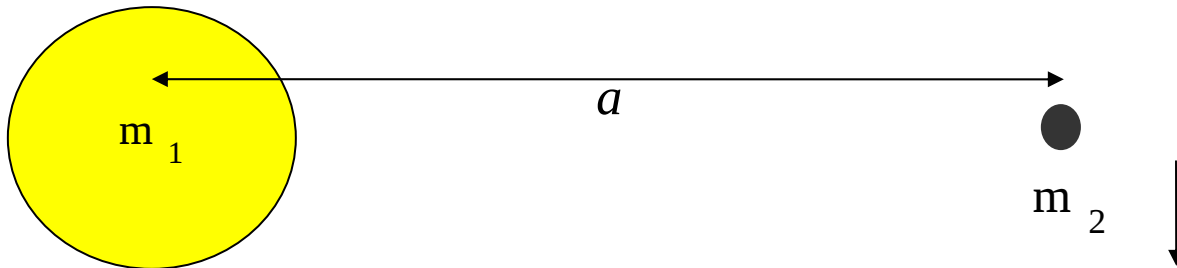
Because the numbers we deal with such as interstellar distances, sizes of stars, temperatures, luminosities are generally very big ! If these are always put in m, kg or seconds then working through the problems will require a lot of painful needless number crunching.

To illustrate the usefulness of forming ratios we will solve a problem first in absolute units and second by forming a ratio.

Q. A comet orbits the Sun with a sidereal period of 64 years. What is the semimajor axis of the orbit ?

To solve this we need to use Kepler's 3rd law

$$P^2 = \left[\frac{4\pi^2}{G(m_1 + m_2)} \right] \times a^3$$



P = sidereal period (time for one full revolution)

a = semi-major axis of elliptical orbit (average distance between the two masses)

$$G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

1. Without Ratios

As $m_2 \ll m_1$ we can replace $(m_1 + m_2)$ by m_1 which equals M_o (mass of the Sun)

$$P^2 = \left[\frac{4\pi^2}{GM_o} \right] \times a^3$$

Isolate variable a : multiply both sides by GM_o

$$P^2 \times GM_o = 4\pi^2 \times a^3$$

Divide by $4\pi^2$

$$a^3 = \frac{P^2}{4\pi^2} \times GM_o$$

$$a = \sqrt[3]{\left[\frac{P^2 G M_o}{4\pi^2} \right]}$$

$$P = 64 \text{ years}$$

$$G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

$$M_o = 2 \times 10^{30} \text{ kg}$$

To use this formula, the units of P and G must match and mass of Sun must be in kg

Convert P to seconds

$$P = 64 \text{ yrs} \times \frac{365 \text{ d}}{1 \text{ yr}} \times \frac{24 \text{ h}}{1 \text{ d}} \times \frac{60 \text{ min}}{1 \text{ h}} \times \frac{60 \text{ s}}{1 \text{ min}} = 2.018 \times 10^9 \text{ s}$$

$$a = \sqrt[3]{\left[\frac{(2.018 \times 10^9)^2 \times 6.67 \times 10^{-11} \times 2 \times 10^{30}}{4\pi^2} \right]} = 2.396 \times 10^{12} \text{ m}$$

I can't even be sure this is right ?? maybe I plugged the numbers in wrong !

2. With ratios

$$\text{Comet:} \quad P^2 = \left[\frac{4\pi^2}{GM_o} \right] \times a^3$$

$$\text{Sun-Earth:} \quad P_{\oplus}^2 = \left[\frac{4\pi^2}{GM_o} \right] \times a_{\oplus}^3$$

$$\text{Divide and cancel:} \quad \frac{P^2}{P_{\oplus}^2} = \frac{a^3}{a_{\oplus}^3}$$

$$\text{Input values:} \quad \frac{(64 \text{ yr})^2}{(1 \text{ yr})^2} = \frac{a^3}{(1 \text{ AU})^3}$$

$$a^3 = (4096 \text{ AU})^3$$

$$a = 16 \text{ AU}$$

The comet is 16 AU from the Sun. *This sounds like a reasonable answer.*