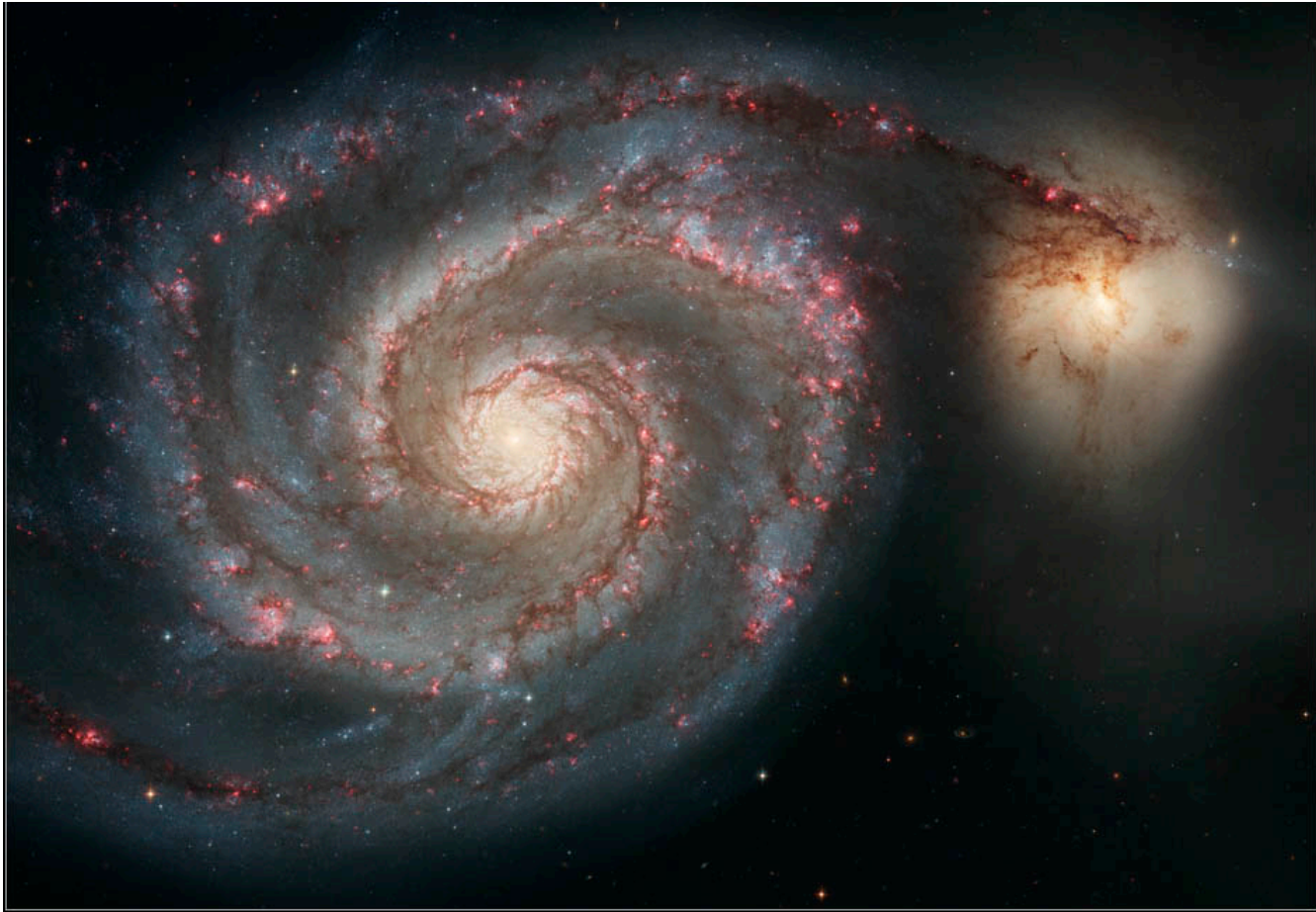


# Astronomy 120



Prof. Jeff Kenney    Class 13    June 13, 2018

# Midterm exam

Sun June 17 1-3pm (classroom) OR

Mon June 18 12:30-2:30pm (classroom)

exam covers:

lectures 1-10 (thru discovery of MW galaxy)

HW 1-4

# Midterm exam

Format: problems, short answer, multiple choice  
(see class website under exams!!)

Formula & constant sheet: to be handed out  
(also on website)

Study: class notes & HWs to prepare for exam.  
use textbook to help you understand  
classnotes & HWs. I won't ask about things in  
textbook that are not mentioned in class! (I  
won't ask about history/dates/people.)

Review: the *mock exam* (now on canvas, files)

# The mass of our galaxy is best found by:

- A. Estimating the number of stars in the northern & southern skies
- B. Radio telescope measurements of the amount of interstellar hydrogen
- C. Measuring how fast the galaxy rotates
- D. Measuring how fast the galaxy expands at large distances from the center
- E. Estimating the numbers and masses of all known components of the Galaxy

# How can astronomers measure the rotational speed of a galaxy?

- A. From a time series of galaxy images taken over many years
- B. From the difference in the Doppler shift from one side of the galaxy to the other
- C. From the motion of the galaxy with respect to the background universe
- D. By measuring the changing curvature of the spiral arms
- E. From the change in periods of Cepheid variables across the galaxy

