Derivation of Newton's form of Keplers $3^{\text {rd }}$ law (this will not be in exam)


Consider 2 masses M1 and M2 orbiting their stationary center

Though they may move at different speeds - they must both have the same sidereal period. Why ?

So they must also have the same angular velocity $\omega$
$r_{1}=$ dist from M1 to stationary center
$r_{2}=$ dist from M2 to stationary center

$$
r_{1}+r_{2}=a=\text { semi-major axis of the orbit }
$$

The centrifugal force $F$ on an object in a circular orbit about its stationary center is

$$
\mathrm{F}=\mathrm{mr} \omega^{2}
$$

Where $r$ is distance to stationary center (center of mass)

Newton's $3^{\text {rd }}$ law requires:

$$
\mathrm{m}_{1} \mathrm{r}_{1} \omega^{2}=\mathrm{m}_{2} \mathrm{r}_{2} \omega^{2}
$$

Rearranging gives

$$
\begin{equation*}
\frac{\mathrm{r}_{1}}{\mathrm{r}_{2}}=\frac{\mathrm{m}_{2}}{\mathrm{~m}_{1}} \tag{1}
\end{equation*}
$$

As $r_{1}+r_{2}=a$ equation (1) can be written as

$$
\begin{equation*}
\mathrm{r}_{2}=\frac{a}{\left(1+\frac{\mathrm{m}_{2}}{\mathrm{~m}_{1}}\right)} \tag{2}
\end{equation*}
$$

The centrifugal force is produced by gravity

$$
\begin{equation*}
\frac{\mathrm{Gm}_{1} \mathrm{~m}_{2}}{a^{2}}=\mathrm{m}_{2} \mathrm{r}_{2} \omega^{2} \tag{3}
\end{equation*}
$$

Now substitute (2) into equation
and use the definition $\omega=\frac{2 \pi}{\mathrm{P}} \quad$ ( $\mathrm{P}=$ period)

Gives: $\quad \frac{P^{2}}{a^{3}}=\frac{4 \pi^{2}}{G\left(m_{1}+m_{2}\right)}$