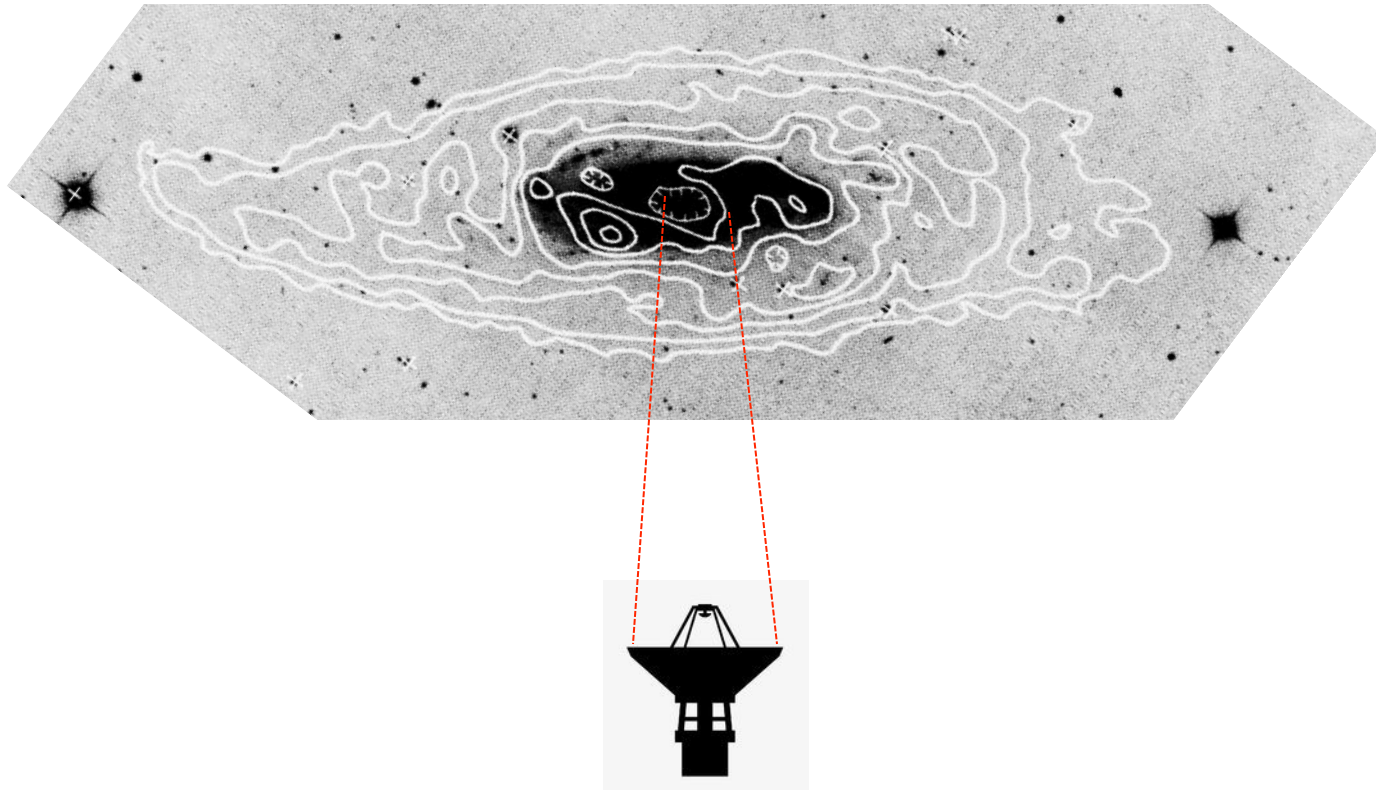
doppler shift for light

$$\Delta\lambda/\lambda = v/c$$

$$c=3.00 \times 10^8 \text{ m/s}$$

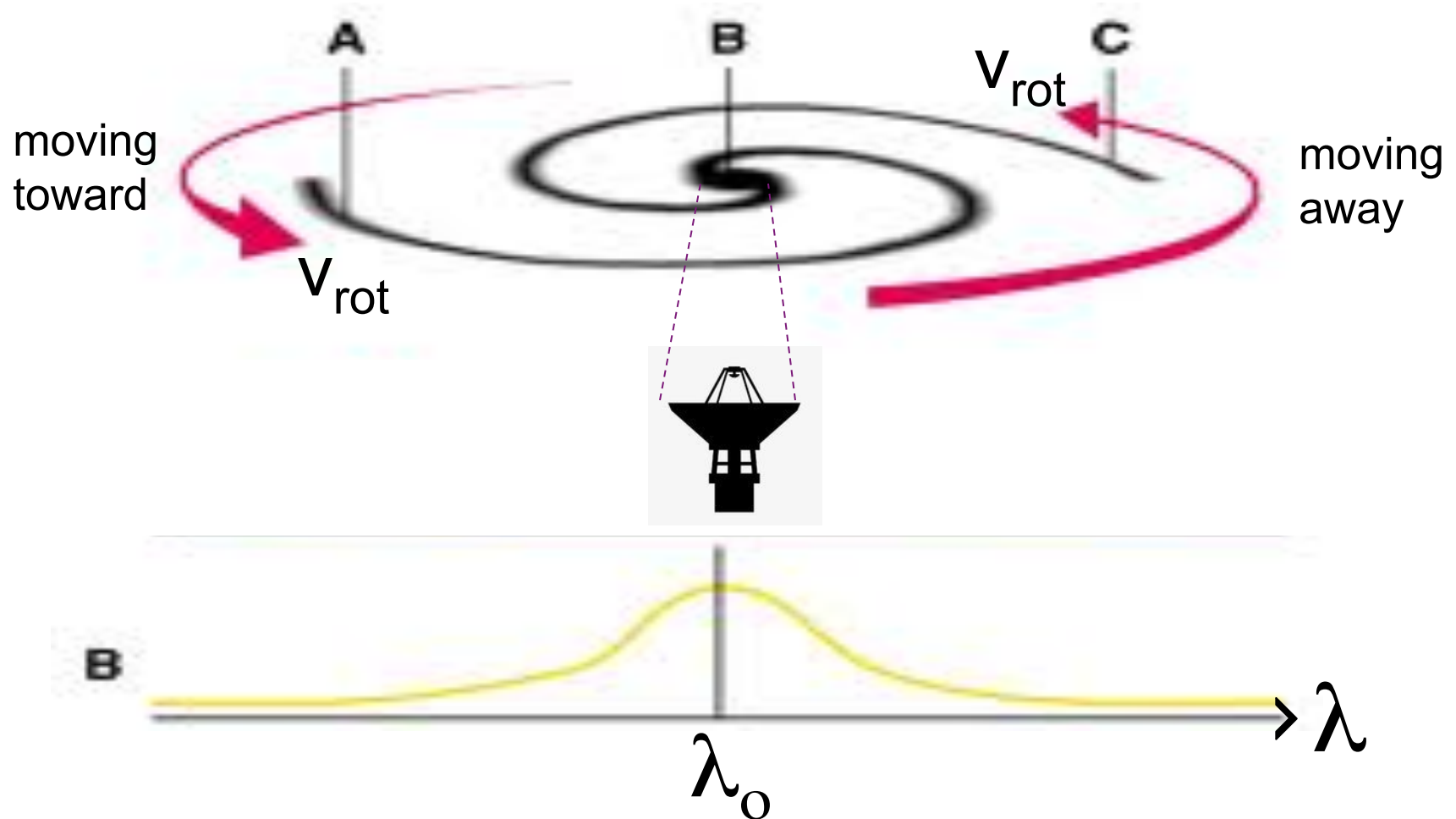
speed of light

# HI map and optical image of inclined spiral galaxy



observe  $\lambda=21\text{cm}$  spectral line of with radio telescope

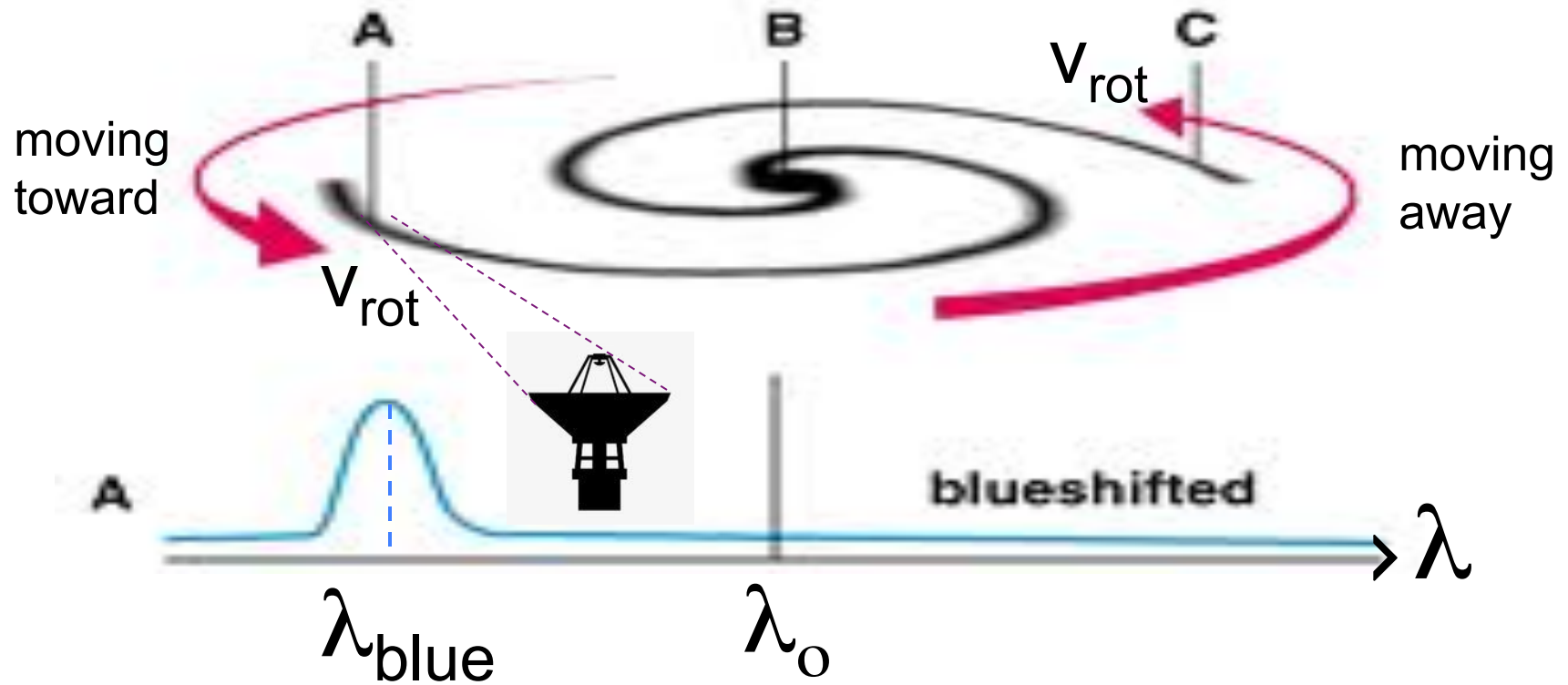
# measuring rotation velocities in spiral galaxy using doppler shifts of spectral lines



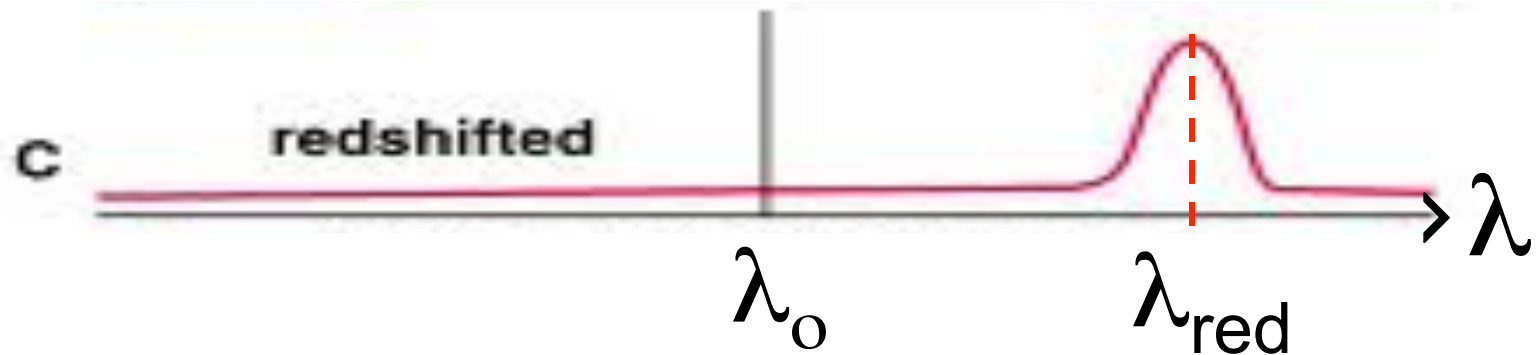
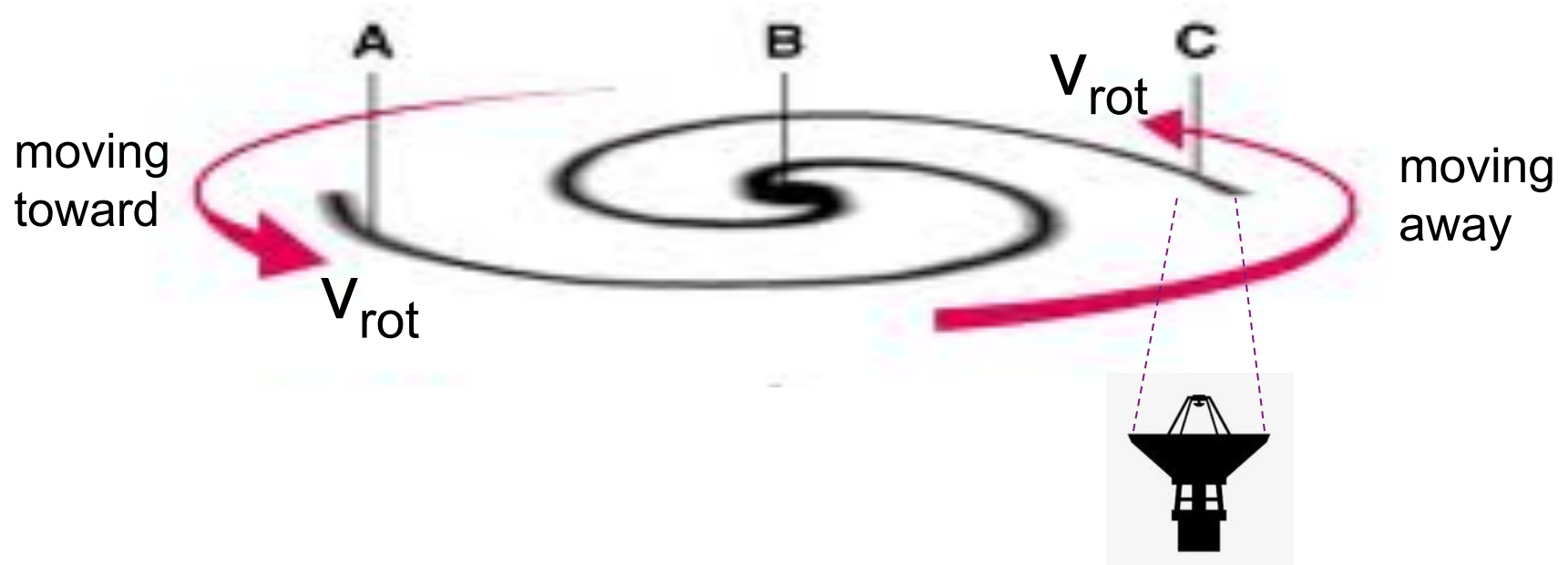
HI in disk of inclined, rotating spiral galaxy



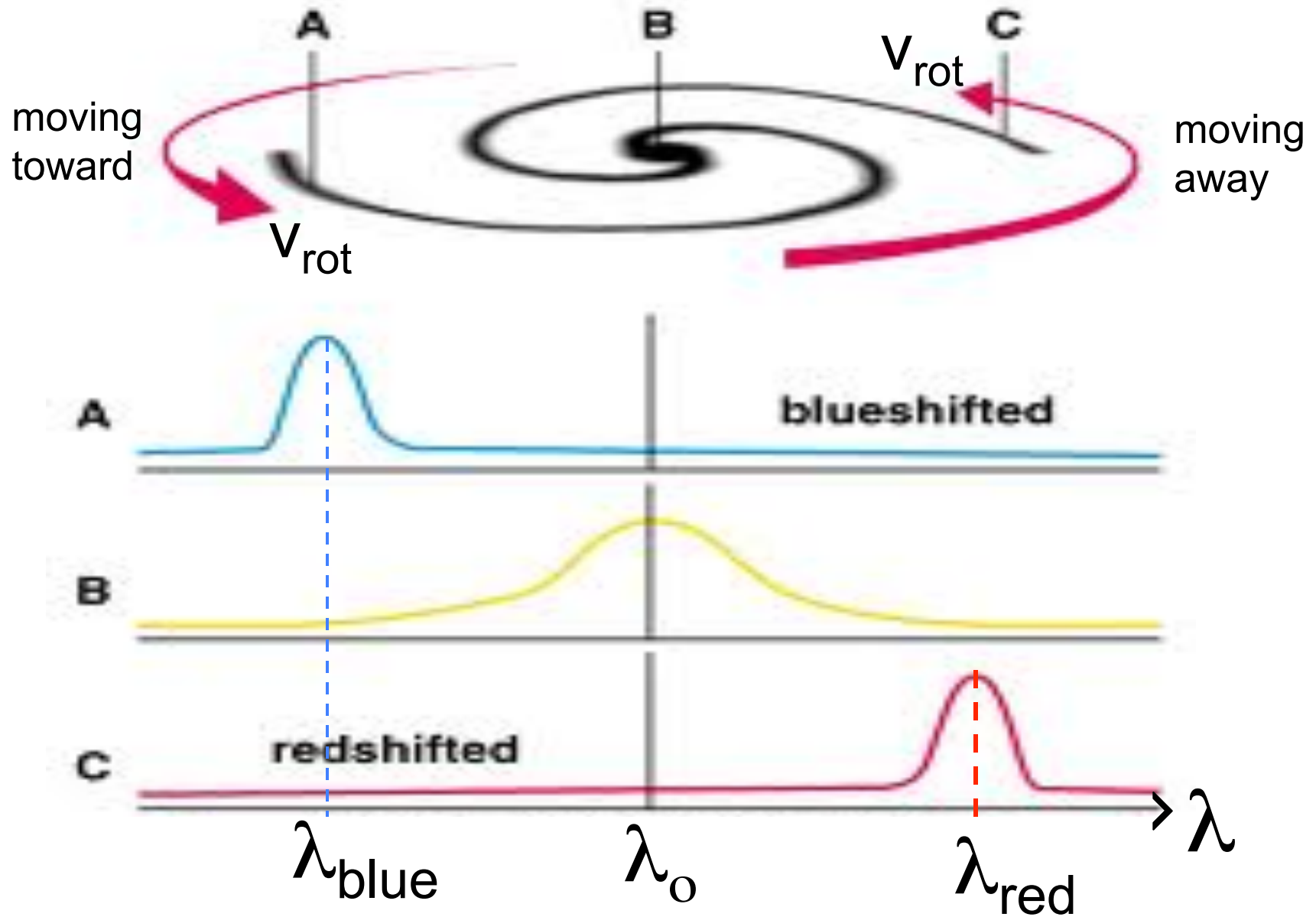
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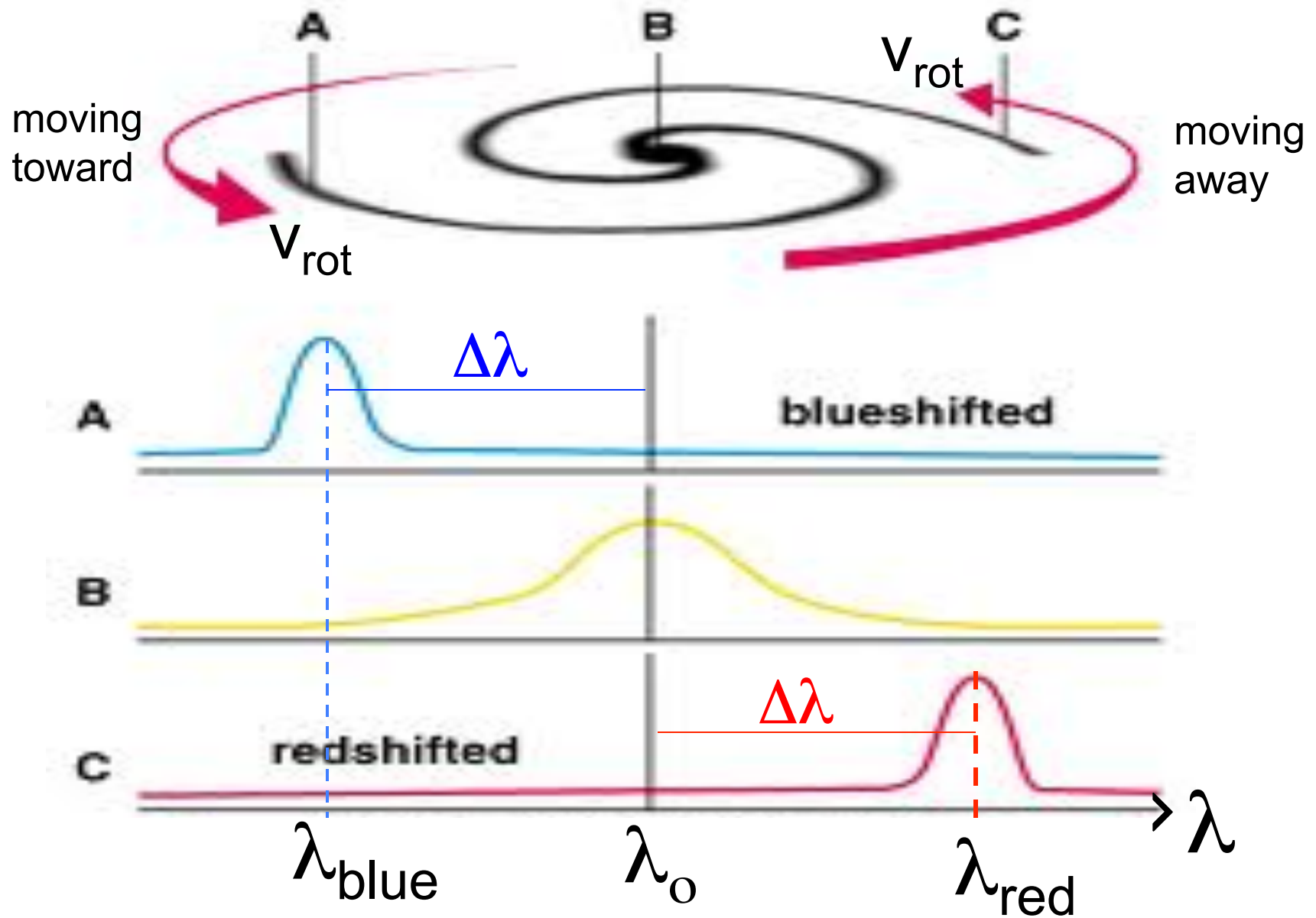
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# measuring rotation velocities in spiral galaxy using doppler shifts of spectral lines



# measuring rotation velocities in spiral galaxy using doppler shifts of spectral lines



3 things which cause  $\lambda$  of light to shift:

**Doppler shift, Cosmological redshift, Gravitational redshift**

**In general shift defined by  $z$ :**

$$z = (\lambda - \lambda_o) / \lambda_o = \Delta\lambda / \lambda_o$$

$\lambda_o$  = rest wavelength

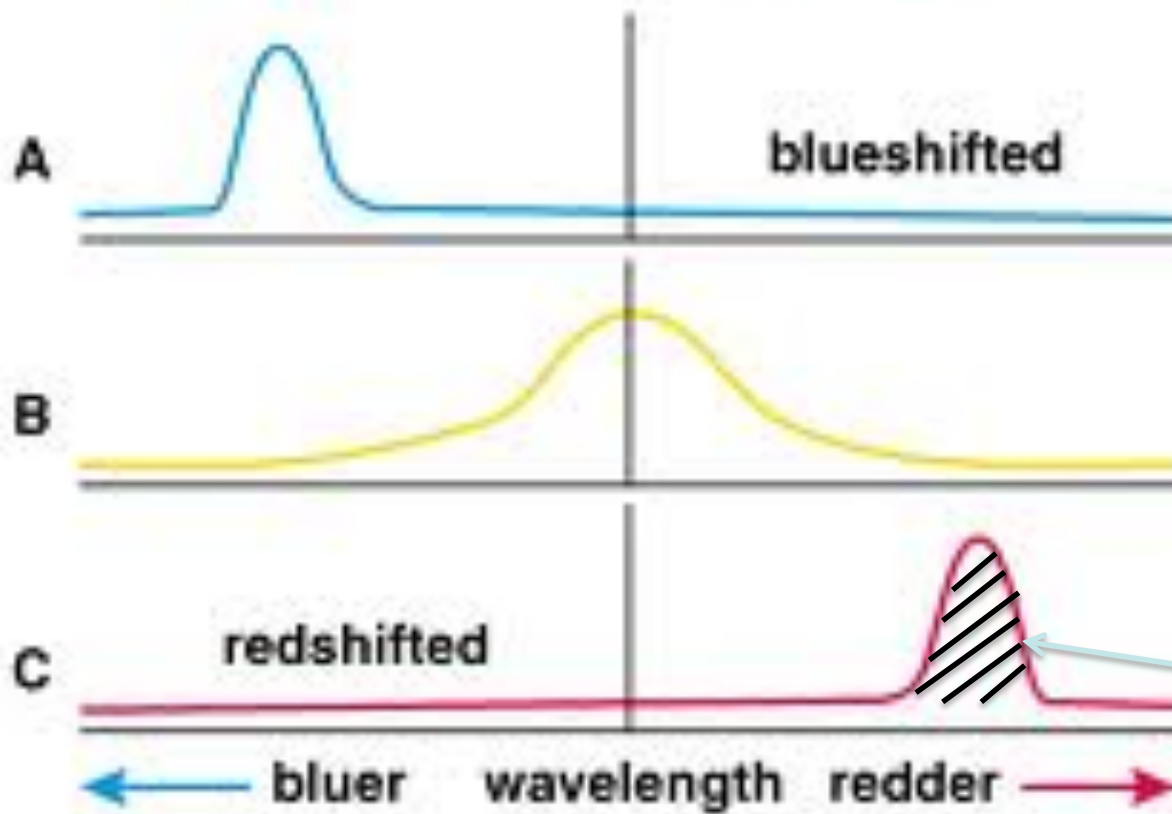
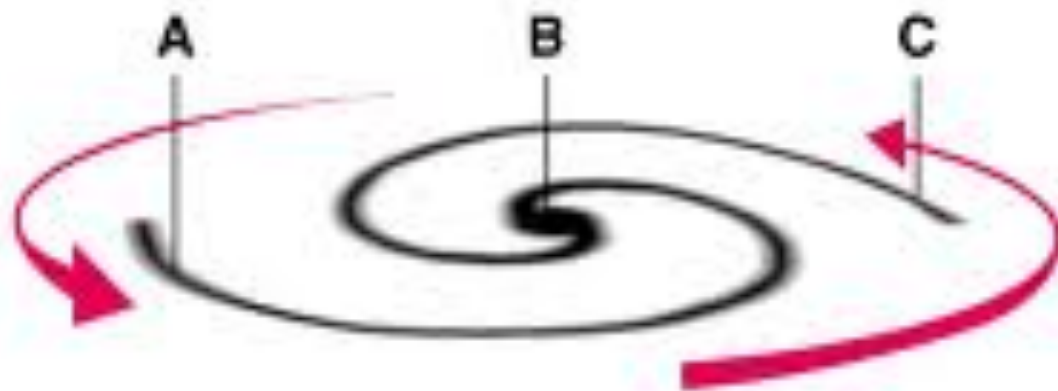
$\lambda$  = detected wavelength

$$\lambda / \lambda_o = (1+z)$$

e.g., if  $\lambda = \lambda_o$  ,  $\Delta\lambda = 0$  ,  $z = 0$  no shift

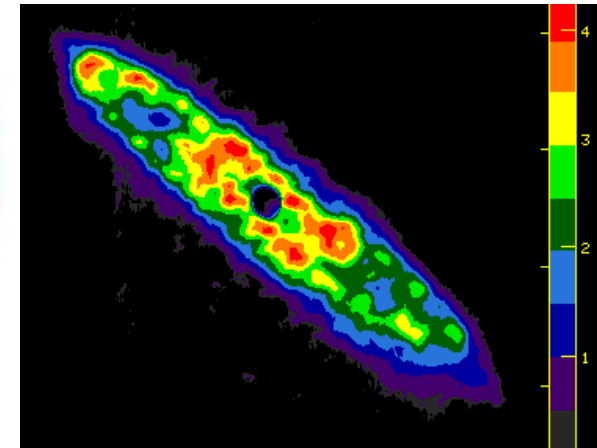
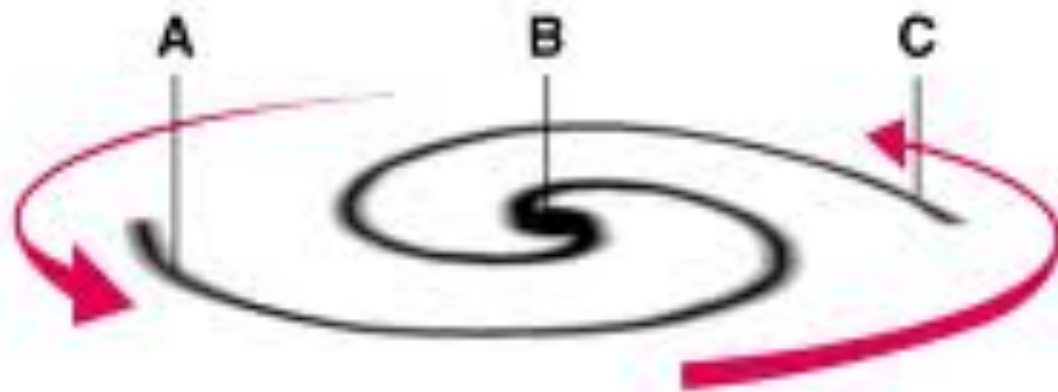
**for Doppler shifts:**

$$z = \Delta\lambda / \lambda_o = v / c \quad (\text{for } v \ll c)$$

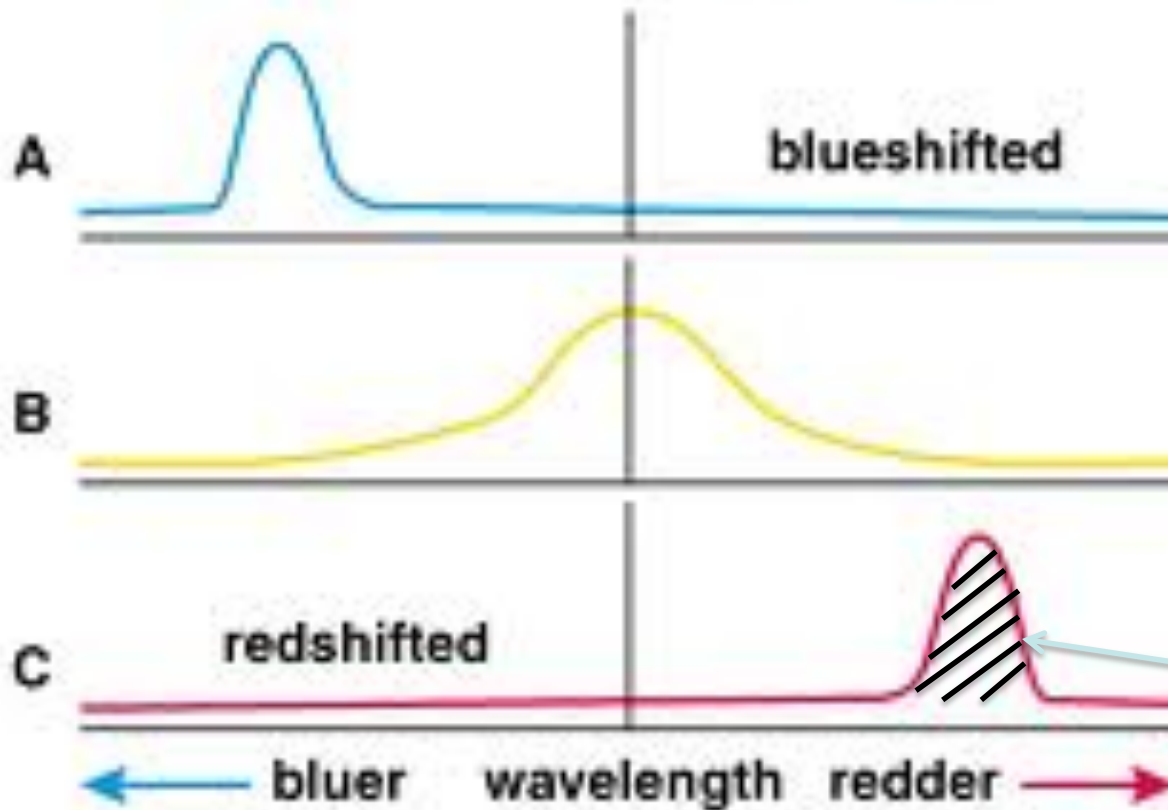


Line intensity (flux)  
gives

HI mass per area  
(surface density)

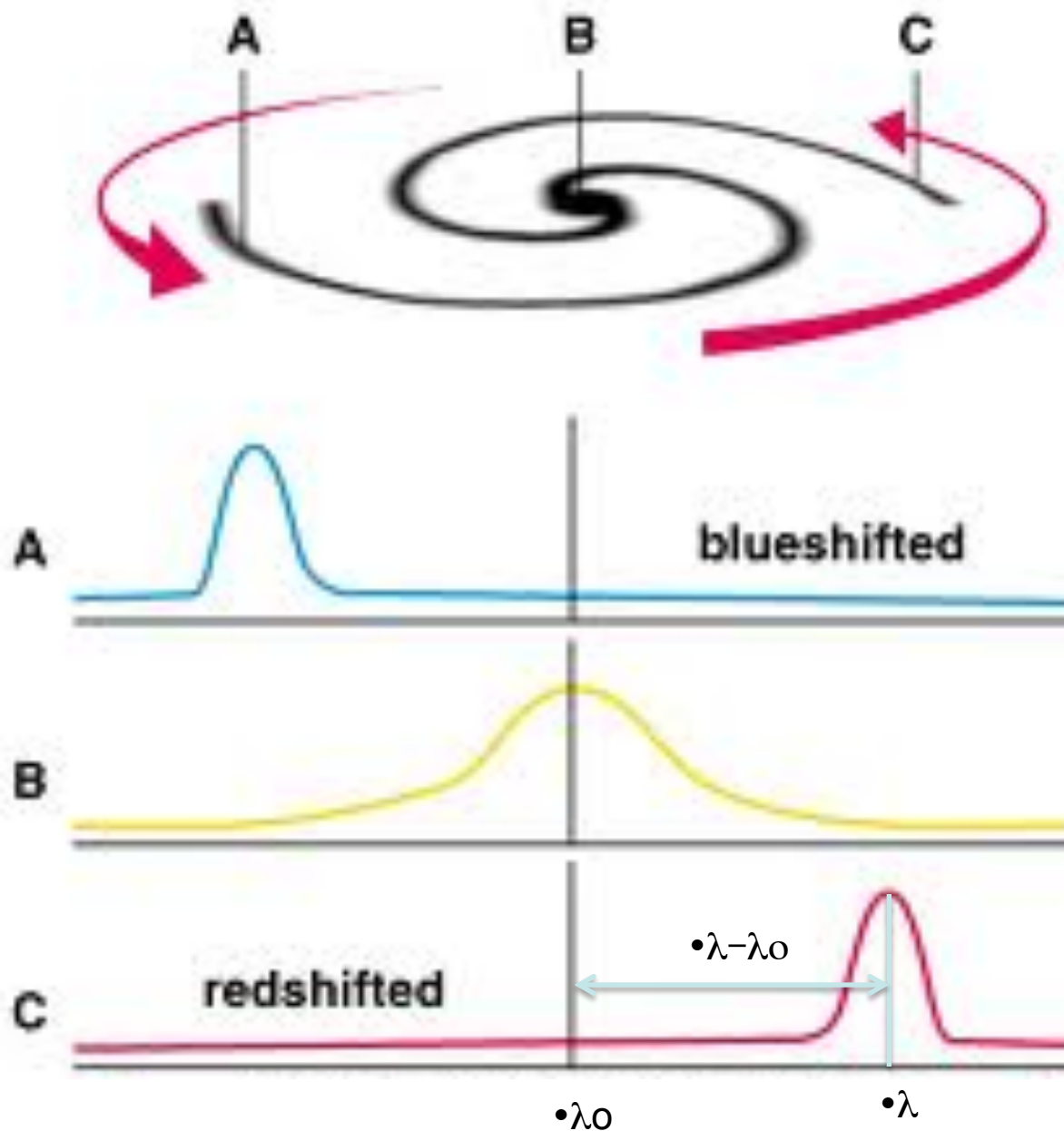


HI surface density map of spiral galaxy NGC 2903  
(color  $\rightarrow$  intensity = amount of HI gas)



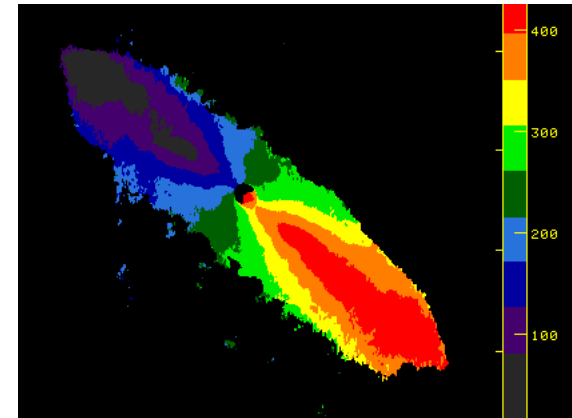
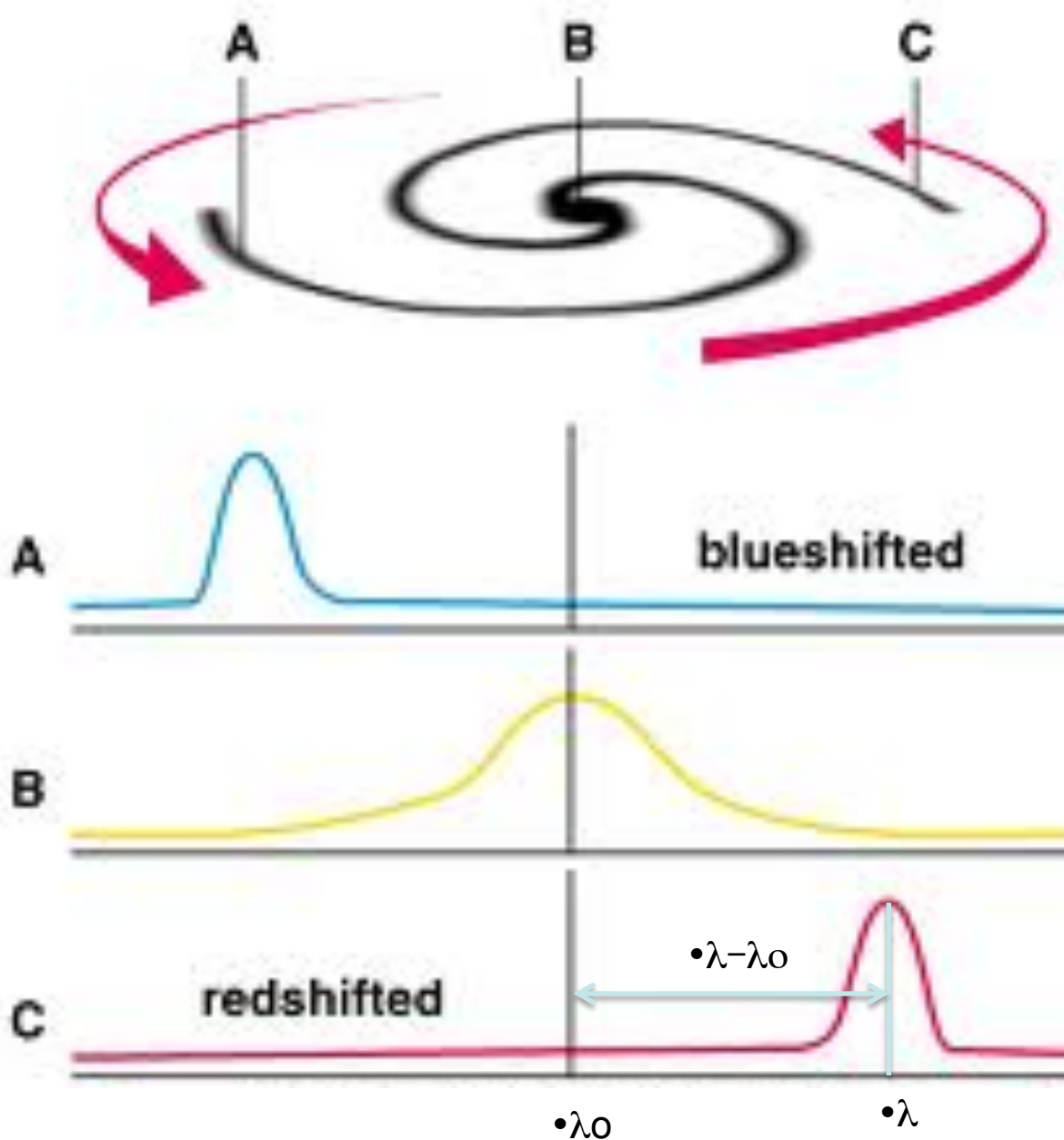
Line intensity (flux)  
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HI mass per area  
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Wavelength shift  
gives **velocity** by  
Doppler shift  
 $(\lambda - \lambda_0) / \lambda_0 = v/c$





HI velocity map of spiral galaxy NGC 2903  
Color= velocity of HI gas

Wavelength shift gives **velocity** by Doppler shift  

$$(\lambda - \lambda_0) / \lambda_0 = v / c$$

# Orbital speed of Sun around center of Milky Way Galaxy

Pretty fast:

- $V_{\text{rot}} = 220 \text{ km sec}^{-1}$  (= 500,000 mph)
- c.f. 60 mph = 88 ft/sec = 27 m/sec = 0.027 km/sec

# Orbital speed of Sun around center of Milky Way Galaxy

Pretty fast:

- $V_{\text{rot}} = 220 \text{ km sec}^{-1}$  (= 500,000 mph)

Pretty slow:

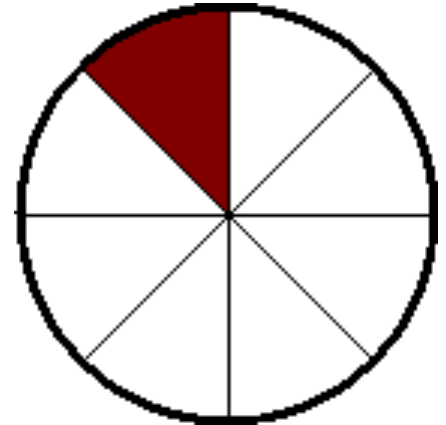
- $P = 2 \times 10^8 \text{ years}$  *c.f. age of sun  $4.5 \times 10^9 \text{ yr}$   
we've gone around only 22 times!*

Period – how long it takes to complete  
one orbit of 360 deg =  $2\pi$  radians

# angular speed

you can also measure the orbital speed in terms of **angular speed  $\Omega$  (angle/time)** – compare with **linear speed  $v$  (distance/time)**

**angular speed  $\Omega$**  : the rate at which something orbits through an angle of 1 radian (57 degrees)



$$\Omega = 2\pi/P = V_{\text{rot}} / r$$

$$\Omega = 2\pi \text{ rad} / 2 \times 10^8 \text{ yr} \sim 3 \times 10^{-8} \text{ rad/yr} \quad \text{“slow”}$$

for galaxies  $V_{\text{rot}}$  “fast” but  $\Omega$  “slow” since  $r$  is “big”

# *relating orbital velocity and mass*

## **Force & orbital velocities**

Newton's 2<sup>nd</sup> law of motion  $F = ma$

Since  $a = v_{\text{rot}}^2 / r$  for a mass in circular orbit

$$F = mv_{\text{rot}}^2 / r$$

## **Newton's law of gravity**

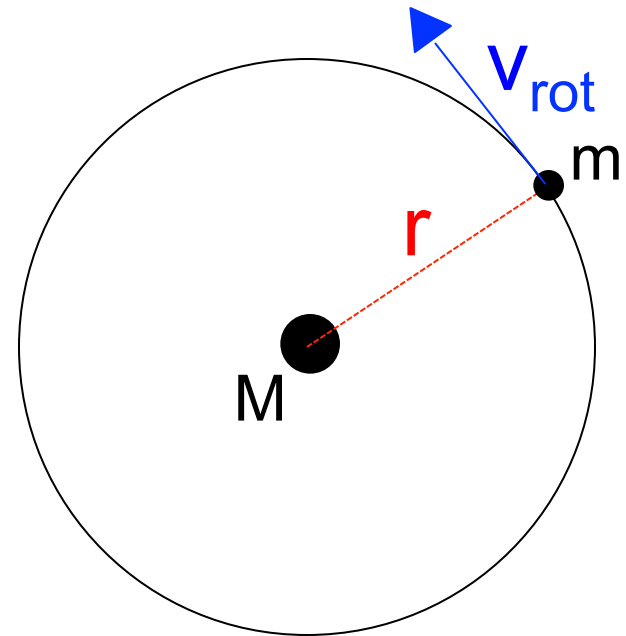
$$F_{\text{grav}} = GMm / r^2$$

if only force is gravity  $F = F_{\text{grav}}$

$$\frac{mv_{\text{rot}}^2}{r} = \frac{GMm}{r^2}$$

$$v_{\text{rot}}^2 = \frac{GM}{r}$$

$$v_{\text{rot}} = \sqrt{\frac{GM}{r}}$$



# *relating orbital velocity and mass*

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## **Newton's law of gravity**

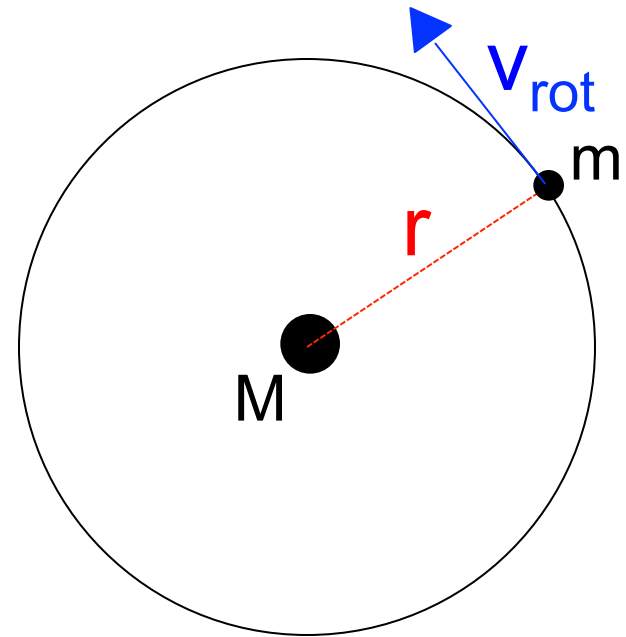
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$$v_{\text{rot}} = \sqrt{\frac{GM}{r}}$$



### **ASIDE:**

this is equivalent to Kepler's 3<sup>rd</sup> law

$$P^2 = 4\pi^2 / G(m_1 + m_2) a^3$$

Where  $a = r$ ,  $P = 2\pi r / v_{\text{rot}}$ ,  $m_2 \ll m_1$ ,  $m_2 = M$

you can prove it yourself if you are interested!!


This derivation was for the case of ***point masses***.

But suppose ***mass distribution is extended***, like in galaxies.

You can use the same relation, as long as you use the right M.

$$v_{\text{rot}} = \sqrt{\frac{GM}{r}}$$

what is this M?



This derivation was for the case of ***point masses***.

But suppose ***mass distribution is extended***, like in galaxies.

You can use the same relation, as long as you use the right M.

$$v_{\text{rot}} = \sqrt{\frac{GM_{\text{in}}}{r}}$$

in case of spherical symmetry,

**M is all of mass *inside* r**      $M_{\text{in}} = M(<r)$

If distribution is spherically symmetric, then gravity from outside mass cancels!!



## *remember this?*

Under the right circumstances, the rotational speed of a star is affected only by the mass within its orbit. What are these circumstances, and in these circumstances why doesn't the mass outside of the star's orbit have an effect on the speed?

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**spherical symmetry**

# *remember this?*

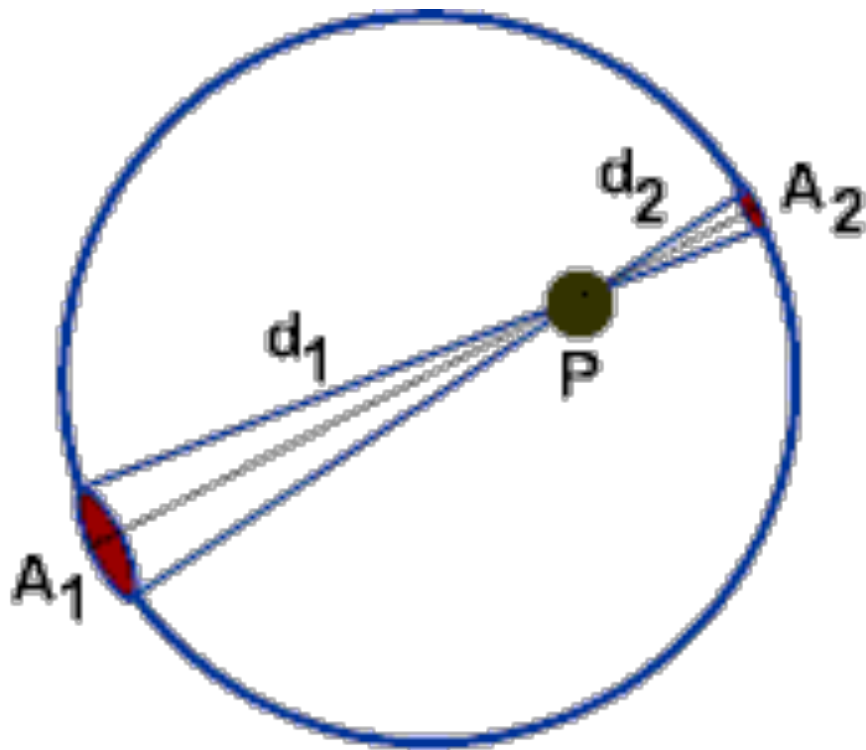
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**forces outside cancel out**

# Newton's theorem #1 for ***spherically symmetric*** gravitational field

*A body that is inside a uniformly dense spherical shell of matter experiences no net gravitational force from that shell.*

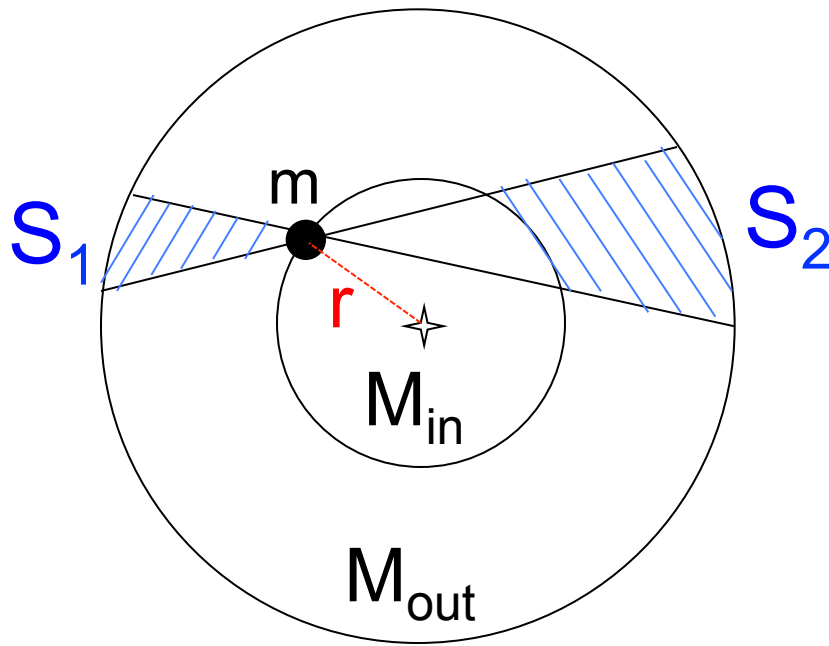


force from  $A_1$  balances  
force from  $A_2$

Geometry of spherical shells precisely compensates for  $1/r^2$  dependence of gravitational force  
(more mass in  $A_1$  but it is further away)

*A body that is inside a uniformly dense spherical shell of matter experiences no net gravitational force from that shell*

for  $M_{\text{out}}$ , force from  $S_1$  cancels force from  $S_2$   
(equal and opposite)



Geometry of spherical shells precisely compensates for  $1/r^2$  dependence of gravitational force  
(more mass in  $S_2$  but it is further away)

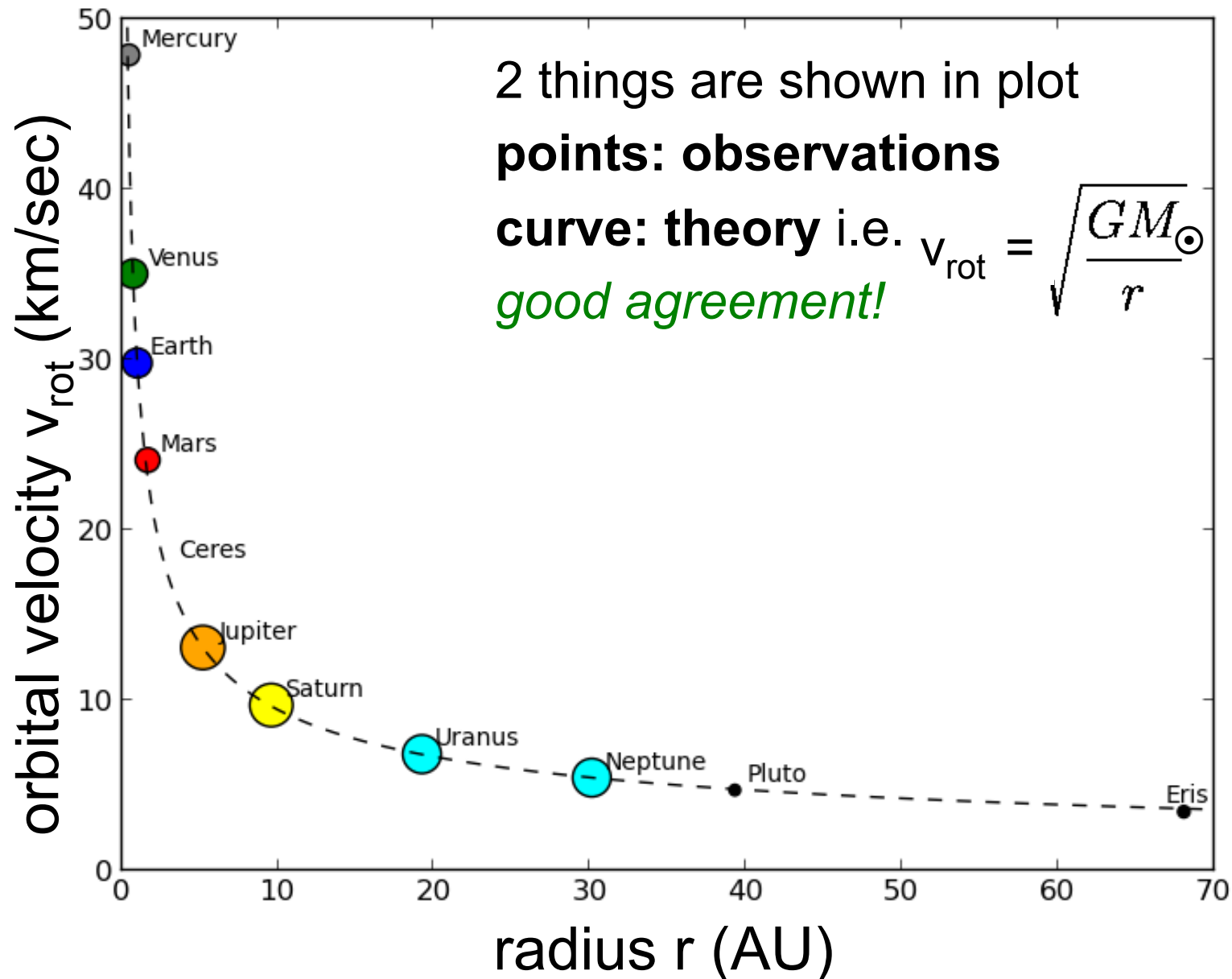
net force on  $m$  is  
all from  $M_{\text{in}}$

let's see how this works in the solar system,  
with the orbits of planets

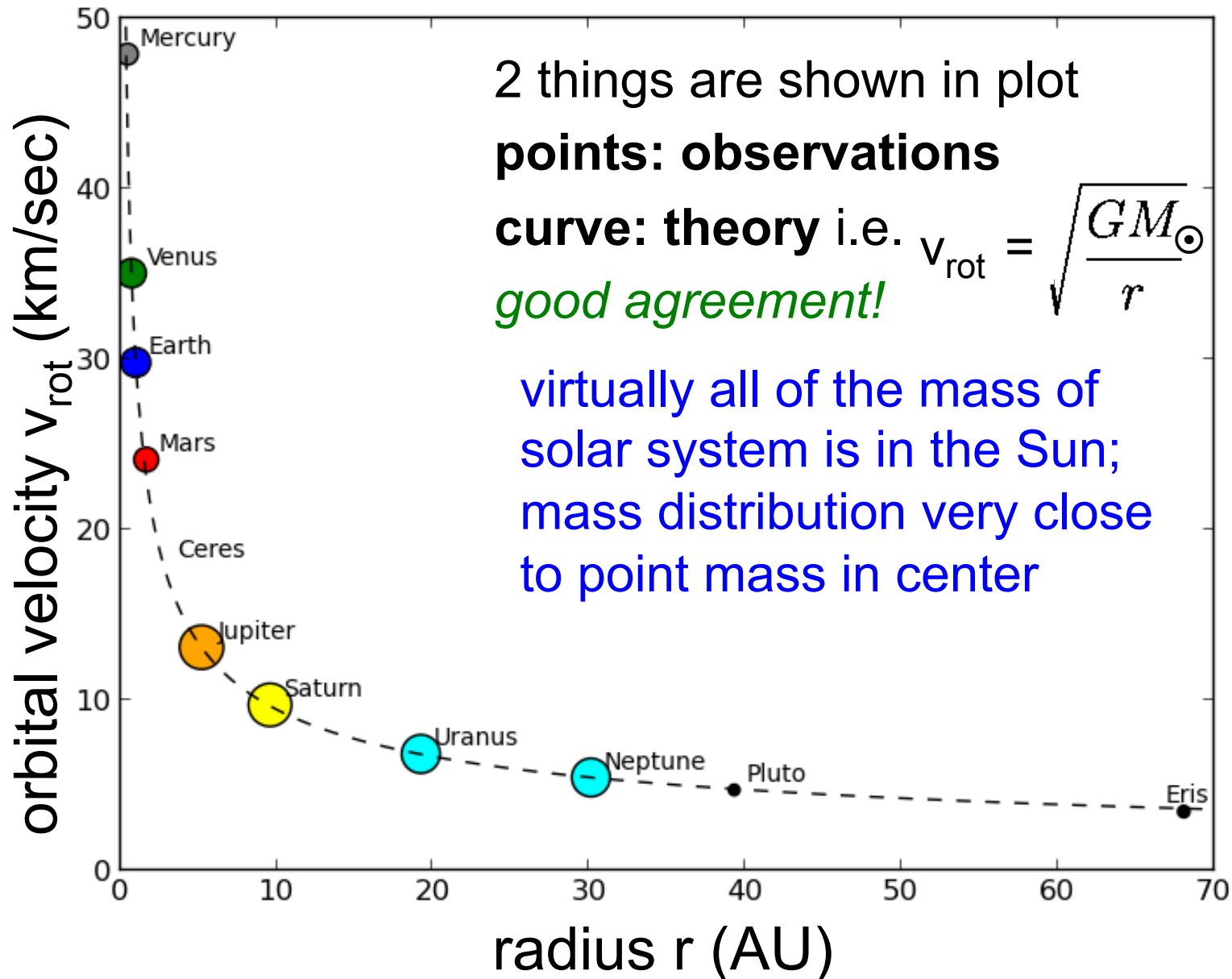
in this case nearly all the mass is  
concentrated in the center

animation 7-1 ...

# solar system rotation curve

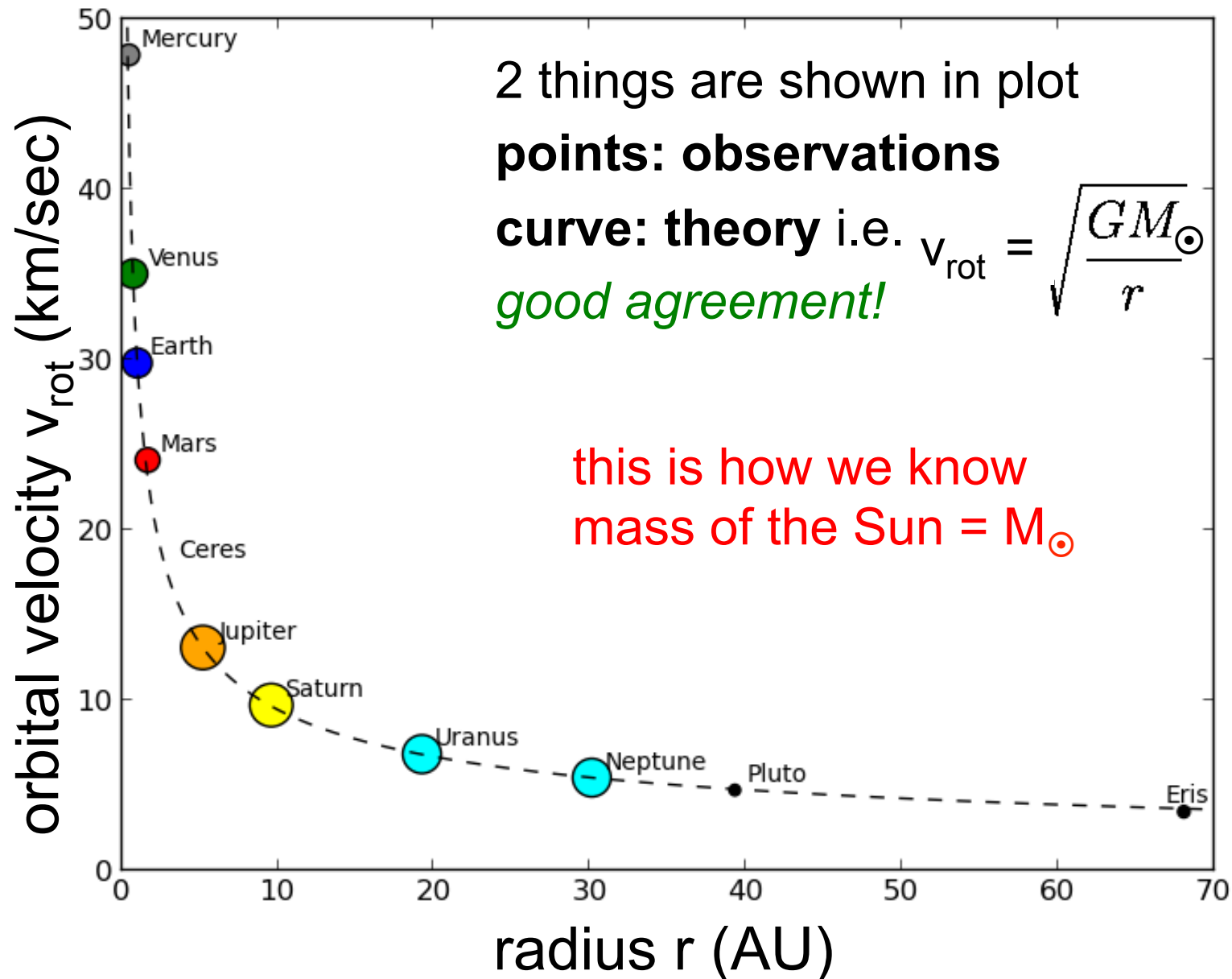


# solar system rotation curve

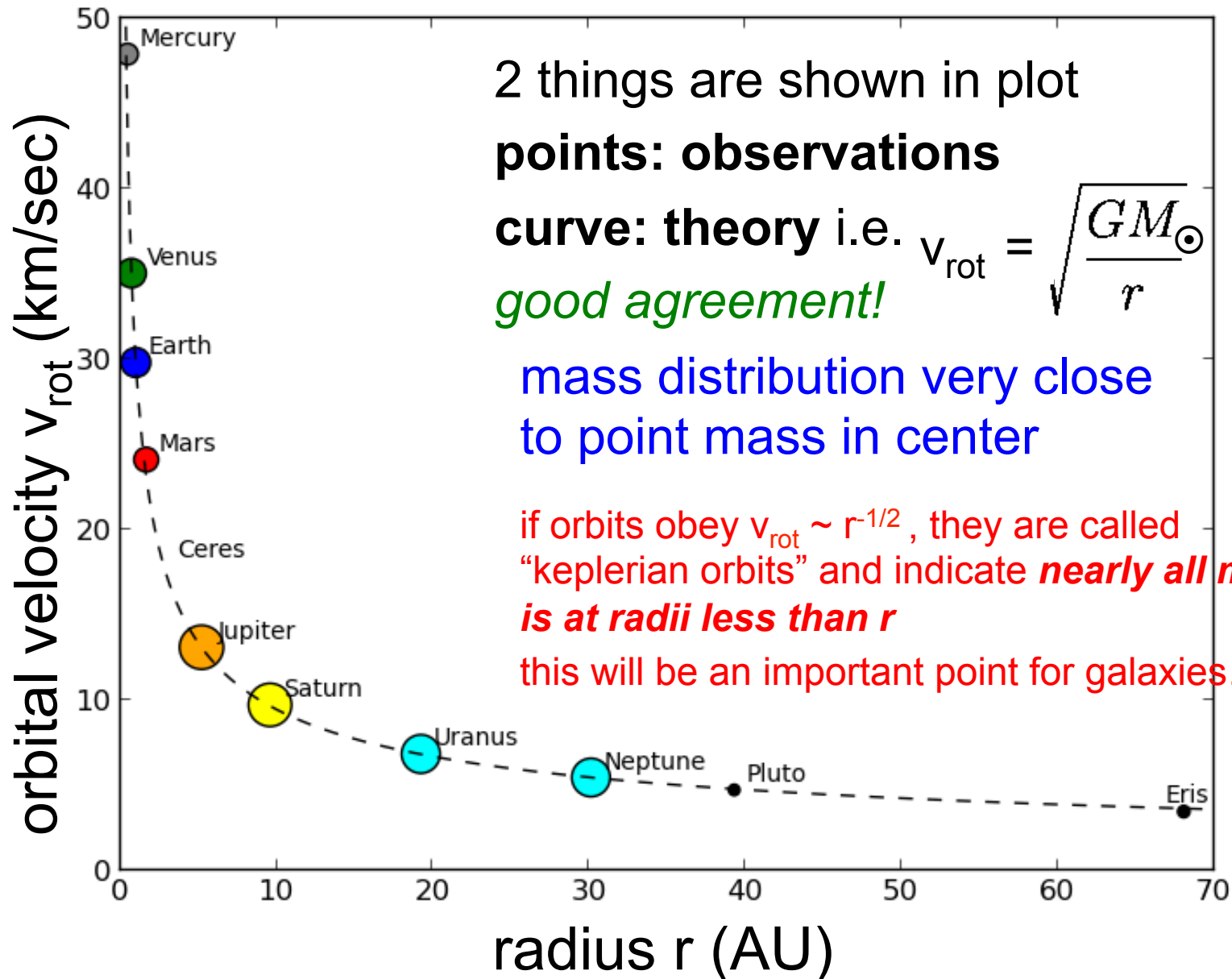




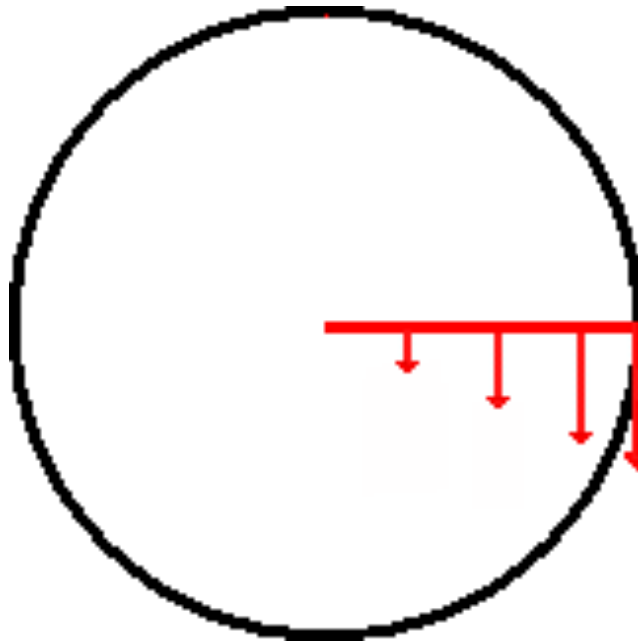
# solar system rotation curve



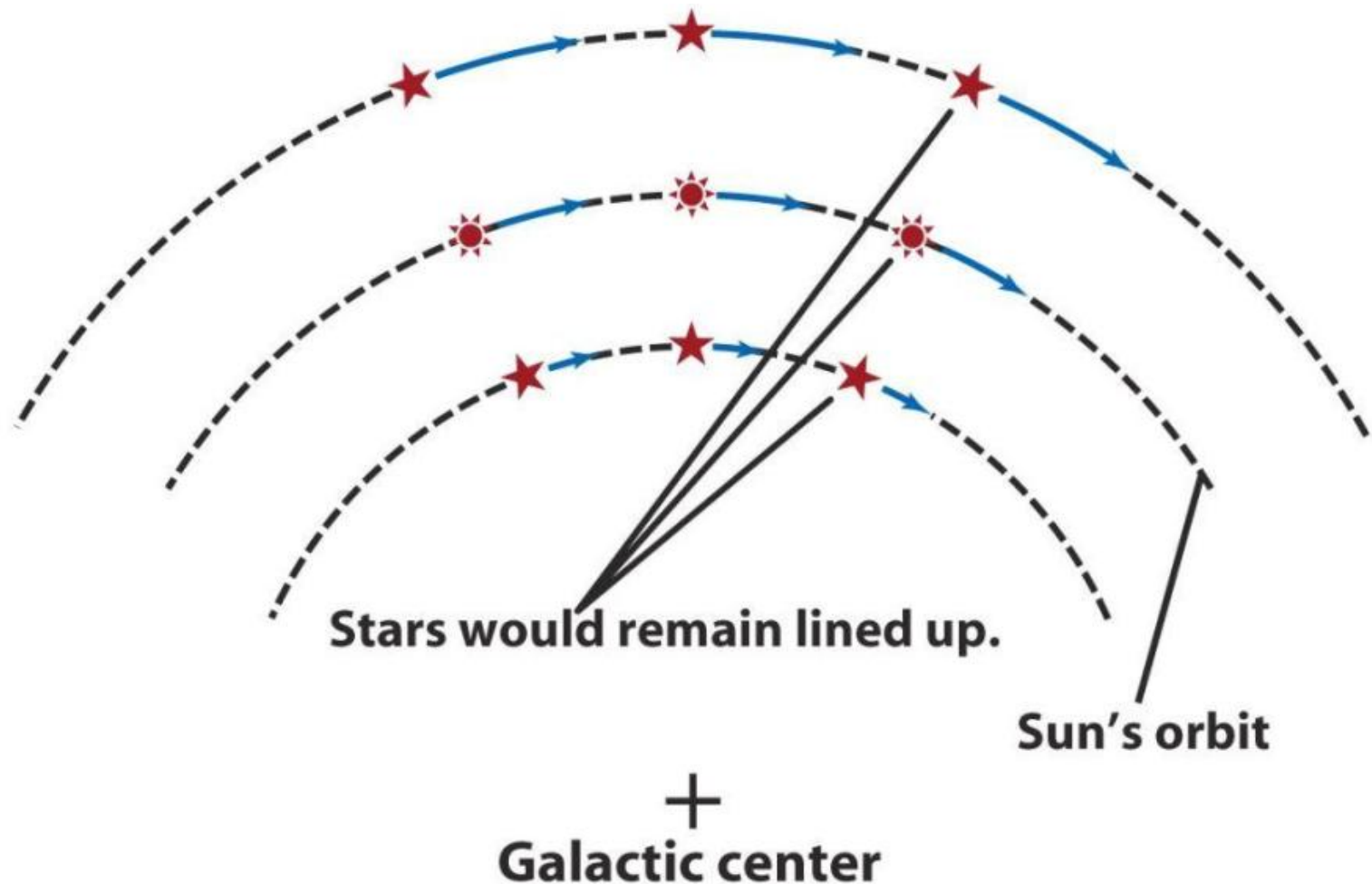
# solar system rotation curve



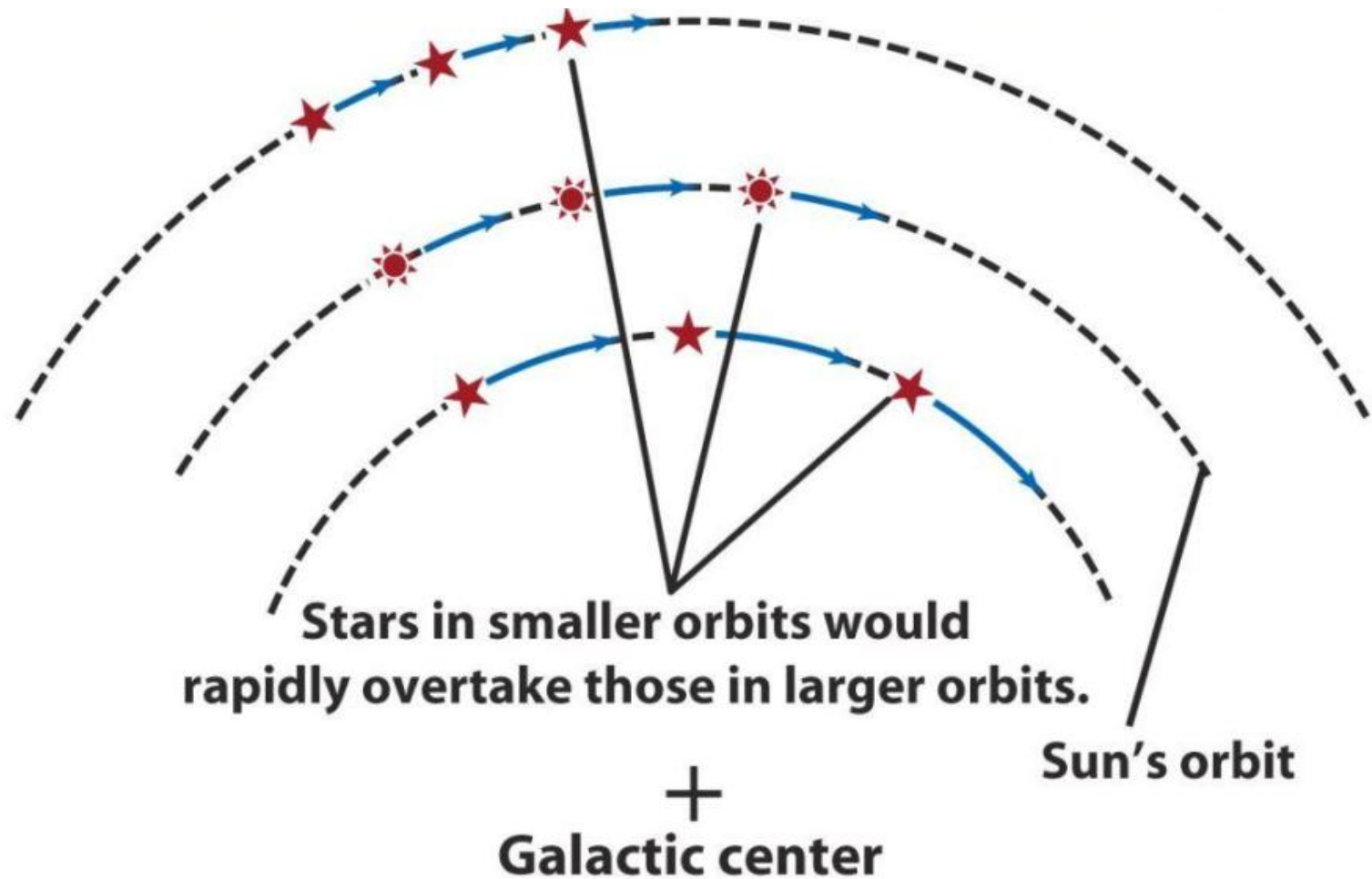
with solid body rotation points further from center move further (in km) every orbit



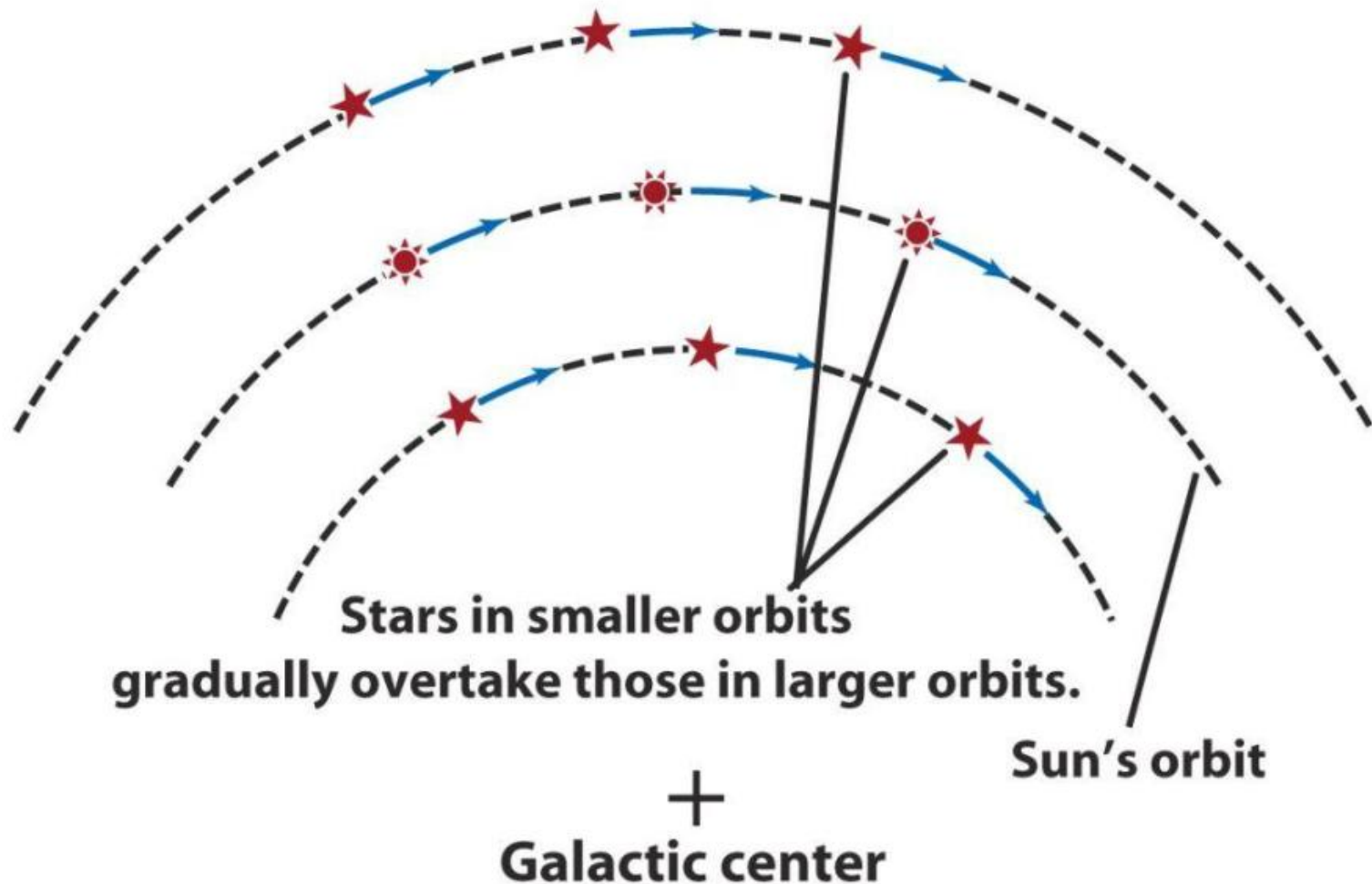
If our galaxy rotated like a solid disk (e.g. frisbee) the orbital speed would be greater for stars in larger orbits



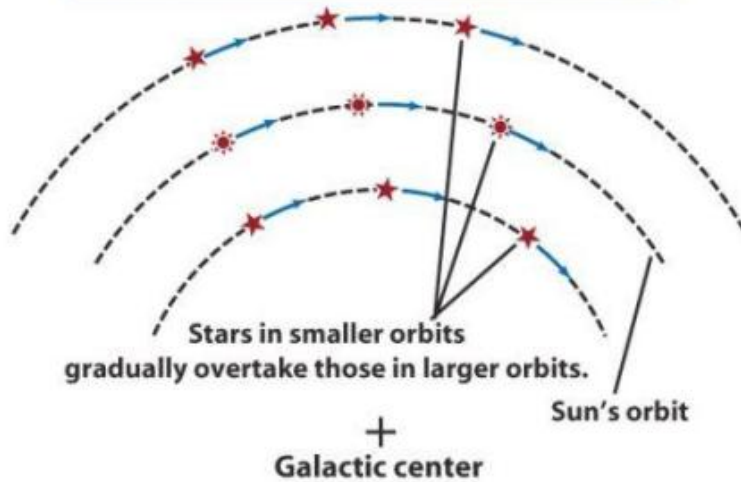
If the sun and stars obeyed Kepler's 3<sup>rd</sup> law, the orbital speed would be less for stars and gas in larger orbits



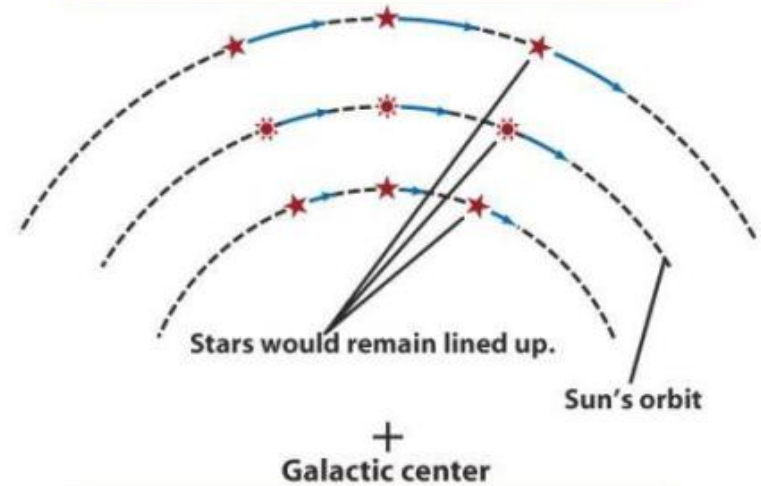
The orbital speed of stars and gas around the galactic center is nearly uniform throughout most of the Galaxy



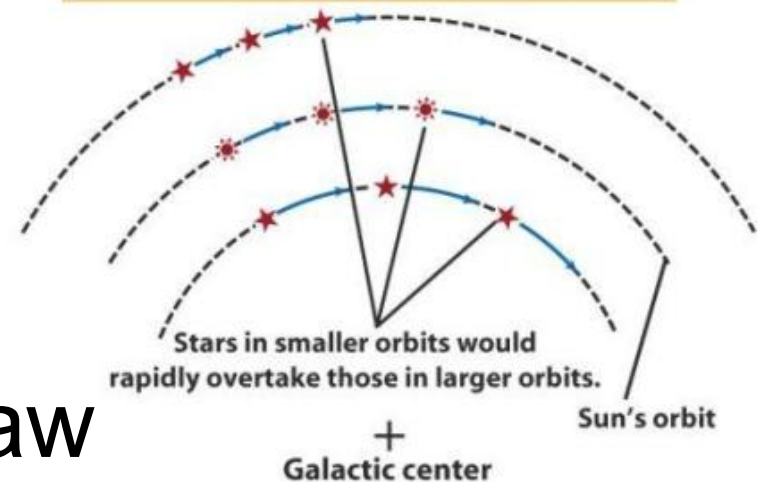
# Actual galaxy



# Solid body



(c) If the Sun and stars obeyed Kepler's third law, the orbital speed would be less for stars and gas in larger orbits.

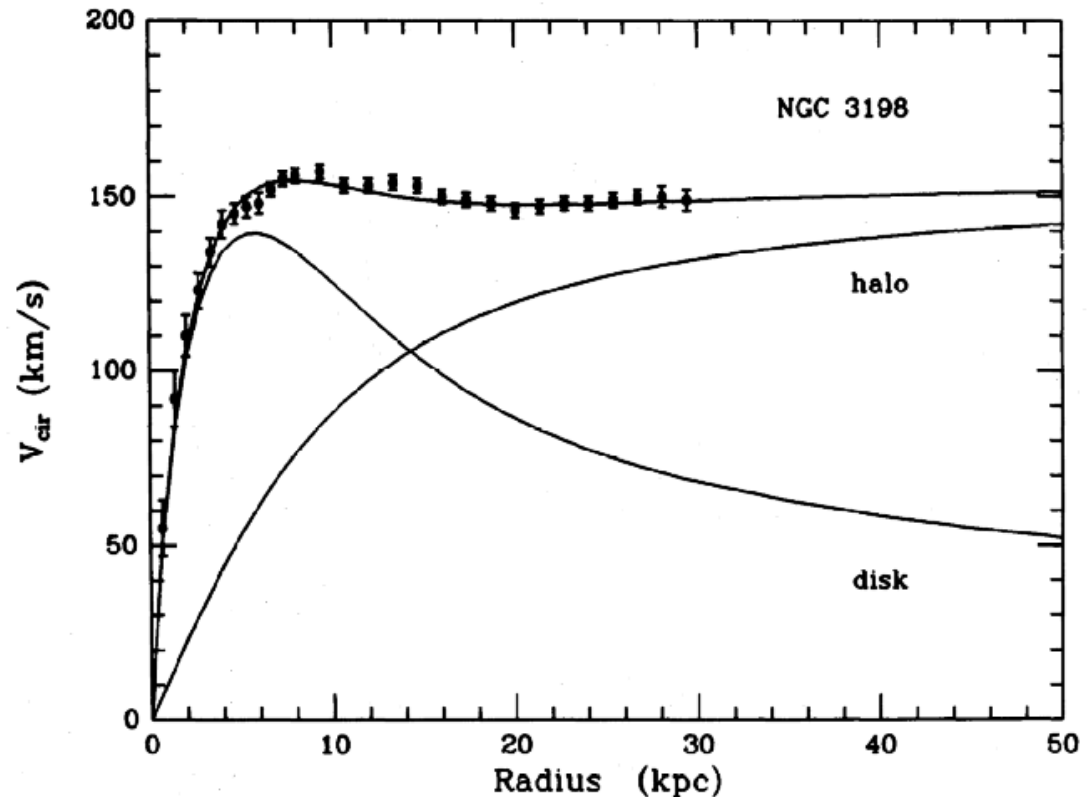
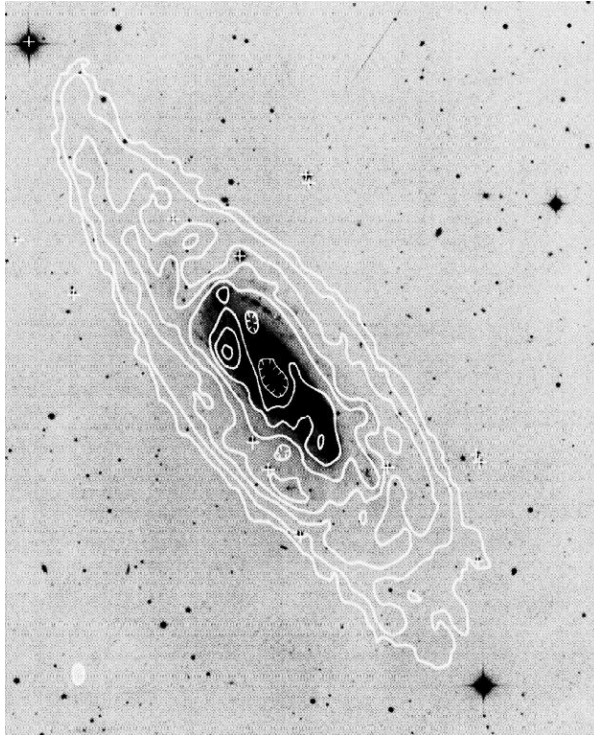


# Kepler's 3<sup>rd</sup> law



# Starlight and HI gas in spiral galaxy

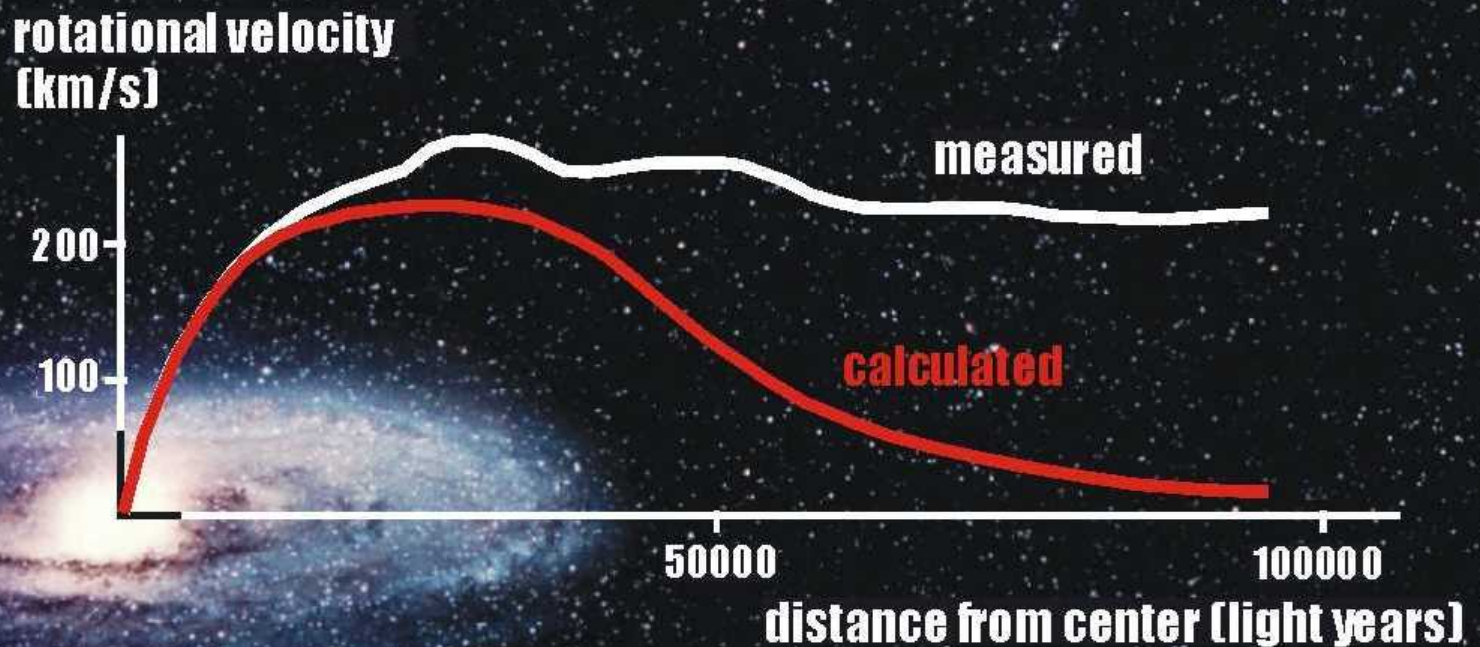
## Rotation curve in this spiral galaxy



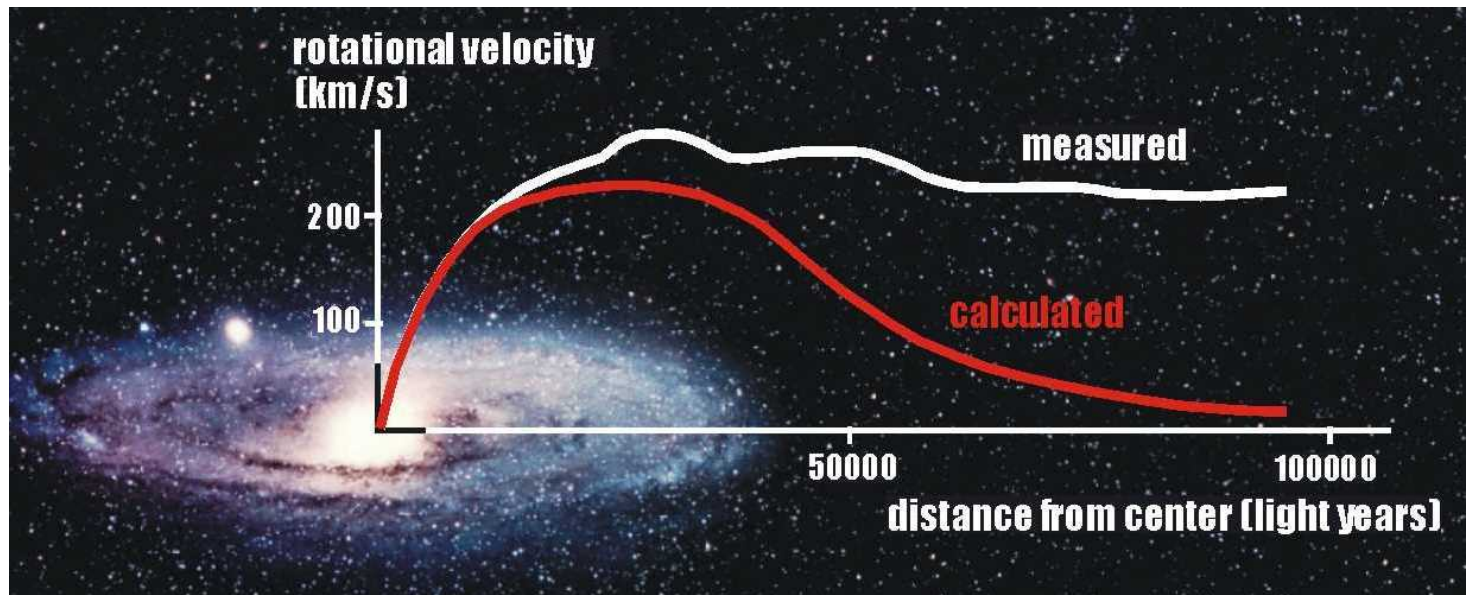
rotation velocities measured from doppler-shifted spectral lines of HI



# Predicted vs. measured rotation curves of galaxies



prediction: assume mass is traced by starlight



beyond most of the stars in galaxies, one might expect the rotation curve to fall roughly as  $v_{\text{rot}} \sim r^{-1/2}$  (“keplerian”)

since *at large radius most of the stars are inside that radius* ( $M_{\text{in}}$  for stars doesn’t increase much with  $r$  in outer parts)

but instead we observe that *rotation curves at large radii are nearly flat!* this implies that ....

According to Newton's & Einstein's laws of gravity, there is not enough mass in (known) stars and gas to account for the high rotational velocities of galaxies.

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So either:

A.) some unknown form of mass contributes most of galaxy mass (called "Dark Matter")

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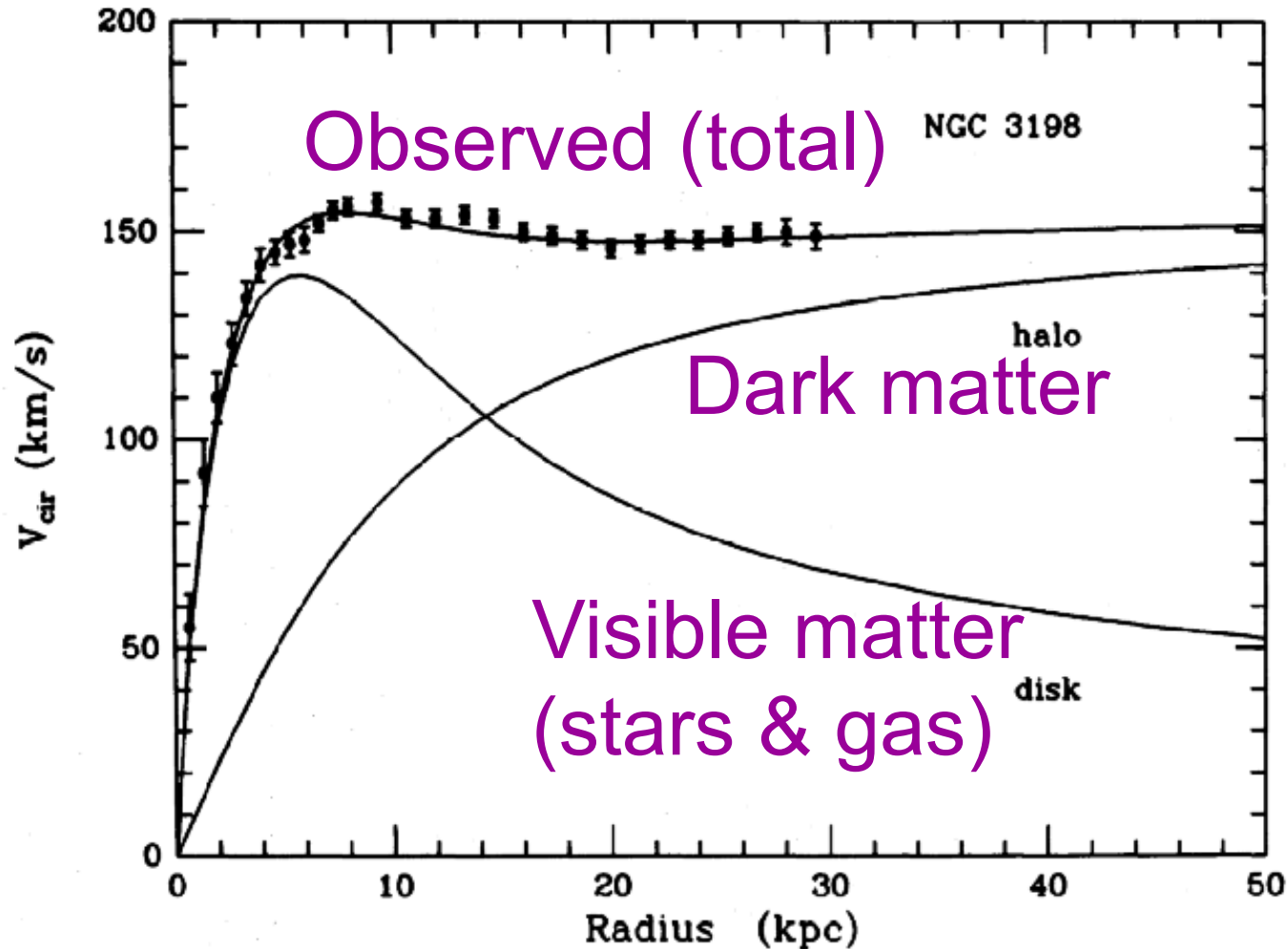
A.) some unknown form of mass contributes most of galaxy mass (called "Dark Matter")

PROBABLY!

B.) law of gravity needs modification

PROBABLY NOT!

# Rotation curve of a spiral galaxy

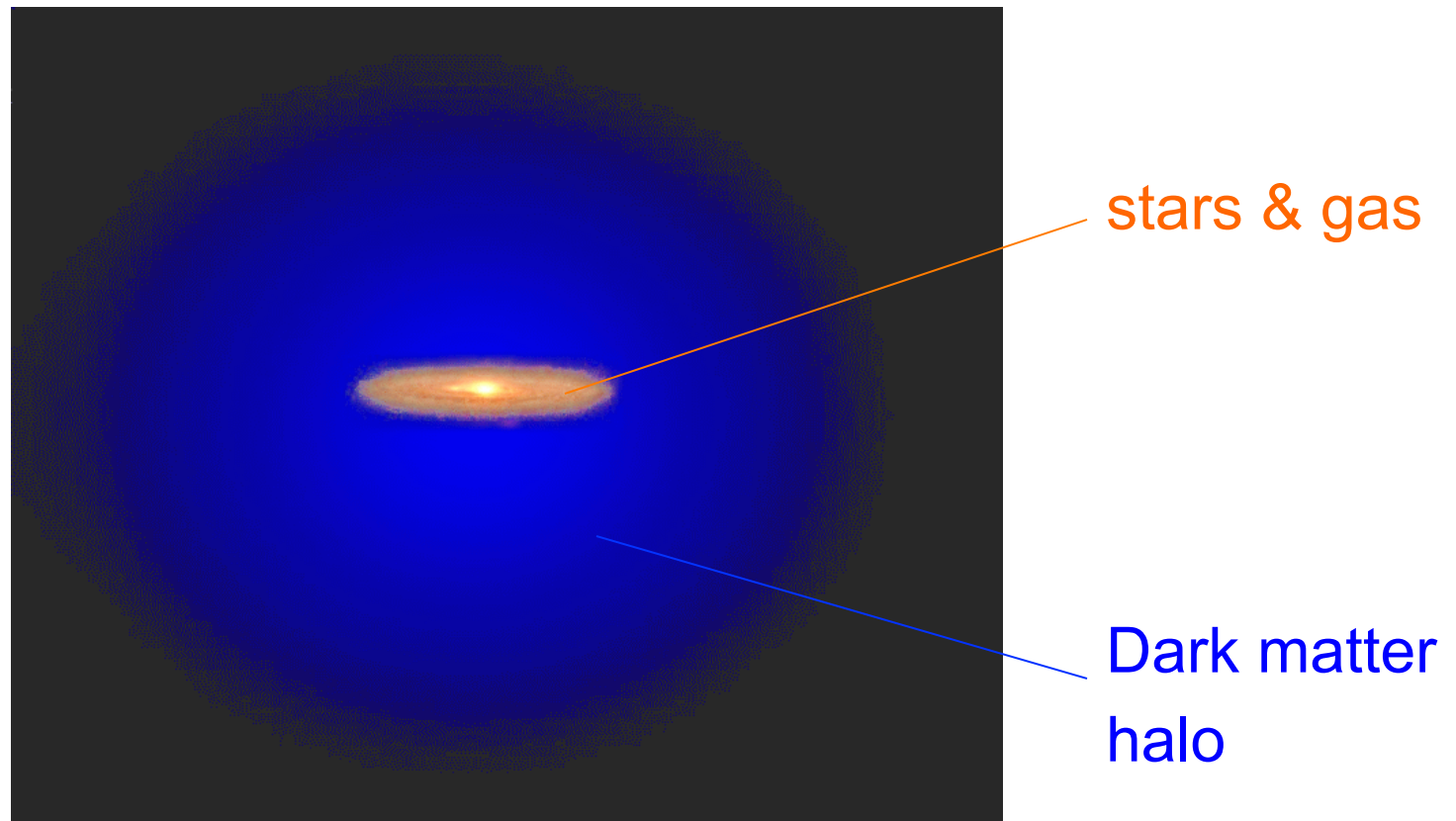


Normal matter is centrally concentrated in galaxies

Dark matter dominates in outer parts



# Cartoon of galaxy with dark matter halo



Normal matter is centrally concentrated in galaxies  
(~10% of total mass)

Dark matter dominates in outer parts (~90% of total mass)



# What is Dark Matter?

## 1. Normal matter?

made from protons & neutrons (baryons)

faint stars, planets, black holes, asteroids, gas,  
atoms, molecules, basketballs

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WIMPs: Weakly Interacting Massive Particles

# What is Dark Matter?

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faint stars, planets, black holes, asteroids, gas,  
atoms, molecules, basketballs

MACHOs

PROBABLY NOT!

## 2. Exotic subatomic particles?

(not baryons)

WIMPs

PROBABLY!

# Why are the effects of Dark Matter felt on scales of galaxies but not within the classroom?

- A. There is no dark matter in the classroom
- B. The atmosphere shields most of the dark matter particles from earth's surface
- C. The density of dark matter relative to normal matter is low in the room but not in the galaxy
- D. They ARE felt in classroom but we are used to it so don't notice it (like atmospheric pressure)
- E. Earth's gravity confines the dark matter to earth's core
- F. I can feel it. What's wrong with you?